EWSB and BSM: Lecture no. 5 and 6

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1. Indirect limits on the Higgs mass from precision measurements
2. Theoretical Limits on the Higgs mass.
3. Instability of the Higgs mass under quantum corrections.

In the Electroweak ($SU(2)_L \times U(1)$) sector we saw how

$$\mathcal{L}_{SM}^{G.I.} = \mathcal{L}_{gauge}^{massless} + \mathcal{L}_{Dirac}^{massless} + \mathcal{L}_{scalar} + \mathcal{L}_{yukawa}$$

after Spontaneous Symmetry Breaking (SSB) $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ getting nonzero vacuum expectation value,

$$\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

gives

$$\mathcal{L}_{SM}^{G.I.} = \mathcal{L}_{gauge}^{massive} + \mathcal{L}_{gauge}^{\gamma} + \mathcal{L}_{Dirac}^{massive} + \mathcal{L}_h$$

Higgs scalar the remnant of the SSB. The Last term contains its interactions with matter as well as gauge bosons.

The SM Higgs, should be a $I_W = 1/2$ doublet.

Must have spin $= 0$, CP $= +1$

• Higgs coupling to all particles is $\propto$ their masses. This is intimately related to the Electroweak symmetry breaking.

• All the masses other than M_h in the SM, predicted in terms of the vacuum expectation value of the Higgs field v. $G_F\sqrt{(2)} = 1/v^2 \Rightarrow v \simeq 246$ GeV.

• Higgs mass not predicted by the theory. $m_h^2 = -2\mu^2 = \lambda v^2$. λ undetermined and hence M_h^2 unpredicted.

• Gauge symmetry predicts precise form of the ZWW coupling.

The interaction of the Higgs bosons with all the other particles is decided by the symmetry breaking mechanism, the interaction of everything with W/Z decided by the symmetry itself!

To produce the Higgs most favourable couplings are $WW\phi, ZZ\phi$ and $f\bar{f}\phi$ where f is a heavy fermion (top).

$$M_W = \left(g_2^2\sqrt{2}/8G_F\right)^{1/2} = \frac{37.4}{\sin\theta_W}\mathrm{GeV}/c^2, \quad M_Z = \frac{M_W}{\cos\theta_W}; \rho = 1.0$$

G_F Fermi coupling constant in the β decay (also called G_μ sometimes). Value extracted using muon life time τ_μ.

These relations change due to quantum corrections. Renormalisabity gurantees that the corrections are finite! The renormalisability in the end is guranteed by Gauge Invariance.

Precision measurements happened at the Large Electron Positron Collider (LEP) and Stanford Linear Collider (SLC). (Z factories!)

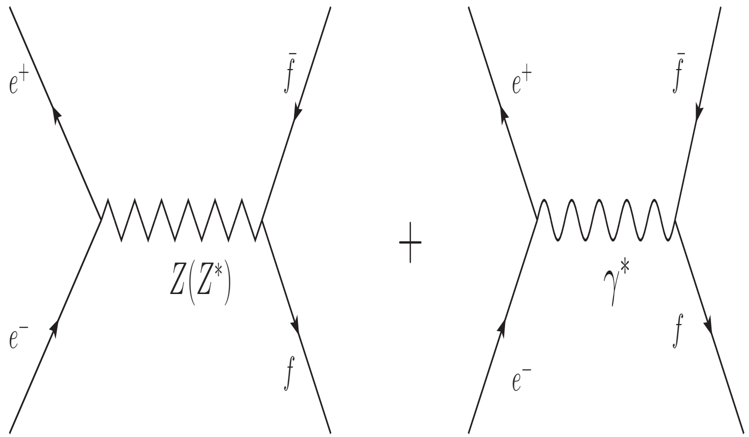
LEP-I: Simple and precision measurement of $e^+e^- \to f\bar{f}$

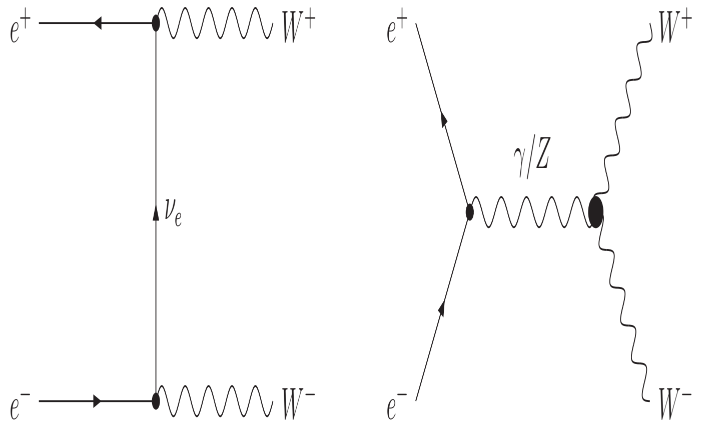
10 million Z's collected by four experiments.

1. Measure the couplings of the Z to all the SM fermions accurately, to establish the nature of the 'weak' neutral current to great accuracy. Study $e^+e^- \to Z^*/\gamma^* \to f\bar{f}$.

2. $SU(2)_L$ symmetry means specific values for ZWW coupling. Can one directly measure ZWW coupling? Need to measure $e^+e^- \to Z^*/\gamma^* \to W^+W^-$.

3. Find the Higgs in $e^+e^- \to \phi Z$: LEP did not do this job!



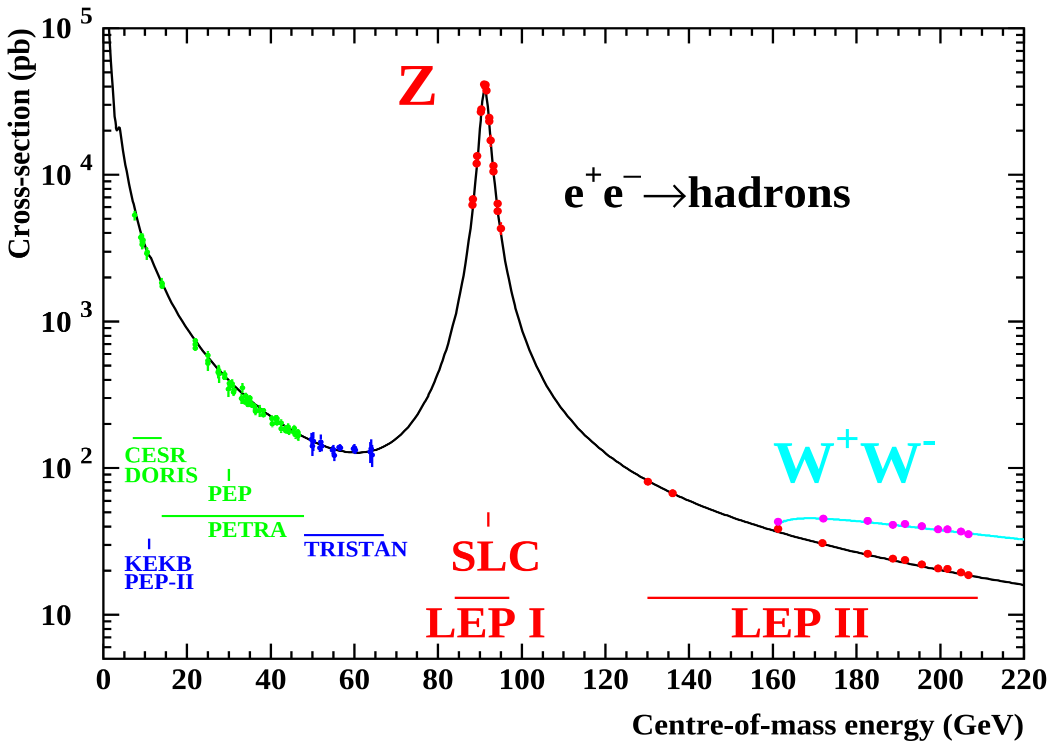


High precision measurements require high precision calculations.

Higher order QED and QCD corrections highly important and non-trivial.

Good understanding of QCD to calculate correctly what the detectors observe: jets.

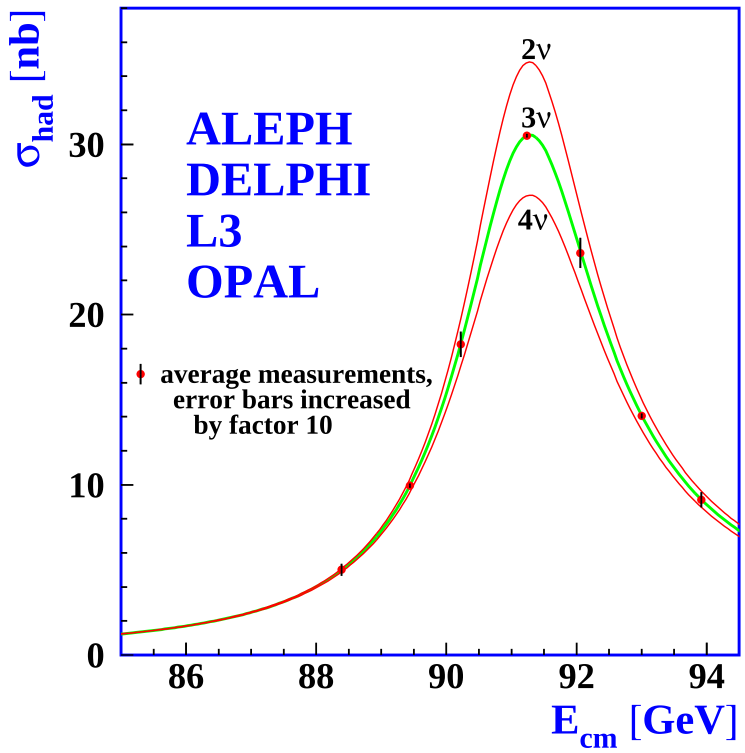
Extensive collaborative studies between experimentalists and theorists LEP Yellow Reports.



Solid line is the SM fit. Phys. Rept. 427, 257 (2006).

Large electromagnetic and QCD radiative corrections,

Initial state radiation makes the curve asymmetric near the resonance.



Width of the Z measured accurately, rules out 4th mass less neutrino generation.

Remember massive gauge bosons have two problems.

1)Mass not compatible with gauge invariance.

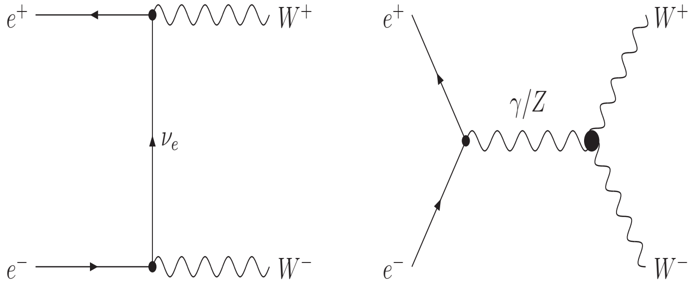
2)Amplitudes like $\nu_e \bar{\nu}_e \to W^+ W^-$ grow with energy and can violate unitarity.

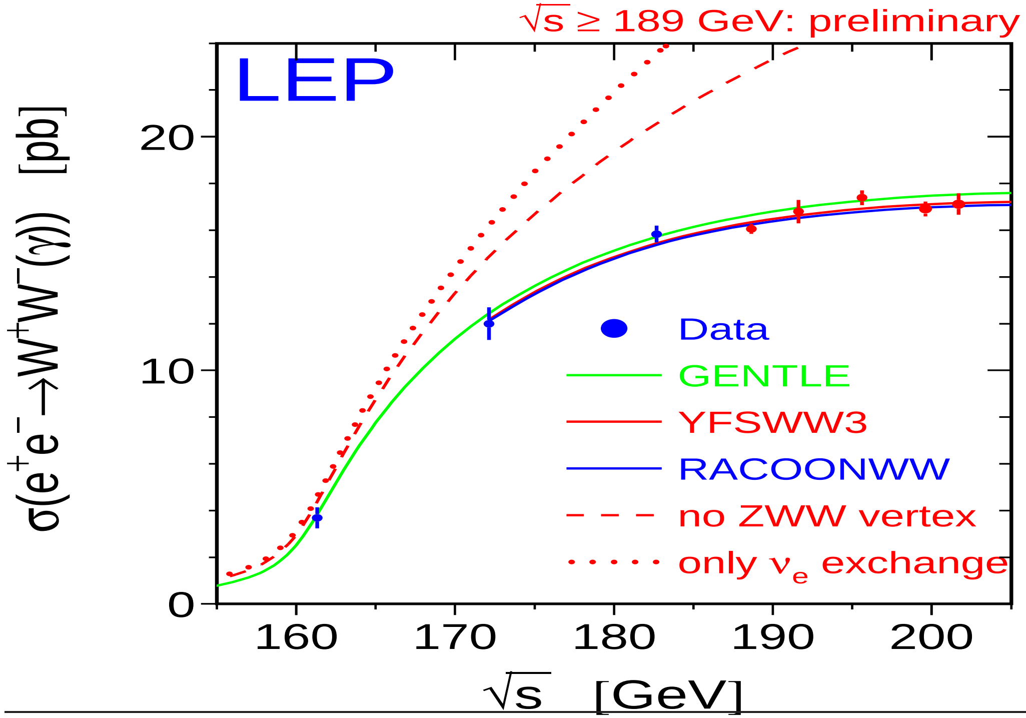
Glashow's model based on $SU(2)_L \times U(1)$ invariance showed that the Gauge Symmetry, which predicts additional Z boson, improved the high energy behaviour of the $e^+ e^- \to W^+ W^-$.

One can show that this violation of unitarity can be tamed by adding a neutral spin 1 boson which ZW^+W^- couplings as expected in the (Glashow) $SU(2)_L \times U(1)_Y$ model!

Cornwall, Tiktopolous (1974, 1975); Llewellyn Smith (1973), S.D. Joglekar (1973)

Directly tested.



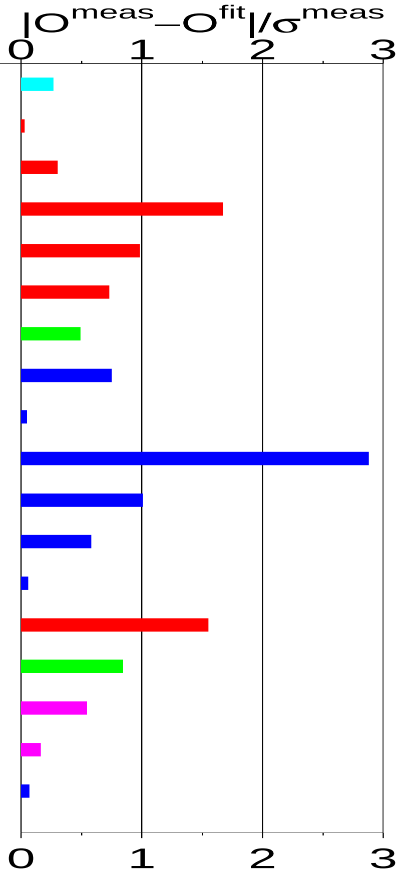


Proof that Electroweak symmetry exists and that it is broken.

The triple gauge boson ZWW coupling tames the bad high energy behaviour of the cross-section caused by the t-channel diagram. Direct proof for the ZWW coupling.

This observation at LEP-II and precision testing at the LEP-I, confirm basics of the SM

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2011

see http://lepewwg.web.cern.ch

Logical steps in Precision testing of the SM and the indirect limits:

• SM has parameters g_2, g_1, v and λ. All the $\bar{f}fV$ and VVV couplings (V =W/Z), M_V and m_h are functions of these. In addition to these there are of course Yukawa couplings.

• A large number of EW observables measured quite accurately.

• M_Z, α_{em} and G_μ are most accurately measured. Trade g_2, g_1 and v for these.

• All observables depend on these three. In addition there is a dependence on m_f (mainly M_t) *and* M_h, and of course α_s.

• Calculate all observables using **1 loop EW** radiative corrections which can be computed in a renormalisable quantum field theory.

• Compare with data, make a SM fit. Tests the SM at loop level.

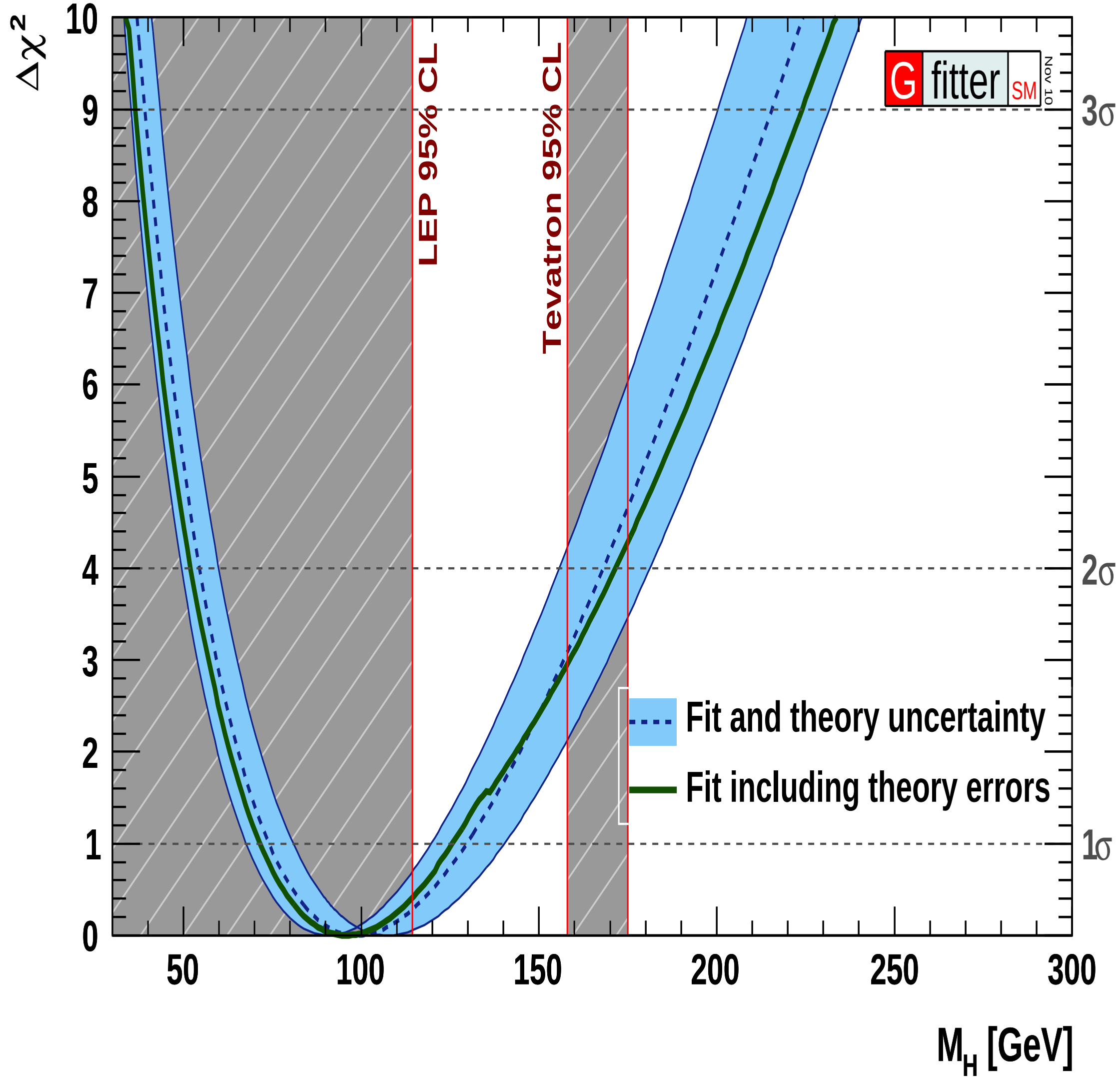
What does this all mean for the Higgs?

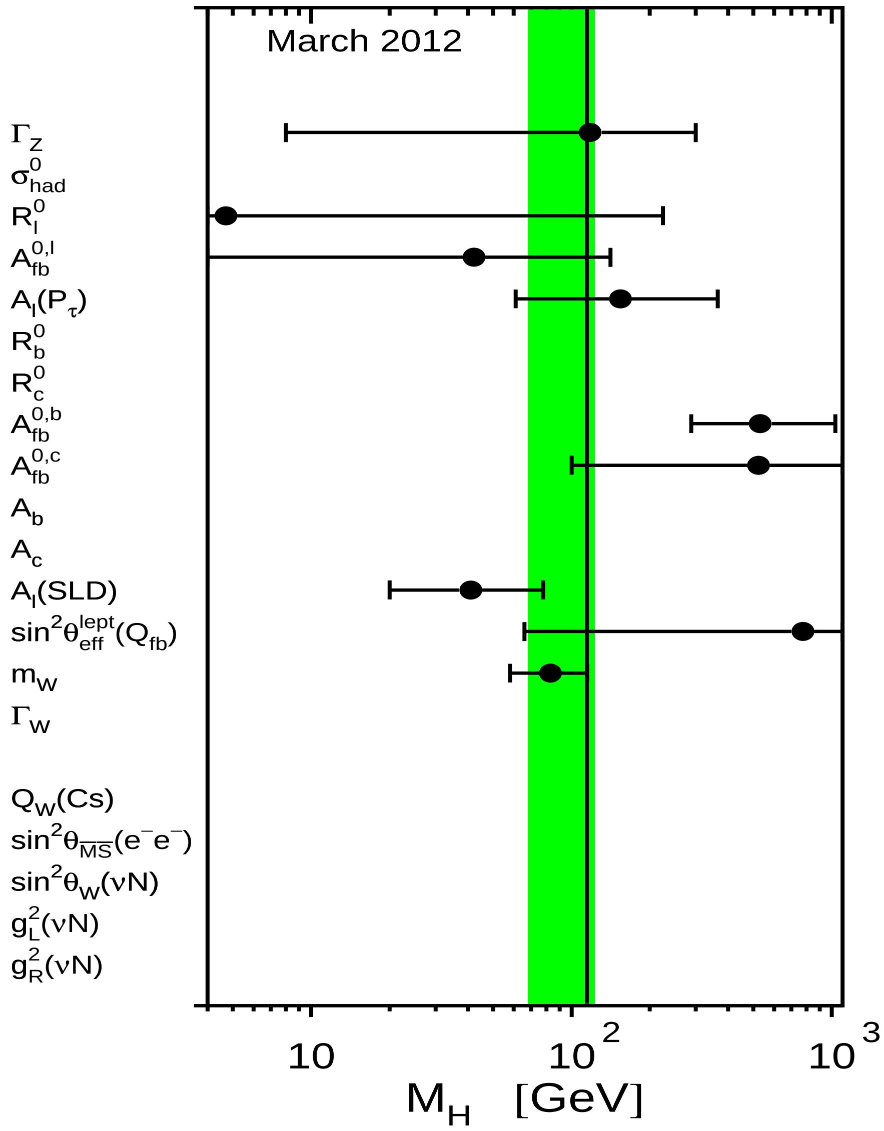
The loop corrections depend on the Higgs mass. Since that is the only unknown these measurements indirectly constrain the Higgs mass.

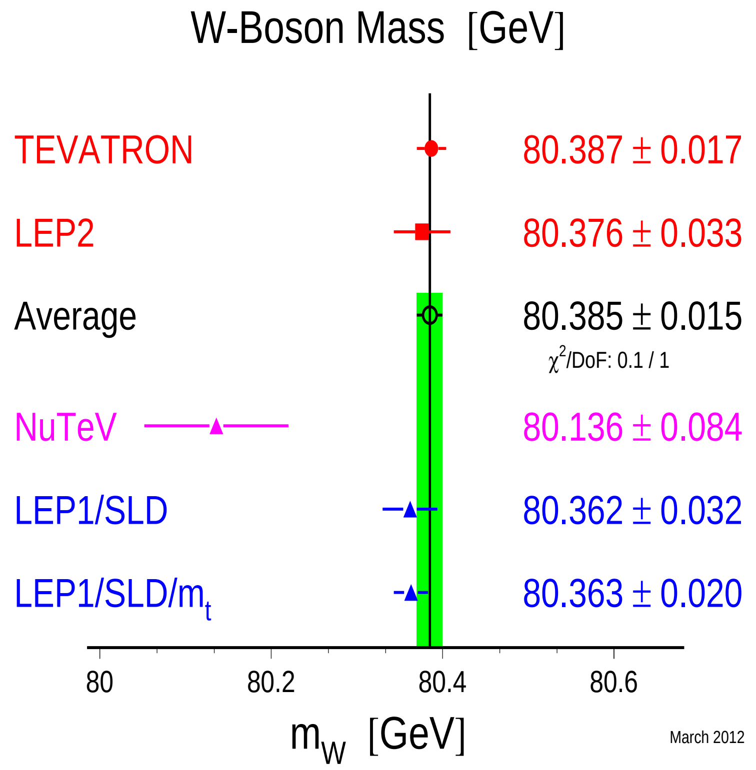
If all the current information is put together the Higgs mass should be less than 150 GeV. (**indirect experimental limit!**)

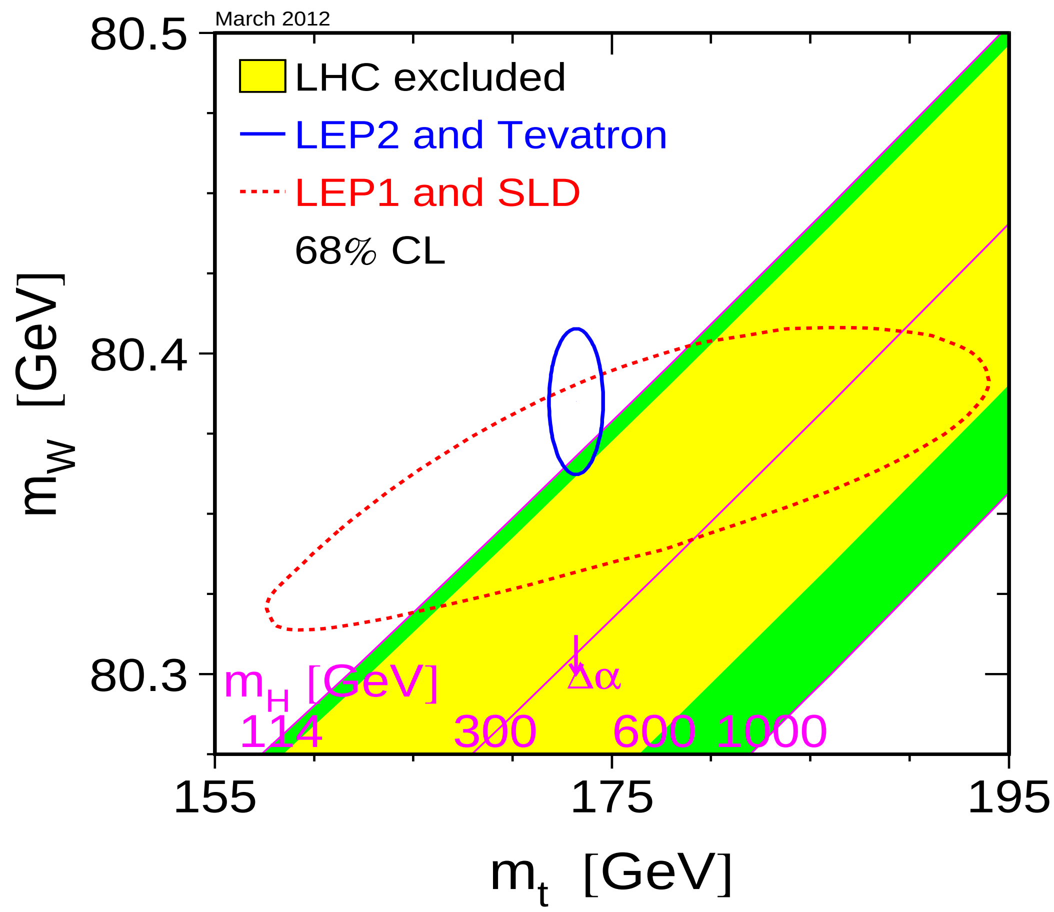
From the Gfitter web page (old pre July 4).

Can be affected by BSM!




March 2012
Γ_Z
σ^0_{had}
R^0_l
$A^{0,l}_{fb}$
$A_l(P_\tau)$
R^0_b
R^0_c
$A^{0,b}_{fb}$
$A^{0,c}_{fb}$
A_b
A_c
A_l(SLD)
$\sin^2\theta^{lept}_{eff}(Q_{fb})$
m_W
Γ_W
Q_W(Cs)
$\sin^2\theta_{\overline{MS}}(e^-e^-)$
$\sin^2\theta_W(\nu N)$
$g^2_L(\nu N)$
$g^2_R(\nu N)$
10
10^2
10^3
M_H [GeV]


W-Boson Mass [GeV]
TEVATRON
80.387 ± 0.017
LEP2
80.376 ± 0.033
Average
80.385 ± 0.015
χ²/DoF: 0.1 / 1
NuTeV
80.136 ± 0.084
LEP1/SLD
80.362 ± 0.032
LEP1/SLD/mt
80.363 ± 0.020
80
80.2
80.4
80.6
mW [GeV]
March 2012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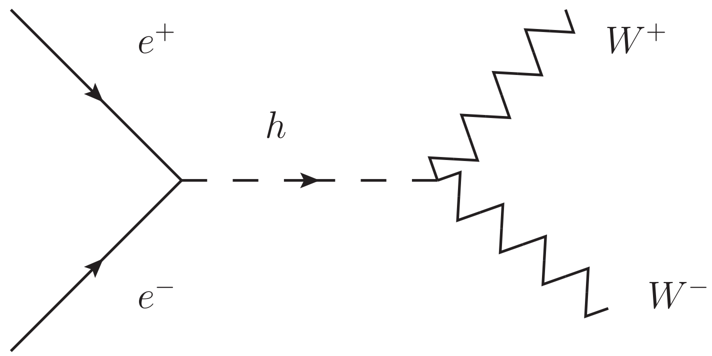
80.5
80.4
80.3
mW [GeV]
LHC excluded
LEP2 and Tevatron
LEP1 and SLD
68% CL
mH [GeV]
114
300
600 1000
Δα
155
175
195
mt [GeV]

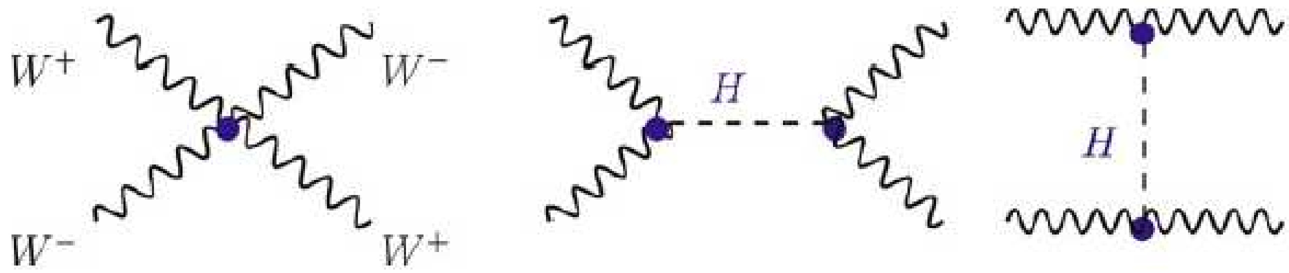


Recall $e^+e^- \to W^+W^-$

In principle at **astonomically** high energies, this will still violate unitarity unless one includes the **Higgs** contribution. I.e the SM is unitary ONLY after the Higgs contributions are included. For, e^-, due to its small mass this is indeed a completely negligible effect.

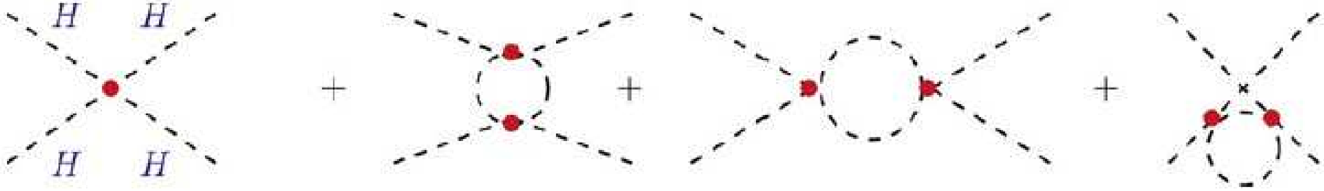
But in general this unitarity is guranteed ONLY if M_h is bounded from above: B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. D16 (1977) 1519

They considered the $W^+W^- \to W^+W^-$ which has diverging high energy behaviour due to longitudinal $W's$.



Obtained a limit $M_h \lesssim 700 - 800$ GeV.

Similar limit comes from demanding that the quartic coupling in the Higgs potential remains perturbative and positive, under loop corrections:



Remember: $M_h^2 = \lambda v^2$. For large λ the loop corrections dominated by the h–loops.

At one loop running of λ given by:

$$\frac{d\lambda(Q^2)}{d\log Q^2} = \frac{3}{4\pi}\lambda^2(Q^2)$$

Solving this, one gets

$$\lambda(Q^2) = \frac{\lambda(v^2)}{[1 - \frac{3}{4\pi^2}\lambda(v^2)\log(\frac{Q^2}{v^2})]}$$

For large $Q^2 \gg v^2$ then $\lambda(Q^2)$ developes a pole (the Landau pole).

Scale at which the pole lies is

$$\Lambda_C = v\ \exp\left(\frac{2\pi^2}{3\lambda}\right) = v \exp\left(\frac{4\pi^2 v^2}{3M_h^2}\right)$$

If e.g. $\Lambda_C = 10^{16}$ GeV, then we will find $M_h \lesssim 200$ GeV. Upper Bound: called triviality bound

Thus just the mass of M_h can give indication of the scale of new physics beyond the SM

When M_h is small and λ not large, the fermion/gauge boson loops are important. Fermions loops come with a negative sign!

Now the RGE for λ is given by

$$\frac{d\lambda(Q^2)}{d\log(Q^2)} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right]$$

$\lambda_t = f^{*(u_3)}$ in our notation.

For $\lambda \ll \lambda_t, g_1, g_2$ drop all the λ terms and solve the RGE. (Exercise)

At small M_h and hence small $\lambda(v)$, at some value of Q, λ can turn negative.

Potential will be unbounded. Vacuum will be unstable

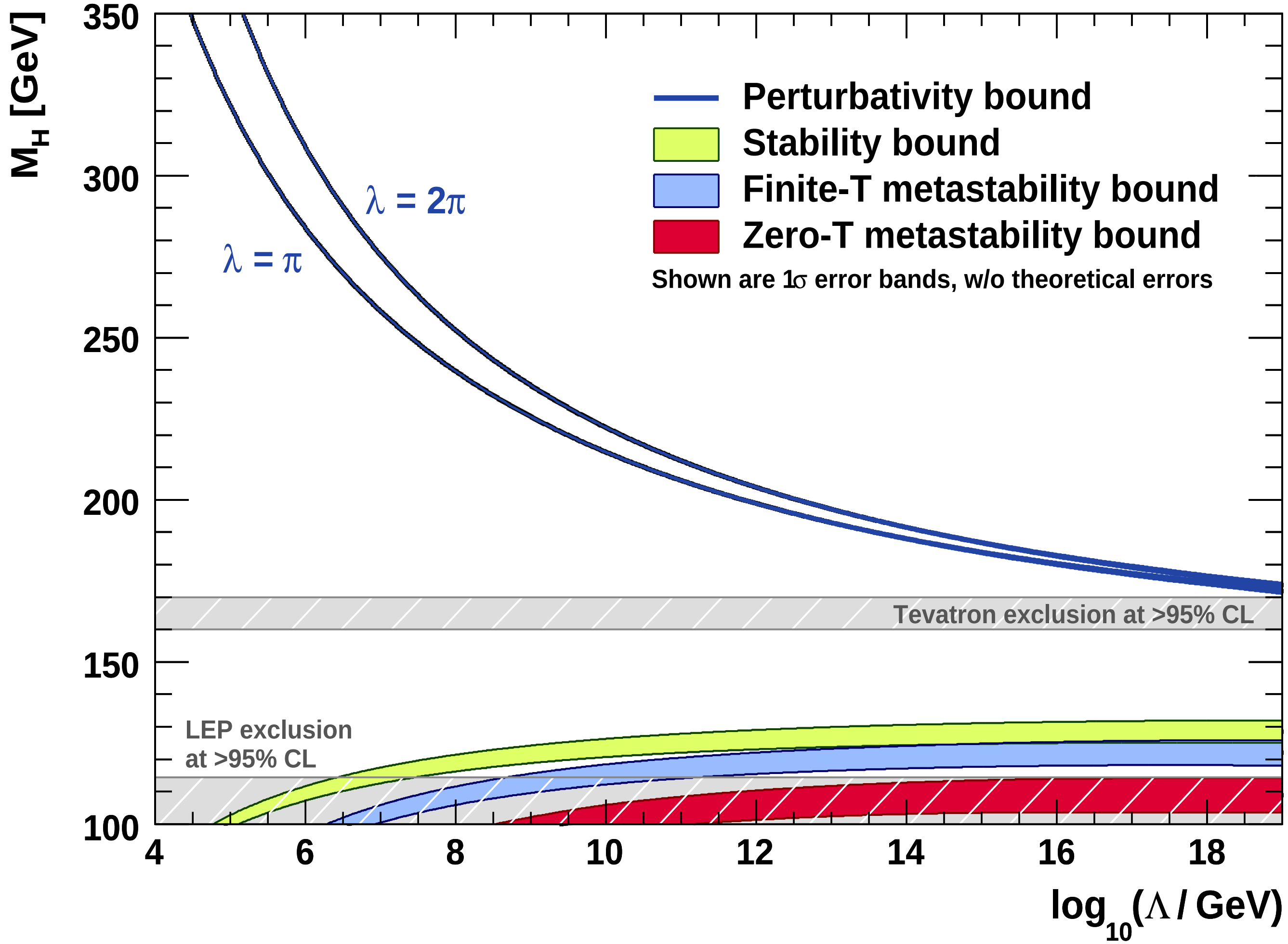
The condition is

$$M_h^2 > \frac{v^2}{8\pi^2} \log(Q^2/v^2) \left[12\frac{m_t^2}{v^4} - \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2) \right].$$

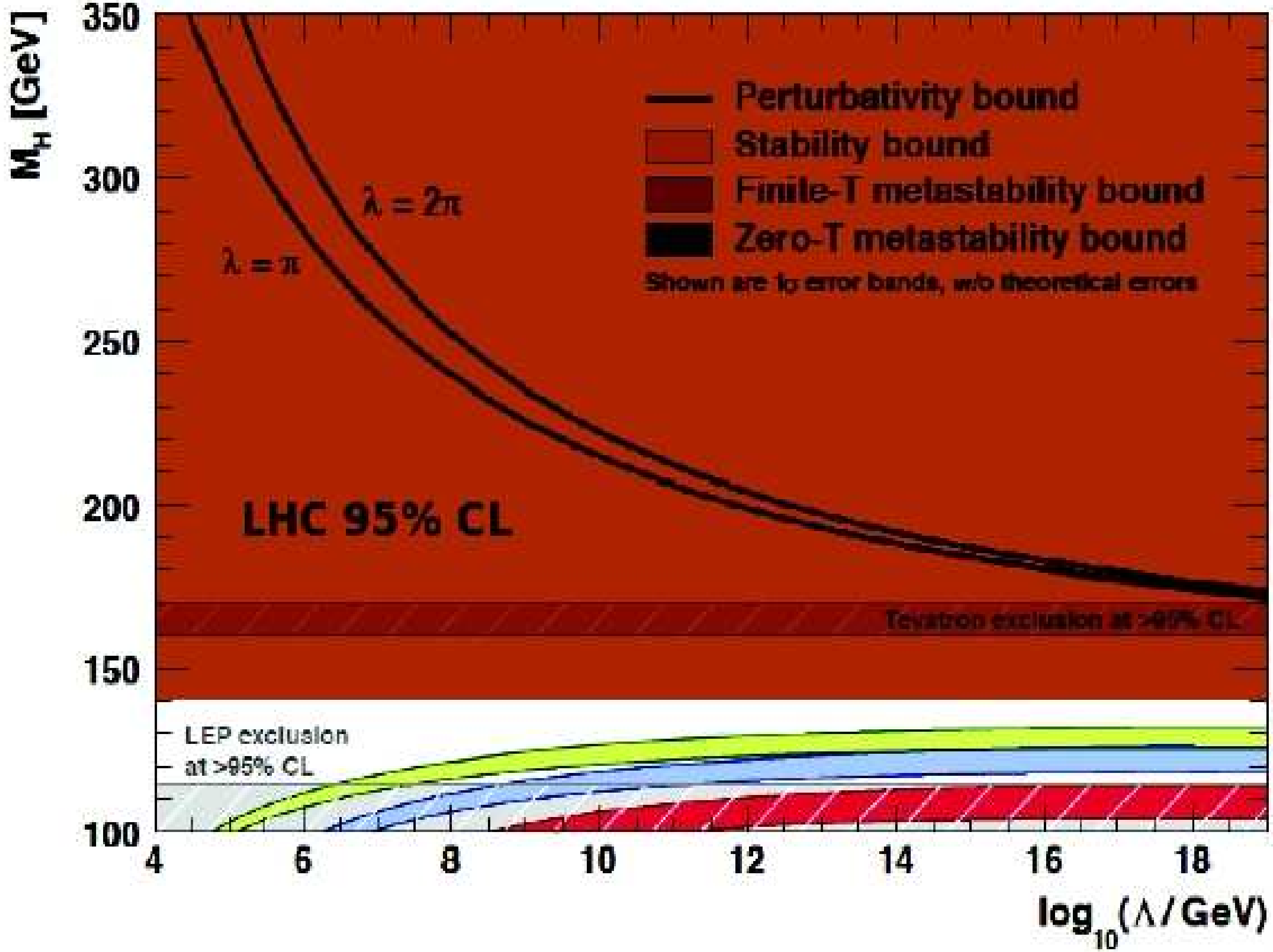
If we demand that the $\lambda(Q)$ is positive upto Λ_C we then get a lower bound.

For example:

$\Lambda_C = 10^3 GeV$, $M_h \gtrsim 70$ GeV



From a paper by Ellis, Giudice et al, PLB 679, 369-375 (2009). Includes higher order effects compared to the forumlae I gave.



M_H [GeV]
350
300
250
200
150
100
Perturbativity bound
Stability bound
Finite-T metastability bound
Zero-T metastability bound
Shown are 1σ error bands, w/o theoretical errors
λ = 2π
λ = π
LHC 95% CL
Tevatron exclusion at >95% CL
LEP exclusion
at >95% CL
4
6
8
10
12
14
16
18
$\log_{10}(\Lambda/\text{GeV})$

So finally the main role of the Higgs is to

1)Make the scattering amplitudes involving gauge bosons in the theory respect unitarity, even for massive gauge bosons.

2)Make gauge theories renormalisable.

LHC searched for the Higgs

We seem to have found it! It is light exactly as we wanted!

Then we will like to explain 'theoretically' why it is light.

Further, almost all the BSM options affect Higgs properties. So study of the Higgs sector is THE LHC goal. May be that will shine the path beyond the SM.

Absence of FCNC $\Rightarrow$ quarks must come in isospin doublets, charm was predicted and top was expected to be present once b was found

Indirect information on M_c, M_t from flavour changing neutral processes. Agreement with experimentally measured values 'proves' gauge theory.

CP violation in meson systems can be explained in terms of the SM parameters and measured CKM mixing in quark sector.

M_W, M_Z predicted in terms of $\sin\theta_W$

M_t predicted from precision measurement of M_W, M_Z.

Once the top was discovered, we used precision measurements to obtain indication on M_h

Can the information on Higgs now give indications on BSM?

• Relationship between M_h^2 and λ

• Dependence of $\frac{M^2}{M_Z^2 \cos\theta_W^2}$ on the Higgs representation. But $\rho = 1$ is expected also on the basis of a symmetry called Custodial Symmetry, which is an unbroken, global symmetry Accidental. Since the data show $\rho \simeq 1$, any BSM must respect this Custodial Symmetry.

• Loop induced decays of the h receive contributions from heavy particles in the loop. This contribution does not vanish in the limit of large masses, nondecoupling, for chiral fermions. Therefore $h \to \gamma\gamma$ and $gg \to h$ can hold a lot of clues about BSM. In fact the LHC observation can now rule out a lot of new physics models.

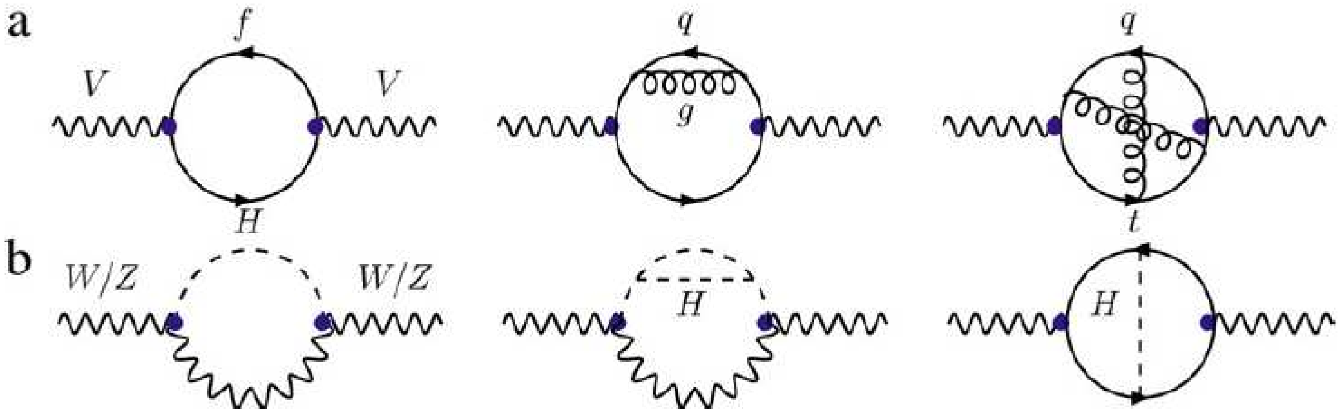
• Observation of a light Higgs means that the SM can be consistent with no BSM upto very large scale ☹.

• If any BSM should determine λ then such a BSM will have a prediction for the Higgs mass! SUSY is such an example!

Gauge boson masses and fermion masses are not allowed by symemtry considerations. I.e in the limit of these masses going to zero, the symmetry of the theory increases.

Small fermion and gauge boson masses are therefore 'natural'.

M_W, M_Z protected from receiving large loop corrections even if there should exist a large scale Λ.

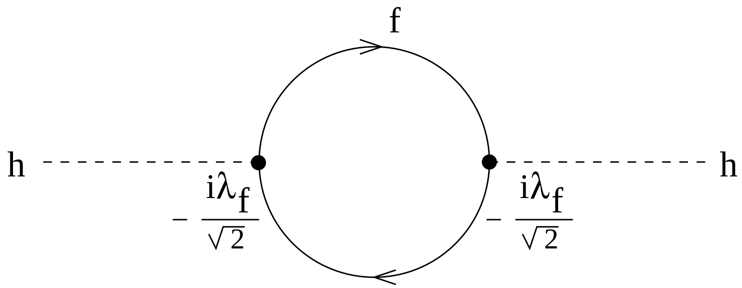


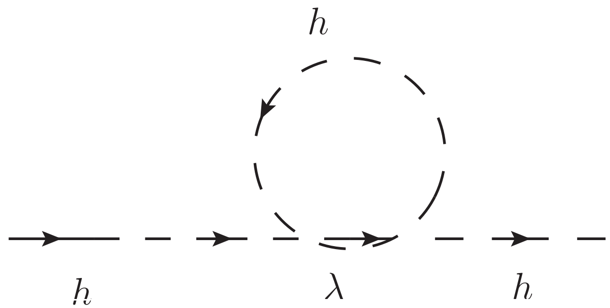
This is not the case for scalar particles. The loop corrections for scalar masses are proportional to the largest mass available.

Consider a model with one massive scalar with mass M_s and coupling to a a fermion of mass $m_f = \lambda_f \frac{v}{\sqrt{2}}$ and self coupling λ, with $v \sim 246$ GeV.

$$\mathcal{L}_{scalar} = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_s^2 h^2 - \frac{\lambda_f v}{\sqrt{2}} \left[\bar{f}_L f_R + \bar{f}_R f_L\right] (1 + \frac{h}{v}) - \frac{\lambda}{4} h^4 - \lambda v h^3$$

Consider self energy of h : loop corrections to the two point function Π_{hh} These measure corrections to the M_s coming from higher orders.





$$\begin{aligned}\Pi^f_{hh}(0) &= (-1)\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f} \\ &= -2\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2+m_f^2}{(k^2-m_f^2)^2} \\ &= -2\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_f^2}+\frac{2m_f^2}{(k^2-m_f^2)^2}\right]. \qquad (1)\end{aligned}$$

$\Pi^f_{hh}(0) = -2\lambda_f^2 I_0 - m_f^2\lambda_f^2\ I_1$

First term is quadratically divergent and is independent of M_S. $\propto \Lambda^2$ where Λ is the cut off of the integral.

Second term $-m_f^2\lambda_f^2\ I_1$ is logarithmically divergent.

$$\Pi^h_{hh} = \Pi^h_{hh}(0)|_{tadpole} = \lambda I_0$$

$$(M_s^2)_{\text{phys}} = (M_s^2)\text{bare} + \Delta M_s^2$$

If $\Lambda = M_{pl}$ (say) then $\delta M_s^2 \sim 10^{36}$ GeV^2. $(M_s^2)_{\text{phys}} \simeq \mathcal{O}(10^4 - -10^6)$ GeV^2 this means that the counterterm has to be adjusted to one part in 10^{32}.☹

If we choose $\lambda = 2\lambda_f^2 \Rightarrow$ the quadratically divergent terms will cancel each other and $\Delta(M_s^2)$ is only logarithmically divergent.

But such an adhoc choice with no symmetry dictating it, would mean that this will be spoiled at higher loop levels.

Further, I_1, though logarithmically divergent is $\propto m_f^2$.

In Grand Unified Theories (GUT) there exist fermions with masses $10^{13} - 10^{15}$, the correction to M_s^2 is still 10^{20} to 10^{25} times larger than the value of M_s^2.

This is called the Gauge Hierarchy problem.

1) Polchinsky, Susskind, Raby: PRL 47,

2) R. Kaul and P. Majumdar, NPB, 199.

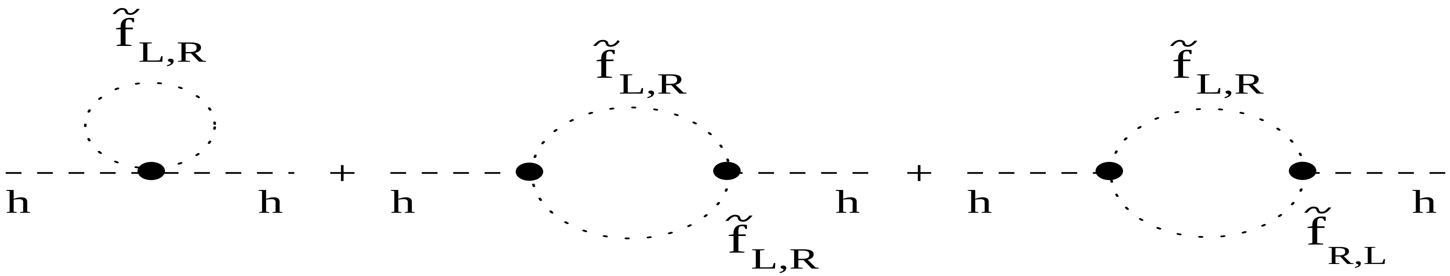
Consider the theory with two more complex scalars.

$$\mathcal{L}_{\tilde{f}\tilde{f}\phi} = \tilde{\lambda}_f |\phi|^2 (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) + (\lambda_f A_f \phi \tilde{f}_L \tilde{f}_R^\star + \text{h.c.})$$

With $\phi = \frac{1}{\sqrt{2}}(v + h(x))$ this becomes:

$$\begin{aligned} \mathcal{L}_{\tilde{f}\tilde{f}h} &= \frac{1}{2}\tilde{\lambda}_f h^2 (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) + v\tilde{\lambda}_f h (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) \\ &\quad + \frac{h}{\sqrt{2}}(\lambda_f A_f \tilde{f}_L \tilde{f}_R^\star + \text{h.c.}) + \cdots . \end{aligned} \tag{2}$$

$\Pi_{hh}^{\tilde{f}}$ receives contributions from $\tilde{f}$ loops.



$$\Pi^{\tilde{f}}_{hh}(0) = -\tilde{\lambda}_f \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right) + (\tilde{\lambda}_f v)^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{(k^2 - m^2_{\tilde{f}_L})^2} + \frac{1}{(k^2 - m^2_{\tilde{f}_R})^2} \right] + |\lambda_f A_f|^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2_{\tilde{f}_L}} \frac{1}{k^2 - m^2_{\tilde{f}_R}} \ . \qquad (3)$$

$\Pi^{\tilde{f}}(0)$ and $\Pi^{f} hh(0)$, both contain quadratically and logarithmically divergent terms.

Quadratically divergent terms are independent of M_s.

In fact the quadratic divergencies in the sum $\Pi^{\tilde{f}}_{hh}(0) + \Pi^{f}_{hh}(0)$ will cancel if $\tilde{\lambda}_f = -\lambda_f^2$. This requires $\tilde{\lambda}_f < 0$.

This is good : it keeps the Hamiltonian is bounded from below.

If apart from this we have $m_f = m_{\tilde{f}_L} = m_{\tilde{f}_R}$ and $A_f = 0$ we can see

$\Pi^{f}_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0) = 0$ ☺

If some symmetry were to 'bless' these equalities then M_s^2 is protected from receiving large corrections.

If we do not make thes extreme assumptions, but assume just

$m_{\tilde{f}_L} = m_{\tilde{f}_R} = m_{\tilde{f}} \neq m_f.$

$$\begin{aligned}\Pi^f_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0) &= i\frac{\lambda_f^2}{16\pi^2}\Big[-2m_f^2\left(1 - \ln\frac{m_f^2}{\mu^2}\right) + 4m_f^2 \ln\frac{m_f^2}{\mu^2} \\ &\quad +2m_{\tilde{f}}^2\left(1 - \ln\frac{m_{\tilde{f}}^2}{\mu^2}\right) - 4m_f^2 \ln\frac{m_{\tilde{f}}^2}{\mu^2} \\ &\quad -|A_f|^2 \ln\frac{m_{\tilde{f}}^2}{\mu^2}\Big]. \qquad (4)\end{aligned}$$

- Only Logarithmic divergencies remain.

- If $A_f = 0, m_f = m_{\tilde{f}}$ and $\lambda_{\tilde{f}} = -\lambda_f^2$ they cancel. No renomalisation of M_s at all ☺

- Indeed in SUSY there exists $\mathcal{L}_{\tilde{f}\tilde{f}\phi}$ and the above eqalities are guranteed by Supersymmetry.

- If SUSY is broken,

$$\Delta(M_s^2) = -\frac{\lambda_f^2}{16\pi^2}\left[4\delta^2 + (|A_{\tilde{f}}^2| + 2\delta^2)\log\left(\frac{m_{\tilde{f}}^2}{\mu^2}\right)\right] + ..$$

where $\delta^2 = m_{\tilde{f}}^2 - m_f^2$

Higgs mass is stabilised against large radiative corrections and the 'Higgs' is naturally small, if $\delta^2 \sim (\text{TeV})^2$.

But the prefactor is not known.

We therefore expected SUSY around TeV scale. But to be honest we do not know the prefactor

Currently we have not seen SUSY upto TeV scale right now BUT we may have seen a 'light' Higgs.

So what does that teach us?