# Generation of  $N$ -atom W-class states in spatially separated cavities

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We propose a feasible and efficient scheme to generate N-atom W-class states in spatially separated cavities without using any classical driving pulses. We adopt the model in which the couplings between different atoms are mediated only by virtual excitations of the cavity and fiber fields, so the scheme is insensitive to the cavity decay and fiber photon leakage. We carry out both theoretical investigation in a decoherence-free subspace and numerical calculation accounting for decoherence due to the atomic spontaneous emission as well as the decay of cavity and fiber modes. The theoretical and numerical results agree in the large atom-cavity detuning regime. Our scheme proves to be useful in scalable distributed quantum networks. © 2013 Optical Society of America

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## 1. INTRODUCTION

Entanglement, a fundamental feature in quantum mechanics, is a key resource for quantum information processing (QIP) [[1](#page-4-0),[2](#page-5-0)], such as quantum teleportation [[3](#page-5-1)], quantum dense coding [\[4](#page-5-2)], quantum cryptography [\[5\]](#page-5-3), and quantum computation [[6](#page-5-4)]. If a composite system is entangled, the whole system cannot be split into independent subsystems. Typical entangled states are the Bell states [[7\]](#page-5-5), the Greenberger–Horne– Zeilinger-class states  $[8,9]$  $[8,9]$  $[8,9]$  and the W-class states  $[10]$  $[10]$ . A state is called an N-qubit W-class state if it is of the form  $x_1|10\cdots0\rangle + x_2|01\cdots0\rangle + \cdots + x_N|00\cdots1\rangle$  with  $\sum_{n=1}^N |x_n|^2$ <br>1 and  $|0\rangle$ ,  $|1\rangle$  being two orthonormal vectors in the two  $\sum_{n=1}^{\infty}$   $\ket{\mu_1|\mathbf{0}\cdots\mathbf{0}}$  +  $\sum_{n=1}^{\infty}$   $\ket{\mu_n}$  = 1 and  $\ket{0}$ ,  $\ket{1}$  being two orthonormal vectors in the two-<br>dimensional (2D) Hilbert space of the qubit. The *N* qubit W dimensional (2D) Hilbert space of the qubit. The N-qubit W state corresponds to  $x_1 = x_2 = \cdots = x_N = 1/\sqrt{N}$ . Compared<br>with other types of entangled states W-class state constitutes with other types of entangled states, W-class state constitutes a very important family of states possessing a high degree of robustness against the qubit loss as they maintain some entanglement when more than two qubits remained [[11,](#page-5-9)[12\]](#page-5-10). Furthermore, deterministic protocols for teleportation and superdense coding [[13\]](#page-5-11) have been designed by utilizing <sup>W</sup>-class entanglements [[14\]](#page-5-12). An asymmetrical N-partite W-class state is also an essential quantum channel for quantum information splitting  $[15,16]$  $[15,16]$  $[15,16]$  and can be converted to another N-partite W-class state via local operations and classical communications  $[17]$  $[17]$ . Therefore, the generation of N-partite W-class states has proved to be an urgent task for the QIP.

However, to our knowledge, there are few studies for the given operationally experimental configurations with multipartite entanglement classes. Bastin et al. proposed an experimental setup to produce arbitrary symmetric long-lived multiqubit W states in the internal ground levels of photon emitters [[18\]](#page-5-16). An and Wang et al. presented protocols to

generate N-party W-class states in a single optical microcavity [[14](#page-5-12)[,19](#page-5-17)]. The *W*-class states in the above protocols are generated locally. For the distributed QIP, Pellizzari [[20](#page-5-18)] first suggested a scheme to realize the reliable transfer of quantum information between two distant cavities connected by an optical fiber in 1997, providing an effective tool for longdistant quantum communication schemes in recent years [[21](#page-5-19)–[26](#page-5-20)]. Here, by using a single-mode integrated optical  $1 \times N$ star coupler  $[27]$  $[27]$ , we propose a scheme to generate N-atom W-class states in a distributed network, which is nonlocally correlative [\[28](#page-5-22)] even in the presence of noise. Another distinct feature of our scheme is that all the bosonic field modes are only virtually populated to overcome the decoherence caused by cavity decays and fiber photon leakages. The excitation exchange among the atoms is caused by the dispersive coupling between the atoms and multiple delocalized bosonic field modes. Therefore, the atom-fiber-cavity system reduces to an effective model that couples only the atomic states while suppresses the states containing real bosonic modes. In addition, no classical pulses are needed so that the scheme is easy to operate. All these features make the scheme very promising for the generation of N-atom W-class states in spatially separated cavities.

## 2. MODEL

We consider  $N(N \geq 3)$  identical atoms trapped in N distant cavities which are connected by a single-mode integrated optical  $1 \times N$  star coupler [\[29](#page-5-23)–[31\]](#page-5-24), as shown in Fig. [1\(a\)](#page-1-0). The optical star coupler is made up of  $N$  identical optical fiber channels and only one resonant field mode interacts simultaneously with N cavity modes with a (real) coupling constant  $\nu$ . Each atom is a two-level one playing the role of a qubit with

<span id="page-1-0"></span>

Fig. 1. (a) Experimental setup. The black dots denote the atoms, which are trapped in  $N$  distant cavities, and these cavities are connected by a  $1 \times N$  single-mode integrated optical star coupler. (b) Level configuration for each atom.

 $|0\rangle \equiv |g\rangle$ , the ground state, and  $|1\rangle \equiv |e\rangle$ , the excited state. The atomic transition frequency  $\Omega$  is detuned from the cavity mode frequency  $\omega$  by a certain amount  $\Delta = \Omega - \omega$ , as shown in Fig. [1\(b\).](#page-1-0) Thus, the atomic transition  $|g\rangle \leftrightarrow |e\rangle$  is dispersively coupled to the corresponding cavity mode with a (real) coupling constant  $f$ . The interaction Hamiltonian of the whole atom-cavity-fiber system under the rotating-wave approximation can be written as  $(h = 1)$ 

<span id="page-1-4"></span>
$$
H = H_1 + H_2,\tag{1}
$$

$$
H_1 = \sum_{l=1}^{N} \nu (a_l^{\dagger} b + b^{\dagger} a_l), \tag{2}
$$

$$
H_2 = \sum_{l=1}^{N} f(a_l^{\dagger} S_l^- e^{-i\Delta t} + S_l^+ a_l e^{i\Delta t}), \tag{3}
$$

where  $a_l^{\dagger}(a_l)$  is the creation (annihilation) operator of the *l*th cavity mode  $b_l^{\dagger}(h)$  is the creation (annihilation) operator of cavity mode,  $b^{\dagger}(b)$  is the creation (annihilation) operator of the fiber mode, and  $S_t^+ = |e_l\rangle\langle g_l|(S_l^- = |g_l\rangle\langle e_l|)$  denotes the region (lowering) operator of the *l*th atom rasing (lowering) operator of the lth atom.

<span id="page-1-1"></span>We introduce new bosonic operators  $c_a$  defined by a linear superposition of  $a_l (l = 1, 2, ..., N)$  and b

$$
c_{\alpha} = \sum_{l=1}^{N} t_{\alpha,l} a_l + t_{\alpha,N+1} b,
$$
 (4)

where  $t_{\alpha,\beta}$  with  $\alpha, \beta \in \{1, 2, ..., N+1\}$  are the elements of a  $(N+1) \times (N+1)$  real unitary matrix T of the form  $(N + 1) \times (N + 1)$  real unitary matrix T of the form

The inverse transformations of Eq. [\(4\)](#page-1-1) are

$$
a_{l} = \sum_{\alpha=1}^{N+1} \chi_{l,\alpha} c_{\alpha},
$$
\n(6)

$$
b = \sum_{\alpha=1}^{N+1} \chi_{N+1,\alpha} c_{\alpha},\tag{7}
$$

<span id="page-1-2"></span>where  $\chi_{\alpha,\beta}$  are the elements of a  $(N + 1) \times (N + 1)$  real unitary matrix

$$
X = T^{-1} = T^T.
$$
 (8)

In terms of the new delocalized bosonic operators in Eq. ([4](#page-1-1)),  $H_1$  and  $H_2$  read

$$
H_1 = -\sqrt{N} \nu (c_N^{\dagger} c_N - c_{N+1}^{\dagger} c_{N+1})
$$
\n(9)

and

$$
H_2 = \sum_{l=1}^{N} \sum_{\alpha=1}^{N+1} f \chi_{l,\alpha} (S_l^+ c_a e^{i\Delta t} + c_a^{\dagger} S_l^- e^{-i\Delta t}). \tag{10}
$$

Switching to the interaction representation,  $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}$ ,<br>with  $\mathcal{H}_c = H$ , we have with  $\mathcal{H}_0 = H_1$ , we have

$$
\mathcal{H}_{\text{int}} = \sum_{l=1}^{N} \sum_{\alpha=1}^{N+1} f \chi_{l,\alpha} (S_l^{\dagger} c_{\alpha} e^{i\Delta_{\alpha} t} + c_{\alpha}^{\dagger} S_l^{-} e^{-i\Delta_{\alpha} t}), \tag{11}
$$

where

$$
\Delta_{\alpha} = \begin{cases}\n\Delta & \text{for } \alpha = 1, 2, ..., N - 1 \\
\Delta + \sqrt{N}\nu & \text{for } \alpha = N \\
\Delta - \sqrt{N}\nu & \text{for } \alpha = N + 1\n\end{cases} (12)
$$

<span id="page-1-3"></span>We assume that all the cavities and the fibers are empty and only one atom is excited initially. Then, in the large detuning regime:  $\Delta$ ,  $|\Delta \pm \sqrt{N} \nu| \gg f$ , the atoms are forbidden to ex-<br>change energy with the bosonic fields but they can exchange change energy with the bosonic fields, but they can exchange energy with each other via virtual field modes. So during the system's evolution no real bosonic modes appear at all, but the atomic excitation can still propagate from atom to atom. The underlying dynamics is thus governed by the effective interaction Hamiltonian

$$
T = \begin{pmatrix} \frac{\sqrt{N-1}}{\sqrt{N}} & \frac{-1}{\sqrt{N(N-1)}} & \frac{-1}{\sqrt{N(N-1)}} & \cdots & \frac{-1}{\sqrt{N(N-1)}} & \frac{-1}{\sqrt{N(N-1)}} & \frac{-1}{\sqrt{N(N-1)}} & 0\\ 0 & \frac{\sqrt{N-2}}{\sqrt{N-1}} & \frac{-1}{\sqrt{(N-1)(N-2)}} & \cdots & \frac{-1}{\sqrt{(N-1)(N-2)}} & \frac{-1}{\sqrt{(N-1)(N-2)}} & \frac{-1}{\sqrt{(N-1)(N-2)}} & 0\\ 0 & 0 & \frac{\sqrt{N-3}}{\sqrt{N-2}} & \cdots & \frac{-1}{\sqrt{(N-2)(N-3)}} & \frac{-1}{\sqrt{(N-2)(N-3)}} & \frac{-1}{\sqrt{(N-2)(N-3)}} & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{3}\sqrt{2}} & \frac{-1}{\sqrt{3}\sqrt{2}} & 0\\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}\sqrt{1}} & 0\\ \frac{-1}{\sqrt{2N}} & \frac{-1}{\sqrt{2N}} & \frac{-1}{\sqrt{2N}} & \cdots & \frac{-1}{\sqrt{2N}} & \frac{-1}{\sqrt{2N}} & \frac{-1}{\sqrt{2N}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2N}} & \frac{1}{\sqrt{2N}} & \frac{1}{\sqrt{2N}} & \cdots & \frac{1}{\sqrt{2N}} & \frac{1}{\sqrt{2N}} & \frac{1}{\sqrt{2N}} & \frac{1}{\sqrt{2N}} \end{pmatrix}
$$
(5)

$$
\mathcal{H}_{\text{eff}} = \sum_{l,m=1}^{N} \xi_{lm} S_l^+ S_m^-, \tag{13}
$$

with

$$
\xi_{lm} = f^2 \sum_{\alpha=1}^{N+1} \frac{\chi_{l,\alpha} \chi_{m,\alpha}}{\Delta_{\alpha}}.
$$
 (14)

<span id="page-2-7"></span>Using the equalities  $\chi_{l,\alpha} = t_{\alpha,l}$  due to Eq. ([8\)](#page-1-2), the unitarity of  $T: \sum_{\alpha=1}^{N+1} t_{\alpha,\beta} t_{\alpha,\delta} = \delta_{\beta\delta}$ , and the properties  $-t_{N,\beta} = t_{N+1,\beta} =$  $1/\sqrt{2N}$  for  $\beta \in \{1, 2, ..., N\}$ , we can verify that

$$
\xi_{lm} = \begin{cases} \frac{f^2}{N} \left( \frac{N-1}{\Delta} + \frac{\Delta}{\Delta^2 - N\nu^2} \right) = \xi_N & \text{for } l = m \\ -\frac{f^2}{N} \left( \frac{1}{\Delta} - \frac{\Delta}{\Delta^2 - N\nu^2} \right) = -\eta_N & \text{for } l \neq m \end{cases} \tag{15}
$$

<span id="page-2-3"></span>Now we turn to the generation of  $N$ -atom W-class states via the effective interaction Hamiltonian in Eq. [\(13](#page-1-3)). In the subspace having only one excited atom and no real bosonic modes, the atoms' state at any time  $t$  can be represented by a linear superposition of N basic states  $\{\phi_n\}; n =$  $1, 2, ..., N$  as

$$
|\Phi_N(t)\rangle = \sum_{n=1}^N C_n^{(N)}(t)|\phi_n\rangle,\tag{16}
$$

<span id="page-2-0"></span>where  $|\phi_n\rangle = |...e_n...\rangle$  denotes a state in which only the *n*th atom is excited while all the other  $N-1$  atoms are in their ground states. From the equation of motion  $i\partial|\Phi_N(t)\rangle/\partial t=$  $\mathscr{H}_{\text{eff}}|\Phi_N(t)\rangle$ , the time-dependent coefficients  $C_n^{(N)}(t)$  must satisfy the differential equations

$$
i\frac{\partial C_n^{(N)}(t)}{\partial t} = \xi_N C_n^{(N)}(t) - \eta_N \sum_{l=1; l \neq n}^N C_l^{(N)}(t) \tag{17}
$$

<span id="page-2-1"></span>for  $n = 1, 2, ..., N$ . Without loss of generality, we assume that at  $t = 0$  the atoms are in the state  $|\phi_1\rangle = |e_1g_2...g_N\rangle$ [i.e., under the initial conditions  $C_1^{(N)}(0) = 1$ ,  $C_2^{(N)}(0) = C_1^{(N)}(0)$  $C_3^{(N)}(0) = \cdots = C_N^{(N)}(0) = 0$ . Then, the solution of Eqs. ([17\)](#page-2-0) can be found in the form

$$
C_1^{(N)}(t) = \frac{1}{N} e^{-i(\xi_N + \eta_N)t} (e^{iN\eta_N t} + N - 1),
$$
 (18)

<span id="page-2-2"></span>
$$
C_2^{(N)}(t) = C_3^{(N)}(t) = \dots = C_N^{(N)}(t) = \frac{1}{N} e^{-i(\xi_N + \eta_N)t} (e^{iN\eta_N t} - 1).
$$
\n(19)

<span id="page-2-4"></span>As is evident from Eqs. [\(18](#page-2-1)) and [\(19](#page-2-2)), at  $t \neq (2k\pi/N\eta_N)$  $(k = 0, 1, 2, ...)$  all the coefficients  $C_n^{(N)}(t) \neq 0$   $(n = 1, 2, ..., N)$ and thus the state  $|\Phi_N(t)\rangle$  of Eq. ([16\)](#page-2-3) is an N-atom W-class state. In particular, omitting an unimportant common phase factor, states of the form

$$
|\Psi_N\rangle = \frac{1}{N} [(N-2)|\phi_1\rangle - 2 \sum_{n=2}^{N} |\phi_n\rangle]
$$
 (20)

<span id="page-2-6"></span>are generated at

$$
t = \frac{(2k+1)\pi}{N\eta_N}.\tag{21}
$$

#### 3. NUMERICAL ANALYSIS

In order to verify the validity of the above theoretical result, we analyze the system's dynamics by numerically solving the Schrödinger equation with the full Hamiltonian  $H$  in Eq. ([1](#page-1-4)). Suppose that we aim at generating the N-atom W-class state  $|\Psi_N\rangle$  given in Eq. [\(20](#page-2-4)). In Fig. [2](#page-2-5) we plot the fidelity  $F_N = \langle \Psi_N | \rho_N(t) | \Psi_N \rangle$  of the atoms' state  $\rho_N(t)$  obtained from the numerical calculation with respect to the state  $|\Psi_N\rangle$ for various values of  $N$  and the parameters chosen as  $\Delta/f = v/f = 10$ . Figure  $2(a)$  shows  $F_N$  versus dimensionless time  $\tau = N\eta_N t$ , showing that  $F_N \simeq 1$  at  $\tau = (2k+1)\pi$  for any  $N$ , which is in agreement with the theoretical result of N, which is in agreement with the theoretical result of Eq.  $(21)$  $(21)$  $(21)$ . Alternatively, we also plot  $F_N$  versus another dimen-sionless time ft in Fig. [2\(b\)](#page-2-5), from which it follows that the time for  $F_N$  to reach 1 is longer if N is larger. This also agrees with the theoretical result of Eq. ([21\)](#page-2-6), because  $N\eta_N$ , with  $\eta_N$ defined by Eq.  $(15)$  $(15)$ , decreases with increasing N. The slow oscillation of  $F_N$  in Fig. [2](#page-2-5) is due to the "hopping" of atomic excitation among the atoms due to the virtual excitation of the field modes, as theoretically predicted by Eqs.  $(18)$  $(18)$  and  $(19)$  when the effective Hamiltonian  $(13)$  $(13)$  is used. As for the fast oscillation in the fidelity  $F_N$ , it results from the atomcavity energy exchange based on the use of the full Hamiltonian ([1](#page-1-4)).

<span id="page-2-5"></span>

Fig. 2. Fidelity  $F_N$  versus dimensionless time (a)  $\tau = N \eta_N t$  and (b) ft, with  $\Delta/f = \Delta/\nu = 10$  for  $N = 3, 4, 5$ , and 6.

To be more concrete, let us deal with a specific situation for  $N = 4$  and  $t = \pi/4\eta_4$ , i.e., the target W-class state is

$$
|\Psi_4\rangle = \frac{1}{2} (|e_1 g_2 g_3 g_4\rangle - |g_1 e_2 g_3 g_4\rangle - |g_1 g_2 e_3 g_4\rangle - |g_1 g_2 g_3 e_4\rangle).
$$
\n(22)

It is necessary to consider the influence of different detunings on the fidelity  $F_4 = \langle \Psi_4 | \rho_4(t) | \Psi_4 \rangle$ . For  $N = 4$ , the condition  $\Delta/\nu$  < 2 should be satisfied according to Eq. ([21\)](#page-2-6). Thus, we assume  $\nu$  and  $\Delta$  such that  $\nu = 10f$  and  $2f < \Delta < 12f$  for plotting the fidelity  $F_4$  against different  $\Delta$  in Fig. [3](#page-3-0), where the oscillatory behavior is mainly caused by the energy exchange between the atoms and the fields. The numerical result shows that the average fidelity becomes closer to 1 for a larger detuning Δ. That is, a larger detuning suits the scheme better under an ideal environment. However, a quantum system interacts with the noisy environment inevitably, which induces unwanted disturbance to the target entangled state. The decoherence originates from physical factors, such as the atomic spontaneous emission, the cavity decay, and the fiber decay. To account for these decoherence factors, we employ the master equation for the density matrix  $\rho$  of the whole system, which is of the well-known form

<span id="page-3-2"></span>
$$
\dot{\rho} = -i[H, \rho] - \sum_{l=1}^{N} \frac{\Gamma_l}{2} (S_l^+ S_l^- \rho - 2S_l^- \rho S_l^+ + \rho S_l^+ S_l^-) \n- \sum_{l+1}^{N} \frac{\gamma_l}{2} (a_l^{\dagger} a_l \rho - 2a_l \rho a_l^{\dagger} + \rho a_l^{\dagger} a_l) \n- \frac{\kappa}{2} (b^{\dagger} b \rho - 2b \rho b^{\dagger} + \rho b^{\dagger} b),
$$
\n(23)

where  $\Gamma_l$  is the spontaneous emission rate from the excited state  $|e\rangle$  to the ground state  $|g\rangle$  of the *l*th atom,  $\gamma_l$  is the decay rate of the *l*th cavity, and  $\kappa$  is the decay rate of the optical star coupler. Assuming  $\Gamma_l = \Gamma$  and  $\gamma_l = \gamma$  for simplicity, the dependence of the fidelity  $F_4$  $F_4$  on  $\Delta/f$  and  $\Delta/\nu$  in Fig. 4 can be obtained by numerically solving the master Eq. [\(23](#page-3-2)). For  $0.8 < \Delta/\nu < 2$  and  $\Delta/f$  being a constant, we find that the fidelity decreases quickly when  $\Delta/\nu \rightarrow 2$ , in which case the large detuning condition  $|\Delta - \sqrt{4\nu}| \gg f$  is not satisfied. This implies<br>that use of the effective Hamiltonian in Eq. (13) is not valid in that use of the effective Hamiltonian in Eq.  $(13)$  $(13)$  is not valid in this case. With the decreasing of  $\Delta/\nu$ , the large detuning

<span id="page-3-0"></span>

Fig. 3. Fidelity  $F_4$  at  $t = \pi/4\eta_4$  versus  $\Delta/f$  when  $\nu/f = 10$ .

<span id="page-3-1"></span>

Fig. 4. Fidelity  $F_4$  at  $t = \pi/4\eta_4$  versus  $\Delta/f$  and  $\Delta/\nu$  when (a)  $\Gamma/f = 0.01$  and  $\gamma = \kappa = 0$ ; (b)  $\Gamma = \kappa = 0$  and  $\gamma/f = 0.3$ , and (c)  $\Gamma = \gamma = 0$  and  $\kappa/f = 0.3$ .

condition is getting satisfied and the dynamics of the whole system will evolve in accordance with that governed by the effective Hamiltonian. As seen from Fig.  $4(a)$ , the fidelity drops slowly when  $\Delta/\nu \leq 1$  and  $\Gamma/f = 0.01$ . This is because the interaction time needed to achieve the target entangled state prolongs according to Eq. [\(21](#page-2-6)), causing more decoherence from atomic spontaneous emission since the probability of population in the atomic excited state is larger. The fidelity in Figs.  $4(b)$   $(4(c))$  keeps very high even when  $\Delta/\nu \leq 1$  and  $\gamma/f = 0.3$  ( $\kappa/f = 0.3$ ), revealing robustness of the fidelity against the cavity decay and fiber decay due to the fact that the probabilities of real population in the cavity and the fiber are negligible in the effective Hamiltonian ([13\)](#page-1-3) that contains no interaction terms among those field modes. Keeping the ratio  $\Delta/\nu$  fixed in Fig. [4,](#page-3-1) we can consider the effect of different  $\Delta$  on the fidelity. From Fig. [4](#page-3-1), we can see that the increase of  $\Delta$  will decrease the fidelity with an oscillatory behavior. This is because although  $\Delta$  is large the interaction time prolongs with the increasing of  $\Delta$  in accordance

<span id="page-4-1"></span>

Fig. 5. Density plots of the fidelity  $F_4$  at  $t = \pi/4\eta_4$  versus (a)  $\Gamma/f$  and  $\gamma/f$  and (b)  $\Gamma/f$  and  $\kappa/f$ .

with Eq.  $(21)$  $(21)$ , where the decoherence dominates the system dynamics. Therefore, the range  $0.8 < \Delta/\nu < 1.2$  is the appropriate choice in our scheme. It also shows that the fidelity is robust against the possible imprecision of the atom-cavity detuning and the coupling strength between the bosonic modes.

Next, by choosing an appropriate values  $\Delta/f = v/f = 10$  in Figs.  $5(a)$ –([5\(b\)\)](#page-4-1), we plot the fidelity versus the ratios  $\Gamma/f$  and  $\gamma$ /f (Γ/f and  $\kappa$ /f). These figures indicate that the atomic spontaneous emission dominates the reduction of fidelity, while the decay rates of the photon leaking out off each cavity and the optical fiber channels just slightly influence  $F_4$ , which is 0.93(0.97) even when  $\gamma/f = 0.3$  ( $\kappa/f = 0.3$ ). Hence, the scheme is remarkably robust against cavity and fiber decays, which can be understood by the virtual excitation of all the bosonic field modes.

Finally, we briefly discuss the basic elements that may be a candidate for further experiments. The requirements of our scheme are the two-level atoms and the cavities resonantly connected by an optical fiber star coupler. The single-mode integrated optical fiber  $1 \times N$  star coupler used as a distributed strain sensor in a white-light interferometer has been reported [\[31](#page-5-24)] and realized by using a 2D arrangement, by using the two confocal arrays of the radial waveguides, which performs with an efficiency 100% under ideal conditions when the waveguides' mutual coupling strange is strong [\[32](#page-5-25)]. A nearperfect fiber-cavity coupling with an efficiency larger than 99.9% can be realized using fiber-taper coupling to high-Q silica microspheres [\[33](#page-5-26)]. The atomic configuration can be achieved with cesium: state  $|g\rangle$  corresponds to  $\{F = 4, m =$ 3} hyperfine state of  $6^{2}S_{1/2}$  electronic ground state and state |e} corresponds to  $\{F = 4, m = 3\}$  hyperfine state of  $6^{2}P_{1/2}$ electronic state. Each single atom can be made localized at a fixed position in each cavity with high  $Q$  for a long time [[34](#page-5-27)–[36](#page-5-28)]. In recent experiments [\[37](#page-5-29)], the parameters  $f = 2\pi \times 750$  MHz,  $\Gamma = 2\pi \times 2.62$  MHz, and  $\gamma = 2\pi \times 3.5$  MHz with the wavelength in the region 630 ∼ 850 nm is predicted achievable. The optical fiber decay at a 852 nm wavelength is about 2.2 dB∕km [[38\]](#page-5-30), which corresponds to fiber decay rate 0.152 MHz. By substituting these experimental parameters into Eq. ([23\)](#page-3-2), we obtain a fidelity higher than 0.9, making our scheme possible to be realized in experiment. The generation of W-class states involving more atoms is also efficient by changing the corresponding experimental parameters.

# 4. CONCLUSION

We have considered a model consisting of any  $N \geq 3$  identical two-level atoms trapped in  $N$  spatially separated cavities. Each cavity has one active mode which is off-resonant with the atomic transition but resonant with a single mode of an integrated optical  $1 \times N$  star coupler (see Fig. [1\)](#page-1-0), so all the atoms are indirectly coupled to each other even though they are far apart. We deduce an effective Hamiltonian in the large atom-cavity mode detuning regime and use it to theoretically study the dynamics of the atoms' system under the initial condition that only one atom is excited while all the cavities and the fiber are empty. The theoretical result shows that as the system evolves the atoms generally appear in an N-atom Wclass state. Of interest are the multiatom entangled states of the form in Eq.  $(20)$  $(20)$ , which are generated periodically at time moments determined by Eq.  $(21)$  $(21)$ . The proposed scheme for generating N-atom W-class states does not require any external classical laser pulses and is insensitive to the cavity decay rate and the rate of photon leakage from the fiber because during the whole evolution no real bosons are to be created due to the large detuning between the atomic transition and the cavity mode. We have also carried out numerical calculations taking into account the effects of decoherence caused by various dissipation mechanisms. The numerical result agrees well with the theoretical one if the atom-cavity detuning is large enough, confirming the validity of the effective Hamiltonian with respect to the full one. Therefore, the present entanglement generation scheme proves to be perspective for wide applications in the scalable distributed quantum networks.

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