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2013 Phys. Scr. 87 025005

(http://iopscience.iop.org/1402-4896/87/2/025005)

Phys. Scr. **87** (2013) 025005 (6pp) doi:10.1088/0031-8949/87/02/025005

Flexible deterministic joint remote state preparation with a passive receiver

Qing-Qin Chen[1], Yan Xia[1,2] and Nguyen Ba An[3]

[1] Department of Physics, Fuzhou University, Fuzhou 350002, China
[2] School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, China
[3] Center for Theoretical Physics, Institute of Physics, 10 Dao Tan, Hanoi, Vietnam

E-mail: xia-208@163.com and nban@iop.vast.ac.vn

Received 19 July 2012
Accepted for publication 19 December 2012
Published 16 January 2013
Online at stacks.iop.org/PhysScr/87/025005

Abstract
We use the Einstein–Podolsky–Rosen pairs as the quantum channel and adopt the adaptive measurement strategy to allow $M > 1$ separate parties to cooperatively prepare an arbitrary two-qubit state for a remote receiver, without leakage of the state's full information. Our protocol is flexible and convenient in the sense that the receiver can be assigned even after the distribution of the qubits of the shared quantum channel and once assigned he/she only needs to perform a simple recovery operation. In addition, our protocol always succeeds and is applicable to a certain realistic situation for which other protocols are not suited.

PACS numbers: 03.67.HK, 03.65.Ud

1. Introduction

Since the first protocol for quantum communication brought out by Bennett *et al* [1] and known as quantum teleportation (QT), research on quantum information processing has grown by degrees. This original protocol with the use of an Einstein–Podolsky–Rosen (EPR) pair as the quantum channel combined with traditional communication in terms of two subsidiary classical bits gives an intriguing way to teleport an unknown qubit state. Gradually, Lo [2] and Pati [3], among others, proposed another protocol called remote state preparation (RSP) which deals with the case when the qubit state is known beforehand. The difference between QT and RSP is that, while in QT the requirement for physical resources (both quantum and classical) is fixed, in RSP one can trade off between the degree of entanglement and the classical communication cost. In recent years, RSP has attracted much attention. Various generalizations of RSP, such as RSP at multiple locations [4], RSP of mixed states [5, 6], high-dimension RSP [7, 8], oblivious RSP [9], continuous variable RSP [10, 11], etc, have been put forward. Of more interest is the fact that some RSP protocols were accomplished experimentally [12–14].

As was designed, RSP involves only one sender, so all the information about the state to be prepared is disclosed to him/her. To avoid such full leakage of information, joint remote state preparation (JRSP) protocols were proposed [15–17]. In JRSP there are two or more senders (later we call them preparers), possibly at different locations, and the information of the to-be-prepared state is secretly split among the preparers in such a way that neither an individual preparer nor a subgroup of them can infer the state. This feature is vital for the transfer of confidential communication between agencies. Most JRSP protocols are probabilistic (see, e.g., [15–24]). However, deterministic JRSP protocols [25–27] have also been suggested very recently. It is interesting that the deterministic protocols employ the same physical resources as in the probabilistic ones, but the ways to perform them are diverse, as should be. Of course, deterministic protocols are most beneficial from the viewpoint of the resources consumed. In [25], Greenberg–Horne–Zeilinger (GHZ) states were used as the quantum channel. However, EPR pairs can be used as well and are even more preferable since, being the simplest kind of entanglement, they can be more easily produced and distributed than the GHZ ones. Deterministic JRSP protocols using EPR pairs have been designed in [26] in which the receiver role is quite hard. Namely, the receiver has to not only apply proper recovery operators at the end but also carry out some measurements, controlled-NOT gates and classical communication at the beginning. Such demands make the protocols in [26] unfulfillable if the receiver's laboratory is not sufficiently well equipped.

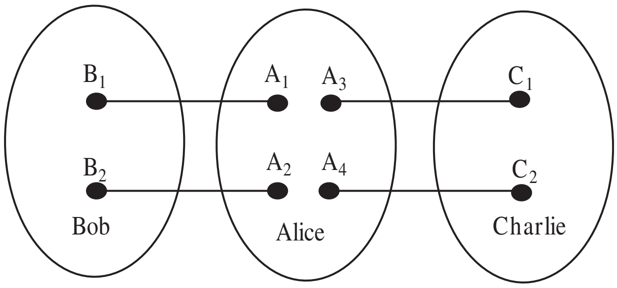

Figure 1. The qubits' distribution for the JRSP of an arbitrary two-qubit state via four EPR pairs in our protocol. Dots represent qubits, while solid lines connect the entangled qubits.

Very recently, Bich *et al* [27] have modified the JRSP of a single-qubit state in [26] so as to considerably release the receiver's function: only the application of recovery operators at the end remains. In addition to that, the protocol in [27] also possesses a flexibility in the sense that assignment of the receiver can be postponed until after the EPR pairs have been distributed, a feature not available in those of [26].

Since two-qubit states also play an important role in quantum information processing, the JRSP of such states deserves explicit consideration. This motivates us in this paper to extend the original idea of [27] to the case of arbitrary two-qubit states so as to retain all the advantageous aspects. That is, the protocol we shall propose should be deterministic, flexible and just requires a passive receiver. In section 2 we consider in detail the case of two preparers. Section 3 deals with $M > 2$ preparers. Finally, the conclusion drawn is presented in section 4.

2. The case with two preparers

Let the state of two qubits C_1 and C_2, which will be remotely prepared for Charlie, have the form

$$|\Psi\rangle_{C_1C_2} = \sum_{n=0}^{3} a_n\, \mathrm{e}^{\mathrm{i}\theta_n} |n\rangle_{C_1C_2}, \qquad (1)$$

where a_n and θ_n are arbitrary real numbers satisfying the normalization condition $\sum_{n=0}^{3} a_n^2 = 1$, while $|0\rangle_{XY}$, $|1\rangle_{XY}$, $|2\rangle_{XY}$ and $|3\rangle_{XY}$ are short for $|00\rangle_{XY}$, $|01\rangle_{XY}$, $|10\rangle_{XY}$ and $|11\rangle_{XY}$, respectively. Obviously, the state (1) is fully identified by the data set $S = \{a_n, \theta_n\}$, which can be split into two subsets $S_1 = \{a_n\}$ and $S_2 = \{\theta_n\}$. In this section, we consider the case with two preparers Alice and Bob. To achieve the deterministic JRSP of $|\Psi\rangle_{C_1C_2}$, the subset S_1 is given to Alice and S_2 to Bob. Clearly, neither Alice nor Bob alone can infer $|\Psi\rangle_{C_1C_2}$. Like in [26], here we also use four EPR pairs as the quantum channel, but unlike in [26], we distribute the qubits in a different manner as shown in figure 1. That is, the quantum channel state is

$$|q\rangle = |\mathrm{EPR}\rangle_{A_1B_1} \otimes |\mathrm{EPR}\rangle_{A_2B_2} \otimes |\mathrm{EPR}\rangle_{A_3C_1} \otimes |\mathrm{EPR}\rangle_{A_4C_2}, \qquad (2)$$

where $|\mathrm{EPR}\rangle = (|0\rangle + |3\rangle)/\sqrt{2}$ with qubits $\{A_1, A_2, A_3, A_4\}$, $\{B_1, B_2\}$ and $\{C_1, C_2\}$ held by Alice, Bob and Charlie, respectively.

To be deterministic, our protocol proceeds in three steps as follows.

Step 1. Alice chooses the basis $\{|\mu_k\rangle_{A_1A_2A_3A_4}; k = 0, 1, \ldots, 15\}$, which is related to the computational basis $\{|0000\rangle, |0101\rangle, |0001\rangle, |0110\rangle, |0010\rangle, |0111\rangle, |0011\rangle, |0100\rangle, |1001\rangle, |1110\rangle, |1000\rangle, |1101\rangle, |1011\rangle, |1100\rangle, |1010\rangle, |1111\rangle\}_{A_1A_2A_3A_4} \equiv \{|00\rangle, |11\rangle, |30\rangle, |01\rangle, |20\rangle, |31\rangle, |10\rangle, |21\rangle, |32\rangle, |03\rangle, |02\rangle, |13\rangle, |12\rangle, |23\rangle, |22\rangle, |33\rangle\}_{A_1A_2A_3A_4}$ as

$$\begin{pmatrix} |\mu_0\rangle_{A_1A_2A_3A_4} \\ |\mu_1\rangle_{A_1A_2A_3A_4} \\ \vdots \\ |\mu_{15}\rangle_{A_1A_2A_3A_4} \end{pmatrix} = U \begin{pmatrix} |00\rangle_{A_1A_2A_3A_4} \\ |11\rangle_{A_1A_2A_3A_4} \\ \vdots \\ |33\rangle_{A_1A_2A_3A_4} \end{pmatrix}, \qquad (3)$$

where U is a 16×16 matrix of the form

$$U = \begin{pmatrix} U_0 & O & O & O & O & O & O & U_1 \\ O & U_0 & O & O & O & O & U_1 & O \\ O & O & U_0 & O & O & U_1 & O & O \\ O & O & O & U_0 & U_1 & O & O & O \end{pmatrix}, \qquad (4)$$

with

$$U_0 = \begin{pmatrix} a_0 & a_1 \\ a_1 & -a_0 \\ a_2 & -a_3 \\ a_3 & a_2 \end{pmatrix}, \quad U_1 = \begin{pmatrix} a_2 & a_3 \\ a_3 & -a_2 \\ -a_0 & a_1 \\ -a_1 & -a_0 \end{pmatrix}, \quad O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \qquad (5)$$

Since Alice knows $S_1 = \{a_0, a_1, a_2, a_3\}$, she is able to manage measuring her qubits in the basis $\{|\mu_k\rangle_{A_1A_2A_3A_4}\}$. Taking the states $|\mu_k\rangle_{A_1A_2A_3A_4}$ into consideration, we rearrange state (2) as

$$|q\rangle = \frac{1}{4} \sum_{k=0}^{15} |\mu_k\rangle_{A_1A_2A_3A_4} |\Phi_k\rangle_{B_1B_2C_1C_2}, \qquad (6)$$

where

$$|\Phi_0\rangle_{B_1B_2C_1C_2} = (a_0|00\rangle + a_1|11\rangle + a_2|22\rangle + a_3|33\rangle)_{B_1B_2C_1C_2}, \qquad (7)$$

$$|\Phi_1\rangle_{B_1B_2C_1C_2} = (a_1|00\rangle - a_0|11\rangle + a_3|22\rangle - a_2|33\rangle)_{B_1B_2C_1C_2}, \qquad (8)$$

$$|\Phi_2\rangle_{B_1B_2C_1C_2} = (a_2|00\rangle - a_3|11\rangle - a_0|22\rangle + a_1|33\rangle)_{B_1B_2C_1C_2}, \ldots \qquad (9)$$

and

$$|\Phi_{15}\rangle_{B_1B_2C_1C_2} = (a_3|10\rangle + a_2|21\rangle - a_1|32\rangle - a_0|03\rangle)_{B_1B_2C_1C_2}. \qquad (10)$$

As follows from equation (6), when carrying out the collective four-qubit measurement Alice obtains a state $|\mu_k\rangle_{A_1A_2A_3A_4}$

Table 1. The collapsed state $|\Psi_{km}\rangle_{C_1C_2}$ of Charlie's qubits C_1, C_2 and the corresponding recovery operators R_{km} related to the measurement outcomes (k, m) from Alice and Bob. I is the identity operator and X (Z) the Pauli bit (phase) flip operator.

(k, m)	$\|\Psi_{km}\rangle_{C_1C_2}$	R_{km}
(0, 0)(4, 0)(8, 0)(12, 0)	$a_0e^{i\theta_0}\|0\rangle+a_1e^{i\theta_1}\|1\rangle+a_2e^{i\theta_2}\|2\rangle+a_3e^{i\theta_3}\|3\rangle$	$I\otimes I$
(0, 1)(4, 1)(8, 1)(12, 1)	$a_0e^{i\theta_0}\|0\rangle-a_1e^{i\theta_1}\|1\rangle+a_2e^{i\theta_2}\|2\rangle-a_3e^{i\theta_3}\|3\rangle$	$I\otimes Z$
(0, 2)(4, 2)(8, 2)(12, 2)	$a_0e^{i\theta_0}\|0\rangle-a_1e^{i\theta_1}\|1\rangle-a_2e^{i\theta_2}\|2\rangle+a_3e^{i\theta_3}\|3\rangle$	$Z\otimes Z$
(0, 3)(4, 3)(8, 3)(12, 3)	$a_0e^{i\theta_0}\|0\rangle+a_1e^{i\theta_1}\|1\rangle-a_2e^{i\theta_2}\|2\rangle-a_3e^{i\theta_3}\|3\rangle$	$Z\otimes I$
(1, 0)(5, 0)(9, 0)(13, 0)	$-a_0e^{i\theta_0}\|1\rangle+a_1e^{i\theta_1}\|0\rangle-a_2e^{i\theta_2}\|3\rangle+a_3e^{i\theta_3}\|2\rangle$	$I\otimes XZ$
(1, 1)(5, 1)(9, 1)(13, 1)	$a_0e^{i\theta_0}\|1\rangle+a_1e^{i\theta_1}\|0\rangle+a_2e^{i\theta_2}\|3\rangle+a_3e^{i\theta_3}\|2\rangle$	$I\otimes X$
(1, 2)(5, 2)(9, 2)(13, 2)	$a_0e^{i\theta_0}\|1\rangle+a_1e^{i\theta_1}\|0\rangle-a_2e^{i\theta_2}\|3\rangle-a_3e^{i\theta_3}\|2\rangle$	$Z\otimes X$
(1, 3)(5, 3)(9, 3)(13, 3)	$-a_0e^{i\theta_0}\|1\rangle+a_1e^{i\theta_1}\|0\rangle+a_2e^{i\theta_2}\|3\rangle-a_3e^{i\theta_3}\|2\rangle$	$Z\otimes XZ$
(2, 0)(6, 0)(10, 0)(14, 0)	$-a_0e^{i\theta_0}\|2\rangle+a_1e^{i\theta_1}\|3\rangle+a_2e^{i\theta_2}\|0\rangle-a_3e^{i\theta_3}\|1\rangle$	$XZ\otimes Z$
(2, 1)(6, 1)(10, 1)(14, 1)	$-a_0e^{i\theta_0}\|2\rangle-a_1e^{i\theta_1}\|3\rangle+a_2e^{i\theta_2}\|0\rangle+a_3e^{i\theta_3}\|1\rangle$	$XZ\otimes I$
(2, 2)(6, 2)(10, 2)(14, 2)	$a_0e^{i\theta_0}\|2\rangle+a_1e^{i\theta_1}\|3\rangle+a_2e^{i\theta_2}\|0\rangle+a_3e^{i\theta_3}\|1\rangle$	$X\otimes I$
(2, 3)(6, 3)(10, 3)(14, 3)	$a_0e^{i\theta_0}\|2\rangle-a_1e^{i\theta_1}\|3\rangle+a_2e^{i\theta_2}\|0\rangle-a_3e^{i\theta_3}\|1\rangle$	$X\otimes Z$
(3, 0)(7, 0)(11, 0)(15, 0)	$-a_0e^{i\theta_0}\|3\rangle-a_1e^{i\theta_1}\|2\rangle+a_2e^{i\theta_2}\|1\rangle+a_3e^{i\theta_3}\|0\rangle$	$XZ\otimes X$
(3, 1)(7, 1)(11, 1)(15, 1)	$a_0e^{i\theta_0}\|3\rangle-a_1e^{i\theta_1}\|2\rangle-a_2e^{i\theta_2}\|1\rangle+a_3e^{i\theta_3}\|0\rangle$	$XZ\otimes XZ$
(3, 2)(7, 2)(11, 2)(15, 2)	$-a_0e^{i\theta_0}\|3\rangle+a_1e^{i\theta_1}\|2\rangle-a_2e^{i\theta_2}\|1\rangle+a_3e^{i\theta_3}\|0\rangle$	$X\otimes XZ$
(3, 3)(7, 3)(11, 3)(15, 3)	$a_0e^{i\theta_0}\|3\rangle+a_1e^{i\theta_1}\|2\rangle+a_2e^{i\theta_2}\|1\rangle+a_3e^{i\theta_3}\|0\rangle$	$X\otimes X$

randomly (i.e. with an equal probability of 1/16). Alice then publicly broadcasts k by means of 4 bits, letting Bob and Charlie be aware of the projection of their qubits onto a corresponding state $|\Phi_k\rangle_{B_1B_2C_1C_2} = {}_{A_1A_2A_3A_4}\langle\mu_k|q\rangle$. Note that the initial pairwise entanglements disappear and a new quadpartite entanglement appears among the qubits B_1, B_2, C_1 and C_2. This phenomenon was referred to as entanglement swapping (see, e.g., [28, 29]).

Step 2. This step is important for achieving complete success. Bob is supposed to measure his two qubits B_1 and B_2 in a delicately chosen basis. That is to say, Bob not only utilizes the subset $S_2=\{\theta_n\}$, which was given to him *a priori*, but also should take into account Alice's measurement outcome in terms of k. After careful consideration we have figured out eight choices $(C_k=0,1,2,\ldots,7)$ for Bob's measurement basis as a function of the value of k, namely

$$C_k=\begin{cases}0 & \text{if } k=0 \text{ or } 10,\\ 1 & \text{if } k=1 \text{ or } 11,\\ 2 & \text{if } k=2 \text{ or } 8,\\ 3 & \text{if } k=3 \text{ or } 9,\\ 4 & \text{if } k=4 \text{ or } 14,\\ 5 & \text{if } k=5 \text{ or } 15,\\ 6 & \text{if } k=6 \text{ or } 12,\\ 7 & \text{if } k=7 \text{ or } 13.\end{cases} \tag{11}$$

Explicitly, the basis $\{|v_m^{(C_k)}\rangle_{B_1B_2};\ C_k=0,1,\ldots,7;\ m=0,1,2,3\}$ for Bob's measurement is given by

$$\begin{pmatrix}|v_0^{(C_k)}\rangle_{B_1B_2}\\|v_1^{(C_k)}\rangle_{B_1B_2}\\|v_2^{(C_k)}\rangle_{B_1B_2}\\|v_3^{(C_k)}\rangle_{B_1B_2}\end{pmatrix}=\frac{1}{2}V^{(C_k)}(\theta_n)\begin{pmatrix}|0\rangle_{B_1B_2}\\|1\rangle_{B_1B_2}\\|2\rangle_{B_1B_2}\\|3\rangle_{B_1B_2}\end{pmatrix}, \tag{12}$$

with

$$V^{(0)}(\theta_n)=\begin{pmatrix}e^{-i\theta_0} & e^{-i\theta_1} & e^{-i\theta_2} & e^{-i\theta_3}\\ e^{-i\theta_0} & -e^{-i\theta_1} & e^{-i\theta_2} & -e^{-i\theta_3}\\ e^{-i\theta_0} & -e^{-i\theta_1} & -e^{-i\theta_2} & e^{-i\theta_3}\\ e^{-i\theta_0} & e^{-i\theta_1} & -e^{-i\theta_2} & -e^{-i\theta_3}\end{pmatrix}, \tag{13}$$

$$V^{(1)}(\theta_n)=\begin{pmatrix}e^{-i\theta_1} & e^{-i\theta_0} & e^{-i\theta_3} & e^{-i\theta_2}\\ e^{-i\theta_1} & -e^{-i\theta_0} & e^{-i\theta_3} & -e^{-i\theta_2}\\ e^{-i\theta_1} & -e^{-i\theta_0} & -e^{-i\theta_3} & e^{-i\theta_2}\\ e^{-i\theta_1} & e^{-i\theta_0} & -e^{-i\theta_3} & -e^{-i\theta_2}\end{pmatrix}, \tag{14}$$

$$V^{(2)}(\theta_n)=\begin{pmatrix}e^{-i\theta_2} & e^{-i\theta_3} & e^{-i\theta_0} & e^{-i\theta_1}\\ e^{-i\theta_2} & -e^{-i\theta_3} & e^{-i\theta_0} & -e^{-i\theta_1}\\ e^{-i\theta_2} & -e^{-i\theta_3} & -e^{-i\theta_0} & e^{-i\theta_1}\\ e^{-i\theta_2} & e^{-i\theta_3} & -e^{-i\theta_0} & -e^{-i\theta_1}\end{pmatrix},\ldots \tag{15}$$

and

$$V^{(7)}(\theta_n)=\begin{pmatrix}e^{-i\theta_2} & e^{-i\theta_1} & e^{-i\theta_0} & e^{-i\theta_3}\\ e^{-i\theta_2} & -e^{-i\theta_1} & e^{-i\theta_0} & -e^{-i\theta_3}\\ e^{-i\theta_2} & -e^{-i\theta_1} & -e^{-i\theta_0} & e^{-i\theta_3}\\ e^{-i\theta_2} & e^{-i\theta_1} & -e^{-i\theta_0} & -e^{-i\theta_3}\end{pmatrix}. \tag{16}$$

For each specific C_k, the states $\{|v_m^{(C_k)}\rangle_{B_1B_2}\}$ comprise an orthonormal complete set in a four-dimensional Hilbert space. It can be checked that the states $|\Phi_k\rangle_{B_1B_2C_1C_2}$ in equations (7)–(10) can be expressed as

$$|\Phi_k\rangle_{B_1B_2C_1C_2}=\frac{1}{2}\sum_{m=0}^{3}|v_m^{(C_k)}\rangle_{B_1B_2}|\Psi_{km}\rangle_{C_1C_2}. \tag{17}$$

Equation (17) indicates a disentanglement of Charlie's qubits from Bob's if Bob measures his qubits in the basis $\{|v_m^{(C_k)}\rangle_{B_1B_2}\}$. When Bob finds a state $|v_m^{(C_k)}\rangle_{B_1B_2}$ (with a probability of 1/4 for any $m\in\{0,1,2,3\}$), he publicly broadcasts m by means of 2 bits.

Step 3. This is the last step which only Charlie takes part in. Upon hearing k and m from Alice and Bob, Charlie is sure that her qubits C_1 and C_2 have collapsed into state $|\Psi_{km}\rangle_{C_1C_2}$ which are listed in table 1. Analyzing these states Charlie can quickly decide what operator should be applied on which qubit to convert them to the intended state (1). The recovery operators R_{km} (i.e. those that satisfy the equation $R_{km}|\Psi_{km}\rangle_{C_1C_2}=|\Psi\rangle_{C_1C_2}$) are also listed in the last column of table 1. By representing k as $k=8p+4q+2r+s$ and m as $m=2d+f$ with $p,q,r,s,d,f\in\{0,\ 1\}$, the operators R_{km} take the general form

$$R_{km}=X^sZ^{d\oplus s}\otimes X^rZ^{d\oplus f\oplus r\oplus s}, \tag{18}$$

with $\oplus$ an addition mod 2.

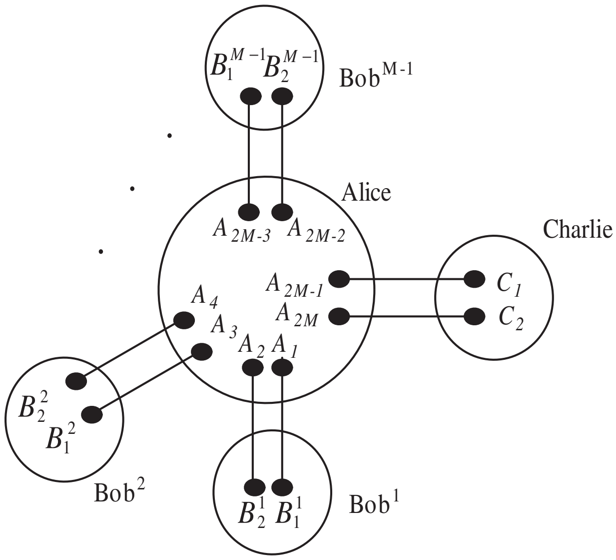

Figure 2. The qubits' distribution for JRSP of an arbitrary two-qubit state via $2M$ EPR pairs in our protocol. Dots represent qubits, while solid lines connect the entangled qubits.

As is clear from steps 1 and 2, the probability for Alice to find state $|\mu_k\rangle_{A_1A_2A_3A_4}$ is $P_k^{(A)} = 1/16\ \forall k$ and that for Bob to find state $|v_m^{(C_k)}\rangle_{B_1B_2}$ is $P_m^{(B)} = 1/4\ \forall m$. So, the probability for a combined outcome $\{k, m\}$ is $P_{km} = P_k^{(A)} P_m^{(B)} = 1/64$. Because, as seen from table 1, for each of the 64 pairs $\{k, m\}$ there always exists a recovery operator R_{km}, the total success probability of our protocol is 1 as we want. We would like to emphasize that, unlike the protocol in [26] where Charlie holds four qubits, here only two qubits are distributed to her, thus relaxing Charlie from any measurements and Controlled-NOT gates. Also, as seen from figure 1, the positions of Bob and Charlie are symmetric so that either Bob or Charlie can be assigned the receiver after the qubits' distribution has been completed. The present protocol is flexible in this sense.

3. The case with $M > 2$ preparers

To raise the security level of JRSP more than two preparers are to be involved. The greater the number of preparers the higher the level of security. In this section we study the case of $M > 2$ preparers. The execution is more complicated, yet the protocol we shall propose remains flexible, deterministic and with a passive receiver.

Suppose that Alice, Bob_1, Bob_2, ... and Bob_{M-1} are the preparers. The number of EPR pairs is now $2M$ which are shared among the participants as depicted in figure 2.

Take $M = 3$ for an explicit demonstration. The quantum channel $|Q\rangle$ is then served by six EPR pairs in the following way:

$$|Q\rangle = |\text{EPR}\rangle_{A_1B_1^1} \otimes |\text{EPR}\rangle_{A_2B_2^1} \otimes |\text{EPR}\rangle_{A_3B_1^2} \otimes |\text{EPR}\rangle_{A_4B_2^2} |\text{EPR}\rangle_{A_5C_1} \otimes |\text{EPR}\rangle_{A_6C_2}. \quad (19)$$

Qubits $\{A_n; n = 1\text{–}6\}$ belong to Alice, qubits $\{B_1^j, B_2^j; j = 1, 2\}$ belong to Bob_j and qubits $\{C_m; m = 1, 2\}$ belong to Charlie, the receiver.

The basis for Alice to measure her qubits in the first step spans a Hilbert space of dimension 64, which is denoted by $\{|\mu_k\rangle_{A_1A_2A_3A_4A_5A_6}; k = 0, 1, \ldots, 63\}$:

$$|\mu_0\rangle_{A_1A_2A_3A_4A_5A_6} = a_0|000\rangle\langle 000| + a_1|111\rangle\langle 111| + a_2|222\rangle\langle 222| + a_3|333\rangle\langle 333|, \quad (20)$$

$$|\mu_1\rangle_{A_1A_2A_3A_4A_5A_6} = a_1|000\rangle\langle 000| - a_0|111\rangle\langle 111| + a_3|222\rangle\langle 222| - a_2|333\rangle\langle 333|, \quad (21)$$

$$|\mu_2\rangle_{A_1A_2A_3A_4A_5A_6} = a_2|000\rangle\langle 000| - a_3|111\rangle\langle 111| - a_0|222\rangle\langle 222| + a_1|333\rangle\langle 333|, \ldots \quad (22)$$

and

$$|\mu_{63}\rangle_{A_1A_2A_3A_4A_5A_6} = a_3|300\rangle\langle 300| + a_2|211\rangle\langle 211| - a_1|122\rangle\langle 122| - a_0|033\rangle\langle 033|, \quad (23)$$

where, for simplicity, we adopt the identification $|000\rangle \equiv |000\rangle_{A_1A_2A_3A_4A_5A_6} \equiv |000000\rangle_{A_1A_2A_3A_4A_5A_6}$, $|001\rangle \equiv |001\rangle_{A_1A_2A_3A_4A_5A_6} \equiv |000001\rangle_{A_1A_2A_3A_4A_5A_6}$, ... and $|333\rangle \equiv |333\rangle_{A_1A_2A_3A_4A_5A_6} \equiv |111111\rangle_{A_1A_2A_3A_4A_5A_6}$. In terms of the 64 basic states, we can rewrite $|Q\rangle$ in the form

$$|Q\rangle = \frac{1}{8}\sum_{k=0}^{63} |\mu_k\rangle_{A_1A_2A_3A_4A_5A_6} |\Phi_k\rangle_{B_1^1B_2^1B_1^2B_2^2C_1C_2}. \quad (24)$$

Obviously, with an equal probability of 1/64, Alice's measurement projects her qubits onto a state $|\mu_k\rangle_{A_1A_2A_3A_4A_5A_6}$, with $k \in \{0, 1, \ldots, 63\}$. This leaves the remaining six unmeasured qubits in a state $|\Phi_k\rangle_{B_1^1B_2^1B_1^2B_2^2C_1C_2}$, which are collected in table 2 for all 64 possible values of k.

In the next step, the Bobs begin their action after hearing the outcome k broadcasted by Alice. Because there are two Bobs, the data subset $S_2 = \{\theta_n\}$ is divided into two pieces $S_2^1 = \{\theta_n^1\}$ and $S_2^2 = \{\theta_n^2\}$ such that $\theta_n^1 + \theta_n^2 = \theta_n$. S_2^1 is given to Bob_1, and S_2^2 to Bob_2. Each Bob independently measures his own qubits in a basis chosen according to k. Namely, the basis for Bob_1 is $\{|v_{l_1}^{(k)}\rangle_{B_1^1B_2^1}; l_1 = 0, 1, 2, 3\}$,

$$\begin{pmatrix} |v_0^{(k)}\rangle_{B_1^1B_2^1} \\ |v_1^{(k)}\rangle_{B_1^1B_2^1} \\ |v_2^{(k)}\rangle_{B_1^1B_2^1} \\ |v_3^{(k)}\rangle_{B_1^1B_2^1} \end{pmatrix} = \frac{1}{2} V^{(k)}(\theta_n^1) \begin{pmatrix} |0\rangle_{B_1^1B_2^1} \\ |1\rangle_{B_1^1B_2^1} \\ |2\rangle_{B_1^1B_2^1} \\ |3\rangle_{B_1^1B_2^1} \end{pmatrix}, \quad (25)$$

while that for Bob_2 is $\{|w_{l_2}^{(k)}\rangle_{B_1^2B_2^2}; l_2 = 0, 1, 2, 3\}$,

$$\begin{pmatrix} |w_0^{(k)}\rangle_{B_1^2B_2^2} \\ |w_1^{(k)}\rangle_{B_1^2B_2^2} \\ |w_2^{(k)}\rangle_{B_1^2B_2^2} \\ |w_3^{(k)}\rangle_{B_1^2B_2^2} \end{pmatrix} = \frac{1}{2} V^{(k)}(\theta_n^2) \begin{pmatrix} |0\rangle_{B_1^2B_2^2} \\ |1\rangle_{B_1^2B_2^2} \\ |2\rangle_{B_1^2B_2^2} \\ |3\rangle_{B_1^2B_2^2} \end{pmatrix}, \quad (26)$$

where $\frac{1}{2}V^{(k)}(\theta_n^j)$ with $j = 1, 2$ is a 4×4 unitary matrix. For example, if $k = 1$, then

$$V^{(1)}(\theta_n^j) = \begin{pmatrix} e^{-i\theta_1^j} & e^{-i\theta_0^j} & e^{-i\theta_3^j} & e^{-i\theta_2^j} \\ e^{-i\theta_1^j} & -e^{-i\theta_0^j} & e^{-i\theta_3^j} & -e^{-i\theta_2^j} \\ e^{-i\theta_1^j} & -e^{-i\theta_0^j} & -e^{-i\theta_3^j} & e^{-i\theta_2^j} \\ e^{-i\theta_1^j} & e^{-i\theta_0^j} & -e^{-i\theta_3^j} & -e^{-i\theta_2^j} \end{pmatrix}. \quad (27)$$

Table 2. The collapsed state $\vert\Phi_k\rangle_{B_1^1B_2^1B_1^2B_2^2C_1C_2}$ for the outcome k broadcasted by Alice in the case of $M = 3$ preparers.

k	$\vert\Phi_k\rangle_{B_1^1B_2^1B_1^2B_2^2C_1C_2}$	k	$\vert\Phi_k\rangle_{B_1^1B_2^1B_1^2B_2^2C_1C_2}$
0	$a_0\vert 000\rangle + a_1\vert 111\rangle + a_2\vert 222\rangle + a_3\vert 333\rangle$	32	$a_0\vert 020\rangle + a_1\vert 131\rangle + a_2\vert 202\rangle + a_3\vert 313\rangle$
1	$-a_0\vert 111\rangle + a_1\vert 000\rangle - a_2\vert 333\rangle + a_3\vert 222\rangle$	33	$-a_0\vert 131\rangle + a_1\vert 020\rangle - a_2\vert 313\rangle + a_3\vert 202\rangle$
2	$-a_0\vert 222\rangle + a_1\vert 333\rangle + a_2\vert 000\rangle - a_3\vert 111\rangle$	34	$-a_0\vert 202\rangle + a_1\vert 313\rangle + a_2\vert 020\rangle - a_3\vert 131\rangle$
3	$-a_0\vert 333\rangle - a_1\vert 222\rangle + a_2\vert 111\rangle + a_3\vert 000\rangle$	35	$-a_0\vert 313\rangle - a_1\vert 202\rangle + a_2\vert 131\rangle + a_3\vert 020\rangle$
4	$a_0\vert 210\rangle + a_1\vert 001\rangle + a_2\vert 332\rangle + a_3\vert 123\rangle$	36	$a_0\vert 130\rangle + a_1\vert 021\rangle + a_2\vert 312\rangle + a_3\vert 203\rangle$
5	$-a_0\vert 001\rangle + a_1\vert 210\rangle - a_2\vert 123\rangle + a_3\vert 332\rangle$	37	$-a_0\vert 021\rangle + a_1\vert 130\rangle - a_2\vert 203\rangle + a_3\vert 312\rangle$
6	$-a_0\vert 332\rangle + a_1\vert 123\rangle + a_2\vert 210\rangle - a_3\vert 001\rangle$	38	$-a_0\vert 312\rangle + a_1\vert 203\rangle + a_2\vert 130\rangle - a_3\vert 021\rangle$
7	$-a_0\vert 123\rangle - a_1\vert 332\rangle + a_2\vert 001\rangle + a_3\vert 210\rangle$	39	$-a_0\vert 203\rangle - a_1\vert 312\rangle + a_2\vert 021\rangle + a_3\vert 130\rangle$
8	$a_0\vert 220\rangle + a_1\vert 331\rangle + a_2\vert 002\rangle + a_3\vert 113\rangle$	40	$a_0\vert 100\rangle + a_1\vert 311\rangle + a_2\vert 022\rangle + a_3\vert 233\rangle$
9	$-a_0\vert 331\rangle + a_1\vert 220\rangle - a_2\vert 113\rangle + a_3\vert 002\rangle$	41	$-a_0\vert 311\rangle + a_1\vert 100\rangle - a_2\vert 233\rangle + a_3\vert 022\rangle$
10	$-a_0\vert 002\rangle + a_1\vert 113\rangle + a_2\vert 220\rangle - a_3\vert 331\rangle$	42	$-a_0\vert 022\rangle + a_1\vert 233\rangle + a_2\vert 100\rangle - a_3\vert 311\rangle$
11	$-a_0\vert 113\rangle - a_1\vert 002\rangle + a_2\vert 331\rangle + a_3\vert 220\rangle$	43	$-a_0\vert 233\rangle - a_1\vert 022\rangle + a_2\vert 311\rangle + a_3\vert 100\rangle$
12	$a_0\vert 330\rangle + a_1\vert 221\rangle + a_2\vert 112\rangle + a_3\vert 003\rangle$	44	$a_0\vert 310\rangle + a_1\vert 201\rangle + a_2\vert 132\rangle + a_3\vert 023\rangle$
13	$-a_0\vert 221\rangle + a_1\vert 330\rangle - a_2\vert 003\rangle + a_3\vert 112\rangle$	45	$-a_0\vert 201\rangle + a_1\vert 310\rangle - a_2\vert 023\rangle + a_3\vert 132\rangle$
14	$-a_0\vert 112\rangle + a_1\vert 003\rangle + a_2\vert 330\rangle - a_3\vert 221\rangle$	46	$-a_0\vert 132\rangle + a_1\vert 023\rangle + a_2\vert 310\rangle - a_3\vert 201\rangle$
15	$-a_0\vert 003\rangle - a_1\vert 112\rangle + a_2\vert 221\rangle + a_3\vert 330\rangle$	47	$-a_0\vert 023\rangle - a_1\vert 132\rangle + a_2\vert 201\rangle + a_3\vert 310\rangle$
16	$a_0\vert 010\rangle + a_1\vert 101\rangle + a_2\vert 232\rangle + a_3\vert 323\rangle$	48	$a_0\vert 030\rangle + a_1\vert 121\rangle + a_2\vert 212\rangle + a_3\vert 303\rangle$
17	$-a_0\vert 101\rangle + a_1\vert 010\rangle - a_2\vert 323\rangle + a_3\vert 232\rangle$	49	$-a_0\vert 121\rangle + a_1\vert 030\rangle - a_2\vert 303\rangle + a_3\vert 212\rangle$
18	$-a_0\vert 232\rangle + a_1\vert 323\rangle + a_2\vert 010\rangle - a_3\vert 101\rangle$	50	$-a_0\vert 212\rangle + a_1\vert 303\rangle + a_2\vert 030\rangle - a_3\vert 121\rangle$
19	$-a_0\vert 323\rangle - a_1\vert 232\rangle + a_2\vert 101\rangle + a_3\vert 010\rangle$	51	$-a_0\vert 030\rangle - a_1\vert 121\rangle + a_2\vert 212\rangle + a_3\vert 303\rangle$
20	$a_0\vert 200\rangle + a_1\vert 011\rangle + a_2\vert 322\rangle + a_3\vert 133\rangle$	52	$a_0\vert 120\rangle + a_1\vert 031\rangle + a_2\vert 302\rangle + a_3\vert 213\rangle$
21	$-a_0\vert 011\rangle + a_1\vert 200\rangle - a_2\vert 133\rangle + a_3\vert 322\rangle$	53	$-a_0\vert 031\rangle + a_1\vert 120\rangle - a_2\vert 213\rangle + a_3\vert 302\rangle$
22	$-a_0\vert 322\rangle + a_1\vert 133\rangle + a_2\vert 200\rangle - a_3\vert 011\rangle$	54	$-a_0\vert 302\rangle + a_1\vert 213\rangle + a_2\vert 120\rangle - a_3\vert 031\rangle$
23	$-a_0\vert 133\rangle - a_1\vert 322\rangle + a_2\vert 011\rangle + a_3\vert 200\rangle$	55	$-a_0\vert 213\rangle - a_1\vert 302\rangle + a_2\vert 031\rangle + a_3\vert 120\rangle$
24	$a_0\vert 230\rangle + a_1\vert 321\rangle + a_2\vert 012\rangle + a_3\vert 103\rangle$	56	$a_0\vert 110\rangle + a_1\vert 301\rangle + a_2\vert 032\rangle + a_3\vert 223\rangle$
25	$-a_0\vert 321\rangle + a_1\vert 230\rangle - a_2\vert 103\rangle + a_3\vert 012\rangle$	57	$-a_0\vert 301\rangle + a_1\vert 110\rangle - a_2\vert 223\rangle + a_3\vert 032\rangle$
26	$-a_0\vert 012\rangle + a_1\vert 103\rangle + a_2\vert 230\rangle - a_3\vert 321\rangle$	58	$-a_0\vert 032\rangle + a_1\vert 223\rangle + a_2\vert 110\rangle - a_3\vert 301\rangle$
27	$-a_0\vert 103\rangle - a_1\vert 012\rangle + a_2\vert 321\rangle + a_3\vert 230\rangle$	59	$-a_0\vert 223\rangle - a_1\vert 032\rangle + a_2\vert 301\rangle + a_3\vert 110\rangle$
28	$a_0\vert 320\rangle + a_1\vert 231\rangle + a_2\vert 102\rangle + a_3\vert 013\rangle$	60	$a_0\vert 300\rangle + a_1\vert 211\rangle + a_2\vert 122\rangle + a_3\vert 033\rangle$
29	$-a_0\vert 231\rangle + a_1\vert 320\rangle - a_2\vert 013\rangle + a_3\vert 102\rangle$	61	$-a_0\vert 211\rangle + a_1\vert 300\rangle - a_2\vert 033\rangle + a_3\vert 122\rangle$
30	$-a_0\vert 102\rangle + a_1\vert 320\rangle + a_2\vert 231\rangle - a_3\vert 013\rangle$	62	$-a_0\vert 122\rangle + a_1\vert 300\rangle + a_2\vert 211\rangle - a_3\vert 033\rangle$
31	$-a_0\vert 013\rangle - a_1\vert 102\rangle + a_2\vert 231\rangle + a_3\vert 320\rangle$	63	$-a_0\vert 033\rangle - a_1\vert 122\rangle + a_2\vert 211\rangle + a_3\vert 300\rangle$

Table 3. The collapsed state $\vert\Psi_{1l_1l_2}\rangle_{C_1C_2}$ of Charlie's qubits C_1, C_2 and the corresponding recovery operators $R_{1l_1l_2}$ related to the measurement outcomes (l_1, l_2) from Bob_1 and Bob_2. I is the identity operator and X (Z) the Pauli bit (phase) flip operator.

(l_1, l_2)	$\vert\Psi_{1l_1l_2}\rangle_{C_1C_2}$	$R_{1l_1l_2}$
(0, 0)(1, 1)(2, 2)(3, 3)	$a_1e^{i\theta_1}\vert 0\rangle - a_0e^{i\theta_0}\vert 1\rangle + a_3e^{i\theta_3}\vert 2\rangle - a_2e^{i\theta_2}\vert 3\rangle$	$I \otimes XZ$
(0, 1)(1, 0)(2, 3)(3, 2)	$a_1e^{i\theta_1}\vert 0\rangle + a_0e^{i\theta_0}\vert 1\rangle + a_3e^{i\theta_3}\vert 2\rangle + a_2e^{i\theta_2}\vert 3\rangle$	$I \otimes X$
(0, 2)(1, 3)(2, 0)(3, 1)	$a_1e^{i\theta_1}\vert 0\rangle + a_0e^{i\theta_0}\vert 1\rangle - a_3e^{i\theta_3}\vert 2\rangle - a_2e^{i\theta_2}\vert 3\rangle$	$Z \otimes X$
(0, 3)(1, 2)(2, 1)(3, 0)	$a_1e^{i\theta_1}\vert 0\rangle - a_0e^{i\theta_0}\vert 1\rangle - a_3e^{i\theta_3}\vert 2\rangle + a_2e^{i\theta_2}\vert 3\rangle$	$Z \otimes XZ$

For a given k, the outcome of Bob_1 is l_1 if he finds $\vert v_{l_1}^{(k)}\rangle_{B_1^1B_2^1}$, while that of Bob_2 is l_2 if he finds $\vert w_{l_2}^{(k)}\rangle_{B_1^2B_2^2}$. The values of $\{l_1, l_2\}$ should be published publicly by the Bobs. In terms of the preparers' basic states $\vert Q\rangle$ can be written as

$$\vert Q\rangle = \frac{1}{32}\sum_{k=0}^{63}\sum_{l,m=0}^{3}\vert\mu_k\rangle_{A_1A_2A_3A_4A_5A_6} \times \vert v_{l_1}^{(k)}\rangle_{B_1^1B_2^1}\vert w_{l_2}^{(k)}\rangle_{B_1^2B_2^2}\vert\Psi_{kl_1l_2}\rangle_{C_1C_2}. \quad (28)$$

In the last step, Charlie is able to determine a right operator conditioned on $\{k, l_1, l_2\}$ to apply on her qubits C_1 and C_2 to obtain the desired state, i.e. to bring $\vert\Psi_{kl_1l_2}\rangle_{C_1C_2}$ into $\vert\Psi\rangle_{C_1C_2}$ of equation (1). Straightforward calculations have shown that any trio $\{k, l_1, l_2\}$ is associated with a recovery operator $R_{kl_1l_2}$, guaranteeing complete success. As an illustration, $\vert\Psi_{1l_1l_2}\rangle_{C_1C_2}$ and $R_{1l_1l_2}$ with all possible l_1 and l_2 are tabulated in table 3.

The case with an arbitrary $M > 3$ is cumbersome. However, the general outline goes as follows. $2M$ EPR pairs are needed for the quantum channel with the qubits' distribution shown in figure 2. The data set $S = \{a_n, \theta_n\}$ should be split into M subsets $S_1 = \{a_n\}$ and $S_2^j = \{\theta_n^j\}$, with $j = 1, 2, \ldots, M-1$ and $\sum_{j=1}^{M-1}\theta_n^j = \theta_n$. S_1 is given to Alice and S_2^j to Bob_j. First, Alice measures $2M$ qubits A_1, A_2, $\ldots$, A_{2M-1} and A_{2M} in a basis suitably determined by S_1, with an outcome $k \in \{0, 1, \ldots, 2^{2M} - 1\}$ to be announced publicly. Next, each Bob_j independently measures qubits B_1^j and B_2^j in a judiciously chosen basis, dependent on both S_2^j and k, obtaining an outcome $l_j \in \{0, 1, 2, 3\}$, which is also announced publicly. Finally, upon hearing k, l_1, l_2, $\ldots$, l_{M-1}, Charlie can always construct a right recovery operator $R_{kl_1l_2\ldots l_{M-1}}$ to be applied on qubits C_1 and C_2 to complete the protocol.

4. Conclusion

In conclusion, we have proposed a useful protocol for joint remote preparation of an arbitrary two-qubit state using only EPR pairs, the simplest kind of entanglement, as the quantum channel. In practice, depending on the need, assignment of the receiver may be decided after sharing the quantum channel and the receiver may not be sufficiently well equipped. Our protocol works deterministically in such situations because it is designed to be flexible and requires only a passive receiver.

Concretely, we have explicitly shown a protocol for $M > 1$ separate parties (Alice, Bob_1, Bob_2, $\ldots$, Bob_{M-1}) to remotely cooperate for preparing an arbitrary two-qubit state $|\Psi\rangle$, equation (1), for a distant receiver (Charlie). The complete information of $|\Psi\rangle$ is split into M independent pieces, each held by a preparer, forbidding any subset of the preparers to fully infer $|\Psi\rangle$. We employ $2M$ EPR pairs as the quantum channel and distribute $2M$ qubits to Alice but just two qubits to each of Bob_j and Charlie (see figure 2). Distribution of qubits in this manner between Bob and Charlie makes our protocol flexible in the sense that either a Bob_j or Charlie can be assigned the receiver even after entanglement sharing. The use of this distribution greatly reduces the role of the receiver in comparison with that in [26]. Key to achieve complete success is the adoption of the adaptive measurement strategy: measurements of Bob_j should be made after that of Alice and, more importantly, in a basis conditioned by Alice's outcome. Needless to say, the merit of the protocol we propose in this work rests on the fact that it is uniquely applicable to a certain realistic situation, which is not the case for other existing protocols.

Acknowledgments

QQC and YX were supported by the National Natural Science Foundation of China under grant numbers 11047122, 11105030 and 10974028, the Foundation of the Ministry of Education of China under grant number 212085 and the China Postdoctoral Science Foundation under grant number 20100471450. NBA was supported by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under project number 103.99-2011.26.

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