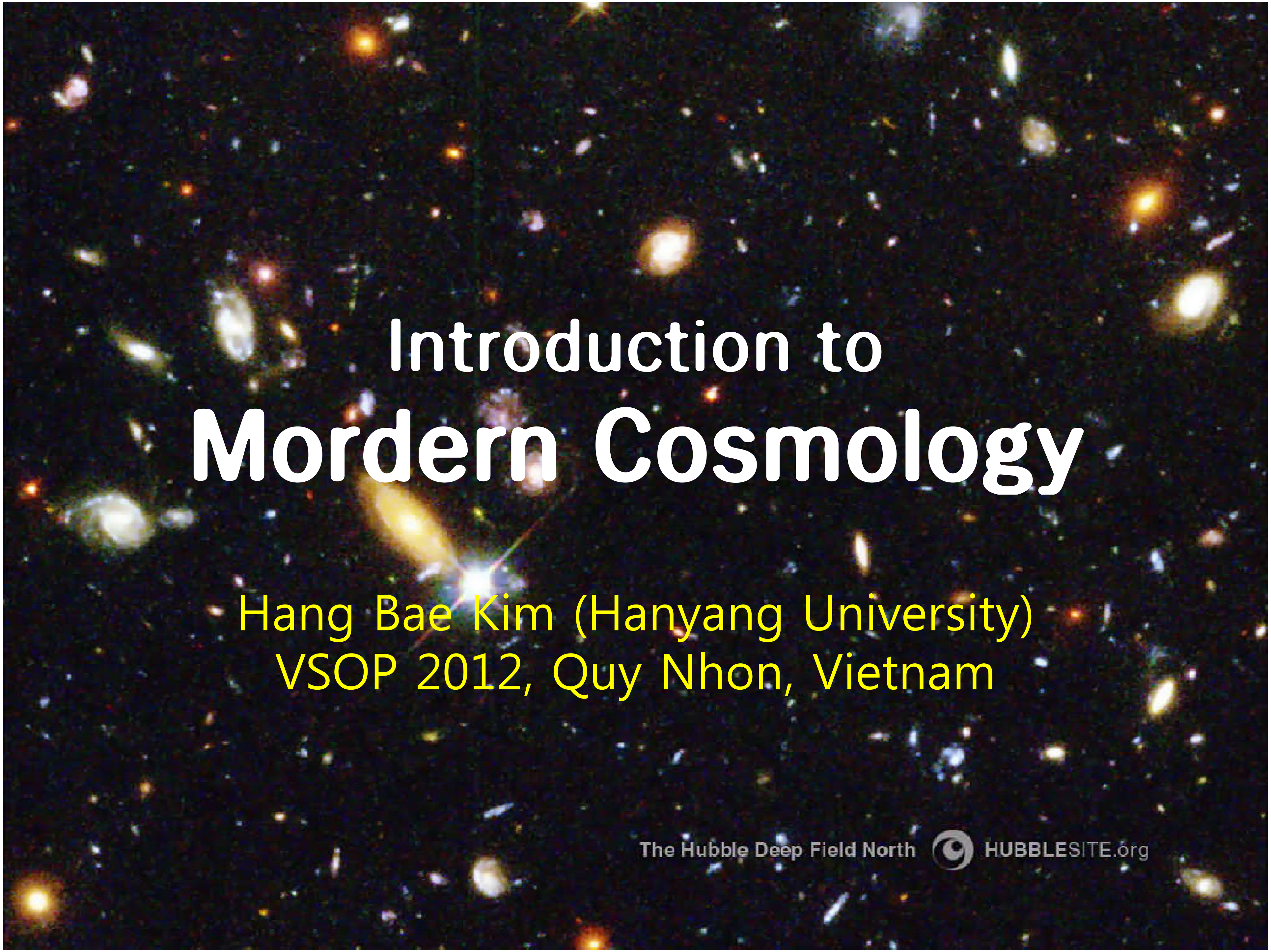
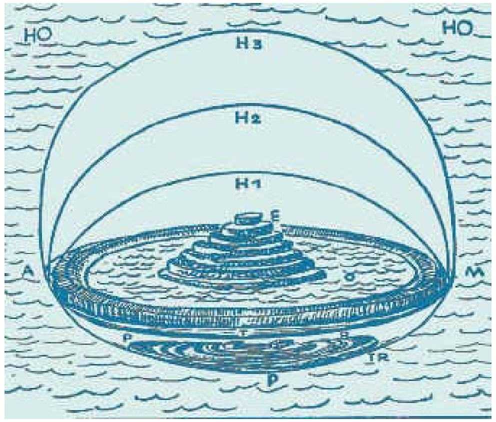
Introduction to
Mordern Cosmology

Hang Bae Kim (Hanyang University)
VSOP 2012, Quy Nhon, Vietnam



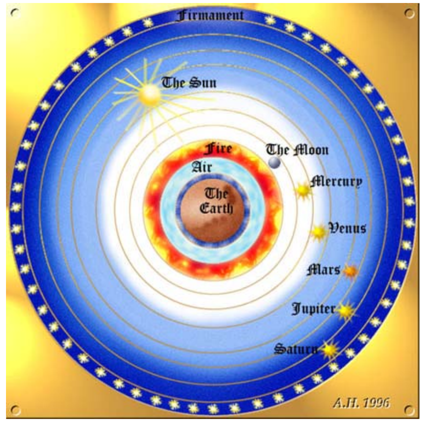
How the world is shaped?

India

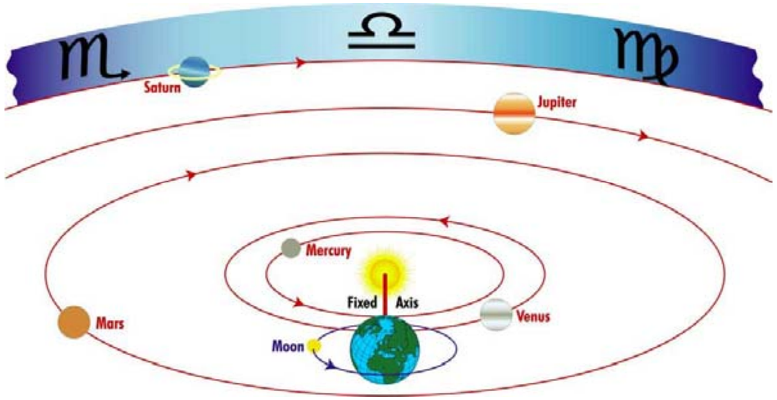


Mesopotamia

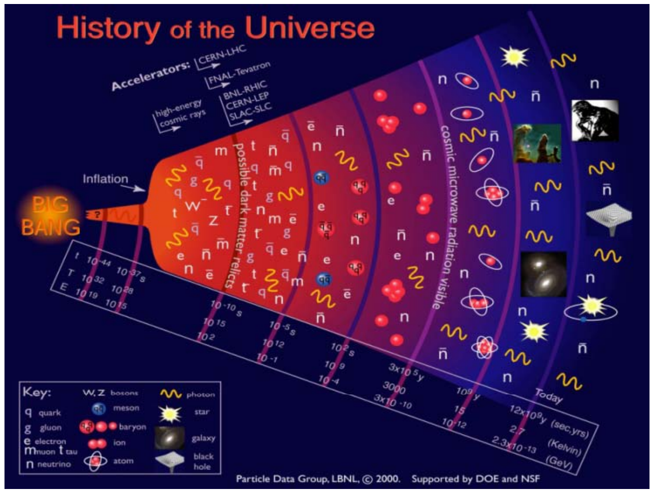
Egypt



Greece-Rome

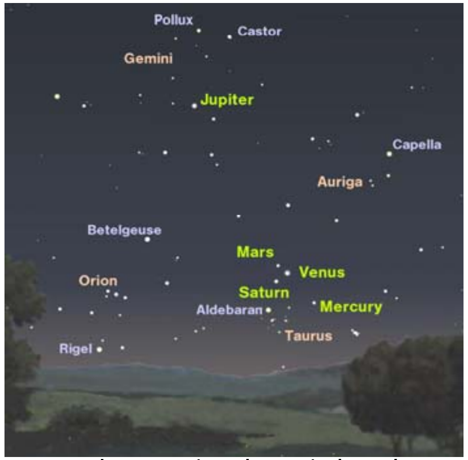


17th C, Western



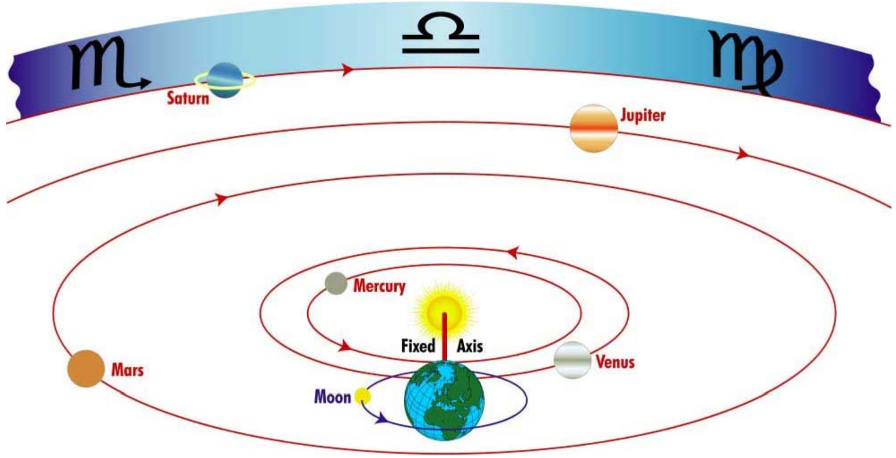
Modern

Understanding the universe
a success story

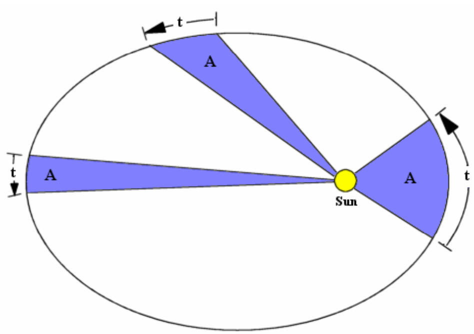


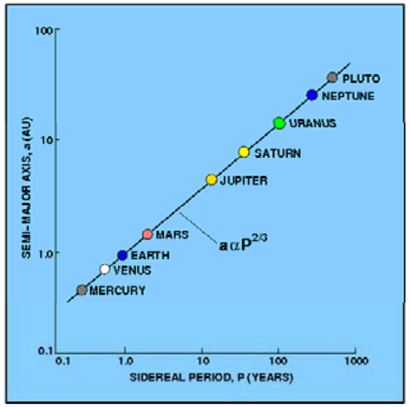
5 planets in the night sky

Tycho's observation



Kepler's Laws



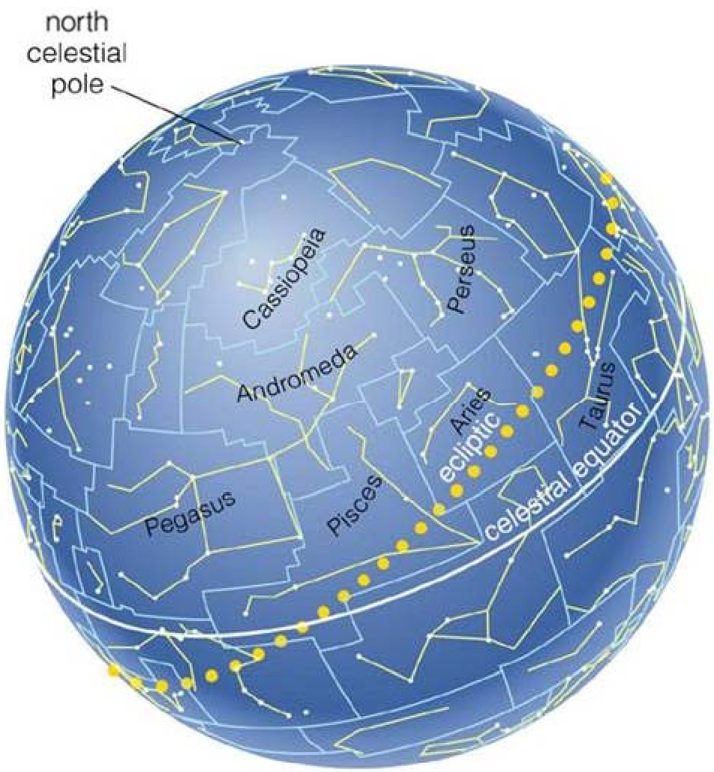


Newton's law of motion and gravitation

$$m\frac{d^2\mathbf{r}}{dt} = -G\frac{mM}{r^2}\hat{\mathbf{r}}$$

Compete understanding of planetary motion

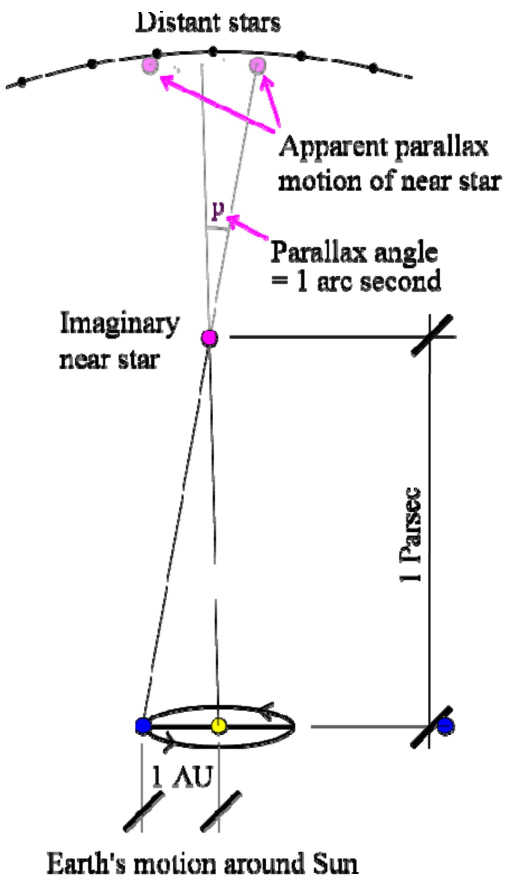
Stars and the Celestial Sphere



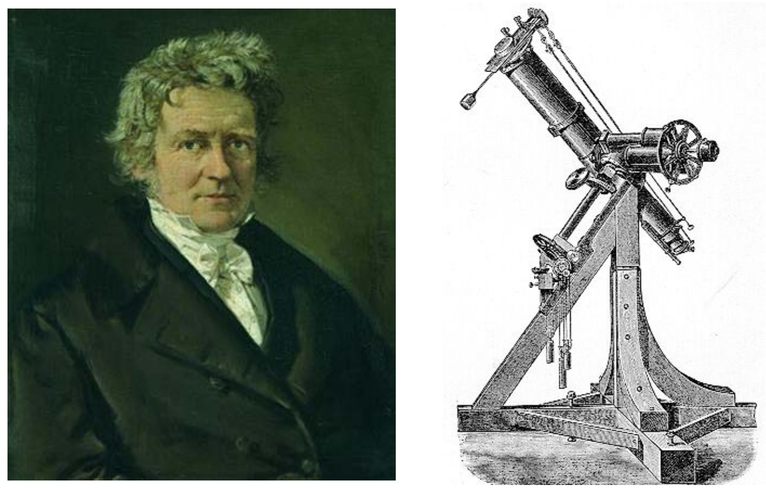
Ancient people could not guess the distance to stars. They thought all stars reside on the celestial sphere.

Measure the distance to stars

- **Annual Parallax**



- F. Bessel, 1838 – measured the annual parallax of Cygni 61 to be 0.314″ (the current value is 0.286″) successfully.



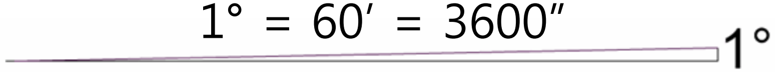
F. Bessel and his heliometer used to measure parallax

- Definition of 1 parsec ⇔ annual parallax 1″

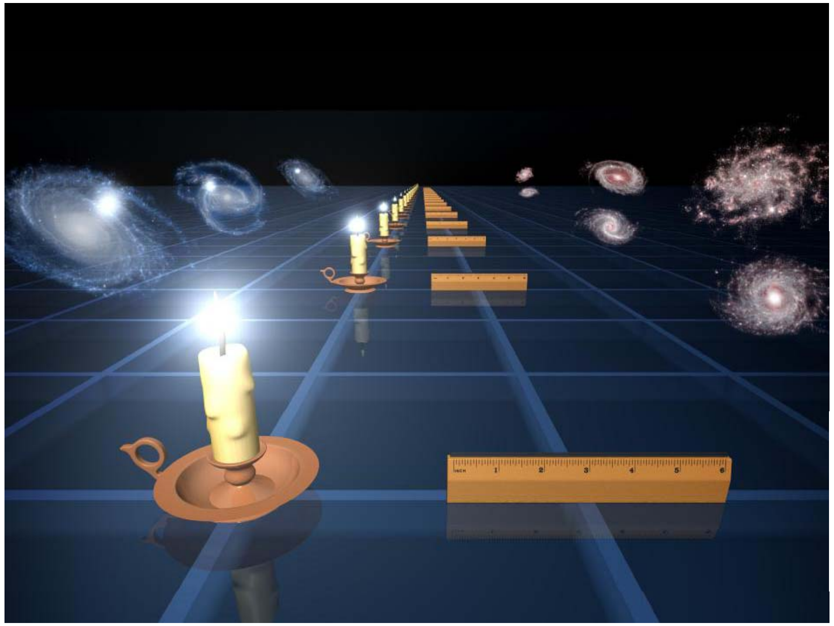
$$1\,\mathrm{pc} = 3.26\,\mathrm{ly} = 3.1 \times 10^{16}\,\mathrm{m}$$

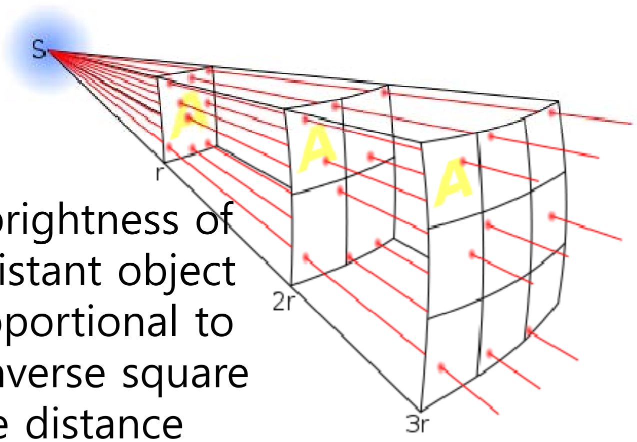
  - Angular diameter of the sun = 32′
  - Annual parallax of the nearest star from the sun = Proxima Centauri, 0.769″

1° = 60′ = 3600″



Standard candles and rulers





The brightness of the distant object is proportional to the inverse square of the distance

How to measure the distance

- **Luminosity Distance**
  - Brightness of the astronomical objects

$$F = \frac{L}{4\pi r^2} \quad \Rightarrow \quad d_L^2 \equiv \frac{L}{4\pi F}$$

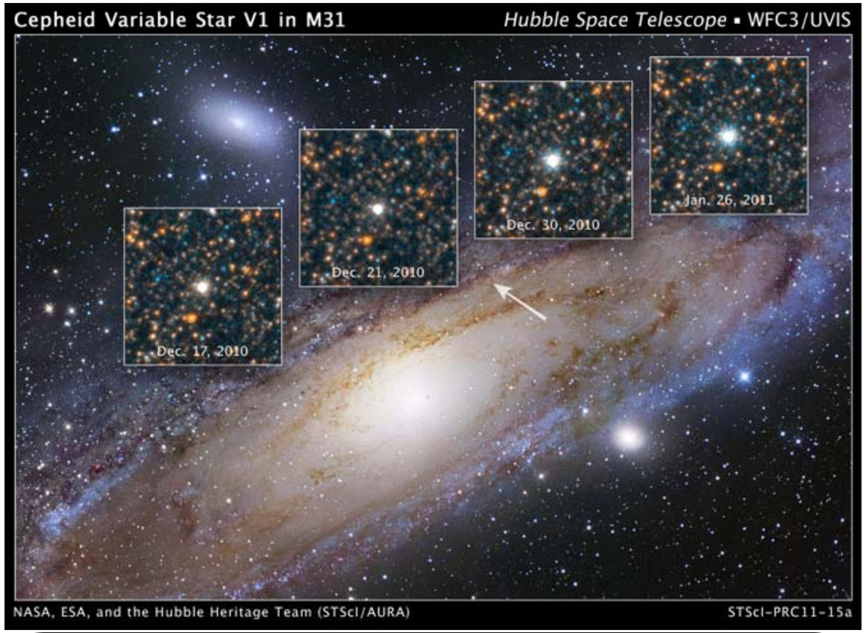
  - Requires the object with known luminosity.
  - Brighter for farther objects

- **Angular Diameter Distance**
  - Angular diameter of the object

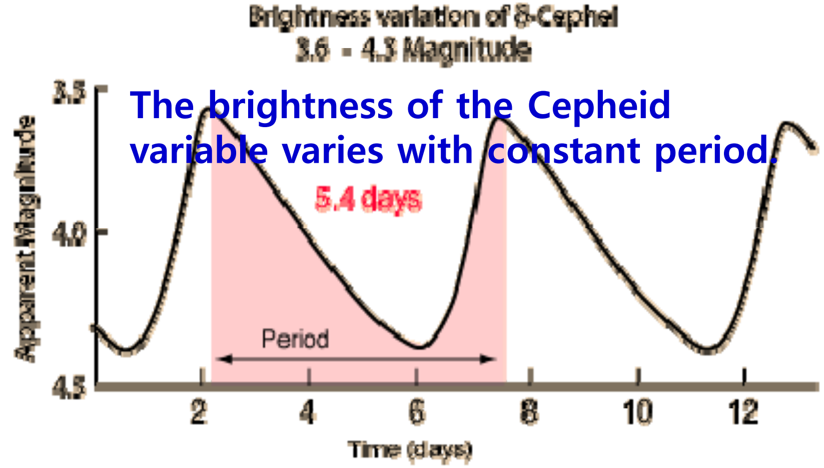
$$\theta = \frac{l}{r} \quad \Rightarrow \quad d_A = \frac{l}{\theta}$$

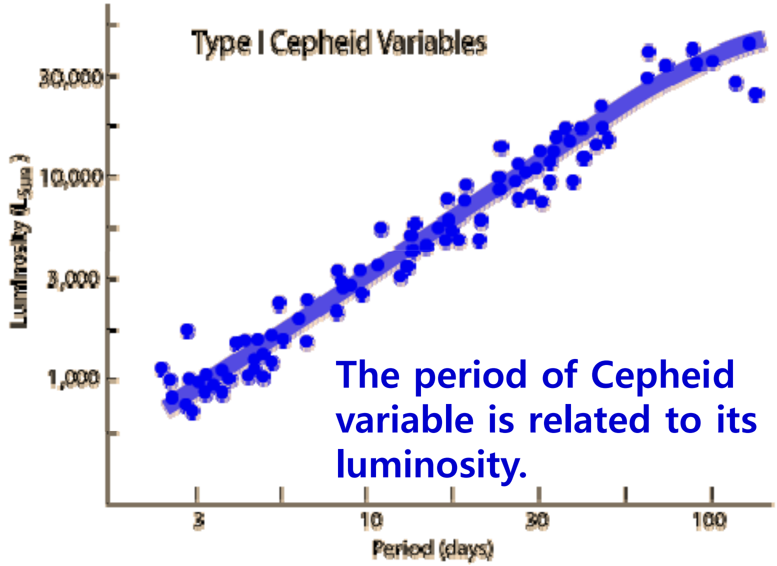
  - Requires the object with known size.
  - Larger for farther objects

Cepheid Variable



- John Goodricke, 1784
  - Discovery of δ-Cepheid variable
- **Henrietta Leavitt, 1908**
  - **Discovery of the period-luminosity relation**
- Harlow Shapley, 1915
  - Size and shape of our galaxy from Cepheid Vs.
- **Edwin Hubble, 1924**
  - Confirmed Andromeda galaxy is extra-galactic
- **Edwin Hubble & Milton Humason, 1929**
  - **Discovery of the expansion of the universe**





Pickering's Harem

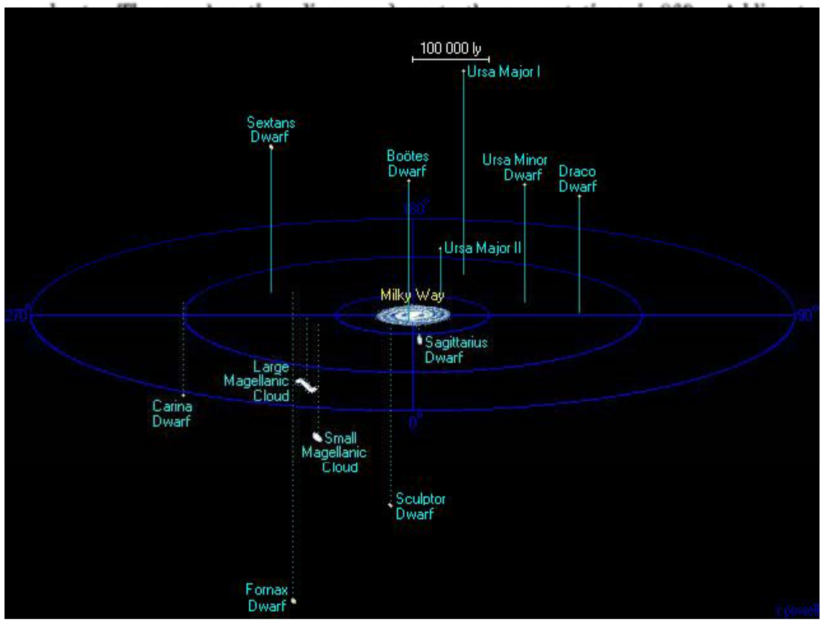
Henrietta Swan Leavitt

In 1908-1912, H. Leavitt studied the variables in Magellanic clouds and found that they can be standard candles.

1777 VARIABLES IN THE MAGELLANIC CLOUDS.

By Henrietta S. Leavitt.

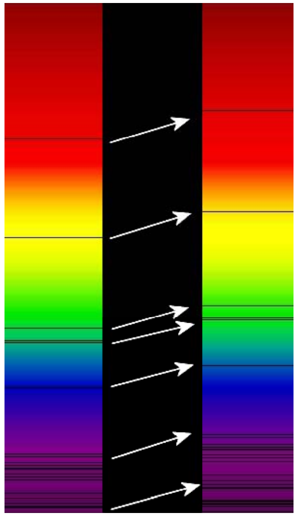
In the spring of 1904, a comparison of two photographs of the Small Magellanic Cloud, taken with the 24-inch Bruce Telescope, led to the discovery of a number of faint variable stars. As the region appeared to be interesting, other plates were examined, and although the quality of most of these was below the usual high standard of excellence of the later plates, 57 new variables were found, and announced in Circular 79. In order to furnish material for determining their periods, a series of sixteen plates, having exposures of from two to four hours, was taken with the Bruce Telescope the following autumn. When they arrived at Cambridge, in January, 1905, a comparison of one of them with an early plate led immediately to the discovery of an extraordinary number of new variable stars. It was found, also, that plates, taken within two or three days of each other, could be compared with equally interesting results, showing that the periods of many of the variables



Discovery of Expansion

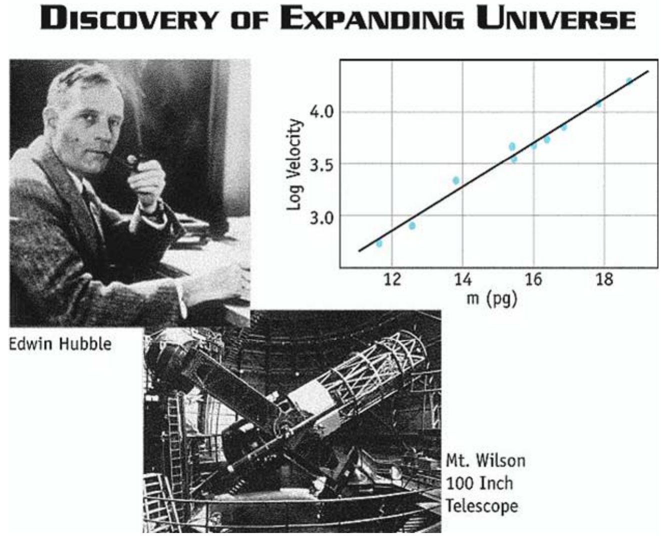
◆ **Redshift**

- Vesto Slipher discovered the redshift of nebula (1912)
- Absorption spectra from distant galaxies are red shifted.
- Interpretation – Distant galaxies are receding from us.



◆ **Evidence for the expansion**

- **Red shift proportional to distance (Hubble's law, 1929)**



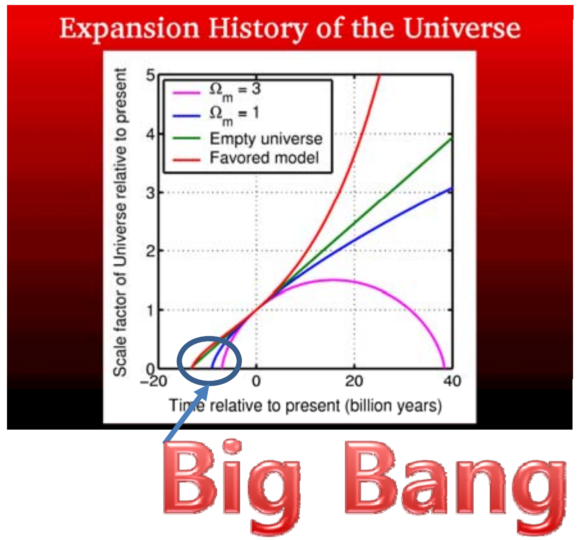
Edwin Hubble discovered that redshift is proportional to distance by observing Cepheid variables in distant galaxies.

◆ Meaning of expansion – Dynamic universe

- Static universe : eternal, no expansion.
- **Dynamic universe : beginning and end, changes.**
- Steady state universe : expanding but no changes.

◆ **Our universe has the beginning.**

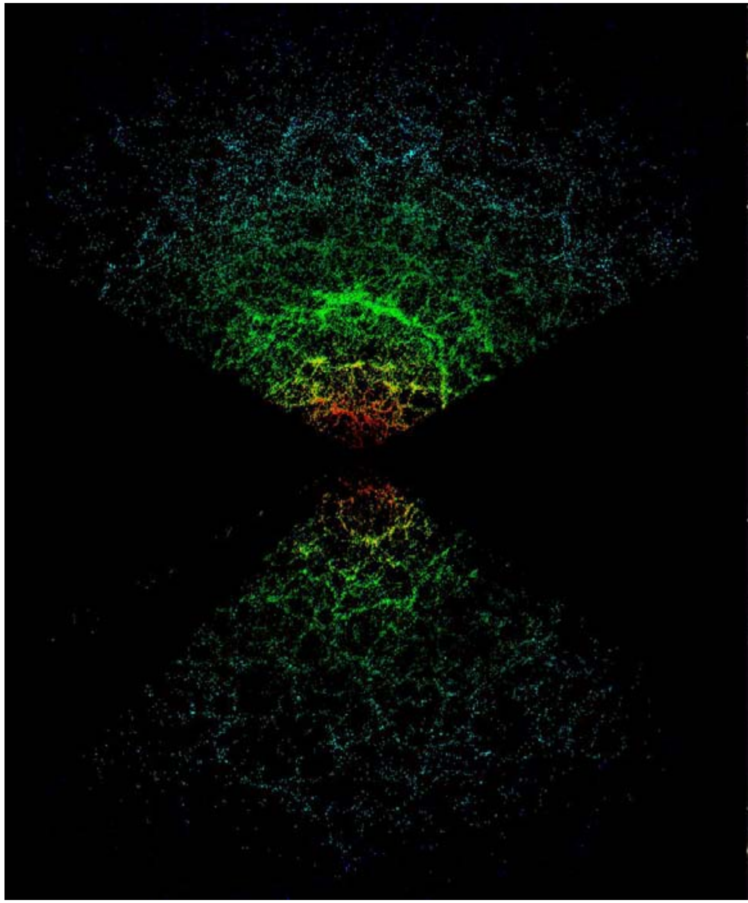
- If we trace back the expansion history, we meet a singular (infinite energy density) point of a=0 in a finite time.



The shape of our universe
from galaxy distribution

- (Relatively) Small Scales - Structures
- Large Scales - Homogeneous

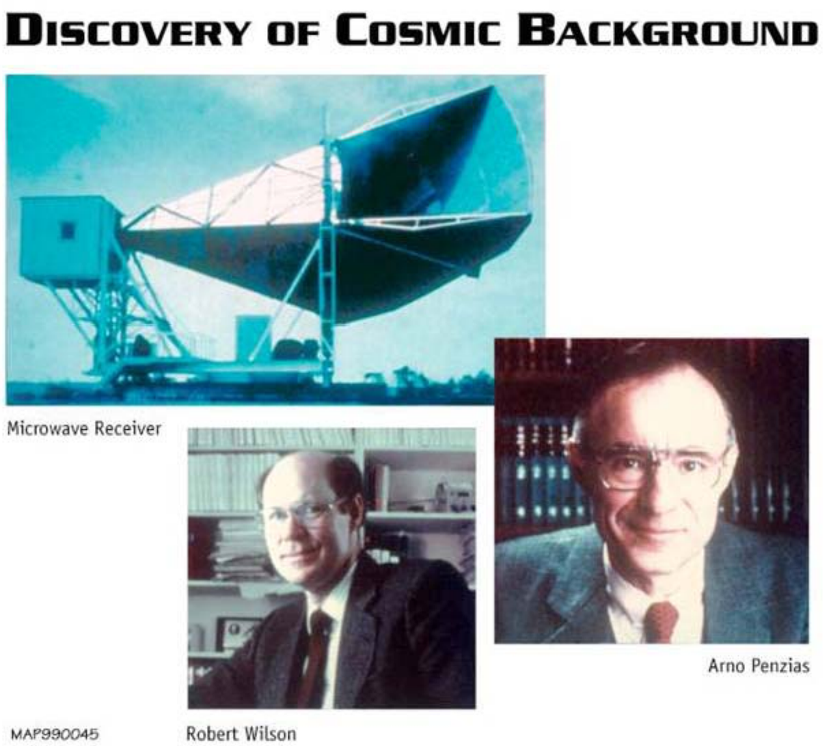
Galaxy map



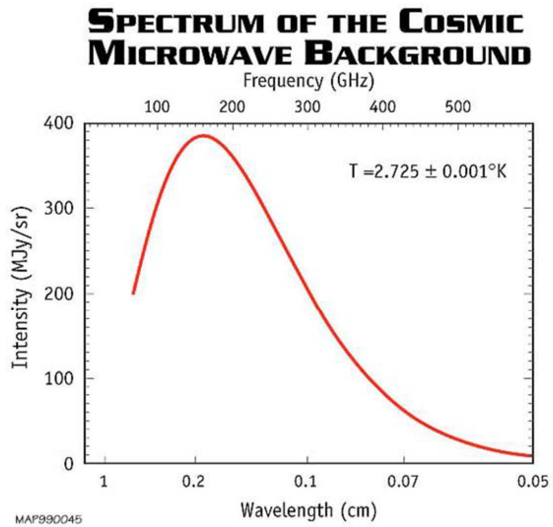
Hubble Deep Field

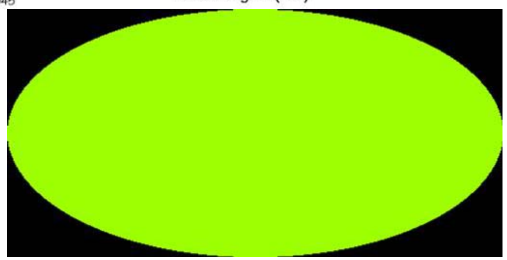
Discovery of CMB

- Matter content of the universe - Radiation
  - Eq. of state $p = \frac{1}{3}\rho$ (Ideal gas of photons)
  - Cosmic Microwave Background Radiation (CMB)
    - George Gamow and Ralph Alpher's prediction (1948)
    - Arno Penzias and Robert Wilson's discovery (1965)
    - Very isotropic, perfect black body spectrum with T=**2.73 K**



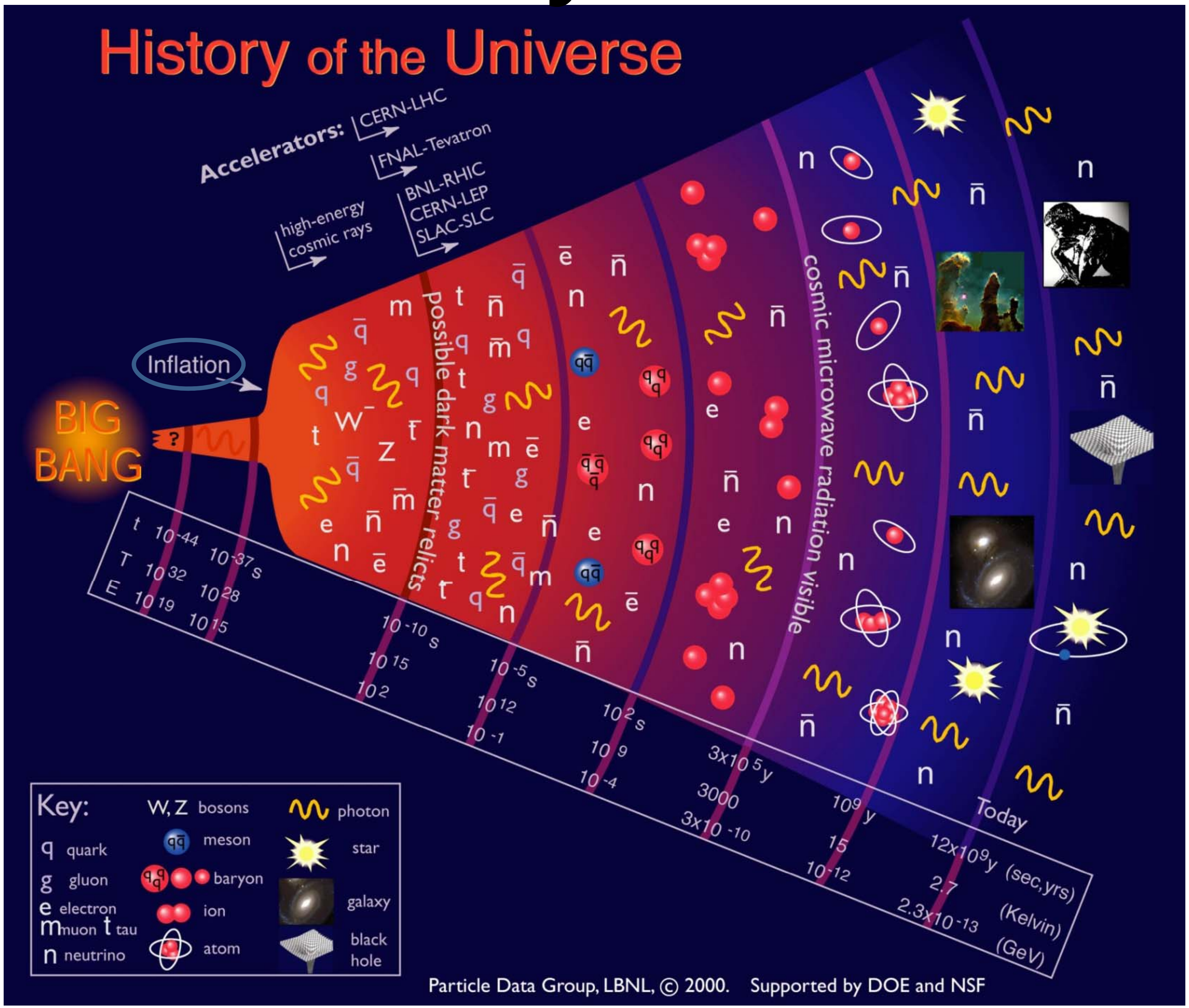
Penzias and Wilson, and the antenna they used in the discovery of CMB





CMB is very isotropic. ($\delta T/T \sim 10^{-5}$)

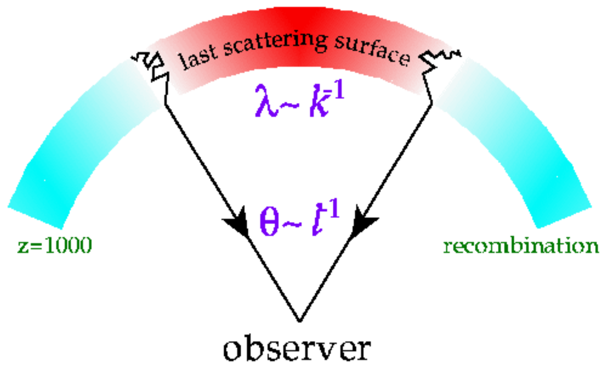
Thermal History of the Universe



Discovery of CMB Anisotropies

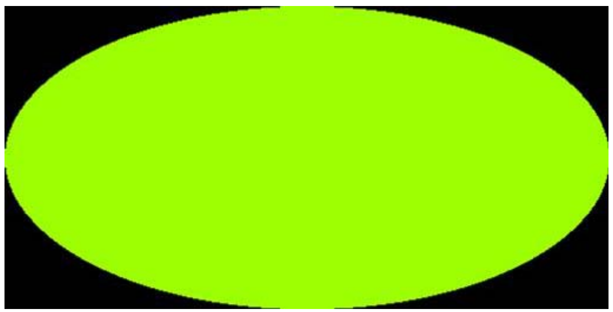
- Is there any other evidence for primordial density perturbation?
- CMB Anisotropies (CMBA)
  - Decoupling of CMB
  - CMB we see today
    – from the last scattering surface
  - Origin of CMBA
    - Gravitational potential due to density perturbation of CDM

$$\delta\rho \longrightarrow \delta\Phi \longrightarrow \delta T$$

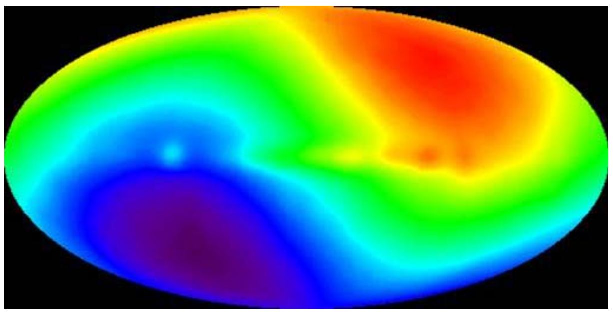


    - Baryon Acoustic Oscillation
      - Oscillation of strongly coupled baryon-photon plasma

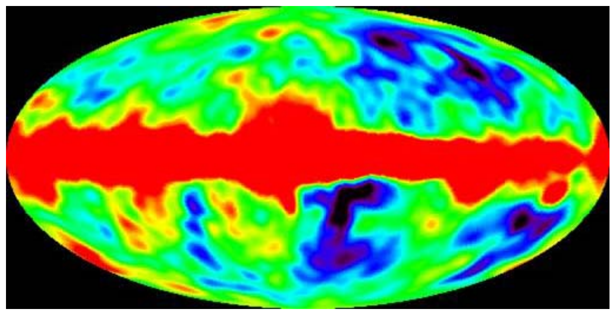
- Observation of CMBA



2.73 K



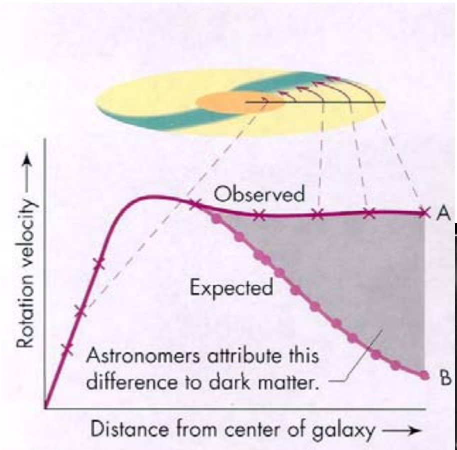
1/1,000 K



1/100,000 K (COBE)

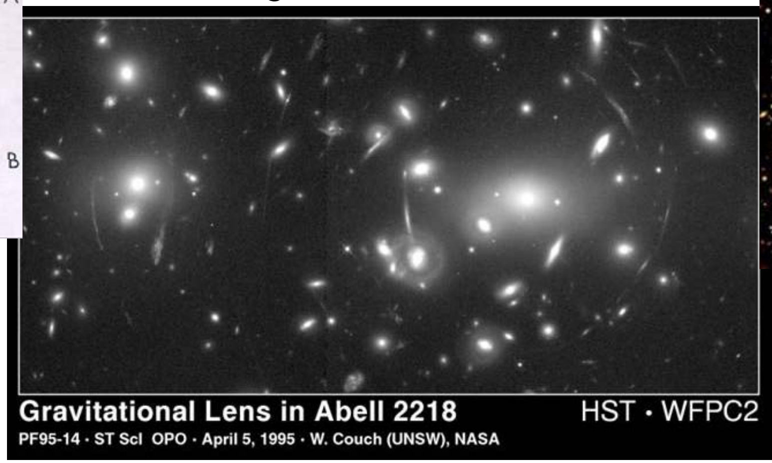
Dark Matter

- To explain the formation of LSS, dark matter is a necessity.
  - The amount of dark matter in our universe : $\Omega_{\rm CDM} \sim 0.25$
- Other evidences for dark matter
  - Rotation curves of galaxies
  - Gravitational lensing, mismatch in baryon and matter distribution



Rotation curve of galaxy shows the existence of dark matter outside the visible disk of galaxy.

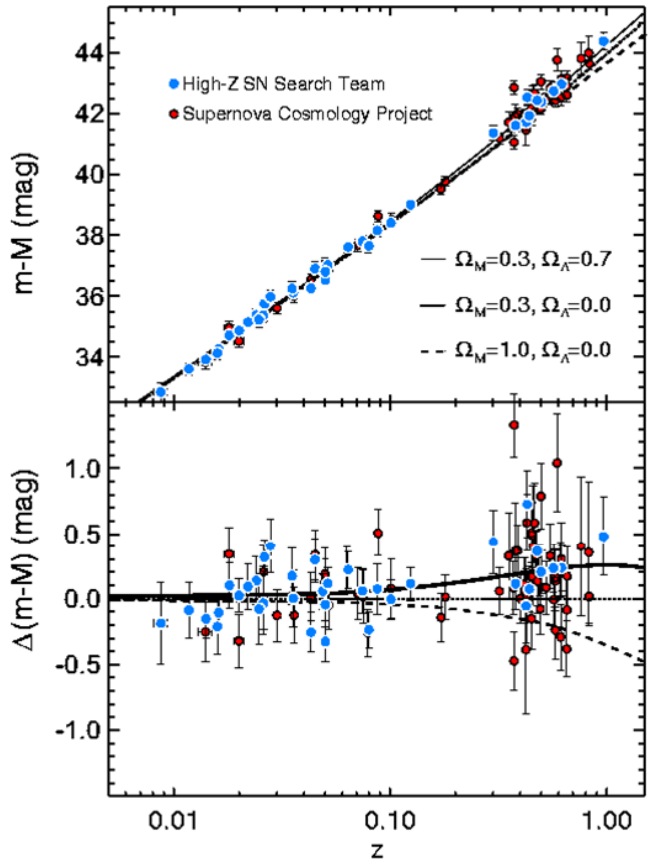
Gravitational lensing effect reveals the existence of matter unseen between far galaxies and us.

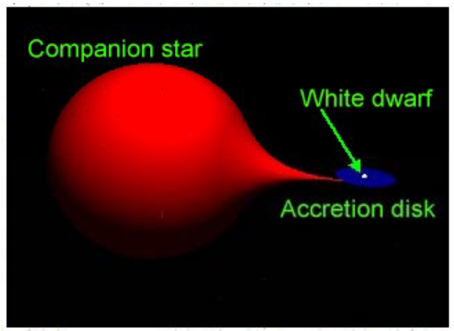


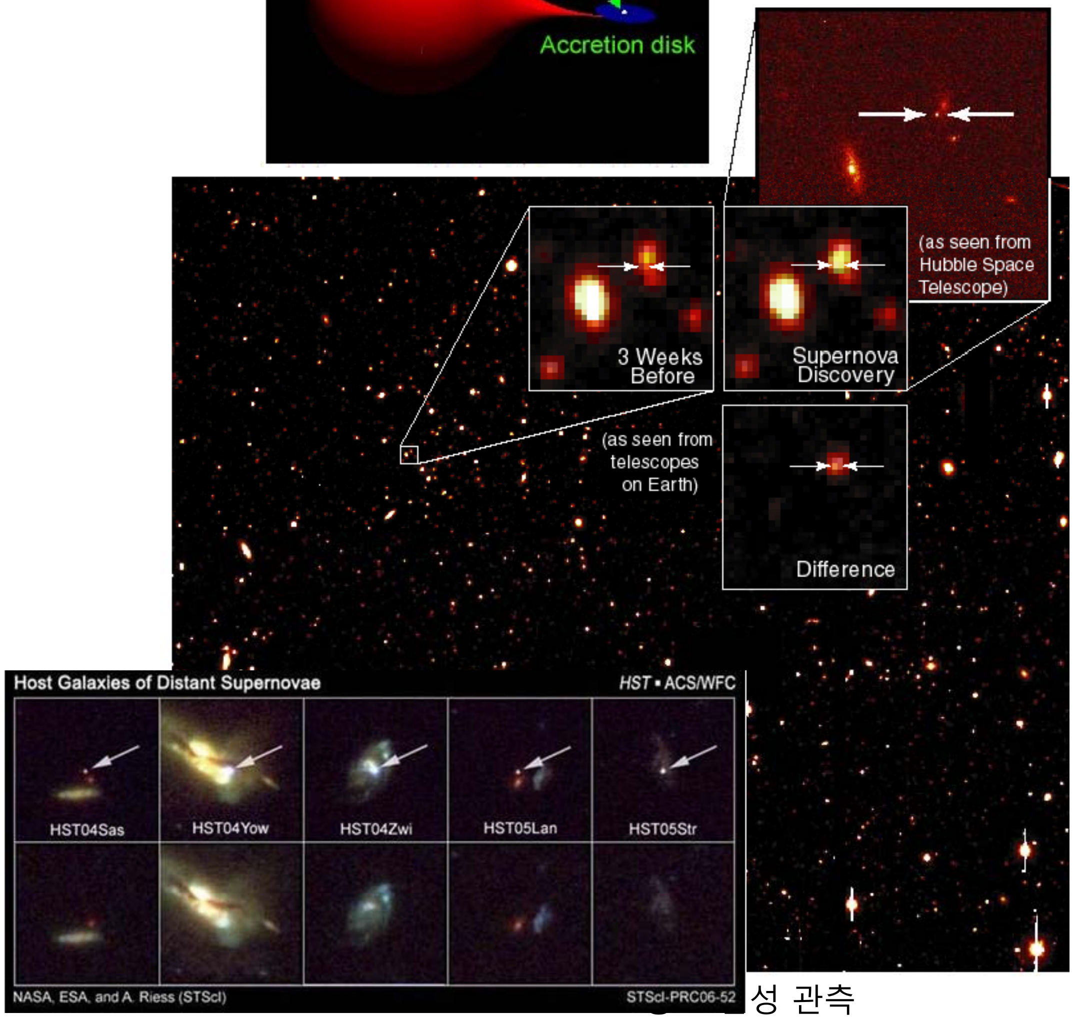
Colliding clusters
Baryon(red, X-ray) and dark matter (blue, gravitational lensing) reside separately. Galaxies follow the dark matter distribution.

Discovery of accelerating expansion

- SN Ia observations
  - Very bright, far distant one can be observed.
  - Uniform luminosity, calibrated by light curve





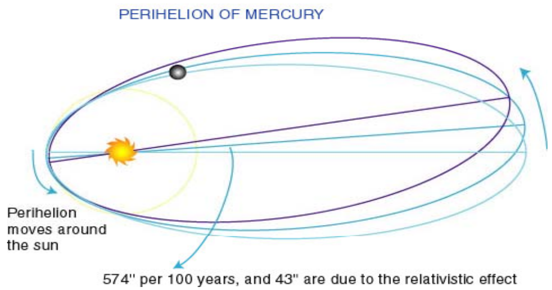


성 관측

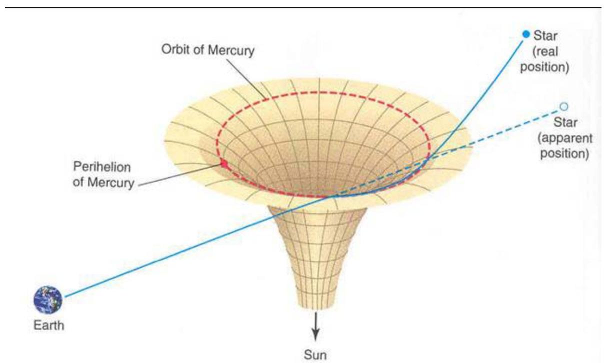
General Relativity

◆ Newton's gravity is replaced by Einstein's gravity (General Relativity)

Newton's Gravity	Einstein's Gravity
Mass → Force	Matter → Geometry
$\vec{F} = \frac{G m_1 m_2}{r^2}\,\hat{r}$	$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$



Success and Failure of Newton's gravity – planetary motion, precession of perihelion of mercury



Even light is bent in the curved spacetime.

◆ Basic Units

- Natural unit $\hbar = c = k_B = 1$
- Planck unit $\hbar = c = k_B = G = 1$
- Reduced Planck mass $M_P = (8\pi G)^{-1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}$
- Solar mass $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$
- parsec $1\,\mathrm{pc} = 3.26\,\text{light-year} = 3.1 \times 10^{16}\,\mathrm{m}$

◆ Conversion Factors in Natural unit

- Energy-Mass $1\,\mathrm{eV}^{-1}/c^2 = 1.78 \times 10^{-36}\,\mathrm{kg}$
- Energy-Time $1\,\mathrm{eV}^{-1}\hbar = 6.58 \times 10^{-16}\,\mathrm{s}$
- Energy-Length $1\,\mathrm{eV}^{-1}\hbar c = 1.97 \times 10^{-7}\,\mathrm{m}$
- Energy-Temperature $1\,\mathrm{eV}/\mathrm{k_B} = 1.16 \times 10^{4}\,\mathrm{K}$

- Most of cosmology can be learned with only a passing knowledge of general relativity : metric, geodesics, Einstein equation, …

  - Metric $g_{\mu\nu}$
  - Connection $\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right)$
  - Curvature $R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda} \Gamma^{\lambda}_{\mu\sigma}$
  - Ricci and Einstein tensor $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}, \quad R = R^{\mu}_{\mu}, \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
  - Einstein equation can be derived from the Einstein-Hilbert action.

$$S_{\rm EH} = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \mathcal{L}_{\rm M} \right) \quad \Rightarrow \quad G_{\mu\nu} = M_P^{-2}\, T_{\mu\nu}$$

  - Geodesic equation – path of a freely falling particle

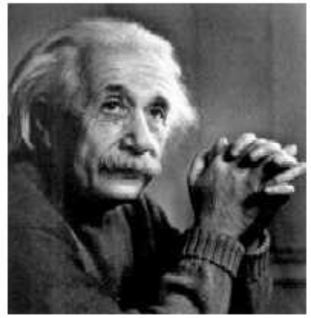
$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^{\mu}_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

  - Isometry

    Symmetry of manifold ⇔ Killing vector

    Maximally symmetric space

FRW Universe


Bilief of Einstein

◆ **Cosmological principle**

- The universe is pretty much the same **everywhere**.

◆ **Observational facts**

- The distribution of matter (galaxies) and radiation (CMB) in the observable universe is homogeneous and isotropic.
- The universe is not static : Distant galaxies are receding.

◆ **Robertson-Walker metric**

Our local Hubble volume during Hubble time

~ **spacetime with homogeneous and isotropic spatial sections**

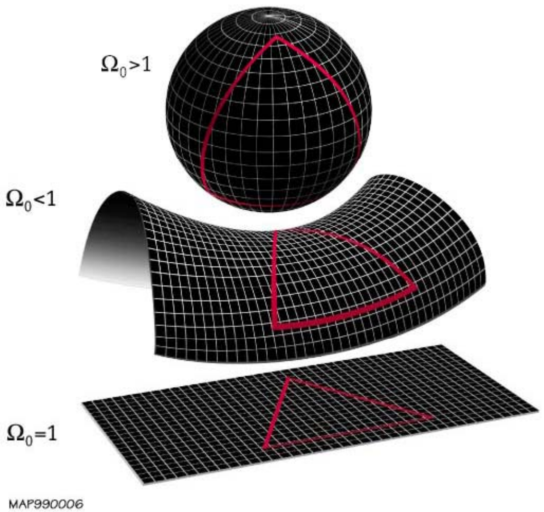
$$M = \mathbf{R} \times \Sigma$$

$\mathbf{R}$ ↑ time Σ ↑ 3D, maximally symmetric space

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right]$$

↑

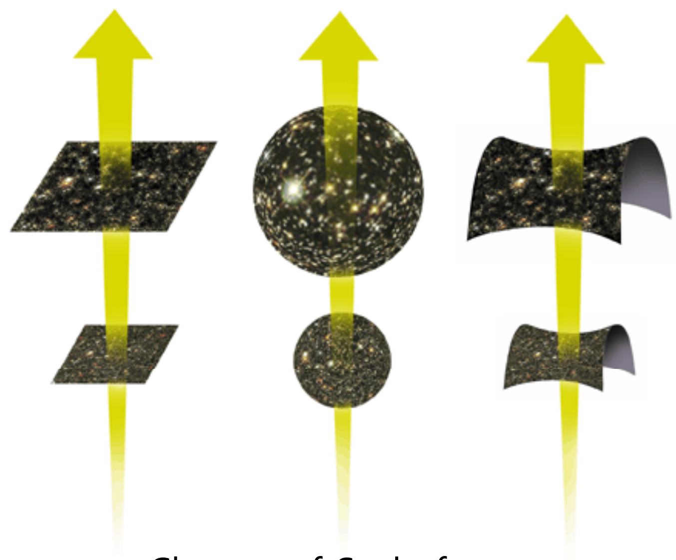
Scale factor, the only dynamical variable of RW metric



2D examples of maximally symmetric space

$K = 0$ $K = +1$ $K = -1$

E^3 S^3 H^3



Change of Scale factor

Kinematics of RW metric

- **Distances**

  Measuring distance in the expanding universe is tricky.

  - Comoving distance – fixed coordinate distance
  - Physical distance – comoving distance x scale factor
  - Luminosity distance
  - Angular diameter distance

- **Particle horizon**

  Consider the light propagation : $ds^2 = 0$

  Total comoving distance light have traveled since t=0

$$\eta = \int_0^{r_H} \frac{dr}{\sqrt{1 - Kr^2}} = \int_0^t \frac{dt'}{a(t')}$$

  No information could have propagated further than this.
  $\Rightarrow$ comoving horizon

  Physical distance to the horizon $d_H(t) = \int_0^{r_H} \sqrt{g_{rr}} = a(t)\eta$

- **Freely falling particle - geodesic equation**
  - Energy-momentum vector $p^\mu = \frac{dx^\mu}{d\lambda} = (E, \vec{p})$
  - 0-component of geodesic eq. $E\frac{dE}{dt} = -\Gamma^0_{ij} p^i p^j = -\delta_{ij} a\dot{a} p^i p^j$

$$\frac{1}{|\vec{p}|}\frac{d|\vec{p}|}{dt} + \frac{\dot{a}}{a} = 0 \quad \Rightarrow \quad |\vec{p}| \propto \frac{1}{a}$$

Momentum red shift
as the scale factor increase

Red shift parameter

$$\frac{\lambda_0}{\lambda} \equiv 1 + z = \frac{a_0}{a}$$

- **Hubble's law**
  - Luminosity distance $F = \frac{L}{4\pi r^2} \quad \Rightarrow \quad d_L^2 \equiv \frac{L}{4\pi F}$
  - Effect of expansion $F = \frac{L}{4\pi (a_0 r_1(z))^2} \frac{1}{(1+z)^2}$

$$d_L = a_0 r_1(z)\,(1+z)$$

$$H_0 d_L = z + \frac{1}{2} q_0 z^2 + \cdots$$