Effects of initial environmental correlations in the dynamics of open systems

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In this work, we study the effects of different forms of correlations of environments on the dynamics of open systems' entanglement and discord. We consider two two-level atoms A and B interacting, respectively with two spatially separated modes a and b, each of which is in turn surrounded by a dissipative reservoir. The two modes may initially be in an entangled or classically correlated or product state with their marginal state being the same in all the cases. We compare the power of different environmental correlation forms in the revival of the atoms' entanglement and discord in the strong atom–mode coupling regime. We also show how the dynamical behavior of the atoms' entanglement and discord nontrivially change by the presence of initial environmental correlation in the weak atom–mode coupling regime. Finally, we reveal that initial entanglement between the modes can induce correlations between initially uncorrelated atoms.

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1. INTRODUCTION

In realizing quantum information technologies, the unavoidable coupling to the environment(s), whether intentional or accidental, is a serious obstacle because it always leads to the loss of an open system's information [1]. The useful quantum properties of the system, such as coherence and entanglement, are sooner or later destroyed in such decoherence processes. Therefore, the dynamics of entanglement (see, e.g., [2–13]) and general quantum correlations (see, e.g., [14–20]) have attracted extensive studies in recent years. One of the notable dynamical features of the system's entanglement is that it may be terminated in a finite time, a phenomenon called entanglement sudden death [21–24]. As for the general quantum correlations in terms of quantum discord [25,26], they are more robust than entanglement in the sense that they never suffer from a sudden death.

The evolution of an open system can be well described in Born and Markov approximations within which the dynamics are given by a semi-group of completely positive dynamical maps. The operatoric structure of the generator of such semi-groups [27,28] establishes the well-known Lindblad master equation, and the resulting dynamical behavior is termed Markovian. Nevertheless, in practice there are many scenarios in which memory effects play a crucial role. That is, during the evolution there exist time windows within which some information that was transferred from the system to the environment flows back into the system. To describe such processes a more general treatment called non-Markovian is required. In recent years, a number of methods for capturing the non-Markovian dynamics have been developed [29–32].

Usually a system of interest consists of several subsystems, each of which is coupled to an independent environment without any system–environment correlation. In this scenario the strong coupling of a subsystem with its own environment that is "turned on" as the total system's evolution starts is a source for non-Markovian dynamics. The assumption of independent environments and zero initial system–environment correlations is however too restrictive in realistic experiments [33]. In fact, nonzero initial system–environment correlations prove to be an important issue, both from theoretical and experimental points of view [34–39]. In particular, as has been shown [40–44], such initial correlations can lead to the growth of the trace distance between two quantum states of an open system over its initial value as time goes on. This means that an open system can acquire some information that was initially outside it. Recently, the trace-distance growth of open systems' states has been experimentally verified [45,46].

In this work, we shall explore the exploitation of environments' initial information for the open system. Namely, we assume that the environments may be correlated initially and investigate the effect of such correlated environments on the system's dynamics of entanglement and discord. As recently been shown [47,48], the correlations of generic dephasing environments may generate global non-Markovian dynamics even though all the local dynamics are Markovian. That is, the information initially possessed by the environments can be exploited by the system in the course of evolution. Here, instead of the dephasing mechanism, we study the dissipative one with energy exchange between the system and the environment. To be concrete, we consider two two-level atoms interacting with two damped modes of a radiation field in two far-apart locations [1]. Within each location a two-level atom (qubit) interacts via the Jaynes–Cummings Hamiltonian with a radiation field mode, which is in turn subjected to damping to a continuous reservoir. Our aim is to reveal how different forms of the modal correlations (quantum correlation, classical correlation, and no correlation at all) influence the atoms' dynamics.

2. MODEL AND CORRELATION MEASURES

The model we choose to consider consists of two two-level atoms (A and B), each interacting locally with a damping mode (a and b). The time-dependent density operators $\rho_{Aa}(t)$ and $\rho_{Bb}(t)$ of the subsystems Aa and Bb obey, respectively, the Lindblad master equations [49]

$$\frac{d\rho_{Aa}(t)}{dt} = -i[H_{Aa}, \rho_{Aa}(t)] - \frac{\Gamma}{2}[a^+ a\rho_{Aa}(t) - 2a\rho_{Aa}(t)a^+ + \rho_{Aa}(t)a^+ a] \tag{1}$$

and

$$\frac{d\rho_{Bb}(t)}{dt} = -i[H_{Bb}, \rho_{Bb}(t)] - \frac{\Gamma}{2}[b^+ b\rho_{Bb}(t) - 2b\rho_{Bb}(t)b^+ + \rho_{Bb}(t)b^+ b], \tag{2}$$

where

$$H_{Aa} = \omega_0 \hat{\sigma}_+^A \hat{\sigma}_-^A + \omega_c \hat{a}^+ \hat{a} + \Omega(\hat{\sigma}_-^A \hat{a}^+ + \hat{\sigma}_+^A \hat{a}) \tag{3}$$

and

$$H_{Bb} = \omega_0 \hat{\sigma}_+^B \hat{\sigma}_-^B + \omega_c \hat{b}^+ \hat{b} + \Omega(\hat{\sigma}_-^B \hat{b}^+ + \hat{\sigma}_+^B \hat{b}). \tag{4}$$

In the above equations H_{Aa} (H_{Bb}) is the interaction Hamiltonian of the subsystem Aa (Bb), $\hat{\sigma}_\pm^{A(B)}$ are the raising and lowering operators of atom $A(B)$, ω_0 is the atomic transition frequency, $a(b)$ and $a^\dagger(b^\dagger)$ are the annihilation and creation operator of mode $a(b)$ with frequency ω_c, and Ω is the atom–mode coupling constant. In Eqs. (1) and (2) Γ is the modal damping rate due to its interaction with a dissipative reservoir. We focus on the resonant case, namely $\omega_0 = \omega_c \equiv \omega$, and discriminate between two atom–mode coupling regimes in terms of Γ and Ω: $\Gamma/2 > \Omega$ implies the weak coupling regime and $\Gamma/2 < \Omega$ the strong one [50]. Note that this master equation, often introduced on the basis of a phenomenological approach, can be microscopically justified for a zero-temperature flat reservoir [51] relying on the Born–Markov approximation. It then provides a description of the atom–mode dynamics on a coarse-grained time scale, which does not resolve the decay time of correlation functions of the damping reservoir.

For the purpose of quantum information processing, the atoms A and B are usually prepared at $t = 0$ in an entangled state $\rho_{AB}(0) = |\psi(0)\rangle_{AB}\langle\psi(0)|$ with

$$|\psi(0)\rangle_{AB} = \alpha|gg\rangle_{AB} + \beta|ee\rangle_{AB}, \tag{5}$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $|g\rangle$ ($|e\rangle$) denotes the atomic ground (excited) state. Because of the dissipation due to the local damping modes, the atoms' correlations (entanglement and discord) degrade as the system evolves. Naturally, the initial conditions of the modes play a significant role in the atoms' dynamics. A question of our concern here is: "how do different forms of initial modal correlations characterized by a given marginal state influence the dynamics of the open atoms' system?" To elucidate this question we consider and compare three types of initial correlations between the modes a and b as follows. The first type expresses the quantum correlation specified by the density matrix

$$\rho_{ab}^{\mathrm{I}}(0) = |\phi(0)\rangle_{ab}\langle\phi(0)|, \tag{6}$$

where

$$|\phi(0)\rangle_{ab} = c_1|01\rangle_{ab} + c_2|10\rangle_{ab} \tag{7}$$

with $|c_1|^2 + |c_2|^2 = 1$. The second type exhibits the classical correlation specified by

$$\rho_{ab}^{\mathrm{II}}(0) = |c_1|^2|01\rangle_{ab}\langle 01| + |c_2|^2|10\rangle_{ab}\langle 10|, \tag{8}$$

and the third type corresponds to totally uncorrelated modes specified by

$$\rho_{ab}^{\mathrm{III}}(0) = |c_1c_2|^2|00\rangle_{ab}\langle 00| + |c_1|^4|01\rangle_{ab}\langle 01| + |c_2|^4|10\rangle_{ab}\langle 10| + |c_1c_2|^2|11\rangle_{ab}\langle 11|. \tag{9}$$

The initial state of the total system can be expressed as

$$\rho_{ABab}^J(0) = \rho_{AB}(0) \otimes \rho_{ab}^J(0) \tag{10}$$

with $J = \mathrm{I}$, II, and III corresponding to the form of initial modal correlations in Eqs. (6), (8), and (9), respectively. As can be verified from Eqs. (6) through (9), all the three types for the initial states of the modes have the same marginal states: $\rho_a(0) = \{\{|c_1|^2, 0\}, \{0, |c_2|^2\}\}$ and $\rho_b(0) = \{\{|c_2|^2, 0\}, \{0, |c_1|^2\}\}$. Thus, the local dynamics of each atom are the same in these three situations. However, as will be clarified, the global dynamics of the atoms' system depend strongly on the concrete situation. Putting it the other way around, the difference in the atoms' dynamical behaviors exclusively reflects the effects of the different forms of the modes' correlations.

To make clear the procedure of obtaining the evolved density matrices of the total system, let us illustrate as an example the situation of $J = \mathrm{I}$, for which we have explicitly

$$\begin{aligned}\rho_{AaBb}^{\mathrm{I}}(0) = {} & |\alpha c_1|^2|g0\rangle_{Aa}\langle g0| \otimes |g1\rangle_{Bb} \\ & \times \langle g1| + |\alpha c_2|^2|g1\rangle_{Aa}\langle g1| \otimes |g0\rangle_{Bb} \\ & \times \langle g0| + |\alpha|^2 c_1 c_2^*|g0\rangle_{Aa}\langle g1| \otimes |g1\rangle_{Bb} \\ & \times \langle g0| + |\alpha|^2 c_2 c_1^*|g1\rangle_{Aa}\langle g0| \otimes |g0\rangle_{Bb} \\ & \times \langle g1| + |\beta c_1|^2|e0\rangle_{Aa}\langle e0| \otimes |e1\rangle_{Bb} \\ & \times \langle e1| + |\beta c_2|^2|e1\rangle_{Aa}\langle e1| \otimes |e0\rangle_{Bb} \\ & \times \langle e0| + |\beta|^2 c_1 c_2^*|e0\rangle_{Aa}\langle e1| \otimes |e1\rangle_{Bb} \\ & \times \langle e0| + |\beta|^2 c_2 c_1^*|e1\rangle_{Aa}\langle e0| \otimes |e0\rangle_{Bb} \\ & \times \langle e1| + \alpha\beta^*|c_1|^2|g0\rangle_{Aa}\langle e0| \otimes |g1\rangle_{Bb} \\ & \times \langle e1| + \alpha\beta^*|c_2|^2|g1\rangle_{Aa}\langle e1| \otimes |g0\rangle_{Bb} \\ & \times \langle e0| + \alpha\beta^* c_1 c_2^*|g0\rangle_{Aa}\langle e1| \otimes |g1\rangle_{Bb} \\ & \times \langle e0| + \alpha\beta^* c_2 c_1^*|g1\rangle_{Aa}\langle e0| \otimes |g0\rangle_{Bb} \\ & \times \langle e1| + \beta\alpha^*|c_1|^2|e0\rangle_{Aa}\langle g0| \otimes |e1\rangle_{Bb} \\ & \times \langle g1| + \beta\alpha^*|c_2|^2|e1\rangle_{Aa}\langle g1| \otimes |e0\rangle_{Bb} \\ & \times \langle g0| + \beta\alpha^* c_1 c_2^*|e0\rangle_{Aa}\langle g1| \otimes |e1\rangle_{Bb} \\ & \times \langle g0| + \beta\alpha^* c_2 c_1^*|e1\rangle_{Aa}\langle g0| \otimes |e0\rangle_{Bb} \\ & \times \langle g1|. \end{aligned} \tag{11}$$

As is obvious from Eq. (11), each subsystem of Aa and Bb may contain zero, one, or two excitations. The trivial case of zero excitation is invariant in time. For the case of one excitation, we should solve the master equation in the basis $\{|g0\rangle, |g1\rangle, |e0\rangle\}$. By means of a computer program, we work out the corresponding coefficients of evolved terms $|g0\rangle\langle g0|$, $|g1\rangle\langle g1|$, $|e0\rangle\langle e0|$, $|g0\rangle\langle g1|$, $|g0\rangle\langle e0|$, $|g1\rangle\langle g0|$, $|g1\rangle\langle e0|$, $|e0\rangle\langle g0|$, $|e0\rangle\langle g1|$ with different initial conditions of the subsystem. For example, the subsystem Aa or Bb may initially be in $|g1\rangle\langle g1|$; we then substitute the term $|g1\rangle\langle g1|$ in Eq. (11) for the evolved terms and the corresponding time-dependent coefficients. It is the same for the other initial conditions of the subsystem with one excitation. For the case of two excitations, we should solve the master equation in the basis $\{|g0\rangle, |g1\rangle, |e0\rangle, |e1\rangle\}$. Following the same process, we can write the evolved terms of Eq. (11) with two excitations initially. Therefore, we can obtain the evolved states of the atoms plus the modes for all the three cases. The interested density operator $\rho^J_{AB}(t)$ of the atoms is derived by tracing out the total evolved state over the modes. Note that $\rho^J_{AB}(t)$ always has the X form; that is, its possible nonzero matrix elements in the basis $\{|1\rangle = |gg\rangle, |2\rangle = |ge\rangle, |3\rangle = |eg\rangle, |4\rangle = |ee\rangle\}$ are $\{\rho^J_{11}(t), \rho^J_{22}(t), \rho^J_{33}(t), \rho^J_{44}(t), \rho^J_{14}(t) = \rho^{J*}_{41}(t), \rho^J_{23}(t) = \rho^{J*}_{32}(t)\}$, with $\rho^J_{mn}(t) \equiv \langle m|\rho^J_{AB}(t)|n\rangle$. The explicit time-dependent expressions of nonzero matrix elements $\rho^J_{mn}(t)$ are collected in Appendix A. Generally, we recognize that for $t > 0$,

$$\rho^{\mathrm{I}}_{nn}(t) = \rho^{\mathrm{II}}_{nn}(t) \neq \rho^{\mathrm{III}}_{nn}(t), \tag{12}$$

$$|\rho^{\mathrm{I}}_{14}(t)| = |\rho^{\mathrm{II}}_{14}(t)| \neq |\rho^{\mathrm{III}}_{14}(t)|, \tag{13}$$

and

$$|\rho^{\mathrm{I}}_{23}(t)| \geq |\rho^{\mathrm{II}}_{23}(t)| = |\rho^{\mathrm{III}}_{23}(t)| = 0. \tag{14}$$

In the following, we use the concurrence and discord as measures of atoms' entanglement and general quantum correlations. The concurrence $\mathcal{C}^J_{AB}(t)$ [52] measures the entanglement amount of the atoms for the case $J = \mathrm{I}$, II, or III, which by virtue of Eqs. (12) through (14) reads

$$\mathcal{C}^{\mathrm{I}}_{AB}(t) = C_1(t) + C_2(t), \tag{15}$$

$$\mathcal{C}^{\mathrm{II}}_{AB}(t) = C_1(t), \tag{16}$$

and

$$\mathcal{C}^{\mathrm{III}}_{AB}(t) = C(t), \tag{17}$$

with

$$C_1(t) = 2\max\left\{0, \left[|\rho^{\mathrm{I}}_{14}(t)| - \sqrt{\rho^{\mathrm{I}}_{22}(t)\rho^{\mathrm{I}}_{33}(t)}\right]\right\}, \tag{18}$$

$$C_2(t) = 2\max\left\{0, \left[|\rho^{\mathrm{I}}_{23}(t)| - \sqrt{\rho^{\mathrm{I}}_{11}(t)\rho^{\mathrm{I}}_{44}(t)}\right]\right\}, \tag{19}$$

and

$$C(t) = 2\max\left\{0, \left[|\rho^{\mathrm{III}}_{14}(t)| - \sqrt{\rho^{\mathrm{III}}_{22}(t)\rho^{\mathrm{III}}_{33}(t)}\right]\right\}. \tag{20}$$

As for the atoms' discord $D^J_{AB}(t)$, we shall be interested in the symmetric modal states with $c_1 = c_2$. Then $D^J_{AB}(t)$ can be expressed analytically as [15]

$$D^J_{AB}(t) = \min\{D^J_1(t), D^J_2(t)\}, \tag{21}$$

where

$$\begin{aligned} D^J_1(t) = {} & S(\rho^J_A) - S(\rho^J_{AB}) - \rho^J_{11}(t)\log_2\left(\frac{\rho^J_{11}(t)}{\rho^J_{11}(t)+\rho^J_{22}(t)}\right) \\ & - \rho^J_{22}(t)\log_2\left(\frac{\rho^J_{22}(t)}{\rho^J_{11}(t)+\rho^J_{22}(t)}\right) \\ & - \rho^J_{44}(t)\log_2\left(\frac{\rho^J_{44}(t)}{\rho^J_{22}(t)+\rho^J_{44}(t)}\right) \\ & - \rho^J_{22}(t)\log_2\left(\frac{\rho^J_{22}(t)}{\rho^J_{44}(t)+\rho^J_{22}(t)}\right) \end{aligned} \tag{22}$$

and

$$\begin{aligned} D^J_2(t) = {} & S(\rho^J_A) - S(\rho^J_{AB}) - \frac{1}{2}(1+T^J)\log_2\left(\frac{1+T^J}{2}\right) \\ & - \frac{1}{2}(1-T^J)\log_2\left(\frac{1-T^J}{2}\right), \end{aligned} \tag{23}$$

with $T^J = \sqrt{[\rho^J_{11}(t) - \rho^J_{44}(t)]^2 + 4[|\rho^J_{14}(t)| + |\rho^J_{23}(t)|]^2}$ and $S(\cdot)$ denoting the von Neumann entropy $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$.

3. DYNAMICS OF ATOMS' ENTANGLEMENT AND DISCORD

First, we compare the time dependence of the atoms' concurrences $\mathcal{C}^J_{AB}(t)$ ($J = \mathrm{I}$, II, III) of Eqs. (15) through (17) in terms of $C_1(t)$, $C_2(t)$, and $C(t)$ defined by Eqs. (18)–(20), respectively, in the strong atom–mode coupling regime for different sets of α and β, keeping the same parameters of the initial modes $c_1 = c_2 = \sqrt{1/2}$; that is, each mode is initially in a maximally mixed state. The sets of α and β are classified into three cases: (1) $|\alpha/\beta| = 1$, (2) $|\alpha/\beta| < 1$, and (3) $|\alpha/\beta| > 1$. As is commonly known, in the strong coupling regime the atoms' entanglement dynamics usually exhibit a sequence of alternate deaths and revivals. Generally, however, it is desirable to figure out the origin of contribution to the revived entanglement: whether it is due to a return of the decayed entanglement initially stored in the atoms or it is due to a transfer of the entanglement initially possessed by the modes. For the classically correlated modes [Eq. (8)] and uncorrelated modes [Eq. (9)], the revivals only come from the first sort of contribution, while for the modes [Eq. (6)] with nonzero initial entanglement, both these sorts contribute during the time evolution. As recognized from Eqs. (18) and (19), for the initial states of the atoms and the entangled modes specified by Eqs. (5) and (6), we can use $C_1(t)$ and $C_2(t)$ to decompose the atoms' entanglement dynamics into two constituent parts: $C_1(t)$ indicates the variations of atomic entanglement that are stored initially by the atoms themselves and thus can signify the first kind of contribution to entanglement revivals, while $C_2(t)$ indicates the variations of atomic entanglement that are induced by the initial entangled modes and thus can denote the second kind of contribution to entanglement revivals. Note that these two kinds of contribution can be sharply distinguished because in Eqs. (18) and (19) $|\rho^{\mathrm{I}}_{14(23)}(t)| \leq \sqrt{\rho^{\mathrm{I}}_{11(22)}(t)\rho^{\mathrm{I}}_{44(33)}(t)}$ at any time, so that $C_1(t)$ and $C_2(t)$ cannot be positive simultaneously, implying that at any given time only one of them, either $C_1(t)$ or $C_2(t)$, contributes. Also, we can observe from Eqs. (15) and (16), besides the additional transfer of modes' entanglement to the atoms, the dynamics of atoms' entanglement are identical with regard to the quantumly and classically correlated modes. Therefore, as far as the ability to return the decayed entanglement back to the atoms, these two forms of correlations are equivalent but differ from the uncorrelated modes.

In Fig. 1 we plot C_1, C_2, and C, Eqs. (18) through (20), as functions of the rescaled time Ωt in the strong atom–mode coupling regime with $\Gamma/R = 0.1$ for (1) $\alpha = \beta = \sqrt{1/2}$; (2) $\alpha = \sqrt{1/10}$, $\beta = \sqrt{9/10}$; and (3) $\alpha = \sqrt{9/10}$, $\beta = \sqrt{1/10}$. At the beginning C_2 does not contribute, but C_1 and C drop quickly to zero at the same time, resulting in the first sudden death of entanglement, regardless of the ratio $|\alpha/\beta|$. Yet the dead entanglement can revive later for some time before the second sudden death of entanglement occurs, and so on. Roughly speaking, the interval between neighboring revivals is shorter for larger ratio $|\alpha/\beta|$. For any α and β, the most pronounced contribution to revivals comes from C_2 (i.e., the gain of initial entanglement of the modes). Although the contribution of C_1 (associated with correlated modes) to the entanglement revivals may be more pronounced than that of C (associated with uncorrelated modes), both the contributions from C_1 and C are less pronounced than that from C_2 for $|\alpha/\beta| \geq 1$ [see Figs. 1(a) and 1(c)] and may disappear for small enough $|\alpha/\beta|$ [see Fig. 1(b)]. Remarkable is the fact that C_2 can be greater than the value of the initial atoms' entanglement if $|\alpha/\beta|$ is large enough [see Fig. 1(c)]. This means that initially entangled modes can amplify the atoms' entanglement. Vice versa, an amplification in atoms' entanglement during the time evolution, if any, would signify presence of initial quantum correlations other than the atom–atom entanglement.

In Fig. 2 we show the dynamics of the atoms' discord D^{J}_{AB}, Eq. (21), using the same parameters as in Fig. 1. We can see that the discord exhibits different dynamical behaviors for different forms of modes' correlations. From Fig. 1(b) it was learned that the dead entanglement of atoms cannot revive if the initial weight of the excited state is much greater than that of the ground state (i.e., $|\alpha/\beta|$ is much less than 1) when there is no entanglement between the modes at the very beginning. However, as shown in Fig. 2(b), the atoms' discord can still be recovered after decaying to zero even when the modes are initially classically correlated or totally independent. Also, as Fig. 2(c) reveals, if initially the atomic ground state weighs more than the excited state (i.e., $|\alpha/\beta| > 1$), then the atoms' discord can grow over its initial value in the time evolution due to the transfer of discord from the modes to the atoms.

Next, we turn to the weak atom–mode coupling regime. It is expected and was confirmed by our numerical calculations that, in this coupling regime, when the initial modes are classically correlated (case of $J = \mathrm{II}$) or independent (case of $J = \mathrm{III}$), the atoms' entanglement can suffer a sudden death,

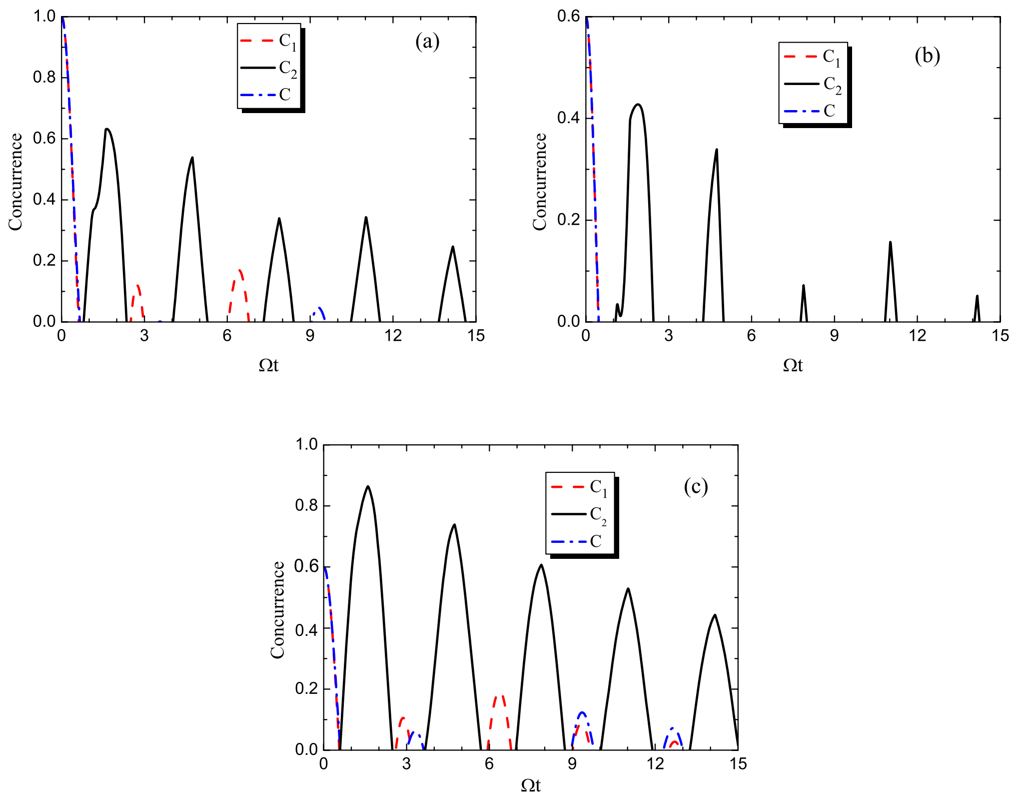

Fig. 1. (Color online) C_1, C_2, and C, Eqs. (18) through (20), versus the rescaled time Ωt in the strong atom–mode coupling regime with $\Gamma/R = 0.1$ for $c_1 = c_2 = \sqrt{1/2}$ and (a) $\alpha = \beta = \sqrt{1/2}$; (b) $\alpha = \sqrt{1/10}$, $\beta = \sqrt{9/10}$; and (c) $\alpha = \sqrt{9/10}$, $\beta = \sqrt{1/10}$. The atoms' concurrences C^{J}_{AB} in terms of C_1, C_2, or C are determined in the text through Eqs. (15) through (17).

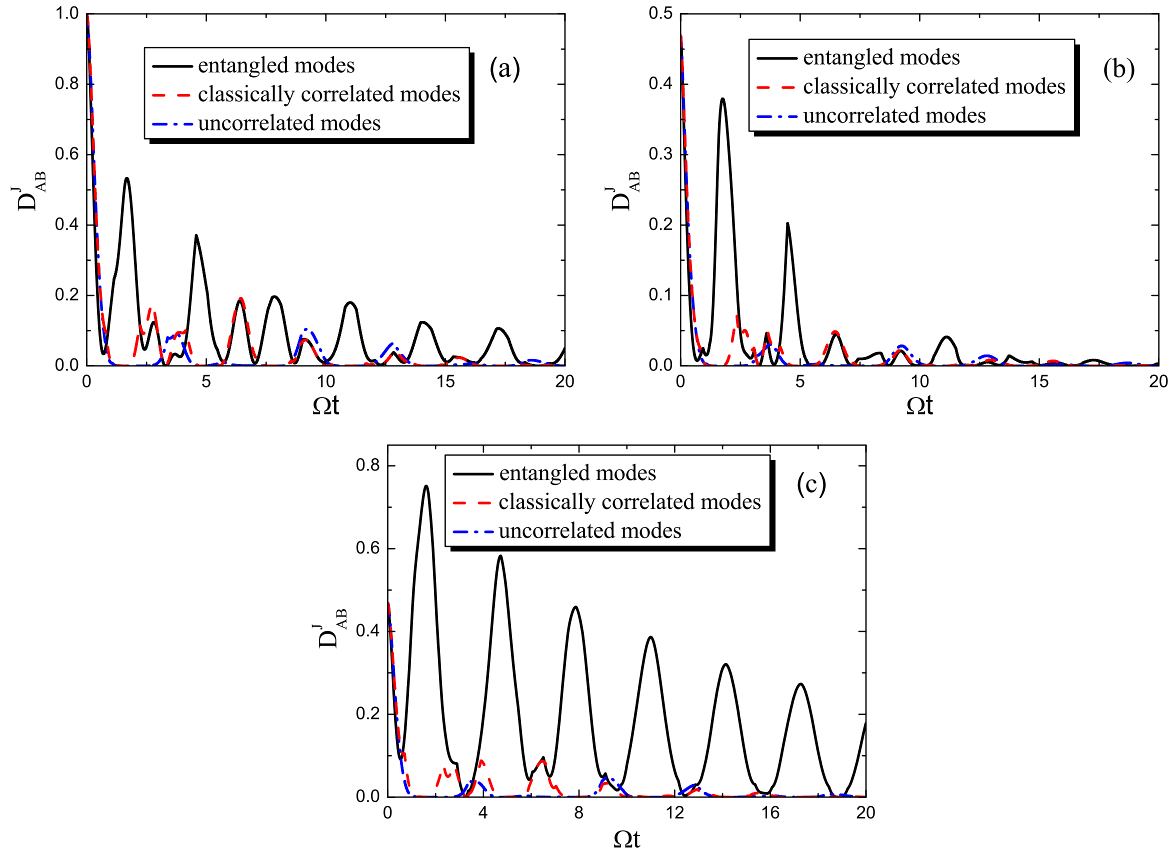

Fig. 2. (Color online) Atoms' discord D^J_{AB}, Eq. (21), versus the rescaled time Ωt in the strong atom–mode coupling regime with $\Gamma/R = 0.1$ for $c_1 = c_2 = \sqrt{1/2}$ and (a) $\alpha = \beta = \sqrt{1/2}$; (b) $\alpha = \sqrt{1/10}$, $\beta = \sqrt{9/10}$; and (c) $\alpha = \sqrt{9/10}$, $\beta = \sqrt{1/10}$.

but after that it cannot revive any more, while the atoms' discord just dies asymptotically. The atoms' dynamical behavior, however, changes remarkably when the initial modes are entangled (case of $J = \mathrm{I}$). Namely, in this case the atoms' entanglement can revive after a sudden death, as shown in Fig. 3(a), while the atoms' discord can slow down the decay rate in a nontrivial manner, as shown in Fig. 3(b). Note that the behaviors in Fig. 3 are striking because they signify the nonlocal effects of the modes that still significantly influence the dynamics of atoms' correlations despite the weakness of local atom–mode couplings.

As the last concern in this paper, we expose another interesting effect in the dynamics of an open system, which is a consequence of the initial entanglement of parts of the composite environment. Because global entanglement cannot be created by local means, two independent atoms in two remote locations can never be entangled in the future if the local modes to which they are coupled are separable or uncorrelated. Nevertheless, the presence of initial entanglement between the modes can make uncorrelated atoms correlated during the total system's evolution, as we shall demonstrate in what follows using the model and the correlation measures

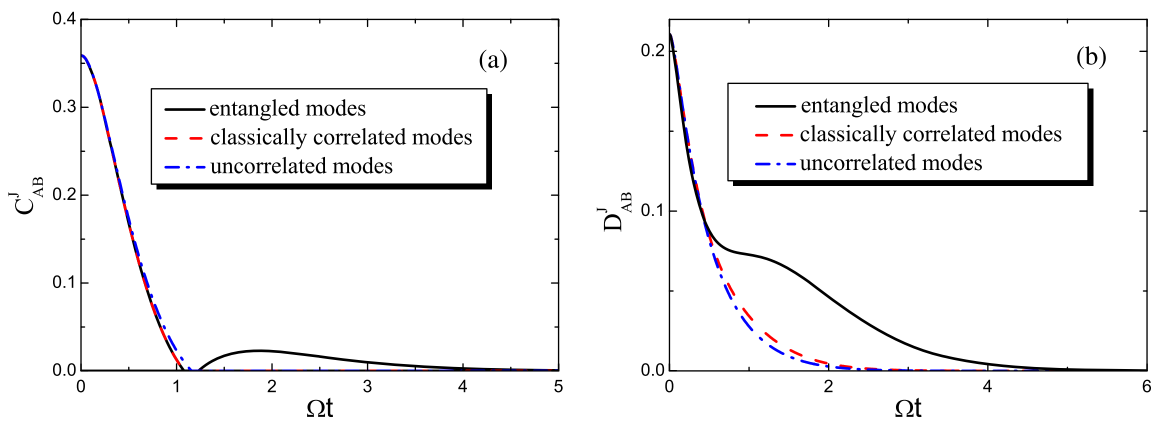

Fig. 3. (Color online) Atoms' (a) entanglement and (b) discord versus the rescaled time Ωt in the weak atom–mode coupling regime with $\Gamma/R = 4$ for $c_1 = c_2 = \sqrt{1/2}$ and $\alpha = \sqrt{29/30}$ and $\beta = \sqrt{1/30}$.

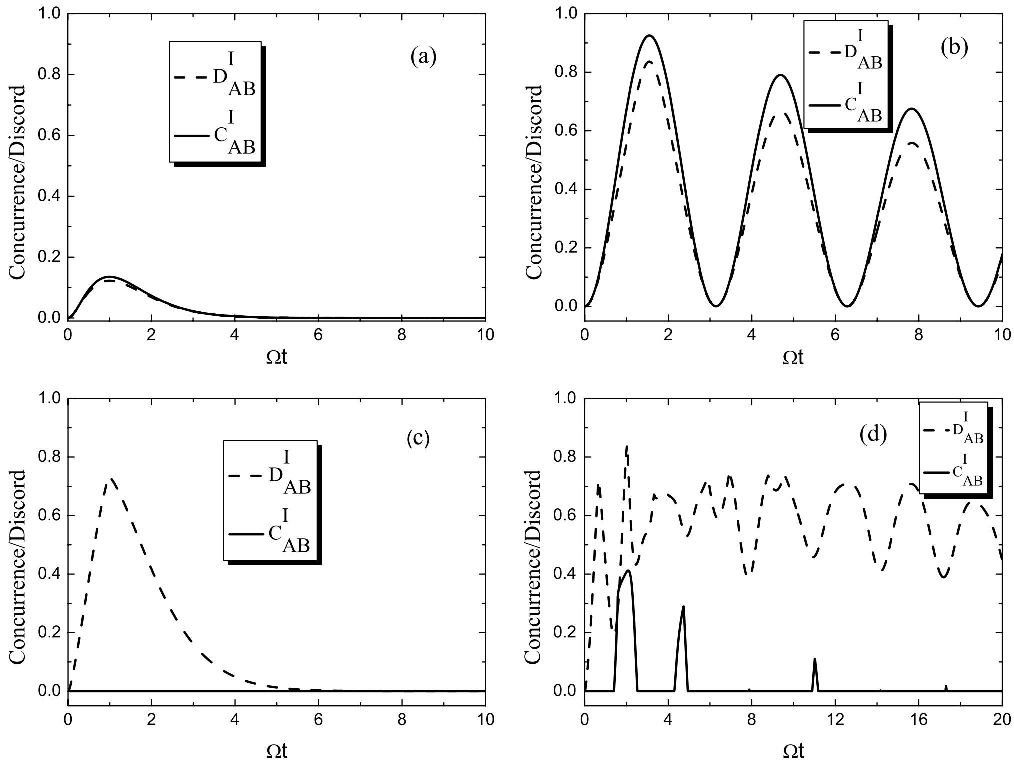

Fig. 4. Atoms' concurrence $C^{\rm I}_{AB}$ and discord $D^{\rm I}_{AB}$ versus the rescaled time Ωt when initially the modes are entangled [Eq. (6)] with $c_1 = c_2 = \sqrt{1/2}$, the atoms are in product state (a), (b) $|gg\rangle_{AB}$ or (c), (d) $|ee\rangle_{AB}$, and the local atom–mode coupling is (a), (c) weak with $\Gamma/R = 4$ or (b), (d) strong with $\Gamma/R = 0.1$.

described in Section 2. In the framework of our model the possibility to entangle the atoms is delicate in the sense that it depends on both the initial atoms' state and the atom–mode coupling regime. For clarity we shall analyze the case when initially the modes possess maximal entanglement specified by the state described by Eq. (6) with $c_1 = c_2 = \sqrt{1/2}$, while the atoms are assumed to be in two possible situations: they are both either in their ground states $|gg\rangle_{AB}$ or in their excited states $|ee\rangle_{AB}$. As for the local atom–mode coupling regime, we assume that it may be either weak or strong. The results of our numerical calculations are summarized in Fig. 4. From the figure the following features can be observed. Figures 4(a) and 4(b) show that two atoms, both of which are initially in their ground state $|gg\rangle_{AB}$, can become correlated right after the system evolves, no matter how strong the atom–mode coupling is, that is, for an arbitrary ratio of Γ/Ω. In this case the role of the coupling regime is that when it is weak ($\Gamma > 2\Omega$), both the atoms' entanglement and discord are going to vanish after reaching a single maximum value, but when it is strong ($\Gamma < 2\Omega$), the atoms' correlations undergo damped oscillations. Differently from the behaviors in Figs. 4(a) and 4(b), two atoms, both of which are initially in their excited state $|ee\rangle_{AB}$, cannot become entangled in the weak atom–mode coupling regime [solid curve in Fig. 4(c)]; they can however become entangled in the strong atom–mode coupling regime, not as soon as the system's evolution begins but with some delay, and the atoms' entanglement suffers several sudden death-revivals before completely vanishing [solid curve in Fig. 4(d)]. As for the atoms' discord, in this case, it can still be induced in both the weak and strong atom–mode coupling regimes [dashed curves in Figs. 4(c) and 4(d)]. Especially if the atom–mode coupling is strong, the atoms' discord can be robust and survive for a quite long time [dashed curve in Fig. 4(d)].

4. CONCLUSION

The dynamics of an open multipartite system depend essentially on the composite environment surrounding it. Different parts of the composite environment that are coupled locally with the corresponding subsystems of the multipartite system can initially be independent or correlated in nature. To explore in detail the difference of correlation in comparison with independence in the initial environment's parts, we have treated two qubits in terms of two two-level atoms A and B as the open system of interest and two damped modes of radiation field a and b as two parts of the composite environment. Atom A (B) and mode a (b) are in one location and interact with each other as the evolution starts. The two modes are assumed initially to be quantum correlated (entangled), classically correlated, or uncorrelated, but their marginal states are the same. The situations we considered in this paper not only represent typical forms of correlations but also allow us to compare their differences in affecting open-system dynamics given the same marginal states. It should be noted that there can be another situation when the initial state of the modes is separable but possesses nonzero discord. In this connection, we notice that the authors of [53] studied the witness for initial system–environment correlations in open-system dynamics, taking into account the condition that initially the system has a certain quantum correlation but null entanglement with the environment. In the following, we briefly summarize the most pronounced signatures of the presence of initial entanglement between the parts of the composite environment.

For initially entangled atoms identified by the state described in Eq. (5), during the course of evolution the atoms' entanglement and discord develop additional revival periods and can be amplified over their initial values if the local atom–mode couplings are strong (see Figs. 1 and 2). As for the weak atom–mode couplings, the atoms' entanglement can revive after a sudden death, if any [see Fig. 3(a)], while the decay rate of the atoms' discord can be slowed down in a nontrivial manner [see Fig. 3(b)], in clear opposition to the cases of classically correlated and uncorrelated composite environment. Furthermore, the presence of initial entanglement between modes of the composite environment can make independent atoms in product state $|gg\rangle_{AB}$ become correlated in terms of both entanglement and discord, no matter how strong are the local atom–mode couplings [see Figs. 4(a) and 4(b)]. By contrast, the atoms in product state $|ee\rangle_{AB}$ can be made correlated in terms of discord in both weak and strong atom–mode coupling regimes; however, their entanglement is induced only when the local couplings are strong [see Figs. 4(c) and 4(d)]. An overall view based on our above-reported results is that initial entanglement among parts of a composite environment can be exploited as a useful source for nonlocal effects in the global dynamics of an open multipartite system not only when the local couplings are strong but also even when all the local couplings are weak.

APPENDIX A: EXPRESSIONS FOR THE ATOMS' MATRIX ELEMENTS

In this Appendix we provide time-dependent expressions for the atoms' matrix elements $\rho^J_{mn}(t)$, with $J=$ I, II, III, in terms of $\mathcal{L}^{-1}\{L(s)\}$, the inverse Laplace transform of $L(s)$.

$$\begin{aligned}\rho^{\mathrm{I}}_{11}(t) &= \rho^{\mathrm{II}}_{11}(t)\\ &= \alpha^2\mathcal{L}^{-1}\{x_1(s)+x_2(s)\}+\beta^2\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\\ &\quad\times\mathcal{L}^{-1}\{x_5(s)+x_6(s)+x_7(s)\},\end{aligned}\tag{A1}$$

$$\begin{aligned}\rho^{\mathrm{I}}_{22}(t) = \rho^{\mathrm{II}}_{22}(t) &= |\alpha c_1|^2\mathcal{L}^{-1}\{x_4(s)\}\\ &\quad+|\beta c_1|^2\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\}\\ &\quad+|\beta c_2|^2\mathcal{L}^{-1}\{x_5(s)\\ &\quad+x_6(s)+x_7(s)\}\times\mathcal{L}^{-1}\{x_{10}(s)\},\end{aligned}\tag{A2}$$

$$\rho^{\mathrm{I}}_{33}(t)=\rho^{\mathrm{II}}_{33}(t)=\rho^{\mathrm{I}}_{22}(t)|_{c_{1,2}\to c_{2,1}}=\rho^{\mathrm{II}}_{22}(t)|_{c_{1,2}\to c_{2,1}},\tag{A3}$$

$$\rho^{\mathrm{I}}_{44}(t)=\rho^{\mathrm{II}}_{44}(t)=\beta^2\mathcal{L}^{-1}\{x_{10}(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\},\tag{A4}$$

$$\rho^{\mathrm{I}}_{14}(t)=\rho^{\mathrm{II}}_{14}(t)=\alpha\beta^*\mathcal{L}^{-1}\{x_{11}(s)\}\times\mathcal{L}^{-1}\{x_{12}(s)+x_{13}(s)\},\tag{A5}$$

$$\begin{aligned}\rho^{\mathrm{I}}_{23}(t) &= |\alpha|^2c_1c_2^*\mathcal{L}^{-1}\{x_{14}(s)\}\times\mathcal{L}^{-1}\{x^*_{14}(s)\}+|\beta|^2c_1c_2\mathcal{L}^{-1}\{x_{15}(s)\\ &\quad+x_{16}(s)\}\times\mathcal{L}^{-1}\{x^*_{15}(s)+x^*_{16}(s)\},\end{aligned}\tag{A6}$$

$$\begin{aligned}\rho^{\mathrm{III}}_{11}(t) &= \alpha^2(|c_1|^4+|c_2|^4)\mathcal{L}^{-1}\{x_1(s)+x_2(s)\}\\ &\quad+\alpha^2|c_1c_2|^2+\alpha^2|c_1c_2|^2\mathcal{L}^{-1}\{x_1(s)\\ &\quad+x_2(s)\}\times\mathcal{L}^{-1}\{x_1(s)+x_2(s)\}\\ &\quad+\beta^2(|c_1|^4+|c_2|^4)\mathcal{L}^{-1}\{x_3(s)\\ &\quad+x_4(s)\}\times\mathcal{L}^{-1}\{x_5(s)+x_6(s)+x_7(s)\}\\ &\quad+\beta^2|c_1c_2|^2\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\times\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\\ &\quad+|\beta|^2|c_1c_2|^2\mathcal{L}^{-1}\{x_5(s)+x_6(s)\\ &\quad+x_7(s)\}\times\mathcal{L}^{-1}\{x_5(s)+x_6(s)+x_7(s)\},\end{aligned}\tag{A7}$$

$$\begin{aligned}\rho^{\mathrm{III}}_{22}(t) &= |\alpha|^2|c_1|^4\mathcal{L}^{-1}\{x_4(s)\}\\ &\quad+|\alpha|^2|c_1c_2|^2\mathcal{L}^{-1}\{x_1(s)+x_2(s)\}\times\mathcal{L}^{-1}\{x_4(s)\}\\ &\quad+|\beta|^2|c_1|^4\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\}\\ &\quad+\beta^2|c_1c_2|^2\mathcal{L}^{-1}\{x_3(s)+x_4(s)\}\times\mathcal{L}^{-1}\{x_{10}(s)\}\\ &\quad+\beta^2|c_1c_2|^2\mathcal{L}^{-1}\{x_5(s)+x_6(s)+x_7(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\}\\ &\quad+\beta^2|c_2|^4\mathcal{L}^{-1}\{x_{10}(s)\}\times\mathcal{L}^{-1}\{x_5(s)+x_6(s)+x_7(s)\},\end{aligned}\tag{A8}$$

$$\rho^{\mathrm{III}}_{33}(t)=\rho^{\mathrm{III}}_{22}(t)|_{c_{1,2}\to c_{2,1}},\tag{A9}$$

$$\begin{aligned}\rho^{\mathrm{III}}_{44}(t) &= \alpha^2|c_1c_2|^2\mathcal{L}^{-1}\{x_4(s)\}\times\mathcal{L}^{-1}\{x_4(s)\}\\ &\quad+\beta^2(|c_1|^4+|c_2|^4)\mathcal{L}^{-1}\{x_{10}(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\}\\ &\quad+\beta^2|c_1c_2|^2\mathcal{L}^{-1}\{x_{10}(s)\}\times\mathcal{L}^{-1}\{x_{10}(s)\}\\ &\quad+\beta^2|c_1c_2|^2\mathcal{L}^{-1}\{x_8(s)+x_9(s)\}\times\mathcal{L}^{-1}\{x_8(s)+x_9(s)\},\end{aligned}\tag{A10}$$

$$\begin{aligned}\rho^{\mathrm{III}}_{14}(t) &= \alpha\beta(|c_1|^4+|c_2|^4)\mathcal{L}^{-1}\{x_{11}(s)\}\times\mathcal{L}^{-1}\{x_{12}(s)+x_{13}(s)\}\\ &\quad+\alpha\beta|c_1c_2|^2\mathcal{L}^{-1}\{x_{11}(s)\}\times\mathcal{L}^{-1}\{x_{11}(s)\}\\ &\quad+\alpha\beta|c_1c_2|^2\mathcal{L}^{-1}\{x_{12}(s)\\ &\quad+x_{13}(s)\}\times\mathcal{L}^{-1}\{x_{12}(s)+x_{13}(s)\}.\end{aligned}\tag{A11}$$

In the above expressions, $x_n(s)$ with $n=1,2,\ldots,16$ are given by

$$x_1(s)=\frac{\Gamma\left[1+\frac{s^2}{s(s+\Gamma)+4\Omega^2}\right]}{s(2s+\Gamma)},\tag{A12}$$

$$x_2(s)=\frac{s(2s+\Gamma)+4\Omega^2}{(2s+\Gamma)[s(s+\Gamma)+4\Omega^2]},\tag{A13}$$

$$x_3(s)=\frac{4\Gamma\Omega^2}{s(2s+\Gamma)[s(s+\Gamma)+4\Omega^2]},\tag{A14}$$

$$x_4(s)=\frac{4\Omega^2}{(2s+\Gamma)[s(s+\Gamma)+4\Omega^2]},\tag{A15}$$

$$x_5(s) = \frac{4\Gamma^2\Omega^2[(2s+\Gamma)(7s+6\Gamma)+24\Omega^2]}{s(2s+\Gamma)(2s+3\Gamma)[s(s+\Gamma)+4\Omega^2][(s+\Gamma)(s+2\Gamma)+8\Omega^2]}, \tag{A16}$$

$$x_6(s) = \frac{4\Gamma\Omega^2[(2s+\Gamma)(7s+6\Gamma)+24\Omega^2]}{(2s+\Gamma)(2s+3\Gamma)[s(s+\Gamma)+4\Omega^2][(s+\Gamma)(s+2\Gamma)+8\Omega^2]}, \tag{A17}$$

$$x_7(s) = \frac{8\Omega^2}{(2s+3\Gamma)[(s+\Gamma)(s+2\Gamma)+8\Omega^2]}, \tag{A18}$$

$$x_8(s) = \frac{\Gamma[4s^4+20s^3\Gamma+35s^2\Gamma^2+25s\Gamma^3+6\Gamma^4+4s(2s+\Gamma)\Omega^2+96\Omega^4]}{(2s+\Gamma)(2s+3\Gamma)[s(s+\Gamma)+4\Omega^2][(s+\Gamma)(s+2\Gamma)+8\Omega^2]}, \tag{A19}$$

$$x_9(s) = \frac{(s+2\Gamma)(2s+3\Gamma)+8\Omega^2}{(2s+3\Gamma)[(s+\Gamma)(s+2\Gamma)+8\Omega^2]}, \tag{A20}$$

$$x_{10}(s) = \frac{(s+\Gamma)(2s+\Gamma)+4\Omega^2}{(2s+\Gamma)(s(s+\Gamma)+4\Omega^2)}, \tag{A21}$$

$$x_{11}(s) = \frac{2s+\Gamma-2i\omega}{(s-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2}, \tag{A22}$$

$$x_{12}(s) = \frac{\Gamma(2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)((s+\Gamma-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2)-16\Gamma\Omega^4}{((s-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2)((s+\Gamma-i\omega)^2((2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24\Omega^2)+4\Omega^4)}, \tag{A23}$$

$$x_{13}(s) = \frac{4s^3+3\Gamma^3+12s^2(\Gamma-i\omega)-11i\Gamma^2\omega+2\Gamma(-6\omega^2+7\Omega^2)+4i(\omega^3-3\omega\Omega^2)}{(s+\Gamma-i\omega)^2(2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24(s+\Gamma-i\omega)^2\Omega^2+4\Omega^4} + \frac{s(11\Gamma^2-24i\Gamma\omega+12(-\omega^2+\Omega^2))}{(s+\Gamma-i\omega)^2(2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24(s+\Gamma-i\omega)^2\Omega^2+4\Omega^4}, \tag{A24}$$

$$x_{14}(s) = \frac{2i\Omega}{(s-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2}, \tag{A25}$$

$$x_{15}(s) = \frac{-2i\Gamma\Omega(4s^3+3\Gamma^3+12s^2(\Gamma-i\omega)-11i\Gamma^2\omega-6\Gamma(2\omega^2+3\Omega^2)+4i(\omega^3+5\omega\Omega^2))}{((s-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2)((s+\Gamma-i\omega)^2((2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24\Omega^2)+4\Omega^4)} - \frac{2i\Gamma\Omega s(11\Gamma^2-24i\Gamma\omega-4(3\omega^2+5\Omega^2))}{((s-i\omega)(2s+\Gamma-2i\omega)+2\Omega^2)((s+\Gamma-i\omega)^2((2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24\Omega^2)+4\Omega^4)}, \tag{A26}$$

$$x_{16}(s) = \frac{2i\Omega[-(s+\Gamma-i\omega)(2s+3\Gamma-2i\omega)+2\Omega^2]}{(s+\Gamma-i\omega)^2(2s+\Gamma-2i\omega)(2s+3\Gamma-2i\omega)+24(s+\Gamma-i\omega)^2\Omega^2+4\Omega^4}. \tag{A27}$$

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