INTRODUCTION AND OVERVIEW

Chapter 1

SUPERSYMMETRY: WHY AND HOW

1.1 History and Motivation

We first give a brief factual account of the history [1.1] of supersymmetry, leaving a more pedagogical development to later sections and chapters. This has evidently been a history of experimenters chasing a theoretically driven idea. The notion of a symmetry transformation between fermionic and bosonic modes emerged [1.2] in connection with string theory. However, this was constructed on a two dimensional world sheet rather than in the real world of $3+1$ dimensions. N=1 supersymmetry in the latter (in terms of supercharges that take fermions into bosons and vice versa) was first proposed and formulated as a graded Lie algebra by Golfand and Likhtman [1.3] in 1971. Akulov and Volkov [1.4] later gave a nonlinear realization of it together with the idea of spontaneous breakdown. Finally, in 1974, Wess and Zumino [1.5] as well as Salam and Strathdee [1.5] constructed field theories with supersymmetry (cf. Ch.5) and the subject immediately attracted attention on a large scale. Supersymmetry was shown by Haag, Łopuzańsky and Sohnius (cf. Ch.3) to be the only possible extension of the known spacetime symmetries of particle interactions. Several important results were derived on the more convergent ultraviolet behavior of supersymmetric field theories, exploiting the cancellation between fermionic and bosonic loops. In particular, a theorem on the nonrenormalization of superpotential terms (cf. Ch.6) was proved [1.6]. Scalar field theories are generically not natural in the sense of Weinberg, Susskind and 't Hooft [1.7]. But it became clear after a while that supersymmetric field theories, despite containing scalar fields, are natural [1.8].

The four momentum operator P^μ is essential to an algebraic formulation of supersymmetry. As elaborated in Ch.3, if a supercharge, carrying spin 1/2, takes a boson to a fermion or vice versa, the anticommutator between two supercharges with arbitrary spinorial components must be proportional to P^μ. If the vacuum is annihilated by a supercharge, a vanishing energy for it is then ensured. In exact supersymmetry, P^μ and $P^2 \equiv P^\mu P_\mu$ commute with the supercharge and one is led to mass degenerate supermultiplets of states differing in spin by 1/2. Since no such mass degeneracy has been seen[1] among particles occurring in Nature,

[1]There were early attempts to put a photon and a neutrino together in a supermultiplet. It soon became

supersymmetry must be a badly broken symmetry. The intra-supermultiplet mass splitting, characteristically denoted as M_s, then becomes a scale of some significance. A question of immediate interest arises in consequence: what is the order of magnitude of M_s? A related issue concerns the existence of **sparticles**, i.e. superpartners of the known particles. The latter cannot make up complete supermultiplets by themselves. Therefore each particle in a broken supersymmetric world must have a new superpartner which we call a sparticle. A sparticle is typically heavier than the corresponding particle by a mass difference $\mathcal{O}(M_s)$ and is lying yet undiscovered. Furthermore, the application of naturalness arguments [1.8] to the weak scale ($\sim$ 100 GeV), generated in the SM by an unnatural scalar field sector, has suggested [1.9] that[2] $M_s \lesssim \mathcal{O}(\text{TeV})$ and that sparticles should be discovered in forthcoming high energy accelerator experiments probing these energies. The highly successful Standard Model (**SM**) of particle interactions has been minimally extended [1.10] to include all these sparticles and is now called the Minimal Supersymmetric Standard Model (**MSSM**). Our aim in this book is to develop this theme concretely to the extent that its links with experiments, now being conducted or planned, become clear.

There have already been major experimental efforts to search for sparticles, undertaken all through the 1980's and 1990's. The production and decays of sparticles are uniquely characterized by large (more than tens of GeV) missing transverse energy at least in R-parity conserving supersymmetric scenarios where the undetected lightest supersymmetric particle (LSP) carries it away. Early hints at the beginning of the eighties in the UA1 experiment, performed at the S$\bar{\text{P}}$PS machine at CERN, did not materialize into believable signals but were later identified with more mundane processes of the Standard Model. Afterwards, the LEP e^+e^- storage ring at CERN and the Tevatron $\bar{p}p$ collider at Fermilab have been heavily deployed in searches for sparticles, but without any success so far. The same can be said for searches in ep collision experiments performed at HERA. The four LEP experimental groups, ALEPH, DELPHI, L3 and OPAL, as well as the two major experimental collaborations at the TEVATRON, CDF and D∅, have published the strongest experimental lower bounds on the masses of numerous sparticles; they have also established exclusion zones in parameter spaces of various supersymmetric extensions of the Standard Model (Ch.15). The TEVATRON experiments are being extended to RUN II with higher integrated luminosity. Two major collaborations in the Large Hadron Collider (LHC), being built at CERN, ATLAS and CMS, are preparing dedicated experiments which will probe sparticles in the TeV mass range. The exploration of sparticles has been stated as a major goal in proposals for e^+e^- linear colliders with CM energies in the range 500 GeV–1.5 TeV, now being pursued vigorously. There are also nonaccelerator experiments trying to detect the very weakly interacting LSP pervading the universe as cold dark matter (Ch.16). Thus we are in for another decade of intense experimental activity full of exciting possibilities.

One criticism, frequently levelled against the supersymmetry idea sketched above, is the

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clear that the supercharge, being the generator of a spacetime symmetry, must commute with all generators of internal symmetries, e.g. electroweak symmetry. Thus all members of a supermultiplet must have identical internal symmetry properties. Such is not the case between the photon and any of the three known neutrinos.

[2]Such a statement is far from obvious. M_s could very well be of the order of other possible scales in Physics. These include the reduced Planck scale $M_{Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV and the speculated grand unification scale $M_U \sim 2 \times 10^{16}$ GeV. As we shall see later, the above conclusion, reached on the basis of naturalness arguments, is quite a deep statement.

"inelegance" in postulating one new state for every known particle. But extended symmetry considerations did lead physicists in the past to postulate new particles which were subsequently discovered. An example, which we elaborate here, is the extension of nonrelativistic quantum electrodynamics of the electron to cover Lorentz invariance. Such an extension requires the existence of the positron, as can be understood from the standpoint of divergences. We know that the classical self energy of an electron of radius r_e, namely

$$E_{\text{self}}^{cl.} = \frac{3}{5}(e^2/4\pi r_e)$$

in rationalized units, is linearly divergent as $r_e \to 0$. One can guess [1.11] that this calculation becomes unreliable for radii less than the "classical electron radius" $R_0 \sim \frac{3}{5}(e^2/4\pi m_e) \simeq 1.7$ fm for which E_{self}^{cl} equals the rest energy of the electron. In the diagrammatic language of old fashioned perturbation theory [1.12], this contribution is given by Fig.1.1a — the solid, wiggly and dashed lines standing for the electron, the photon and a time slice respectively.

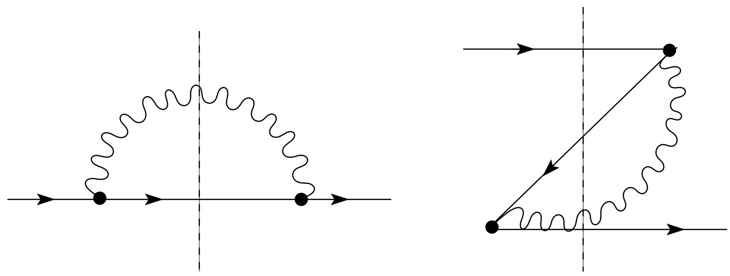

Fig.1.1. Electron self energy diagrams in old fashioned perturbation theory.

Of course, in a relativistic quantum description of the electron, it has been possible to probe r_e far below R_0 (indeed below 10^{-3} fm). This is because the linear divergence, mentioned earlier, gets cured here by the presence of the positron. Owing to the latter, there is also[3] Fig.1-1b now. In this contribution the electron annihilates the positron, created in a pair from a vacuum fluctuation, while the remaining electron goes out. As a result, there is an intrinsic uncertainty in the electron's position of the order of its Compton wavelength $r_e \sim 1/m_e$. The linear divergence cancels in the sum of these two contributions and the self energy becomes [1.13]

$$E_{\text{self}}^{quant.} = \frac{3e^2 m_e}{16\pi^2} \ln(m_e r_e) \ .$$

The expression for E_{self} is still logarithmically divergent as $r_e \to 0$, but this mild divergence[4] can be easily tackled within the renormalization program. One postulates a bare mass of the electron which is also logarithmically divergent, owing to a counterterm inserted in the Lagrangian, so that the renormalized mass $m = m^{\text{bare}} + E_{\text{self}}$ is finite.

[3]This reasoning has been highlighted by H. Murayama, hep-ph/9410285, *ibid*/0002242.

[4]The mildness of this divergence can be seen as follows. Even if r_e is replaced by the smallest length known in Physics, namely the Planck length $\lambda_{Pl} = M_{Pl}^{-1}$, the above expression becomes only 10% of the rest energy of the electron.

The cancellation of the linear divergence is really a consequence of chiral symmetry. The latter refers to an invariance under the transformation of the electron field $\psi_e \to e^{i\varphi\gamma_5}\psi_e$ (with φ being a real parameter), which becomes a symmetry of relativistic electrodynamics in the limit when $m_e \to 0$. Taking this limit in which $E_{\rm self} \to 0$ enhances the symmetry of the theory which then includes chiral invariance. This makes the smallness of the mass of the electron *natural* in the sense of Weinberg, Susskind and 't Hooft [1.7] since the electron is protected by this symmetry from acquiring a huge mass due to self energy corrections. According to those authors, a small parameter in a theory is natural if, and only if, setting it to zero enhances the symmetry of the system. Being a symmetry breaking parameter, its smallness then gets protected against large radiative corrections by the concerned symmetry. Indeed, this criterion can be extended to the theory itself. In the modern Wilsonian view, every Lagrangian density $\mathcal{L}$ should be defined with a cutoff Λ and should be written as $\mathcal{L}(\Lambda)$, keeping renormalization in mind. Λ represents the highest energy scale upto which $\mathcal{L}(\Lambda)$ is the appropriate Lagrangian density and can be perceived as the energy scale where new physics comes into play. Now, a Lagrangian density $\mathcal{L}(\Lambda)$ is "natural" upto and below the energy scale Λ if any set of small parameters $\{\delta_n\}$, appearing in $\mathcal{L}(\Lambda)$, is associated with some approximate symmetry of $\mathcal{L}(\Lambda)$ which is exactly recovered in the $\delta_n \to 0$ limit. In this case quantum corrections – characterized by the scale Λ – will also vanish as $\{\delta_n\} \to 0$ and will remain small for nonvanishing but small $\{\delta_n\}$.

In order to make the above discussion more quantitative, let us write the tree level Lagrangian density of the low energy theory as $\mathcal{L}_{\rm tree} = \sum_n \delta_n O_n$. Here the $\{O_n\}$ are a set of general operators, indexed by nonnegative integers n with δ_n as the corresponding coefficients, while the summation covers all such n that occur. The inclusion of quantum effects, characterized by the scale Λ, then leads to the following general form for the low energy effective Lagrangian density[5]:

$$\mathcal{L}_{\rm eff} = \sum_{n,i} c_{n,i} (\delta_n)^i \Lambda^{[(i-1)(d_n-4)]} O_n \ . \tag{1.1}$$

In (1.1) the summation over nonnegative integers i can, in principle, go from zero to infinity; however, in practice, only a few terms matter. Moreover, d_n is the mass dimension of the operator O_n and $c_{n,i}$ are dimensionless coefficients which can depend only logarithmically on Λ. Since we take the low energy tree level Lagrangian density to be renormalizable, $\delta_n = 0$ whenever O_n has $d_n > 4$. For such operators, the sum over i in (1.1) collapses to the single term with $i = 0$ and (1.1) describes the usual expansion of the low energy effective Lagrangian density with higher dimensional operators suppressed by appropriate powers of Λ^{-1}. These latter terms are irrelevant to any discussion of naturalness, since they disappear when $\Lambda \to \infty$.

Turning to operators with $d_n \leq 4$, we can distinguish two cases. (1) A small coefficient δ_n is "naturally small" only if $c_{n,o} = 0$, since then the corresponding coefficient will remain small in the full effective low energy Lagrangian density as well. As already noted above, in all known examples, a symmetry is needed to ensure that $c_{n,o}$ vanishes to all orders in

[5]There could be additional terms in (1.1) involving powers of products of different δ_n's, but they do not change the discussion in substance.

perturbation theory. Illustrative examples are gauge symmetries "protecting" gauge couplings and chiral symmetries "protecting" fermion masses or Yukawa couplings. (2) On the other hand, if $c_{n,o} \neq 0$, there is no reason to assume δ_n to be small or zero in the tree level low energy Lagrangian density; such a choice would be "unnatural". This may not lead to serious problems for operators with $d_n = 4$. In this case our argument only shows that the natural scale for $\sum_i c_{n,i}(\delta_n)^i$ should be at least $\delta_n + O(\alpha/\pi)$, α being $(4\pi)^{-1}$ times the square of some (typically gauge) coupling strength. Thus even if, at the tree level, δ_n is chosen in magnitude to be much less than $O(\alpha/\pi)$, the coefficient of O_n in $\mathcal{L}_{\text{eff}}$ will naturally become of that order. An example is the quartic Higgs self coupling in the SM which is "naturally" at least $O(10^{-2})$ in $\mathcal{L}_{\text{eff}}$. (In this particular case, however, the experimental lower bound on the mass of the physical Higgs boson leads to a much stronger lower limit). The problem of "naturalness" becomes really severe only for operators with dimension $d_n < 4$. As shown in (1.1), the corresponding coefficients in the low energy effective Lagrangian density *diverge* like Λ^{4-d_n} if $c_{n,0} \neq 0$. The lowest dimensional relevant operator in the SM is the Higgs mass term, which has dimension two. Since the relevant coefficient is not protected by a symmetry, we expect it to receive *quadratically divergent* quantum corrections. In the next section we shall show explicitly that such divergences do indeed occur in the SM. An exorbitant degree of fine tuning between the bare mass and the radiative correction becomes necessary to keep the renormalized Higgs mass near the weak scale. We shall then show in §1.3 how supersymmetry removes this quadratic divergence (i.e. makes the corresponding $c_{n,o}$ vanish) and solves the problem by protecting the renormalized Higgs mass.

Let us return to the question of the mass of the electron. On dimensional grounds, one might naively expect m_e to grow like Λ after loop corrections. As already noted, it does not do so. The fact that it grows instead as $\ln(\Lambda/m_e)$ is because of chiral symmetry which makes m_e a "naturally small" parameter of the theory. Notice that chiral symmetry can be formulated only within a relativistic framework where a positron is obligatory. The existence of a new particle here is therefore linked to the greater convergence of the theory at short distances (or high energies) and is ultimately related to a symmetry. A similar motivation can be given for supersymmetry. Supersymmetry, or more specifically the existence of sparticle superpartners with masses near the weak scale, cures the problem of quadratic divergences through cancellations between fermionic and bosonic loops. This can be understood on the basis of symmetries as follows. Supersymmetry links boson masses to fermion masses, which are "protected" by chiral symmetry[6]. The weak scale M_W can then be naturally chosen to be many orders of magnitude below the Planck scale M_{Pl} or the hypothetical scale M_U of grand unification and kept protected. Operatively, the nonrenormalization theorem of supersymmetry (cf. Ch.6) provides this protection. Thus supersymmetry holds the key to the stability and naturalness of the weak scale vis-à-vis M_U or M_{Pl}. This really is the *raison d'être* for the extension of the phenomenologically successful Standard Model of particle interactions to the Minimal Supersymmetric Standard Model to which a large part of this book will be devoted. In the next sections we shall illustrate this main argument through explicit calculations at the one loop level.

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[6]We note in passing that supersymmetry also allows one to "naturally" choose arbitrarily small, even vanishing, scalar self couplings, by relating them either to gauge or to Yukawa couplings.

1.2 Quadratic Divergence and Unnaturalness

We illustrate the problem of the quadratic divergence in the Higgs sector of the SM through an explicit calculation. The example studied is that of the two point function (inverse propagator) of the Higgs scalar at vanishing external momentum, computed at the one loop level. This quantity is roughly $-i$ times the squared scalar mass appearing in the Lagrangian. This particular object has been chosen since its calculation is simple and yet suffices to highlight the problem. Let ϕ be the SM neutral Higgs field with $v = \sqrt{(1/\sqrt{2}G_F)} \simeq 246$ GeV defined to be $\sqrt{2}\langle\phi\rangle$ so that the shifted physical field h is given by

$$\Re e\ \phi = \frac{1}{\sqrt{2}}(h+v)\ . \tag{1.2}$$

Take f to be a generic matter fermion field (of one species) with a Yukawa coupling to ϕ via the term (we follow the conventions of Bjorken and Drell [1.14])

$$\begin{aligned} \mathcal{L}_{\bar{f}f\phi} &= -\lambda_f \bar{f}_L f_R \phi + \text{h.c.} \\ &= -\frac{\lambda_f}{\sqrt{2}} h\bar{f}f - \frac{\lambda_f v}{\sqrt{2}}\bar{f}f, \end{aligned} \tag{1.3}$$

where $f_{L,R}$ are left, right chiral components of f. Thus, on account of spontaneous symmetry breaking, the fermion develops a tree level mass $m_f = \lambda_f v/\sqrt{2}$.

Let us now proceed to compute the one loop f-$\bar{f}$ contribution to the scalar two point function, as illustrated in Fig.1.2. We have

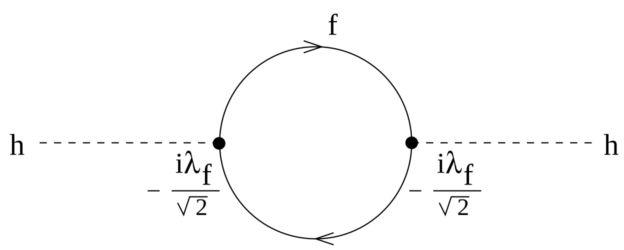

Fig.1.2. Fermionic loop contribution to the scalar two point function.

$$\begin{aligned} \Pi^f_{hh}(0) &= (-1)\int \frac{d^4k}{(2\pi)^4}\text{Tr}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f} \\ &= -2\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2+m_f^2}{(k^2-m_f^2)^2} \\ &= -2\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_f^2}+\frac{2m_f^2}{(k^2-m_f^2)^2}\right]. \end{aligned} \tag{1.4}$$

The first term in the final RHS of (1.4) is quadratically divergent and is moreover independent of the scalar mass m_S. First of all, this divergence is very severe. Suppose the integral is cut off by a Λ parameter which is then set equal to the Planck mass $M_{Pl} \simeq 2.4 \times 10^{18}$ GeV, the highest scale known in physics. Then the one loop correction to m_S^2 would be 30 orders of magnitude larger than m_S^2 itself since m_S is restricted [1.15], by the requirement of perturbative unitarity in the amplitude $W^+W^- \to W^+W^-$, to be $\leq \mathcal{O}$ (1 TeV). Furthermore, the correction (1.4) being independent of m_S is an indication of the fact that m_S is an *unnatural* parameter in the SM. Setting $m_S = 0$ does not increase the symmetry of that theory. That means that there exists no symmetry in the SM which protects the Higgs mass. For simplicity, we have dealt with only the fermion antifermion loop contribution to the Higgs self energy and ignored the gauge boson loop and (self-coupled) Higgs loop contributions. Each of the latter contains a quadratic divergence and has the same problem as above[7].

Of course, one could simply renormalize such quadratic divergences away in the same way that logarithmic divergences are disposed of. But the legacy of the severity of the quadratic divergence would still remain. Thus the residual finite correction in (1.4) would be or order $m_f^2\lambda_f^2/(8\pi)$. Such a correction would be managably small for a standard model fermion like the top quark. However, the SM is expected to give way to a more fundamental theory, e.g. a Grand Unified one [1.16] unifying all forces in it, at a high energy scale $M_U \sim 10^{16}$ GeV. In this case the leading contribution will come from a fermion-antifermion pair which can couple to h and have the highest mass, with m_f expected to be $O(M_U)$, causing the loop correction to the scalar mass squared, i.e. δm_S^2, to be $O(M_U^2)$. One would have to do an unnatural amount of fine tuning (1 in 10^{26}) between the bare scalar mass squared $m_{S,0}^2$ and the renormalization δm_S^2 in order to keep the renormalized mass squared

$$m_S^2 = m_{S,0}^2 + \delta m_S^2 \tag{1.5}$$

to less than a $(\text{TeV})^2$. The argument can be amplified through the consideration of quartic scalar couplings.

To make the above discussion more concrete, let us take the Grand Unifying group to be [1.16] $SU(5)$ with Σ $(H, \bar{H})$ representing Higgs fields in the **24** $(\mathbf{5},\bar{\mathbf{5}})$ representation. While the mass of Σ is expected to be $O(M_U)$, that of the weak doublet parts of H and $\bar{H}$ should be of $O(M_W)$. At the tree level, the unifying scale M_U is generated via $M_U = g_U\langle\Sigma\rangle$, where g_U is the unified gauge coupling strength. The one loop effective action contains an interaction term $\lambda\bar{H}\Sigma^2 H$, from the graph of Fig.1.3, where the wiggly lines represent gauge bosons of the $SU(5)$ theory. If we take momentum scales at the two external H and Σ lines to be of order M_W and M_U respectively, we shall have

$$\lambda(M_W^2) \sim \lambda(M_U^2) + \frac{g_U^4}{16\pi^2}\ln\frac{M_U^2}{M_W^2} \ . \tag{1.6}$$

The induced mass of all components of H is $\lambda\langle\Sigma\rangle^2$. Even if λ is taken to be very small at the

[7]In principle, one can cancel the total one loop quadratic divergences by explicitly cancelling bosonic and fermionic contributions through some postulated relation between the boson and fermion masses. However, because such a cancellation is 'accidential', rather than being enforced by a symmetry, it will not work in higher loop order.

tree level, $\lambda(M_W^2)$ becomes $O(g_U^4)$ after the one loop correction. Without an extreme fine

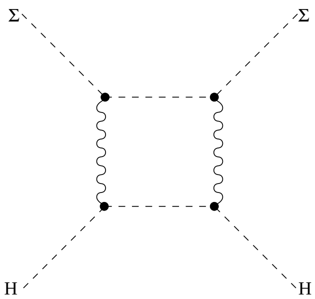

Fig.1.3. One loop graph for the $\bar{H}\Sigma^2 H$ vertex in an $SU(5)$ Grand Unified theory.

tuning of $\lambda(M_W^2)$, the induced mass of the weak scale Higgs doublet will then be at an unacceptably large level. Moreover, the fine tuning would be very different in different orders of perturbation theory. This, basically, is the gauge hierarchy problem arising out of the radiative instability of scalar masses: the latter like to be close to the highest mass scale in the theory.

1.3 Naturalness, Nonrenormalization, Supersymmetry

Loops induced by other scalar fields, contributing to the Higgs two point function, can also be considered. Let us construct a toy model [1.17] by introducing to the system of §1.2 two additional complex scalar ("sfermion") fields $\tilde{f}_L$, $\tilde{f}_R$ with the following coupling to the Higgs field:

$$\begin{aligned}\mathcal{L}_{\tilde{f}\tilde{f}\phi} &= \tilde{\lambda}_f|\phi|^2(|\tilde{f}_L|^2+|\tilde{f}_R|^2)+(\lambda_f A_f\phi\tilde{f}_L\tilde{f}_R^\star+\text{h.c.})\\ &= \frac{1}{2}\tilde{\lambda}_f h^2(|\tilde{f}_L|^2+|\tilde{f}_R|^2)+v\tilde{\lambda}_f h(|\tilde{f}_L|^2+|\tilde{f}_R|^2)\\ &\quad+\frac{h}{\sqrt{2}}(\lambda_f A_f\tilde{f}_L\tilde{f}_R^\star+\text{h.c.})+\cdots. \end{aligned}\tag{1.7}$$

In the second step of (1.7), we have rewritten, by means of (1.2), the interaction in terms of the h-field and have displayed only the h-dependent terms. The coefficient of the last RHS term is, in fact, arbitrary; the factor λ_f, multiplying the new unknown coupling strength A_f, has been put in only by convention. (1.7) makes the following additional contribution to the two point function via the loops of Fig.1.4:

$$\begin{aligned}\Pi_{hh}^{\tilde{f}}(0) &= -\tilde{\lambda}_f\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2-m_{\tilde{f}_L}^2}+\frac{1}{k^2-m_{\tilde{f}_R}^2}\right)+\\ &\quad(\tilde{\lambda}_f v)^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2-m_{\tilde{f}_L}^2)^2}+\frac{1}{(k^2-m_{\tilde{f}_R}^2)^2}\right]\\ &\quad+|\lambda_f A_f|^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_{\tilde{f}_L}^2}\frac{1}{k^2-m_{\tilde{f}_R}^2}\ . \end{aligned}\tag{1.8}$$

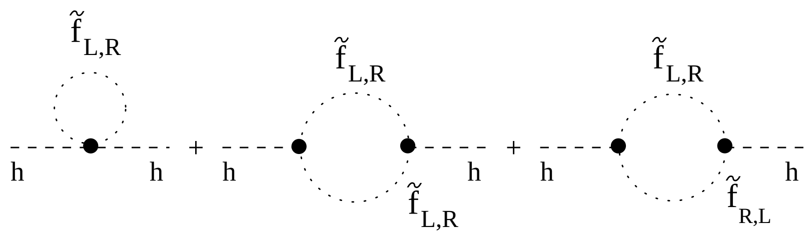

Fig.1.4. Sfermion loop contributions to Higgs self energy

Only the first line in (1.8), which comes from the leftmost diagram of Fig.1.4, contains a quadratic divergence. This, however, can be cancelled with that in the fermionic contribution (1.4), i.e. $\Pi^f_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0)$ becomes free of any quadratic divergence, provided the following coupling constant equality is obeyed:

$$\tilde{\lambda}_f = -\lambda_f^2. \tag{1.9}$$

Note that the inequality $\tilde{\lambda}_f < 0$ is required in (1.7) to keep the Hamiltonian bounded from below. Another important point to note is that the above cancellation of the quadratic divergence is independent of the masses $m_{\tilde{f}_L}$, $m_{\tilde{f}_R}$ or the coupling strength A_f.

Now that the quadratic divergence has disappeared from $\Pi^f_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0)$, the remaining logarithmic ones can be cancelled by contributions from logarithmically infinite counterterms introduced in the Lagrangian density as part of the renormalization procedure. In the $\overline{\mathrm{MS}}$ renormalization scheme[8] [1.18], one can replace the logarithmic divergence in our loop integrals by the logarithm of the square of the **renormalization scale** μ. Utilizing the B_0-function of Passarino and Veltman, we can then make [1.19] the following types of replacements:

$$\int \frac{d^4k}{i\pi^2}\left(\frac{1}{k^2-m_1^2} - \frac{1}{k^2-m_2^2}\right) \equiv (m_1^2-m_2^2)B_0(0,m_1^2,m_2^2) m_1^2\left(1-\ln\frac{m_1^2}{\mu^2}\right) - m_2^2\left(1-\ln\frac{m_2^2}{\mu^2}\right), \tag{1.10a}$$

$$\int \frac{d^4k}{i\pi^2}\frac{1}{(k^2-m^2)^2} \to -\ln\frac{m^2}{\mu^2}\ . \tag{1.10b}$$

[8] A caveat is in order here. The $\overline{\mathrm{MS}}$ scheme was originally proposed with dimensional regularization which has a problem in supersymmetry since the numbers of bosons and fermions do not match as one goes off four dimensions. For supersymmetric loop computations, one needs to adopt the modified dimensional reduction or $\overline{\mathrm{DR}}$ scheme where the momentum integrals are evaluated in continued dimensions and the subtraction is performed as in $\overline{\mathrm{MS}}$, but the Dirac algebra in the numerator is done strictly in four dimensions. A more extensive discussion of the $\overline{\mathrm{DR}}$ scheme will come in Ch.6.

The consequent expression for the sum of (1.4) and (1.8) can be simplified by choosing

$$m_{\tilde{f}_L} = m_{\tilde{f}_R} = m_{\tilde{f}} \ . \tag{1.11}$$

The choices (1.9) and (1.11) as well as the substitutions (1.10) lead to the result:

$$\begin{aligned}\Pi^f_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0) &= i\frac{\lambda_f^2}{16\pi^2}\Bigg[-2m_f^2\left(1 - \ln\frac{m_f^2}{\mu^2}\right) + 4m_f^2 \ln\frac{m_f^2}{\mu^2} \\ &\quad +2m_{\tilde{f}}^2\left(1 - \ln\frac{m_{\tilde{f}}^2}{\mu^2}\right) - 4m_f^2 \ln\frac{m_{\tilde{f}}^2}{\mu^2} - |A_f|^2 \ln\frac{m_{\tilde{f}}^2}{\mu^2}\Bigg] . \end{aligned} \tag{1.12}$$

Thus, if along with (1.11), we also require the relations

$$m_f = m_{\tilde{f}} \ , \tag{1.13a}$$

$$A_{\tilde{f}} = 0 \ , \tag{1.13b}$$

we will have

$$\Pi^f_{hh}(0) + \Pi^{\tilde{f}}_{hh}(0) = 0 \ . \tag{1.14}$$

Eq. (1.14) can be restated as follows. If the fermion Yukawa coupling strength squared equals the quartic coupling between the Higgs and the scalars $\tilde{f}_{L,R}$, if the masses of the fermion f and of the scalars $\tilde{f}_{L,R}$ are identical and if the A_f parameter is zero, the *entire* one loop renormalization of the Higgs self energy $\Pi_{hh}(0)$ vanishes.

We are now ready to give a supersymmetric interpretation of the above. In an exactly supersymmetric theory, the two scalars $\tilde{f}_{L,R}$ are the left and right superpartners (sfermions) of the fermion f. Moreover, the coupling strength equality (1.9), the mass equalities (1.11) and (1.13a) and the required null value of the (supersymmetry breaking) parameter A_f (1.13b) are all ensured by supersymmetry. Indeed, with these conditions, the vanishing of the renormalization of the Higgs self energy holds in all perturbation orders as a consequence of the **nonrenormalization theorem** (cf. §6.7) valid in supersymmetric theories. This is the essence of naturalness due to supersymmetry. The naturalness aspect is also made clear by the introduction of a certain kind of small supersymmetry breaking, namely that the breaking is confined to the masses m_f and $m_{\tilde{f}}$ being different and to A_f being nonzero but does *not* change the coupling equality (1.9). These are specific instances of parameters typical of softly broken supersymmetry, i.e. as coefficients of supersymmetry breaking operators of mass dimension less than four in the Hamiltonian. Suppose we characterize this supersymmetry breaking in terms of two small parameters A_f and δ, with

$$\delta^2 = m_{\tilde{f}}^2 - m_f^2 \ . \tag{1.15}$$

(Here we have chosen to maintain[9] (1.11) while relaxing (1.13) via a small mass splitting.) Thus δ characterizes the mass splitting within the f-$\tilde{f}$ supermultiplet. With the assumption

[9] In a more general discussion, one could introduce another supersymmetry breaking mass parameter, splitting m_{f_L} and m_{f_R}, which would then enter the RHS of (1.16). But the basic conclusion would still be the same.

that $|\delta|,\ |A_f| \ll m_f$, we can approximate $\ln(m_{\tilde{f}}^2/\mu^2) \simeq \ln(m_f^2/\mu^2) + \delta^2/m_f^2$ and rewrite (1.12) as

$$\Pi_{hh}^{f}(0) + \Pi_{hh}^{\tilde{f}}(0) \simeq -i\frac{\lambda_f^2}{16\pi^2}\left[4\delta^2 + (2\delta^2 + |A_f|^2)\ln\frac{m_f^2}{\mu^2}\right] + \mathcal{O}(\delta^4, |A_f|^2\delta^2)\ . \tag{1.16}$$

Hence the one loop renormalization of the Higgs self energy is linearly proportional[10] to the small supersymmetry breaking parameters δ^2 and $|A_f|^2$, restricting the correction to one of modest magnitude, though m_f may be quite large.

Thus the introduction of the superpartners $\tilde{f}_{L,R}$ with the interactions of (1.7) has served two purposes: (1) the quadratic divergence in the scalar self energy is cancelled; (2) the scalar mass is shielded from large loop corrections involving heavy particles *so long as* the mass splitting between the heavy fermion and boson superpartners is itself of the order of the scalar mass. This then is a toy model example of how naturalness is restored by supersymmetry in the scalar sector of the SM. We have confined ourselves here to discussing fermion and sfermion loop contributions to the Higgs self energy. But the same conclusions follow *mutatis mutandis* if loop contributions from gauge bosons and their superpartners are combined or Higgs bosons and their superpartners are added together.

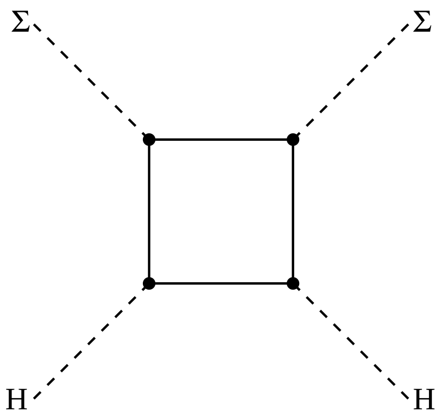

Fig.1.5. Additional one loop graph for the $\bar{H}\Sigma^2 H$ vertex in the supersymmetric $SU(5)$ theory.

Returning to the discussion, given at the end of §1.2, of the supersymmetric $SU(5)$ Grand Unified Theory, there will now be a new one loop diagram contributing to the $\bar{H}\Sigma^2 H$ vertex in addition to Fig.1.3. This is shown in Fig.1.5 where the solid lines represent appropriate fermionic superpartners of $SU(5)$ gauge bosons and Σ-fields. The two graphs cancel in the leading terms and the quartic coupling strengths λ at those two different scales are now related by

$$\lambda(M_W^2) \sim \lambda(M_U^2)\ln\frac{M_W^2}{M_U^2}\ .$$

[10]The persistence of the renormalization scale μ in (1.16) need not worry us since the LHS is not a physically measurable quantity.

Hence the previous hierarchical instability does not materialize and the problem of the radiative instability of the gauge hierarchy is solved[11] [1.20] as a consequence of the nonrenormalization theorem of supersymmetry (cf. Ch.6). The earlier additive term in the RHS of (1.6), proportional to g_U^4, has got cancelled. The multiplicative logarithmic factor comes in the following way. The scalar quartic coupling is a coupling in the superpotential (cf. Ch.5). Since the latter is not renormalized (cf. Ch.6), the renormalization of such a coupling has to be balanced by the wavefunction renormalizations of the multiplying superfields. Owing to dimensional reasons, the latter can at most have a logarithmic dependence on the two mass scales. The same must then be true of λ. These issues will become much clearer after the discussion in §6.6.

In our introduction and overview, as given in this chapter, we have tried to provide a motivation for softly broken supersymmetry other than just its mathematical beauty. It is needed as a stabilizer of the weak scale M_W. The latter is radiatively unstable in the Standard Model; the instability of the Higgs mass m_h accrues via the Higgs VEV to M_W. Stabilization within the Standard Model can be achieved only by fine tuning. As a result, despite its logical consistency and impressive experimental support, the Standard Model is an unnatural theory. Supersymmetry with soft breaking makes the theory radiatively stable and natural, provided the sparticles are not much heavier than a few TeV.

[11] This is a far cry, however, from explaining the *origin* of the hierarchy, namely the ratio of the magnitudes of the weak and the unification scales within a supersymmetric grand unified framework.

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Chapter 8

BASIC STRUCTURE OF THE MSSM

8.1 Brief Review of the Standard Model

We discuss in this chapter the minimal extension of the Standard Model (SM) [8.1] that is needed to incorporate softly broken N=1 global supersymmetry in the latter. This is called the *Minimal Supersymmetric Standard Model* (MSSM) [8.2]. The prefix "minimal" is used to distinguish from nonminimal extensions which we shall come to in Ch.14. In order to supersymmetrize the SM, we need (cf. Chs. 1,3) to introduce for every particle a superpartner. The latter differs from the former in spin by half and in mass generally by some positive amount $O(M_s)$, but with all other internal quantum numbers kept identical. In the SM all matter fields (pertaining to quarks and leptons) are spin half fermionic fields while gauge bosons have spin one. The superpartners of the former cannot have spin one. Since they are supposed to be matter fields, they are not gauge bosons while the only known consistent relativistic field theories of spin one particles are those of gauge bosons. Thus superpartners of matter fermions are taken to be spin zero scalars and are described, along with the latter, by chiral superfields. These scalars are called **sfermions** and they can be classfied into scalar leptons or **sleptons** and scalar quarks or **squarks**. Similarly, since even at the classical level, the only consistent interacting field theory of spin 3/2 particles has to include [8.3] gravity, the superpartner fields of the SM gauge bosons are chosen to have spin 1/2; they are called **gauginos**. Gauge bosons and gauginos are described by vector superfields. Gauginos can be further classified into the strongly interacting **gluinos** as well as the electroweak **zino** (corresponding to the Z boson) and **winos** (corresponding to the W bosons). Spin zero Higgs bosons are described, along with their spin half superpartners (called **higgsinos**), by chiral superfields. We shall later see that electroweak symmetry breaking mixes the EW gauginos with the higgsinos making physical **charginos** and **neutralinos**.

To begin with, let us set up the notation by briefly summarizing some basic ingredients of the SM itself. The gauge symmetry group is $SU(3)_C \times SU(2)_L \times U(1)_Y$, with subscripts C, L, Y referring respectively to color, left chirality and weak hypercharge. All matter (quark and lepton) fields are fermion fields with left chiral ones transforming as doublets and right chiral ones as singlets of $SU(2)_L$. The hypercharge Y_f of each fermion field is related to its electromagnetic charge Q_f and the third component of its left chiral weak isospin T^f_{3L} by

$$Q_f = T^f_{3L} + \frac{Y_f}{2} \ . \tag{8.1}$$

The electroweak gauge transformation properties of the left chiral, right chiral fermion fields $f_L = \frac{1}{2}(1-\gamma_5)f$, $f_R = \frac{1}{2}(1+\gamma_5)f$ are:

$$f_L(x) \to e^{-ig_Y\alpha_Y(x)Y/2} \ e^{-ig_2\vec{\alpha}_2(x)\cdot\vec{\tau}/2} f_L(x) \ , \tag{8.2a}$$

$$f_R(x) \to e^{-ig_Y\alpha_Y(x)Y/2} f_R(x) \ , \tag{8.2b}$$

where g_Y, $\alpha_Y(x)$ and g_2, $\vec{\alpha}_2(x)$ are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, functions respectively. Moreover, Y is the hypercharge operator and the Pauli matrices $\vec{\tau}$ act in the weak isospin doublet representation space.

Fields for the three generations (generation index $i = 1, 2, 3$) of leptons and quarks, along with the dimension of the corresponding $SU(2)_L$ representation and the Y quantum number are listed below.

$$\begin{aligned}
\ell_{iL} &= \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L , \text{ i.e. } \ell_{1L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \ \ell_{2L} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \ \ell_{3L} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \ : \ (\mathbf{2}, -1) \, , \\
e_{1R} &= e^-_R, \ e_{2R} = \mu^-_R, \ e_{3R} = \tau^-_R \ : \ (\mathbf{1}, -2), \\
q_{iL} &= \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L , \text{ i.e. } q_{1L} = \begin{pmatrix} u \\ d \end{pmatrix}_L , \ q_{2L} = \begin{pmatrix} c \\ s \end{pmatrix}_L , \ q_{3L} = \begin{pmatrix} t \\ b \end{pmatrix}_L \ : \ \left(\mathbf{2}, \frac{1}{3}\right), \\
u_{1R} &= u_R, \ u_{2R} = c_R, \ u_{3R} = t_R \ : \ \left(\mathbf{1}, \frac{4}{3}\right), \\
d_{1R} &= d_R, \ d_{2R} = s_R, \ d_{3R} = b_R \ : \ \left(\mathbf{1}, -\frac{2}{3}\right).
\end{aligned} \tag{8.3}$$

The color gauge transformations of quark (q) and lepton (ℓ) fields are:

$$q_{L,R}(x) \to e^{-ig_s\alpha^a_s(x)\lambda^a/2} q_{L,R}(x), \ \ \ell_{L,R}(x) \to \ell_{L,R}(x) \ , \tag{8.4}$$

with g_s, α^a_s being the $SU(3)_C$ gauge coupling, functions and λ^a being the Gell-Mann $SU(3)$ lambda matrices acting in the triplet ($\mathbf{3}$) representation space. The quark fields of (8.4) transform as color triplets ($\mathbf{3}$) of $SU(3)_C$ whereas the lepton fields of (8.3) are color singlets. The $SU(2)_L$ singlet right chiral fermion fields can be converted into left chiral ones by charge conjugation. For instance, $u^C_R = (u^C)_L$ is such a field with $T^L_f = 0$, $Y = -\frac{4}{3}$ and transforming as a color antitriplet ($\bar{\mathbf{3}}$). Again, $e^-_R = (e^+_L)^C$ and so on.

The gauge fields g^a_μ ($a = 1, \cdots, 8$), $\vec{W}_\mu$ and B_μ transform according to the adjoint representations of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ respectively. The eight gluons g^a are always massless while the three $SU(2)_L$ gauge bosons $W_{1,2,3}$ and the one $U(1)_Y$ gauge boson B are massless only in the limit of exact electroweak symmetry. At the weak scale, the $SU(2)_L \times U(1)_Y$ electroweak (EW) symmetry gets spontaneously broken to $U(1)_{em}$. The unbroken symmetry group at energies lower than the weak scale is thus $SU(3)_C \times U(1)_{em}$. This spontaneous symmetry breakdown is driven by an $SU(2)_L$ doublet of scalar Higgs fields $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ with $Y = 1$ and is signaled by a real nonzero vacuum expectation value (VEV) for this field, arising from the minimization of the Higgs potential term $V(\phi)$ and given by

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{8.5}$$

While the photon γ remains massless, the weak bosons $W^\pm$ and Z acquire masses though the VEV v in (8.5). The latter is related to the masses $M_{W,Z}$ and the couplings $g_{2,Y}$ as well as to the Fermi constant G_F by

$$M_W = \frac{1}{2} g_2 v, \ M_Z = \frac{1}{2}\sqrt{g_Y^2 + g_2^2}\, v, \ v = \left(\frac{1}{\sqrt{2} G_F}\right)^{1/2} \simeq 246 \text{ GeV}. \tag{8.6}$$

The fields $W_\mu^\pm$, Z_μ and A_μ, which are mass eigenstates, are given respectively in terms of the fields $\vec{W}_\mu$ and B_μ, introduced earlier, as

$$W^{\mu,\pm} = \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu) \ , \tag{8.7a}$$

$$\begin{aligned} Z^\mu &= \frac{g_2}{\sqrt{g_Y^2 + g_2^2}} W_3^\mu - \frac{g_Y}{\sqrt{g_Y^2 + g_2^2}} B^\mu \\ &= -\sin\theta_W B^\mu + \cos\theta_W W_3^\mu \ , \end{aligned} \tag{8.7b}$$

$$A^\mu = \cos\theta_W B^\mu + \sin\theta_W W_3^\mu \ , \tag{8.7c}$$

with

$$e = g_2 \sin\theta_W = g_Y \cos\theta_W \ . \tag{8.8}$$

The nonzero VEV, introduced in (8.5), is also responsible in the SM for generating fermion masses through Yukawa interaction terms characterized by coupling strengths f and generation indices[1] i, j. For the latter, we can write:

$$\mathcal{L}_Y^1 = -f_{ij}^{e\star} \overline{\ell_{iL}} \phi e_{jR} - f_{ij}^{d\star} \overline{q_{iL}} \phi d_{jR} + \text{h.c.} \tag{8.9}$$

in case of "down type" right chiral fermions (e_{jR}, d_{jR}) and

$$\mathcal{L}_Y^2 = -f_{ij}^{u\star} \overline{q_{iL}} \phi^C u_{jR} + \text{h.c.} \tag{8.10}$$

for "up type" right chiral fermions u_{jR}. The complex conjugate $f^\star$ has been chosen here for convenience in later supersymmetric generalization (cf. 8.33) and i, j are summed on repetition. Furthermore,

$$\phi^C = i\tau_2 \phi^\star = \begin{pmatrix} \phi^{0\star} \\ -\phi^- \end{pmatrix}$$

[1]We shall not use here the type subspace formalism, introduced in Ch.5, with both superscripts and subscripts. Thus all generation indices will henceforth be subscripts.

is the "charge conjugated" Higgs doublet field. Note that leptonic couplings are absent from (8.10) since there is no ν_R. The substitution of (8.5) into (8.9) and (8.10) leads to the fermion mass terms. Suppose, for a set of Dirac fermions ψ_i, we define the mass matrix m_{ij} by writing the fermion mass term in the Lagrangian density as

$$\mathcal{L}_{FMT} = -\left(\overline{\psi_{iL}} m_{ij} \psi_{Rj} + \text{h.c.}\right) .$$

Then one can write the charged lepton, down type quark, up type quark mass matrices [8.1] in generation space as

$$(\mathbf{m}_e)_{ij} = \frac{1}{\sqrt{2}} f^{e\star}_{ij} v = m_{e_i}\delta_{ij},\ (\mathbf{m}_d)_{ij} = \frac{1}{\sqrt{2}} f^{d\star}_{ij} v,\ (\mathbf{m}_u)_{ij} = \frac{1}{\sqrt{2}} f^{u\star}_{ij} v\ , \tag{8.11}$$

the first being brought into a real diagonal form without loss of generality on account of the assumed masslessness[2] of the neutrinos. However, the up type and down type quark mass matrices do not have this advantage and can be put into real diagonal forms only by biunitary transformations. Thus if the mass eigenstate left, right u- and d-quark fields are unitarily transformed to the corresponding flavor eigenstate ones by $\mathbf{U}^{u_L}$, $\mathbf{U}^{u_R}$ and $\mathbf{U}^{d_L}$, $\mathbf{U}^{d_R}$, the quark mass matrices transform as

$$(\mathbf{U}^{u_L^\dagger} \mathbf{m}_u \mathbf{U}^{u_R})_{ij} = [\mathbf{m}_u^{(D)}]_{ij} \equiv m_{u_i}\delta_{ij}\ , \tag{8.12a}$$

$$(\mathbf{U}^{d_L^\dagger} \mathbf{m}_d \mathbf{U}^{d_R})_{ij} = [\mathbf{m}_d^{(D)}]_{ij} \equiv m_{d_i}\delta_{ij}\ . \tag{8.12b}$$

In (8.12) $\mathbf{m}_u^{(D)}$ and $\mathbf{m}_d^{(D)}$ are the physical real diagonal mass matrices for up and down type quarks respectively.

Baryon number B and lepton type numbers $L_{e,\mu,\tau}$ (and hence lepton number $L \equiv L_e + L_\mu + L_\tau$) are conserved in the SM. These 'accidental' global symmetries are a consequence of the particle content and the gauge group. As will be discussed in more detail later, the situation is quite different for the MSSM. The latter can accommodate several types of renormalizable interactions which violate some or all of these symmetries. For the time being, let us nonetheless restrict ourselves to a version of the MSSM where these symmetries are conserved by the assumption of R-parity invariance (cf. §4.5).

8.2 Superfields of the MSSM

We now proceed to introduce a chiral superfield for every chiral fermion of the SM. Apart from these chiral fermions and auxiliary fields, such superfields will contain new scalar fields. For the first generation, these scalar fields can be enumerated as

$$\tilde{\ell}_{1L} = \begin{pmatrix} \tilde{\nu} \\ \tilde{e}^- \end{pmatrix}_L,\ \tilde{e}_{1R} = \tilde{e}_R,\ \tilde{q}_{1L} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L,\ \tilde{u}_{1R} = \tilde{u}_R,\ \tilde{d}_{1R} = \tilde{d}_R\ . \tag{8.13}$$

[2]An important example of a term that violates the lepton number symmetry is a Majorana neutrino mass. Since neutrino masses can be introduced without significantly altering the specific supersymmetric aspects of particle phenomenology, we postpone a detailed discussion of this point to Ch.14.

Here $\tilde{\ell}_{1L}$ are called left sleptons (more specifically, left selectron and sneutrino) while $\tilde{e}_R$ is called the right selectron. Let us denote by L_1 (Q_1) and $\bar{E}_1$ ($\bar{U}_1$, $\bar{D}_1$) the left chiral lepton (quark) doublet and antilepton (antiquark) singlet chiral superfields respectively. Thus, for the first generation of leptons and sleptons, we can take the superfields

$$L_1 = \begin{pmatrix} L_{\nu_e} \\ L_e \end{pmatrix}, \ \bar{E}_1 \ . \tag{8.14}$$

Contained in these are the fields ℓ_{1L}, $\tilde{\ell}_{1L}$, ${e_{1R}}^C = e_R^C$ and $\tilde{e}_{1R}^\star = \tilde{e}_R^\star$ corresponding[3] respectively to $\psi_{\ell 1L}$, $\phi_{\ell 1+}$, ${\psi_{eR}}^C$ and ϕ_{e-} in the notation of Ch.5. There is no singlet neutrino superfield since the SM does not contain any left chiral antineutrino. Similarly, the first quark (and squark) generation is represented by the superfields

$$Q_1 = \begin{pmatrix} Q_u \\ Q_d \end{pmatrix}; \ \bar{U}_1, \ \bar{D}_1 \ . \tag{8.15}$$

These contain the fields q_{1L}, $\tilde{q}_{1L}$, ${u_{1R}}^C = {u_R}^C$, ${d_{1R}}^C = {d_R}^C$, $\tilde{u}_{1R}^\star = \tilde{u}_R^\star$ and $\tilde{d}_{1R}^\star = \tilde{d}_R^\star$ corresponding to ψ_{q1L}, ϕ_{q1+}, ${\psi_{uR}}^C$, ${\psi_{dR}}^C$, ϕ_{u-} and ϕ_{d-} respectively.

The above procedure can be repeated for the second and third generations. Thus we denote matter superfields corresponding to these generations by L_i, $\bar{E}_i$, Q_i, $\bar{U}_i$ and $\bar{D}_i$ with $i = 2, 3$. So we have

$$L_2 = \begin{pmatrix} L_{\nu_\mu} \\ L_\mu \end{pmatrix}, \ \bar{E}_2, \ Q_2 = \begin{pmatrix} Q_c \\ Q_s \end{pmatrix}, \ \bar{U}_2, \ \bar{D}_2 \ , \tag{8.16}$$

respectively containing the fields ℓ_{2L}, $\tilde{\ell}_{2L}$, ${e_{2R}}^C = {\mu_R}^C$, $\tilde{e}_{2R}^\star = \tilde{\mu}_R^\star$, q_{2L}, $\tilde{q}_{2L}$, ${u_{2R}}^C = {c_R}^C$, $\tilde{u}_{2R}^\star = \tilde{c}_R^\star$, ${d_{2R}}^C = {s_R}^C$, $\tilde{d}_{2R}^\star = \tilde{s}_R^\star$. Furthermore, there are

$$L_3 = \begin{pmatrix} L_{\nu_\tau} \\ L_\tau \end{pmatrix}, \ \bar{E}_3, \ Q_3 = \begin{pmatrix} Q_t \\ Q_b \end{pmatrix}, \bar{U}_3, \bar{D}_3 \ , \tag{8.17}$$

respectively containing the fields ℓ_{3L}, $\tilde{\ell}_{3L}$, ${e_{3R}}^C = {\tau_R}^C$, $\tilde{e}_{3R}^\star = \tilde{\tau}_R^\star$, q_{3L}, $\tilde{q}_{3L}$, ${u_{3R}}^C = {t_R}^C$, $\tilde{u}_{3R}^\star = \tilde{t}_R^\star$, ${d_{3R}}^C = {b_R}^C$, $\tilde{d}_{3R}^\star = \tilde{b}_R^\star$.

Supersymmetry, by itself, does not provide any clear answer to the generation or family problem and, in the MSSM, one simply replicates the superfields thrice for the three generations. Within each family, however, the counting of fermionic and bosonic degrees of freedom must match for every supermultiplet, as described by a chiral superfield. Corresponding to a massive Dirac fermion field, f_u say, with four on-shell degrees of freedom (two spin states for the particle and two for the antiparticle, as embodied in the complex chiral fields f_{uL} and f_{uR}), there are two corresponding complex scalar fields $\tilde{f}_{uL}$ and $\tilde{f}_{uR}$. Each of the latter, together with its complex conjugate, stands for particle and antiparticle fields; thus the components match. Note further that $\tilde{f}_{uL}$ and $\tilde{f}_{uR}$ have different $SU(2)_L \times U(1)_Y$ quantum numbers just as f_{uL} and f_{uR} do. Another point needs to be emphasized here. Since the superpotential $\mathcal{W}$ can contain only left chiral superfields, one is obliged to use the left chiral

[3]Cf. §5.6, except that we have dropped the $+, -$ subscripts and used overbars for singlets.

charge conjugates of the $SU(2)_L$ singlet right chiral fermion fields, i.e. ${f_{uR}}^C = (f_u^C)_L$ etc., and the complex conjugates of their superpartner right sfermion fields, i.e. $\tilde{f}^\star_{uR}$ etc. These are contained in left chiral superfields with quantum numbers of the conjugate representations. Finally, all matter superfields are taken to have *odd* matter parity (cf. §4.5).

In the gauge sector we introduce one vector superfield corresponding to each gauge field in the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Thus we have the $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ gauge fields B_μ, $\vec{W}_\mu$, g^a_μ and the corresponding spin half (four component) Majorana gaugino fields $\tilde{\lambda}_0$, $\tilde{\vec{\lambda}}$, $\tilde{g}^a$ contained in the superfields

$$\{V^Y, \vec{V}^W, V^a_g\} \tag{8.18}$$

respectively. Every gaugino field, like its gauge boson partner, transforms as the adjoint representation of the corresponding gauge group. Moreover, each such field has left chiral and right chiral components which are charge conjugates of each other:

$$(\tilde{\lambda}_{0L})^C = \tilde{\lambda}_{0R} \; . \tag{8.19}$$

Next, we turn to the supersymmetrization of the Higgs sector of the SM. The latter has only one $SU(2)_L$ doublet field ϕ with a hypercharge $Y_\phi = 1$. As discussed earlier, the same Higgs VEV v can be used to give masses to the $T_{3L} = 1/2$ and $T_{3L} = -1/2$ fermions via the Yukawa interaction terms of (8.9) and (8.10). In particular, (8.10) has been made possible only by use of the conjugate Higgs field ϕ^C which has $Y_{\phi^C} = -1$. Such a term, however, will not be allowed in a supersymmetric theory. There the Yukawa interactions are derived from the superpotential $\mathcal{W}$ which has to be an *analytic* function of left chiral superfields (see §5.1). Hence interaction terms, derived from the same superpotential, cannot contain both ϕ and ϕ^C. Therefore, in order to make the $T_{3L} = -1/2$ fermions massive, a second Higgs doublet is needed. We must then have – in a supersymmetric theory – two Higgs doublets with hypercharges $Y = -1$ and 1 which we shall denote by h_1 (down type) and h_2 (up type) respectively. If the superscript D is an $SU(2)$ doublet index taking values $1, 2$, we can write for $D = 1$, $h_1^1 = h_1^0$ and $h_2^1 = h_2^+$ while, for $D = 2$, we can write $h_1^2 = h_1^-$, $h_2^2 = h_2^0$:

$$h_1 \equiv \begin{pmatrix} h_1^1 \\ h_1^2 \end{pmatrix} = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}; \; h_2 \equiv \begin{pmatrix} h_2^1 \\ h_2^2 \end{pmatrix} = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}. \tag{8.20}$$

Their Yukawa interactions can be written down simply by replacing ϕ and ϕ^C by $-i\tau_2 h_1^\star$ and $i\tau_2 h_2^\star$ respectively in (8.9) and (8.10). The Higgs VEVs, after the spontaneous breakdown of electroweak symmetry, are now given by real, positive quantities (cf. §10.2) $v_{1,2}$ which arise from the minimization of the Higgs potential term $V(h_1, h_2)$ and are shown below:

$$\langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \; \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{8.21}$$

It is well known [8.4] that this two Higgs doublet extension of the SM, with the up and down type fermions coupling to separate Higgs doublets, is perfectly compatible with all FCNC constraints[4] since it obeys the Glashow-Weinberg/Paschos condition [8.1]. The only change

[4]This is true even including one loop corrections.

is that (8.6) and (8.11) are now respectively modified to

$$M_W = \frac{1}{2} g_2 \sqrt{v_1^2 + v_2^2},\ M_Z = \frac{1}{2}\sqrt{g_Y^2 + g_2^2}\sqrt{v_1^2 + v_2^2},\ \sqrt{v_1^2 + v_2^2} = \left(\frac{1}{\sqrt{2}G_F}\right)^{1/2} \simeq 246 \text{ GeV} \tag{8.22}$$

and

$$(\mathbf{m}_e)_{ij} = m_{e_i}\delta_{ij} = \frac{1}{\sqrt{2}} f_{ij}^{e\star} v_1,\ (\mathbf{m}_d)_{ij} = \frac{1}{\sqrt{2}} f_{ij}^{d\star} v_1,\ (\mathbf{m}_u)_{ij} = \frac{1}{\sqrt{2}} f_{ij}^{u\star} v_2\ , \tag{8.23a}$$

$$f_{ij}^{e\star} = \frac{g_2}{\sqrt{2}M_W \cos\beta}(\mathbf{m}_e)_{ij},\ f_{ij}^{d\star} = \frac{g_2}{\sqrt{2}M_W \cos\beta}(\mathbf{m}_d)_{ij},\ f_{ij}^{u\star} = \frac{g_2}{\sqrt{2}M_W \sin\beta}(\mathbf{m}_u)_{ij}\ . \tag{8.23b}$$

The relations in (8.23b) have been obtained by inverting those in (8.23a). The ratio

$$\frac{v_2}{v_1} = \tan\beta \tag{8.24}$$

becomes a free parameter of the theory in so far as fermion masses are concerned.

The left chiral fermionic partners of the Higgs bosons of (8.20) are given by

$$\tilde{h}_{1L} \equiv \begin{pmatrix} \tilde{h}_1^1 \\ \tilde{h}_1^2 \end{pmatrix} = \begin{pmatrix} \tilde{h}_1^0 \\ \tilde{h}_1^- \end{pmatrix}_L;\ \tilde{h}_{2L} \equiv \begin{pmatrix} \tilde{h}_2^1 \\ \tilde{h}_2^2 \end{pmatrix} = \begin{pmatrix} \tilde{h}_2^+ \\ \tilde{h}_2^0 \end{pmatrix}_L . \tag{8.25}$$

In (8.25) we have defined higgsino fields $\tilde{h}_{1L}^0, \tilde{h}_{1L}^-, \tilde{h}_{2L}^+$ and $\tilde{h}_{2L}^0$, which are two component spinorial fields in the $(\frac{1}{2}, 0)$ representation (cf. §3.2) and identified with $\tilde{h}_1^1, \tilde{h}_1^2, \tilde{h}_2^1$ and $\tilde{h}_2^2$ respectively. Generalizing, we can denote the left chiral superfields containing $h_1, \tilde{h}_{1L}$ and $h_2, \tilde{h}_{2L}$ by H_1, H_2 respectively. So we have

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix},\ H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} \tag{8.26}$$

as the down type, up type Higgs superfields with $Y = -1, 1$ respectively. They are assigned *even* matter parity since they are perceived to be quantalike (cf. Table 4.1). Note that, for quarks and leptons, the need to have a massive Dirac fermion makes it necessary for us to introduce $SU(2)_L$ doublet and singlet chiral superfields. This is unnecessary in the case of the Higgs superfields since $\tilde{h}_{1L}^0$ and $(\tilde{h}_{2L}^0)^C$ can combine to form a four component spinorial field and ditto $\tilde{h}_{1L}^-$ and $(h_{2L}^+)^C$. There is therefore only one four component neutral higgsino field and similarly only one four component charged higgsino field[5]. The two Higgs superfields of (8.26) are thus sufficient. These, together with those in (8.14) – (8.18), comprise all the superfields of MSSM. They are all listed in Tables 8.1a and 8.1b.

[5]This is true with unbroken electroweak symmetry. The broken symmetric case is more complicated and will be discussed later.

LEFT CHIRAL MATTER SUPERFIELDS			
Lepton doublets	(color multiplet, T_{3L}, Y)	Quark doublets	(color multiplet, T_{3L}, Y)
$L_1 = \begin{pmatrix} L_{\nu_e} \\ L_e \end{pmatrix}$	$\left(1, \frac{1}{2}, -1\right)$ $\left(1, -\frac{1}{2}, -1\right)$	$Q_1 = \begin{pmatrix} Q_u \\ Q_d \end{pmatrix}$	$\left(3, \frac{1}{2}, \frac{1}{3}\right)$ $\left(3, -\frac{1}{2}, \frac{1}{3}\right)$
$L_2 = \begin{pmatrix} L_{\nu_\mu} \\ L_\mu \end{pmatrix}$	$\left(1, \frac{1}{2}, -1\right)$ $\left(1, -\frac{1}{2}, -1\right)$	$Q_2 = \begin{pmatrix} Q_c \\ Q_s \end{pmatrix}$	$\left(3, \frac{1}{2}, \frac{1}{3}\right)$ $\left(3, -\frac{1}{2}, \frac{1}{3}\right)$
$L_3 = \begin{pmatrix} L_{\nu_\tau} \\ L_\tau \end{pmatrix}$	$\left(1, \frac{1}{2}, -1\right)$ $\left(1, -\frac{1}{2}, -1\right)$	$Q_3 = \begin{pmatrix} Q_t \\ Q_b \end{pmatrix}$	$\left(3, \frac{1}{2}, \frac{1}{3}\right)$ $\left(3, -\frac{1}{2}, \frac{1}{3}\right)$
Antilepton singlets	(color multiplet, T_{3L}, Y)	Antiquark singlets	(color multiplet, T_{3L}, Y)
$\bar{E}_e$	$(1, 0, 2)$	$\bar{U}_1, \bar{D}_1$	$\left(\bar{3}, 0, -\frac{4}{3}\right), \left(\bar{3}, 0, \frac{2}{3}\right)$
$\bar{E}_\mu$	$(1, 0, 2)$	$\bar{U}_2, \bar{D}_2$	$\left(\bar{3}, 0, -\frac{4}{3}\right), \left(\bar{3}, 0, \frac{2}{3}\right)$
$\bar{E}_\tau$	$(1, 0, 2)$	$\bar{U}_3, \bar{D}_3$	$\left(\bar{3}, 0, -\frac{4}{3}\right), \left(\bar{3}, 0, \frac{2}{3}\right)$

Table 8.1a. Matter superfield content of the MSSM.

GAUGE SUPERFIELDS		LEFT CHIRAL HIGGS SUPERFIELDS		
Notation	Name	Doublets	Name	Y
V^Y	Hypercharge	$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$	Down type	–1
$\vec{V}^W$	Weak isospin			
V_g^a	Color	$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	Up type	1

Table 8.1b. Gauge and Higgs Superfield content of the MSSM.

A question can be raised at this point as to whether one could have been more economical with the contents of superfields in the MSSM. The requirement that all the component fields in each superfield must carry the same internal quantum numbers would quickly convince anyone that the above is necessarily the minimum set. The components of H_1 and L_i, for instance, have the same electromagnetic charges, but they differ in lepton number (including lepton type) and matter parity. We have already given the *raison d'être* for the existence of two Higgs superfield doublets with $Y = -1$ and $Y = 1$, namely the generation of masses for both $T_{3L} = -1/2$ and $T_{3L} = 1/2$ fermions respectively. In fact, even in the supersymmetric extension of a matterless SM (with only gauge and Higgs fields), the two Higgs doublet superfields H_1 and H_2 are necessary for self-consistency. The condition of anomaly cancellation [8.5] in the higgsino sector, a requirement of renormalizability, demands in particular that $\Sigma_{\tilde{h}} Y_{\tilde{h}}^3 = 0$ where $Y_{\tilde{h}}$ is the hypercharge of each higgsino field $\tilde{h}$. Thus one doublet $\tilde{h}_2$ with $Y_{\tilde{h}_2} = 1$ has to be compensated by another $\tilde{h}_1$ doublet with $Y_{\tilde{h}_1} = -1$. (Gauginos, which are another set of new fermions in the supersymmetric theory, are in the safe adjoint representations and do not cause anomaly problems.) We see finally that all the superfields, introduced above and tabulated in Tables 8.1a,b are indeed necessary for the minimal extension of the SM keeping intact its local symmetries, such as electromagnetic charge and color, as well as its global symmetries through the conservation of baryon (B) and lepton (L) number (including lepton type). As stated earlier, the exact conservation of R-parity is an assumed additional requirement. Within the MSSM the assumption of B and L (including lepton type L_i) conservation[6] is equivalent to that of R-parity conservation[7]. But, for superpotential terms and supersymmetry breaking operators in the Lagrangian density, this is a highly constraining requirement.

Of course, states corresponding to all component fields of the superfields, described above, are only 'interaction' eigenstates. In the real world, the absence of mass degenerate particle-sparticle pairs requires supersymmetry to be broken. We shall discuss in the next chapter why such a breaking cannot be spontaneous within the framework of the MSSM itself. Suffice it to say here that it has to be explicit and soft (cf. §7.7). This breaking of supersymmetry in the MSSM can be parametrized in terms of a few explicit soft terms added to the Lagrangian density. We choose the most general terms of this kind. But they are first introduced in an ad hoc manner, though some rationale for them will be given in Chs.12 and 13 on the basis of high scale physics. The contents of these terms will be discussed in detail in Ch.9. Let us remark, for the moment, that they can induce mixing between different sparticles with the same charge and color. Indeed, even without supersymmetry breaking, electroweak symmetry breaking alone causes mixings between gauginos and higgsinos (cf. 5.30). Thus, for instance, charged gauginos mix with charged higgsinos through a 2×2 mixing matrix. The two physical mass eigenstates from that are called **charginos** $\tilde{\chi}^{\pm}_{1,2}$, the subscript 1 (2) conventionally referring to the lighter (heavier) sparticle. A more elaborate discussion will appear in §9.2.

We can immediately see yet another need for two Higgs doublets in this theory. The two doublet superfields H_1, H_2 are left chiral ones and they contain the left chiral higgsinos of (8.25); the conjugate superfields $H_1^\dagger, H_2^\dagger$ contain the corresponding right chiral ones. The left chiral charginos comprise four orthogonal states: the positively charged $\tilde{\chi}^+_{1L}, \tilde{\chi}^+_{2L}$ and the negatively charged $\tilde{\chi}^-_{1L}, \tilde{\chi}^-_{2L}$. Let us define charged gaugino (wino) fields

$$\tilde{\lambda}^{\pm} = \frac{1}{\sqrt{2}} \left(\tilde{\lambda}_1 \mp i \tilde{\lambda}_2 \right) = \tilde{\lambda}^{\pm}_L + \tilde{\lambda}^{\pm}_R \,, \tag{8.27}$$

where the superscripts 1, 2 are Cartesian $SU(2)_L$ indices. The massive $\tilde{\chi}^+_{1L}$ and $\tilde{\chi}^+_{2L}$ are orthogonal linear combinations of $\tilde{\lambda}^+_L$ and $\tilde{h}^+_{2L}$ while $\tilde{\chi}^-_{1L}$ and $\tilde{\chi}^-_{2L}$ are formed by similarly

[6] Strictly speaking, even B and L (also L_i) are violated at the loop level through anomalies both in the SM and the MSSM, only $\frac{1}{3}B - L_i$ is exactly conserved. But these violations are very tiny in a zero temperature field theory.

[7] This equivalence is not necessarily valid in extensions of the MSSM. Of course, supersymmetric Grand Unified Theories usually violate B and L but may respect R-parity.

combining $\tilde{\lambda}_L^-$ and $\tilde{h}_{1L}^-$. (N.B. there is *no* $\tilde{h}_{2L}^-$ or $\tilde{h}_{1L}^+$!) Correspondingly, the right chiral charginos $\tilde{\chi}_{1R}^-$, $\tilde{\chi}_{2R}^-$ and $\tilde{\chi}_{1R}^+$, $\tilde{\chi}_{2R}^+$ are orthogonal linear combinations of the charge conjugates of the above pairs of gauginos and higgsinos, viz. $\tilde{\lambda}_R^-$, $\tilde{h}_{2R}^-$ and $\tilde{\lambda}_R^+$, $\tilde{h}_{1R}^+$ respectively. Evidently, we require both $\tilde{h}_{2L}^+$ and $\tilde{h}_{1L}^-$, as appear in the two Higgs doublets, otherwise some chargino field, lacking a partner to make a Dirac mass term in the Lagrangian density, would remain massless. Thus we see how the two higgsino doublet fields in the MSSM are used, in combination with the charged winos, to generate two massive Dirac charginos.

Similarly, there is mixing among the neutral gauginos, which can be described by four component Majorana fields. There are two, namely $\tilde{\lambda}_0$ and $\tilde{\lambda}_3$, which mix with the neutral higgsinos $\tilde{h}_2^0$ and $\tilde{h}_1^0$ through a 4×4 mixing matrix. In this case the four physical mass eigenstate Majorana fermions are called **neutralinos** $\tilde{\chi}_i^0$ $(i = 1, \cdots, 4)$, the subscripts being monotonically ordered in the direction of increasing mass, by convention. Once again, a detailed description of the mixing among charge neutral gauginos and higgsinos, forming mass eigenstate neutralinos, will be given in §9.2. In fact, similar mixings can occur among different squark generations or among different slepton generations (if lepton type number gets violated) as well. Also, one can (and does) have left right sfermion mixing. Not much more can be said a priori about mixing between different interaction eigenstates in the sparticle sector. These depend on the detailed structure of the supersymmetry breaking terms and their relationship with EW symmetry breaking. Such details about sparticle mass eigenstates will be taken up in the next chapter after we have discussed the soft supersymmetry breaking terms at length.

An enumeration has been given below (Table 8.2) of sparticle fields in the minimal globally supersymmetric extension of the SM which follows from the construction described earlier.

Sfermions		Gauginos and higgsinos	
Name	Symbol	Name	Symbol
(left, right) selectron	$\tilde{e}_{L,R}$	gluinos	$\tilde{g}^a$
(left, right) smuon	$\tilde{\mu}_{L,R}$		
(left, right) stau	$\tilde{\tau}_{L,R}$	lighter charginos	$\tilde{\chi}_1^\pm$
e-sneutrino	$\tilde{\nu}_e$		
μ-sneutrino	$\tilde{\nu}_\mu$	heavier charginos	$\tilde{\chi}_2^\pm$
τ-sneutrino	$\tilde{\nu}_\tau$		
(left, right) u-squark	$\tilde{u}_{L,R}$	lightest neutralino	$\tilde{\chi}_1^0$
(left, right) d-squark	$\tilde{d}_{L,R}$		
(left, right) c-squark	$\tilde{c}_{L,R}$	next-to-lightest neutralino	$\tilde{\chi}_2^0$
(left, right) s-squark	$\tilde{s}_{L,R}$		
(left, right) stop	$\tilde{t}_{L,R}$	next-to-heaviest neutralino	$\tilde{\chi}_3^0$
(left, right) sbottom	$\tilde{b}_{L,R}$	heaviest neutralino	$\tilde{\chi}_4^0$

Table 8.2. List of sparticle fields in the MSSM. Antisfermion fields have not been listed.

Sfermions of the third generation are likely to have strong L-R mixing; the mass eigenstate sfermion fields are denoted as $\tilde{\tau}_{1,2}$, $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$. Antisfermionic fields are denoted by conjugations of sfermionic fields, e.g. $\tilde{e}_{L,R}^\star$ from $\tilde{e}_{L,R}$ and $\tilde{q}_{L,R}^\dagger$ from $\tilde{q}_{L,R}$. However, this is a notation that we shall use for fields only, while an antisfermionic particle – the superpartner of an antifermion – will be labeled $\overline{\tilde{f}}$, i.e. $\overline{\tilde{e}_L}$ for the right spositron and $\overline{\tilde{u}_L}$ for the right u-antisquark. Additional particles and sparticles may be needed by theoretical schemes which go beyond this minimal extension. For instance, the gravitino $\tilde{G}$, which is needed in a spontaneously broken N=1 supergravity (SUGRA) theory, has not been included here.

8.3 Supersymmetric Part of the MSSM

In this section we will introduce and discuss those interaction and mass terms in the Lagrangian density $\mathcal{L}_{\rm MSSM}$ which come from the exact supersymmetrization of the SM. Soft interaction terms with mass dimensions less than four as well as mass terms, which describe the heavier masses of sparticles as different from those of their particle partners, arise from supersymmetry breaking and will be addressed in a later section. The general form of the Lagrangian density is

$$\mathcal{L}_{\rm MSSM} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm SOFT} \tag{8.28}$$

and in this section we will give explicit expressions for $\mathcal{L}_{\rm SUSY}$ only. In order to write down the supersymmetric interactions among the dynamical fields enumerated in §8.1, we will essentially use the forms of the Lagrangian densities of SQED, SQCD and SχGT of Chapter 5, but covering three families of quarks and leptons. The only really new addition is the contribution from the Higgs sector. The gauge couplings are the same as in the SM. There is no need to give the explicit gauge transformations of the matter superfields enumerated in §8.1. These can be obtained by a straightforward extension of (5.15) and (5.38). But we can decompose the supersymmetric part of the MSSM Lagrangian density as follows:

$$\mathcal{L}_{\rm SUSY} = \mathcal{L}_g + \mathcal{L}_M + \mathcal{L}_H \ , \tag{8.29}$$

where $\mathcal{L}_g, \mathcal{L}_M$ and $\mathcal{L}_H$ are the pure gauge, matter and Higgs-Yukawa parts respectively.

The **pure gauge part** of $\mathcal{L}_{\rm SUSY}$ can be written, in terms of field strength spinorial superfields W_g^a, $\vec{W}_W$ and W_Y, constructed respectively via (4.39) and (5.45) from V_g^a, $\vec{V}_W$ and V^Y, according to (5.17) and (5.54):

$$\mathcal{L}_g = \frac{1}{4}\int d^2\theta \left(W_g^{aA} W_{gA}^a + \vec{W}_W^A \cdot \vec{W}_{WA} + W_Y^A W_{YA} \right) + \text{h.c.} \ , \tag{8.30}$$

where the color index a has been summed on repetition. Similarly, the matter contribution can be given by the generalization of (5.62) as

$$\begin{aligned} \mathcal{L}_M \ = \int d^4\theta \Big[& L_i^\dagger \ e^{\left(g_2 \vec{V}^W \cdot \vec{\tau} + g_Y V^Y Y\right)} L_i + \bar{E}_i^\dagger \ e^{g_Y V^Y Y} \bar{E}_i + \bar{U}_i^\dagger \ e^{\left(g_s V_g^a \bar{\lambda}^a + g_Y V^Y Y\right)} \bar{U}_i \\ & + \bar{D}_i^\dagger \ e^{\left(g_s V_g^a \bar{\lambda}^a + g_Y V^Y Y\right)} \bar{D}_i + Q_i^\dagger \ e^{\left(g_s V_g^a \lambda^a + g_2 \vec{V}^W \cdot \vec{\tau} + g_Y V^Y Y\right)} Q_i \Big] . \end{aligned} \tag{8.31}$$

In (8.31) the Pauli matrices $\vec{\tau}$ act in the weak isospin doublet representation space while the Gell-Mann matrices λ^a (and their complex conjugates $\bar{\lambda}^a$) act in the color triplet **3** (and antitriplet $\bar{\mathbf{3}}$) representation spaces. The subscript i is a family index, summed over $1, 2, 3$ on repetition. Finally, the Higgs contribution can be written as

$$\mathcal{L}_H = \sum_{p=1}^{2} \int d^4\theta \left[H_p^\dagger \, e^{(g_2 \vec{V}_W \cdot \vec{\tau} + g_Y V^Y Y)} H_p + \mathcal{W}_{\rm MSSM}\delta^{(2)}(\bar{\theta}) + \mathcal{W}^\dagger_{\rm MSSM}\delta^{(2)}(\theta) \right], \quad (8.32)$$

where the superpotential $\mathcal{W}_{\rm MSSM}$ is given by

$$\mathcal{W}_{\rm MSSM} = \mu H_1 \cdot H_2 - f^e_{ij} H_1 \cdot L_i \bar{E}_j - f^d_{ij} H_1 \cdot Q_i \bar{D}_j - f^u_{ij} Q_i \cdot H_2 \bar{U}_j \ . \quad (8.33)$$

(We use the notation $A \cdot B \equiv \epsilon_{DE} A^D B^E$ for two $SU(2)$-doublet superfields or fields A, B with D, E being indices in the doublet representation space with the same superscript/subscript conventions as for two component spinors in Ch.3). The signs in (8.33) have been chosen so that the f_{ij}'s here are the same as of those in (8.9) and (8.10), as can be checked by use of (5.3) and (3.28a,b). The second, third and fourth terms in the RHS of (8.33) are just the supersymmetric generalizations of the Yukawa couplings in (8.9) and (8.10). Only the first RHS term of (8.33) is new. This term, containing the parameter μ, which has the dimension of mass, can be thought of as a supersymmetric generalization of a higgsino mass term. We shall later see that a consistent incorporation of spontaneous electroweak symmetry breakdown requires μ to be of the order of the weak scale. The choice of terms in $\mathcal{W}_{\rm MSSM}$ has been constrained by the **requirement** of R-parity (R_p) conservation (cf. §4.5) which is one of the assumptions of the MSSM. Let us remark here that, since baryon number B and lepton number L are conserved in the SM Lagrangian, the conservation of R_p may be posited as a natural assumption in a minimal supersymmetric extension of the SM which may be expected to preserve the conservation laws of the latter. Additional terms, that are gauge invariant with respect to SM gauge transformations, could be admitted to the RHS of (8.33) if R-parity were violated explicitly. We postpone a discussion of this possibility to Ch.14. For the moment, we take the **conservation of R-parity to be a central assumption** of the MSSM. The terms in $\mathcal{L}_{\rm MSSM}$, that are generated from $\mathcal{W}_{\rm MSSM}$, are obtained from a generalization of (5.5) with the Higgs VEVs from (8.21) taken into account to properly incorporate spontaneous electroweak symmetry breaking.

Let us concentrate first on the auxiliary F and D fields following from (8.31)-(8.33). By use of (5.56c), we can identify seventeen (including $i = 1, 2, 3$) F fields from (8.33). For the $SU(2)$ doublet representation space, we can employ the two spinor subscript/superscript notation of Ch.3, i.e. $H_{1D} = \epsilon_{DE} H_1^E$ and $F^{\star D}_{H_1} = -\partial\mathcal{W}/\partial H_{1D}|$ etc. This enables us to write

$$F^{\star D}_{H_1} = -\mu h_2^D + f^e_{ij} \tilde{e}^\star_{jR} \tilde{\ell}^D_{iL} + f^d_{ij} \tilde{d}^\dagger_{jR} \tilde{q}^D_{iL} \ , \quad (8.34a)$$

$$F^{\star D}_{H_2} = \mu h_1^D - f^u_{ij} \tilde{u}^\dagger_{jR} \tilde{q}^D_{iL} \ , \quad (8.34b)$$

$$F^{\star D}_{L_i} = -f^e_{ij} h_1^D \tilde{e}^\star_{jR} \ , \quad (8.34c)$$

$$F^\star_{\bar{E}_i} = f^e_{ji} h_1 \cdot \tilde{\ell}_{jL} \ , \quad (8.34d)$$

$$F^{\star D}_{Q_{i\alpha}} = -f^d_{ij} h_1^D \tilde{d}^\dagger_{jR\alpha} + f^u_{ij} h_2^D \tilde{u}^\dagger_{jR\alpha} \ , \quad (8.34e)$$

$$F^\star_{\bar{D}_{i\alpha}} = f^d_{ji} h_1 \cdot \tilde{q}_{jL\alpha} \ , \quad (8.34f)$$

$$F^\star_{\tilde{U}_{i\alpha}} \quad = \quad f^u_{ji}\tilde{q}_{jL\alpha}\cdot h_2 \ . \tag{8.34g}$$

In (8.34e-g) the subscript α is the floating color index, whereas in (8.34a,b) appropriate color contractions are implied. Now the three D fields, corresponding to the three factors $U(1)_Y, SU(2)_L$ and $SU(3)_C$ of the gauge group and ignoring a possible field independent term in D_Y, cf. (5.22b), are given respectively from (5.56c) by

$$\begin{aligned} D^Y \quad &= -\frac{1}{2}g_Y\Big(h_2^\dagger h_2 - h_1^\dagger h_1 + \frac{1}{3}\tilde{q}^\dagger_{iL}\tilde{q}_{iL} - \frac{4}{3}\tilde{u}_{iR}\tilde{u}^\dagger_{iR} + \frac{2}{3}\tilde{d}_{iR}\tilde{d}^\dagger_{iR} \\ &\qquad -\tilde{\ell}^\dagger_{iL}\tilde{\ell}_{iL} + 2\tilde{e}_{iR}\tilde{e}^\star_{iR}\Big), \end{aligned} \tag{8.35a}$$

$$\vec{D} \quad = -\frac{1}{2}g_2\left(h_1^\dagger\vec{\tau}h_1 + h_2^\dagger\vec{\tau}h_2 + \tilde{q}^\dagger_{iL}\vec{\tau}\tilde{q}_{iL} + \tilde{\ell}^\dagger_{iL}\vec{\tau}\tilde{\ell}_{iL}\right), \tag{8.35b}$$

$$\begin{aligned} D^a \quad &= -\frac{1}{2}g_s\left(\tilde{q}^\dagger_{iL}\lambda^a\tilde{q}_{iL} + \tilde{u}^T_{iR}\bar{\lambda}^a\tilde{u}^\star_{iR} + \tilde{d}^T_{iR}\bar{\lambda}^a\tilde{d}^\star_{iR}\right) \\ &= -\frac{1}{2}g_s\left(\tilde{q}^\dagger_{iL}\lambda^a\tilde{q}_{iL} + \tilde{u}^\dagger_{iR}\lambda^a\tilde{u}_{iR} + \tilde{d}^\dagger_{iR}\lambda^a\tilde{d}_{iR}\right). \end{aligned} \tag{8.35c}$$

Here a is a color index and we have utilized the hermiticity of λ^a in the last step. It may be noted that, in both (8.35a) and (8.35c), $\tilde{u}_{iR}{}^\star$, $\tilde{d}_{iR}{}^\star$ and $\tilde{e}_{iR}{}^\star$ are the equivalents of ϕ in (5.56c). Finally, the supersymmetric scalar potential is given (cf. 5.56b) by

$$V_{\mathrm{SUSY}} = F^\star_k F_k + \frac{1}{2}\left[\vec{D}^2 + (D^Y)^2 + D^a D^a\right]. \tag{8.36}$$

k referring to the type of superfield (including any internal symmetry index) and repeated k and a being summed.

The interaction part can be written down in terms of component fields in four component notation in much the same way as shown in Ch.5. The major difference now is that we want to incorporate the spontaneous electroweak symmetry breakdown $SU(2)_L \times U(1)_Y \to U(1)_{em}$ and obtain the consequent mass terms and mass eigenstates, i.e. equivalents of (8.5) to (8.10). Let us consider non-Higgs vertices for the moment. We postpone all discussions of interactions involving Higgs bosons to Ch.10; in particular, these include Yukawa, Higgs-gauge and Higgs-Higgs interactions and some of their supersymmetric generalizations. Furthermore, those vertices with physical sparticles, which involve supersymmetry breaking, will be treated in Ch.9. In the next section of this chapter we consider (A) fermion-fermion-gauge boson, (B) triple gauge boson, and (C) quadruple gauge boson vertices in the Standard Model. We also discuss from the MSSM those (D) sfermion-sfermion-gauge boson, (E) gauge boson-gaugino-gaugino, (F) fermion-sfermion-gaugino, (G) gauge boson-gauge boson-sfermion-sfermion and (H) sfermion quartic vertices which have to do with only the purely supersymmetric part of $\mathcal{L}_{\mathrm{MSSM}}$ and without left right mixing. Some subsets of these as well as other non-Higgs vertices crucially involve supersymmetry breaking and both generation as well as left right mixing in a physical situation. Those will be covered in Ch.9, which will contain the corresponding final physical vertices with the said mixings.

8.4 Some Non-Higgs Vertices of the MSSM

First, we recount the non-Higgs SM vertices in (A), (B) and (C). Subsections (D), (E), (F) and (G) contain the new supersymmetric extensions.

(A) *Fermion-fermion-gauge boson vertices*
We can discuss the strong and electroweak vertices separately.

(i) **Quark-quark-gluon vertices**
These are the same as in QCD, vide §5.5. The only additional remark is that all six flavors of quarks ($p = u, d, c, s, t, b$) have to be included with all interactions being diagonal in flavour space. Thus, with p henceforth summed on repetition, we have

$$\mathcal{L}_{q\bar{q}g} = -g_s g^a_\mu \bar{q}_p T^a \gamma^\mu q_p \ .$$

This form is valid in any basis for the quarks that can be reached from the current basis by a unitary rotation in generation space. Thus flavor mixings of quark mass eigenstates are inconsequential here.

(ii) **Fermion-fermion-electroweak vector boson vertices**
These follow exactly those given in §5.6. The only differences arise on account of $\gamma - Z$ mixing, cf. (8.7) and (8.8). Furthermore, one has to replicate for three generations. We can now employ the notation of (5.65), understanding f_{u_i,d_i} to be either a quark or a lepton of generation i with $f_{u_iL} = P_L f_{u_i}$, $f_{u_iR} = P_R f_{u_i}$, $f_{d_iL} = P_L f_{d_i}$, $f_{d_iR} = P_R f_{d_i}$ and $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. Then, with f, f' and v respectively chosen as two fermions and one EW vector boson generically, we can write

$$\begin{aligned} \mathcal{L}_{ff'\mathrm{v}} \;=\; & -\frac{g_2}{\sqrt{2}}\left(W^+_\mu \bar{f}_{u_i}\gamma^\mu P_L f_{d_i} + W^-_\mu \bar{f}_{d_i}\gamma^\mu P_L f_{u_i}\right) \\ & - eA_\mu\left(Q_{f_u}\bar{f}_{u_i}\gamma^\mu f_{u_i} + Q_{f_d}\bar{f}_{d_i}\gamma^\mu f_{d_i}\right) - \frac{g_2}{2\cos\theta_W} Z_\mu \cdot \\ & \Big[\bar{f}_{u_i}\gamma^\mu\left\{(1 - 2Q_{f_u}\sin^2\theta_W)P_L - 2Q_{f_u}\sin^2\theta_W P_R\right\} f_{u_i} \\ & - \bar{f}_{d_i}\gamma^\mu\left\{(1 + 2Q_{f_d}\sin^2\theta_W)P_L + 2Q_{f_d}\sin^2\theta_W P_R\right\} f_{d_i}\Big]. \end{aligned} \tag{8.37}$$

In (8.37) Q_{f_u} and Q_{f_d} are the electromagnetic charges of the up type and down type fermions f_{u_i} and f_{d_i} respectively in units of the charge of the positron. Thus $Q_{u_i} = \frac{2}{3}$, $Q_{d_i} = -\frac{1}{3}$, $Q_{e_i} = -1$, $Q_{\nu_i} = 0$. Referring back to (8.3) and comparing with (5.65), we note that for quarks, we can write

$$q_{iL} = \begin{pmatrix} f_{u_{iL}} \\ f_{d_{iL}} \end{pmatrix}, \ u_{iR} = f_{u_{iR}}, \ d_{iR} = f_{d_{iR}} \ . \tag{8.38}$$

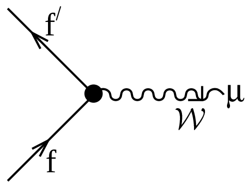
Similarly, for leptons, the notation is

$$\ell_{iL} = \begin{pmatrix} f_{u_{iL}} \\ f_{d_{iL}} \end{pmatrix}, \ e_{iR} = f_{d_{iR}} \ . \tag{8.39}$$

However, the above quarks are gauge interaction or "current" basis eigenstates. When we go to physical mass eigenstates, we will need to incorporate the Cabibbo-Kobayashi-Maskawa (CKM) matrix for charged current couplings in the quark sector. This part can be written as

$$\mathcal{L}_{q\bar{q}'W^{\pm}} = -\frac{g_2}{\sqrt{2}} \left(W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij}^{q_L} d_j + \text{h.c.} \right), \tag{8.40}$$

where the u_i, d_j etc. are now *understood to be mass eigenstate* quark fields. In (8.40), $V_{ij}^{q_L}$ are the elements of the CKM matrix [8.1] $\mathbf{V}^{q_L} = \mathbf{U}^{u_L^\dagger}\mathbf{U}^{d_L}$ in the notation of (8.12). Electromagnetic and neutral current vertices, of course, do not involve these on account of the GIM mechanism [8.1]. Finally, the vertices and Feynman rules for $i\mathcal{L}_{ff'v}$ can be written as in Fig. 8.1 below.

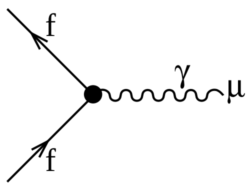


$\mathcal{W} = W^-, f = d_j, f' = u_i \quad -\frac{ig_2}{\sqrt{2}}\gamma_\mu P_L V_{ij}^{q_L}$

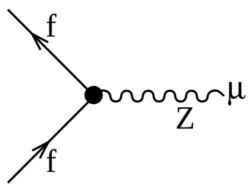
$\mathcal{W} = W^+, f = u_j, f' = d_i \quad -\frac{ig_2}{\sqrt{2}}\gamma_\mu P_L V_{ij}^{q_L^\dagger}$

$\mathcal{W} = W^+, f = \nu_j, f' = e_i \quad -\frac{ig_2}{\sqrt{2}}\gamma_\mu P_L \delta_{ij}$

$\mathcal{W} = W^-, f = e_j, f' = \nu_i \quad -\frac{ig_2}{\sqrt{2}}\gamma_\mu P_L \delta_{ij}$



$-ieQ_f\gamma_\mu$



$$-\frac{ig_2}{\cos\theta_W}T_{3L}^f\left[(1-4T_{3L}^f Q_f \sin^2\theta_W)\gamma_\mu P_L - 4T_{3L}^f Q_f \sin^2\theta_W \gamma_\mu P_R\right]$$
$$\equiv -\frac{ig_2}{\cos\theta_W}(g_L^f\gamma_\mu P_L - g_R^f\gamma_\mu P_R)$$

Fig. 8.1. Fermion-fermion-electroweak vector boson vertices with T_{3L}^f, Q_f as in (8.1).

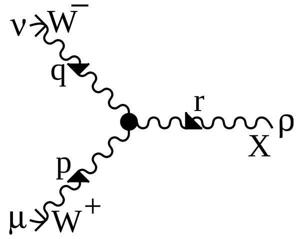
Note that in the lowermost vertex g_L^f stands for $T_{3L}^f(1 - 4T_{3L}^f Q_f \sin^2\theta_W)$ and g_R^f for $4(T_{3L}^f)^2 Q_f \sin^2\theta_W$.

(B) *Triple gauge boson vertices*

Again, the strong and electroweak cases can be distinguished.

(i) **Triple gluon vertex**
This is exactly the same as in QCD, vide (5.60) and Fig. 5.2.

(ii) **Triple electroweak vector boson vertices**
These are generalized from the $W^+W^-W^3$ vertex of SχGT in §5.6, as shown in Fig. 8.2.



$$p + q + r = 0$$

$$X = \gamma \quad ie\,[(r-p)_\nu \eta_{\rho\mu} + (p-q)_\rho \eta_{\mu\nu} + (q-r)_\mu \eta_{\nu\rho}]$$

$$X = Z \quad ig_2 \cos\theta_W\,[(r-p)_\nu \eta_{\rho\mu} + (p-q)_\rho \eta_{\mu\nu} + (q-r)_\mu \eta_{\nu\rho}]$$

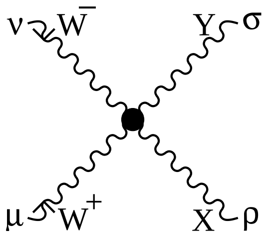
Fig. 8.2. Triple electroweak vector boson vertices

(C) *Quadruple gauge boson vertices*

Once more, we can consider the strong and electroweak vertices in different categories.

(i) **Quadruple gluon vertex**
This is identical to that in QCD, vide §5.5 and Fig. 5.2.

(ii) **Quadruple electroweak vector boson vertices**
The $W^+W^-W^+W^-$ vertex is identical to that given in Fig. 5.3. The $W^+W^-W^3W^3$ vertex, shown there, generalizes to three cases here, as given in Fig. 8.3.



$$X = \gamma, Y = \gamma \quad -ie^2\,[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\nu\rho}\eta_{\mu\sigma}]$$

$$X = \gamma, Y = Z \quad -2ieg_2\cos\theta_W\,[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\nu\rho}\eta_{\mu\sigma}]$$

$$X = Z, Y = Z \quad -ig_2^2\cos^2\theta_W\,[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\nu\rho}\eta_{\mu\sigma}]$$

Fig. 8.3. Quadruple electroweak vector boson vertices

(D) *Sfermion-sfermion-gauge boson vertices*

We shall work in the $\tilde{f}_L$-$\tilde{f}_R$ basis, deferring a discussion of left right sfermion mixing to Ch.9. Again, we can consider two cases, pertaining to strong and electroweak interactions.

(i) **Squark-squark-gluon vertices**
These are the same as the SQCD (vide §5.5) except for the generalization to six diagonal flavors (index p summed on repetition). Then, in the notation of §5.5, we can write

$$\mathcal{L}_{\tilde{q}\tilde{q}g} = -2ig_S A^a_\mu \tilde{q}^\star_p T^a [\partial^\mu] \tilde{q}_p \ ,$$

where the operator $[\partial^\mu]$ is as defined in (4.28). Each vertex is precisely the same as that in Fig. 5.2 with $\tilde{q}$ generalized to $\tilde{q}_p$. As with quarks, neither flavor nor left right mixing in squark mass eigenstates will matter here.

(ii) **Sfermion-sfermion-electroweak vector boson vertices**
Once more, we generalize from the corresponding interactions of SχGT in §5.6 and write (with v $= W, Z, \gamma$ and $Q_{f_{u,d}}$ as electric charge of $f_{u,d}$ in units of the positron charge) the terms containing an electroweak vector boson as follows:

$$\begin{aligned}
\mathcal{L}_{\bar{\tilde{f}}\tilde{f}'\mathrm{v}} &= -i\sqrt{2}g_2\left\{W^+_\mu \tilde{f}^\star_{u_iL}[\partial^\mu]\tilde{f}_{d_iL} + W^-_\mu \tilde{f}^\star_{d_iL}[\partial^\mu]\tilde{f}_{u_iL}\right\} \\
&\quad -2ieA_\mu\left\{q_{f_u}\left(\tilde{f}^\star_{u_iL}[\partial^\mu]\tilde{f}_{u_iL} + \tilde{f}^\star_{u_iR}[\partial^\mu]\tilde{f}_{u_iR}\right) + q_{f_d}\left(\tilde{f}^\star_{d_iL}[\partial^\mu]\tilde{f}_{d_iL} + \tilde{f}^\star_{d_iR}[\partial^\mu]\tilde{f}_{d_iR}\right)\right\} \\
&\quad -\frac{ig_2}{\cos\theta_W}Z_\mu\Big\{\tilde{f}^\star_{u_iL}\left(1-2Q_{f_u}\sin^2\theta_W\right)[\partial^\mu]\tilde{f}_{u_iL} - 2Q_{f_u}\sin^2\theta_W\tilde{f}^\star_{u_iR}[\partial^\mu]\tilde{f}_{u_iR} \\
&\qquad -\tilde{f}^\star_{d_iL}\left(1+2Q_{f_d}\sin^2\theta_W\right)[\partial^\mu]\tilde{f}_{d_iL} - 2Q_{f_d}\sin^2\theta_W\tilde{f}^\star_{d_iR}[\partial^\mu]\tilde{f}_{d_iR}\Big\}, \quad (8.41)
\end{aligned}$$

with the repeated generation index i summed. As done for quarks and leptons, the expressions

$$\tilde{q}_{iL} = \begin{pmatrix}\tilde{f}_{u_iL}\\ \tilde{f}_{d_iL}\end{pmatrix},\ \tilde{u}_{iR} = \tilde{f}_{u_iR},\ \tilde{d}_{iR} = \tilde{f}_{d_iR} \quad (8.42)$$

can be written for squark fields and

$$\tilde{\ell}_{iL} = \begin{pmatrix}\tilde{f}_{u_iL}\\ \tilde{f}_{d_iL}\end{pmatrix},\ \tilde{e}_{iR} = \tilde{f}_{d_iR} \quad (8.43)$$

for slepton ones. For charged current couplings of mass eigenstate squarks, we can, in analogy with (8.40), use left chiral flavor rotation matrix elements $V^{\tilde{q}_L}_{ij}$ in generation space. Here we have used a symbol different[8] from that of the CKM matrix $\mathbf{V}^{q_L}$ to take account of the general situation with supersymmetry breaking which may make $\mathbf{V}^{\tilde{q}_L} \neq \mathbf{V}^{q_L}$. Again, in the coupling of the charged W to two sleptons of different flavor too, to account for different generation dependent masses for charged sleptons and sneutrinos, we put in the left chiral flavor rotation matrix element $V^{\tilde{\ell}_L}_{ij}$ in generation space, though such a matrix element is absent in the leptonic sector. Thus we have

$$\mathcal{L}_{\bar{\tilde{q}}\tilde{q}'W} = -i\sqrt{2}g_2\left\{W^+_\mu\tilde{u}^\star_{iL}V^{\tilde{q}_L}_{ij}[\partial^\mu]\tilde{d}_{jL} + \text{h.c.}\right\}, \quad (8.44\text{a})$$

$$\mathcal{L}_{\bar{\tilde{\ell}}\tilde{\ell}'W} = -i\sqrt{2}g_2\left\{W^+_\mu\tilde{\nu}^\star_{iL}V^{\tilde{\ell}_L}_{ij}[\partial^\mu]\tilde{e}_{jL} + \text{h.c.}\right\}, \quad (8.44\text{b})$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}\gamma} = -2ieA_\mu Q_{\tilde{f}}\left(\tilde{f}^\star_{iL}[\partial^\mu]\tilde{f}_{iL} + \tilde{f}^\star_{iR}[\partial^\mu]\tilde{f}_{iR}\right), \quad (8.44\text{c})$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}Z} = -\frac{ig_2}{\cos\theta_W}Z_\mu\left\{2T^{\tilde{f}}_{3L}\left(1-4T^{\tilde{f}}_{3L}Q_{\tilde{f}}\sin^2\theta_W\right)\tilde{f}^\star_{iL}[\partial^\mu]\tilde{f}_{iL} - 2\sin^2\theta_W Q_{\tilde{f}}\tilde{f}^\star_{iR}[\partial^\mu]\tilde{f}_{iR}\right\}. \quad (8.44\text{d})$$

[8] Of course, in the limit of exact supersymmetry, $\mathbf{V}^{\tilde{q}_L}$ equals $\mathbf{V}^{q_L}$ of (8.40) and $\mathbf{V}^{\tilde{\ell}_L}$ becomes the unit matrix.

Here $\tilde{f}_{i(L,R)}$ covers $\tilde{u}_{i(L,R)}$, $\tilde{d}_{i(L,R)}$, $\tilde{e}_{i(L,R)}$ and $\tilde{\nu}_{iL}$. Eq. (8.44) describes the sfermion-sfermion-EW vector boson couplings with the left chiral or right chiral squark and slepton fields understood as mass eigenstates in the limit of no left right sfermionic mixing. The latter, to be treated in Ch.9, will generate additional complications in these equations except for the photon vertex. We defer an enumeration of the final physical vertices and Feynman rules in this case till that discussion.

(E) *Gauge boson-gaugino-gaugino vertices*

Here also strong and electroweak vertices are distinctly separate.

(1) **Gluon-gluino-gluino vertices**

These are identical to those in SQCD, as discussed in §5.5 (vide Fig. 5.2).

(2) **EW gauge boson-neutralino/chargino-neutralino/chargino vertices**

Even in the supersymmetric limit, these will not be similar to those of SχGT, Fig. 5.3. This is because mass eigenstate charginos and neutralinos will involve combinations of gauginos and higgsinos on account of the breakdown of EW symmetry. Moreover, in reality, supersymmetry breaking has a significant influence. We shall discuss those aspects in detail in Ch.9, and give the final physical vertices there.

(F) *Fermion-sfermion-gaugino vertices*

(1) **Quark-squark-gluino vertex**

In the supersymmetric approximation of neglecting the differences between squark flavor rotations and quark flavor rotations from eigenstates of mass to those of gauge interactions, these vertices will be the same as in SQCD (§5.5, Fig. 5.2). In reality, however, these differences need to be recognized. In a broken supersymmetric world the two flavor rotation matrices will be different. We have already introduced the matrices $\mathbf{U}^{u_{L,R}}$ and $\mathbf{U}^{d_{L,R}}$ in (8.12) for quarks. Let us define analogous flavor rotation matrices $\mathbf{U}^{\tilde{u}_{L,R}}$ and $\mathbf{U}^{\tilde{d}_{L,R}}$ for u- and d-squarks respectively. These take mass diagonal squarks to flavor eigenstate ones. If we define u, d as three component column vectors in color space, then the relevant terms in (5.60) can be generalized to

$$\mathcal{L}_{\bar{q}\tilde{q}\tilde{g}} = -\sqrt{2}g_s\left[\bar{u}_i P_R T^a \tilde{g}^a \left(\mathbf{U}^{u_L^\dagger}\mathbf{U}^{\tilde{u}_L}\right)_{ij}\tilde{u}_{jL} - \tilde{u}^\dagger_{iR}\left(\mathbf{U}^{\tilde{u}_R^\dagger}\mathbf{U}^{u_R}\right)_{ij}\bar{\tilde{g}}^a T^a P_R u_j\right]$$
$$-\sqrt{2}g_s\left[\bar{d}_i P_R T^a \tilde{g}^a \left(\mathbf{U}^{d_L^\dagger}\mathbf{U}^{\tilde{d}_L}\right)_{ij}\tilde{d}_{jL} - \tilde{d}^\dagger_{iR}\left(\mathbf{U}^{\tilde{d}_R^\dagger}\mathbf{U}^{d_R}\right)_{ij}\bar{\tilde{g}}^a T^a P_R d_j\right] + \text{h.c.}, \quad (8.45)$$

i, j being generation indices. The interaction $\mathcal{L}_{\bar{q}\tilde{q}\tilde{g}}$ is obviously the hermitian conjugate of (8.45). These forms are valid in the *absence of left-right mixing* for squarks. The corresponding physical vertices will be given Ch.9 after accounting for the latter.

(2) **Fermion-sfermion-neutralino/chargino vertices**

Once again, we postpone a treatment of these to Ch.9 because of their essential dependence on supersymmetry breaking via both flavor rotations and gaugino higgsino mixings.

(G) *Gauge boson-gauge boson-sfermion-sfermion vertices*

These will be given in three categories since there are mixed strong and electroweak vertices apart from purely strong and purely electroweak ones.

(i) **Gluon-gluon-squark-squark vertex**
Since the two squarks at this four point vertex have the same flavor, the corresponding flavor rotations cancel out. Thus this vertex is exactly the same as in SQCD (§5.5, Fig. 5.2) with a trivial flavor generalization $\tilde{q} \to \tilde{q}_i$.

(ii) **Electroweak vector boson-electroweak vector boson-sfermion-sfermion vertices**
These are present in SχGT and can, therefore, be read off from (5.64) with $\tilde{f}$ generalized to cover three generations of sleptons and squarks. The neutral gauge bosons W^3 and B get transformed to Z and A via (8.7b,c). As before, we work in the $\tilde{f}_L$-$\tilde{f}_R$ basis, neglecting left right sfermion mixing for the moment. The corresponding interaction terms in the Lagrangian density can be written as

$$
\begin{aligned}
&\mathcal{L}_{\tilde{f}\bar{\tilde{f}}'\mathrm{vv}'} = \\
&\frac{g_2^2}{2} W_\mu^+ W^{\mu^-} \left(\tilde{f}^\star_{u_{iL}} \tilde{f}_{u_{iL}} + \tilde{f}^\star_{d_{iL}} \tilde{f}_{d_{iL}} \right) \\
&+ \frac{g_2}{\sqrt{2}} \left(eA_\mu - \frac{g_2 \sin^2\theta_W}{\cos\theta_W} Z_\mu \right) Y_{f_L} \left(\tilde{f}^\star_{u_{iL}} \tilde{f}_{d_{iL}} W^{\mu^+} + \tilde{f}^\star_{d_{iL}} \tilde{f}_{u_{iL}} W^{\mu^-} \right) \\
&+ e^2 A_\mu A^\mu \left\{ Q^2_{fu} \left(\tilde{f}^\star_{u_{iL}} \tilde{f}_{u_{iL}} + \tilde{f}^\star_{u_{iR}} \tilde{f}_{u_{iR}} \right) + Q^2_{\tilde{f}d} \left(\tilde{f}^\star_{d_{iL}} \tilde{f}_{d_{iL}} + \tilde{f}^\star_{d_{iR}} \tilde{f}_{d_{iR}} \right) \right\} \\
&+ \frac{g_2^2}{4\cos^2\theta_W} Z_\mu Z^\mu \Big\{ \tilde{f}^\star_{u_{iL}} \left(1 - 2Q_{f_u} \sin^2\theta_W\right)^2 \tilde{f}_{u_{iL}} + 4Q^2_{f_u} \sin^4\theta_W \tilde{f}^\star_{u_{iR}} \tilde{f}_{u_{iR}} \\
&\qquad + \tilde{f}^\star_{d_{iL}} \left(1 + 2Q_{f_d} \sin^2\theta_W\right)^2 \tilde{f}_{d_{iL}} + 4Q^2_{f_d} \sin^4\theta_W \tilde{f}^\star_{d_{iR}} \tilde{f}_{d_{iR}} \Big\} \\
&+ \frac{g_2 e}{\cos\theta_W} A_\mu Z^\mu \Big[Q_{f_u} \left\{ \tilde{f}^\star_{u_{iL}} \left(1 - 2Q_{f_u} \sin^2\theta_W\right) \tilde{f}_{u_{iL}} - 2Q_{f_u} \sin^2\theta_W \tilde{f}^\star_{u_{iR}} \tilde{f}_{u_{iR}} \right\} \\
&\qquad - Q_{f_d} \left\{ \tilde{f}^\star_{d_{iL}} \left(1 + 2Q_{f_d} \sin^2\theta_W\right) \tilde{f}_{d_{iL}} + 2Q_{f_d} \sin^2\theta_W \tilde{f}^\star_{d_{iR}} \tilde{f}_{d_{iR}} \right\} \Big].
\end{aligned}
\tag{8.46}
$$

A summation over the generation index i is understood. The relations between $\tilde{f}_{u_{iL,R}}$, $\tilde{f}_{d_{iL,R}}$ and the corresponding squark/slepton flavor eigenstate fields are given in (8.42-3). Elements of the CKM type matrix $\mathbf{V}^{\tilde{q}_L}$ arising out of flavor rotations between sfermionic mass and flavor eigenstate fields (cf. 8.44), enter the $\bar{\tilde{f}}\tilde{f}'\gamma W$ and $\bar{\tilde{f}}\tilde{f}'ZW$ interactions but not the[9] $\bar{\tilde{f}}\tilde{f}\gamma\gamma$, $\bar{\tilde{f}}\tilde{f}ZZ$ and $\bar{\tilde{f}}\tilde{f}\gamma Z$ ones. All these can be rewritten in terms of mass eigenstate sfermion fields. They then read

$$\mathcal{L}_{\bar{\tilde{q}}\tilde{q}'\gamma W} = \frac{g_2 e}{3\sqrt{2}} A_\mu \left(\tilde{u}^\dagger_{iL} V^{\tilde{q}_L}_{ij} \tilde{d}_{jL} W^{\mu^+} + \tilde{d}^\dagger_{iL} V^{\tilde{q}^\dagger_L}_{ij} \tilde{u}_{jL} W^{\mu^-} \right), \tag{8.47a}$$

$$\mathcal{L}_{\bar{\tilde{q}}\tilde{q}'ZW} = -\frac{g_2^2 \sin^2\theta_W}{3\sqrt{2}\cos\theta_W} Z_\mu \left(\tilde{u}^\dagger_{iL} V^{\tilde{q}_L}_{ij} \tilde{d}_{jL} W^{\mu^+} + \tilde{d}^\dagger_{iL} V^{\tilde{q}^\dagger_L}_{ij} \tilde{u}_{jL} W^{\mu^-} \right), \tag{8.47b}$$

$$\mathcal{L}_{\bar{\tilde{\ell}}\tilde{\ell}'\gamma W} = -\frac{g_2 e}{\sqrt{2}} A_\mu \left(\tilde{\nu}^\star_{iL} V^{\tilde{\ell}_L}_{ij} \tilde{e}_{jL} W^{\mu^+} + \tilde{e}^\star_{iL} V^{\tilde{\ell}^\dagger_L}_{ij} \tilde{\nu}_{jL} W^{\mu^-} \right), \tag{8.47c}$$

$$\mathcal{L}_{\bar{\tilde{\ell}}\tilde{\ell}'ZW} = \frac{g_2^2 \sin^2\theta_W}{\sqrt{2}\cos\theta_W} Z_\mu \left(\tilde{\nu}^\star_{iL} V^{\tilde{\ell}_L}_{ij} \tilde{e}_{jL} W^{\mu^+} + \tilde{e}^\star_{iL} V^{\tilde{\ell}^\dagger_L}_{ij} \tilde{\nu}_{jL} W^{\mu^-} \right), \tag{8.47d}$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}\gamma\gamma} = e^2 A_\mu Q^2_{\tilde{f}} \left(\tilde{f}^\dagger_{iL} \tilde{f}_{iR} + \tilde{f}^\dagger_{iL} \tilde{f}_{iR} \right) A^\mu, \tag{8.47e}$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}ZZ} = \frac{g_2^2}{4\cos^2\theta_W} Z_\mu \Big\{ \left(1 - 4T^{\tilde{f}}_{3L} Q_{\tilde{f}} \sin^2\theta_W\right)^2 \tilde{f}^\dagger_{iL}\tilde{f}_{iL} + 4Q^2_{\tilde{f}} \sin^4\theta_W \tilde{f}^\dagger_{iR}\tilde{f}_{iR} \Big\} Z^\mu, \tag{8.47f}$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}\gamma Z} = \frac{2g_2 e}{\cos\theta_W} A_\mu Q_{\tilde{f}} \Big\{ T^{\tilde{f}}_{3L} \left(1 - 4T^{\tilde{f}}_{3L} Q_{\tilde{f}} \sin^2\theta_W\right) \tilde{f}^\dagger_{iL}\tilde{f}_{iL} - Q_{\tilde{f}} \sin^2\theta_W \tilde{f}^\dagger_{iR}\tilde{f}_{iR} \Big\} Z^\mu, \tag{8.47g}$$

$$\mathcal{L}_{\bar{\tilde{f}}\tilde{f}WW} = \frac{g_2^2}{2} W^+_\mu W^{\mu^-} \tilde{f}^\dagger_{iL}\tilde{f}_{iL}. \tag{8.47h}$$

In (8.47e–h) $\tilde{f}_{i(L,R)}$ covers[10] $\tilde{u}_{i(L,R)}$, $\tilde{d}_{i(L,R)}$, $\tilde{e}_{i(L,R)}$ and $\tilde{\nu}_{iL}$ which, as in (8.44), are now understood as mass diagonal fields *in the limit of no left right mixing.* Since we have yet to include the left right mixing of sfermions and this will be done in Ch.9, we postpone a listing of the vertices and Feynman rules till then.

(iii) **Electroweak vector boson-gluon-squark-squark vertices**
These mixed terms can be written, with the CKM-type matrix elements $V^{\tilde{q}_L}_{ij}$ put in and with $Q_q = 2/3$ or $-1/3$ for $q = u$ or d respectively, as

$$\begin{aligned}\mathcal{L}_{\tilde{q}\bar{\tilde{q}}'g\mathrm{v}} = {} & \sqrt{2} g_2 g_s A^a_\mu (W^{\mu+} \tilde{u}^\dagger_{iL} T^a V^{\tilde{q}_L}_{ij} \tilde{d}_{jL} + W^{\mu^-} \tilde{d}^\dagger_{iL} T^a V^{\tilde{q}^\dagger_L}_{ij} \tilde{d}_{jL}) \\ & + 2 g_s e Q_q A^\mu A^a_\mu \tilde{q}^\dagger_i T^a \tilde{q}_i \\ & + 2 g_s g_2 (\cos\theta_W)^{-1} Z^\mu A^a_\mu \tilde{q}^\star_i (T^{\tilde{q}}_{3L} - Q_q \sin^2\theta_W) T^a \tilde{q}_i .\end{aligned} \tag{8.48}$$

[9]Sfermion flavor mixings do not matter here because of the GIM-mechanism [8.1].
[10]Read $\tilde{f}^\star$ for $\tilde{f}^\dagger$ in case of sleptons.

In (8.48) g_2, g_s are the $SU(2)_L$, $SU(3)_C$ coupling strengths, as before, while v can be W or Z or γ, and i in $\tilde{q}_i$ is summed over all flavors as well as left and right chiral fields. Here all squark fields are supposed to be mass eigenstates in the limit of no L-R mixing. The corresponding vertices and Feynman rules will be listed in Ch.9 along with the proper L-R mixing factors put in.

(H) *Scalar quartic vertices without Higgs*

These can be picked out from the supersymmetric potential (8.36) and the detailed expressions for the F- and D-terms given in (8.34) and (8.35) respectively. One can then write the corresponding interaction term in the Lagrangian density in the limit of zero left right mixing as follows.

$$\begin{aligned}
\mathcal{L}_{\tilde{f}_1\tilde{f}_2\tilde{f}_3\tilde{f}_4} = \\
&-\frac{g_2^2}{2M_W^2\sin^2\beta}\Big(|\tilde{u}_L^\dagger \mathbf{U}^{\tilde{u}_L^\dagger}\mathbf{U}^{u_L}\mathbf{m}_u^{(D)}\mathbf{U}^{u_R^\dagger}\mathbf{U}^{\tilde{u}_R}\tilde{u}_R|^2 \\
&\qquad +|\tilde{d}_L^\dagger \mathbf{U}^{\tilde{d}_L^\dagger}\mathbf{U}^{u_L}\mathbf{m}_u^{(D)}\mathbf{U}^{u_R^\dagger}\mathbf{U}^{\tilde{u}_R}\tilde{u}_R|^2\Big) \\
&-\frac{g_2^2}{2M_W^2\cos^2\beta}\Big(|\tilde{e}_L^\dagger \mathbf{U}^{\tilde{e}_L^\dagger}\mathbf{m}_e^{(D)}\mathbf{U}^{\tilde{e}_R}\tilde{e}_R+\tilde{d}_L^\dagger \mathbf{U}^{\tilde{d}_L^\dagger}\mathbf{U}^{d_L}\mathbf{m}_d^{(D)}\mathbf{U}^{d_R^\dagger}\mathbf{U}^{\tilde{d}_R}\tilde{d}_R|^2 \\
&\qquad +|\tilde{\nu}^\dagger \mathbf{U}^{\tilde{\nu}^\dagger}\mathbf{m}_e^{(D)}\mathbf{U}^{\tilde{e}_R}\tilde{e}_R+\tilde{u}_L^\dagger \mathbf{U}^{\tilde{u}_L^\dagger}\mathbf{U}^{d_L}\mathbf{m}_d^{(D)}\mathbf{U}^{d_R^\dagger}\mathbf{U}^{\tilde{d}_R}\tilde{d}_R|^2\Big) \\
&-\frac{g_s^2}{4}\Big[\sum_{i,j}\Big(|\tilde{u}_{iL}^\dagger\tilde{u}_{jL}|^2+|\tilde{d}_{iL}^\dagger\tilde{d}_{jL}|^2+|\tilde{u}_{iR}^\dagger\tilde{u}_{jR}|^2+|\tilde{d}_{iR}^\dagger\tilde{d}_{jR}|^2+2|\tilde{u}_{iL}^\dagger\tilde{d}_{jL}|^2-2|\tilde{u}_{iL}^\dagger\tilde{u}_{jR}|^2 \\
&\qquad -2|\tilde{u}_{iL}^\dagger\tilde{d}_{jR}|^2-2|\tilde{d}_{iL}^\dagger\tilde{u}_{jR}|^2-2|\tilde{d}_{iL}^\dagger\tilde{d}_{jR}|^2+2|\tilde{u}_{iR}^\dagger\tilde{d}_{jR}|^2\Big) \\
&\qquad -\frac{1}{3}\Big\{\sum_i\Big(|\tilde{u}_{iL}|^2+|\tilde{d}_{iL}|^2-|\tilde{u}_{iR}|^2-|\tilde{d}_{iR}|^2\Big)\Big\}^2\Big] \\
&-\frac{g_2^2}{8}\Big[\Big\{\sum_i\Big(|\tilde{u}_{iL}|^2-|\tilde{d}_{iL}|^2+|\tilde{\nu}_i|^2-|\tilde{e}_{iL}|^2\Big)\Big\}^2+4|\tilde{u}_L^\dagger\mathbf{V}^{\tilde{q}_L}\tilde{d}_L+\tilde{\nu}^\dagger\mathbf{V}^{\tilde{\ell}_L}\tilde{e}_L|^2\Big] \\
&-\frac{g_2^2\tan^2\theta_W}{8}\Big\{\sum_i\Big(\frac{1}{3}|\tilde{u}_{iL}|^2+\frac{1}{3}|\tilde{d}_{iL}|^2-\frac{4}{3}|\tilde{u}_{iR}|^2+ \\
&\qquad \frac{2}{3}|\tilde{d}_{iR}|^2-|\tilde{\nu}_i|^2-|\tilde{e}_{iL}|^2+2|\tilde{e}_{iR}|^2\Big)\Big\}^2.
\end{aligned} \tag{8.49}$$

In (8.49) i, j are generation indices. Moreover, $\mathbf{m}_e^{(D)}$ is the physical real diagonal charged lepton mass matrix of (8.11) in the generation space; the unitary flavor rotation matrix $\mathbf{U}^{\tilde{e}_R}$ transforms the mass eigenstate charged right slepton fields to the corresponding flavor eigenstate ones, while $\mathbf{U}^{\tilde{e}_L}$ and $\mathbf{U}^{\tilde{\nu}}$ do the same for charged and neutral left slepton fields

$\tilde{e}_L$ and $\tilde{\nu}$ respectively. Similarly, $\mathbf{m}_u^{(D)}$ and $\mathbf{m}_d^{(D)}$ are the real diagonal quark mass matrices of (8.12). Thus we needed to take into account not only the unitary left, right squark flavor rotation matrices $\mathbf{U}^{\tilde{u}_L}$, $\mathbf{U}^{\tilde{u}_R}$, $\mathbf{U}^{\tilde{d}_L}$, $\mathbf{U}^{\tilde{d}_R}$, defined in analogy with $\mathbf{U}^{\tilde{\ell}_{L,R}}$ and $\mathbf{U}^{\tilde{\nu}}$, but also the corresponding ones for quarks, namely $\mathbf{U}^{u_L}$, $\mathbf{U}^{u_R}$, $\mathbf{U}^{d_L}$, $\mathbf{U}^{d_R}$ of (8.12). Finally, $\mathbf{V}^{\tilde{q}_L}$ and $\mathbf{V}^{\tilde{\ell}_L}$ are respectively the left squark and left slepton versions of the Kobayashi-Maskwa matrix $\mathbf{V}^{q_L}$, cf (8.40). Thus

$$\mathbf{V}^{\tilde{q}_L} = \mathbf{U}^{\tilde{u}_L^\dagger}\mathbf{U}^{\tilde{d}_L}, \tag{8.50a}$$

$$\mathbf{V}^{\tilde{\ell}_L} = \mathbf{U}^{\tilde{\nu}^\dagger}\mathbf{U}^{\tilde{e}_L}. \tag{8.50b}$$

These may be called 'super-CKM' matrices. We shall enumerate the physical quartic sfermion vertices in Ch.9 with left right mixing taken into account.

This brings us to the end of our discussion of supersymmetric vertices in the MSSM except for the ones which are significantly affected by supersymmetry breaking parameters as well as L-R mixing and possible gaugino-higgsino mixing. Those will be discussed extensively in the next chapter.

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Chapter 9

SOFT SUPERSYMMETRY BREAKING IN THE MSSM

9.1 The Content of $\mathcal{L}_{\rm SOFT}$

We now turn to that part of $\mathcal{L}_{\rm MSSM}$ through which supersymmetry breaking is explicitly introduced. But first we need to demonstrate [9.1] the impossibility of effecting a spontaneous breakdown of global supersymmetry purely within the framework of the MSSM. We follow the *reductio ad absurdum* procedure in assuming such a spontaneous breaking and applying the supertrace mass sum rule (7.50). Let us separately consider the mass squared matrices M_e^2 for the charge -1, M_u^2 for the charge $+\frac{2}{3}$, M_d^2 for the charge $-\frac{1}{3}$ and M_ν^2 for the neutral matter fermion sfermion supermultiplets of any generation. Assuming charge and color conservation, the RHS of (7.50) now can receive possible contributions from the generators T_3 and $Y/2$ only. We can sum over all possible left and right chiral supermultiplets in the supertrace, except that the latter have to be conjugated since (7.50) has been written for a left chiral supermultiplet. We can then use the results $(T_3)_{e_L} = -\frac{1}{2}$, $(Y/2)_{e_L} = -\frac{1}{2}$, $(T_3)_{e^C_L} = 0$, $(Y/2)_{e^C_L} = 1$, $(T_3)_{u_L} = \frac{1}{2}$, $(Y/2)_{u_L} = \frac{1}{6}$, $(T_3)_{u^C_L} = 0$, $(Y/2)_{u^C_L} = -\frac{2}{3}$, $(T_3)_{d_L} = -\frac{1}{2}$, $(Y/2)_{d_L} = \frac{1}{6}$, $(T_3)_{d^C_L} = 0$, $(Y/2)_{d^C_L} = \frac{1}{3}$, $(T_3)_{\nu_L} = \frac{1}{2}$, $(Y/2)_{\nu_L} = -\frac{1}{2}$. Thus we have[1]

$$S{\rm Tr}M_e^2 \;=\; g_2\langle D^3\rangle - g_Y\langle D^Y\rangle, \tag{9.1a}$$

$$S{\rm Tr}M_u^2 \;=\; -g_2\langle D^3\rangle + g_Y\langle D^Y\rangle, \tag{9.1b}$$

$$S{\rm Tr}M_d^2 \;=\; g_2\langle D^3\rangle - g_Y\langle D^Y\rangle, \tag{9.1c}$$

$$S{\rm Tr}M_\nu^2 \;=\; -g_2\langle D^3\rangle + g_Y\langle D^Y\rangle. \tag{9.1d}$$

Two positive combinations of the above four supertraces are seen to have vanishing RHS, namely

[1] These M's are the mass matrices of (5.10) and (7.50) taken for each generation and summed over left and right chiral supermultiplets. Thus $S{\rm Tr}M_e^2 \equiv S{\rm Tr}M_{e_L}^2 + S{\rm Tr}M_{e_R}^2 = m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 - 2m_e^2$ etc. Though we have not included generation mixing in this argument, (9.2) can be generalized to cover generation space.

$$S\mathrm{Tr}M_e^2 + S\mathrm{Tr}M_\nu^2 = S\mathrm{Tr}M_u^2 + S\mathrm{Tr}M_d^2 = 0 \ . \tag{9.2}$$

Eq. (9.2) can be satisfied only if in each family *some* slepton/squark is lighter than the corresponding fermion. This is manifestly contrary to observation, except possibly for the third squark family. Hence the starting assumption is wrong and, if one sticks to MSSM fields alone, supersymmetry has to be explicitly broken.

In principle, one could introduce an extra $U(1)_{Y'}$ factor in the gauge group in a way such that all left chiral fermionic fields carried the quantum number $Y' = 1$. The $D_{Y'}$-term (cf. 7.50), corresponding to this $U(1)_{Y'}$, could be given a nonzero VEV, spontaneously breaking supersymmetry. (9.2) would now have a nonzero RHS and the compulsion of having some sfermions lighter than the corresponding fermions could be evaded. But then there will be an additional weak neutral gauge boson Z', mixing with the Z, whereas such mixing is now severely constrained by experiment. Moreover, an extra $U(1)_{Y'}$ gauge factor would introduce [9.2] uncancelled ABJ anomalies [9.3] and make the theory nonrenormalizable. A great many extra superfields would be needed to cancel all anomalies and it would be difficult in general to keep all sfermions heavier than extant lower mass bounds. Furthermore, gauginos would not acquire masses at the tree level.

We can then conclude that, though the spontaneous breakdown of supersymmetry is a theoretically desirable feature, such a mechanism will have to involve fields beyond those of the MSSM. Phenomenological constraints point to such fields being significantly heavier than the electroweak scale and hence carrying masses much larger than those of the MSSM sparticles. Much theoretical speculation has taken place so far regarding the specifics of such a mechanism and the current wisdom on it will be elaborated in Chs.12 and 13. Two broad characteristics can, however, be mentioned at this juncture. Spontaneous Supersymmetry Breakdown (SSB) needs to be effected in a sector of fields which are singlets with respect to the SM gauge group and known as the **hidden** or **secluded sector**. SSB can take place there at a distinct scale denoted by Λ_s, say. Supersymmetry breaking is then transmitted to the gauge nonsinglet **observable** or **visible sector** by a messenger sector (associated with a typical mass scale M_M that could, but need not, be as high as the Planck mass M_{Pl}); this may or may not require the introduction of additional gauge nonsinglet messenger superfields. Fig. 9.1 is a cartoon depicting this.

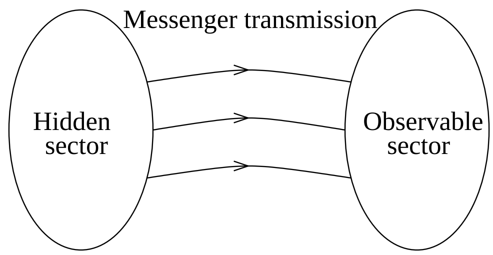

Fig. 9.1. Cartoon showing the transmission of supersymmetry breaking from the hidden to the observable sector.

It is nonetheless true that this messenger scale must be at least two (and perhaps many more) orders of magnitude above the mass of the MSSM fields. Hence, when the former are integrated out at lower (electroweak) energies, the residual theory is described (cf. 7.40) by the supersymmetric Lagrangian density of the MSSM, namely $\mathcal{L}_{SUSY}$ plus some soft explicit supersymmetry breaking terms, collected in $\mathcal{L}_{SOFT}$ and characterized by the supermultiplet splitting mass parameter M_s (cf. Ch.1). In Chs. 12 and 13 we shall discuss in detail two alternative broad scenarios in which the messenger sector consists of

(1) higher dimensional operators [9.4] suppressed by inverse powers of the Planck mass,

or,

(2) fields with gauge interactions [9.5] at lower energy scales.

For (1), the mechanism of Fig. 9.1 can generally proceed at the tree level leading to $M_s \sim \Lambda_s^2/M_{Pl}$. For (2), the origin of M_s may be seen in terms of a one loop supergraph such as that of Fig. 9.2, in which the letters V, M and H refer to superfields in the visible, messenger and hidden sectors respectively, yielding $M_s \sim$ (gauge coupling)^2 Λ_s^2/M_M. The occurrence of the square of Λ_s in the numerator in either case is easy to understand if supersymmetry breaking in the hidden sector arises through the VEV of an auxilary F- or D-field (cf. §7.4–§7.6). Finally, then, a total Lagrangian density of the form of (7.40) can provide a phenomenologically realistic description at least for a range of energies above the EW scale. That will be our starting point here.

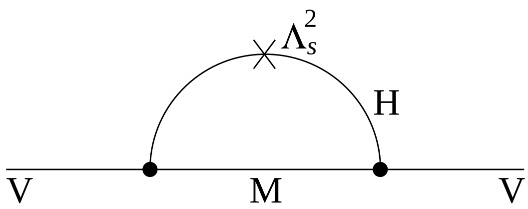

Fig. 9.2. Possible one loop supergraph implementing the scheme of Fig. 9.1.

We wrote the most general form of $\mathcal{L}_{SOFT}$ in (7.42) for a supersymmetric gauge theory. An appraisal of the different terms in it shows that, for the MSSM, $\mathcal{L}_{SOFT}$ can have no C_i-type terms. This is due to the fact that the model does not contain any scalar field that is invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge transformations. All other types of terms, shown in (7.42), are possible. Thus we can write

$$\begin{aligned}
-\mathcal{L}_{\rm SOFT} &= \tilde{q}^\star_{iL}(\mathcal{M}^2_{\tilde q})_{ij}\tilde{q}_{jL} + \tilde{u}^\star_{iR}(\mathcal{M}^2_{\tilde u})_{ij}\tilde{u}_{jR} + \tilde{d}^\star_{iR}(\mathcal{M}^2_{\tilde d})_{ij}\tilde{d}_{jR} + \tilde{\ell}^\star_{iL}(\mathcal{M}^2_{\tilde \ell})_{ij}\ell_{jL} \\
&\quad +\tilde{e}^\star_{iR}(\mathcal{M}^2_{\tilde e})_{ij}\tilde{e}_{jR} + \Big[h_1\cdot\tilde{\ell}_{iL}(f^eA^e)_{ij}\tilde{e}^\star_{jR} + h_1\cdot\tilde{q}_{iL}(f^dA^d)_{ij}\tilde{d}^\star_{jR} \\
&\quad +\tilde{q}_{iL}\cdot h_2(f^uA^u)_{ij}\tilde{u}^\star_{jR} + {\rm h.c.}\Big] + m_1^2|h_1|^2 + m_2^2|h_2|^2 + (B\mu h_1\cdot h_2 + {\rm h.c.}) \\
&\quad +\frac{1}{2}(M_1\bar{\tilde\lambda}_0P_L\tilde\lambda_0 + M_1^\star\bar{\tilde\lambda}_0P_R\tilde\lambda_0) + \frac{1}{2}(M_2\bar{\vec{\tilde\lambda}}P_L\vec{\tilde\lambda} + M_2^\star\bar{\vec{\tilde\lambda}}P_R\vec{\tilde\lambda}) \\
&\quad +\frac{1}{2}(M_3\bar{\tilde g}^aP_L\tilde g^a + M_3^\star\bar{\tilde g}^aP_R\tilde g^a) \\
&\equiv V_{\rm SOFT} + \text{gaugino mass terms.}
\end{aligned} \tag{9.3}$$

In (9.3) $M_{1,2,3}$ are the (generally complex) gaugino (Majorana) mass parameters in the Lagrangian density pertaining to $\tilde{\lambda}_0$, $\tilde{\vec{\lambda}}$ and $\tilde{g}^a$ which are (cf. Ch.8) the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gaugino fields respectively while $m_{1,2}$ are the real Higgs scalar mass parameters. Furthermore, i, j are generation indices with summation implied by repetition. Thus the squared left squark mass $\mathcal{M}^2_{\tilde{q}}$ and the squared right squark masses $\mathcal{M}^2_{\tilde{u}}$, $\mathcal{M}^2_{\tilde{d}}$ as well as those for left sleptons $\mathcal{M}^2_{\tilde{\ell}}$ and those for charged right sleptons $\mathcal{M}^2_{\tilde{e}}$ are all 3×3 hermitian matrices in generation space. The products $f^e A^e, f^d A^d$ and $f^u A^u$, which form coefficients of the trilinear scalar terms in (9.3), are general 3×3 complex matrices in the same space. These are the soft supersymmetry breaking $\mathcal{A}$ terms of (7.43), each written as a product of a superpotential coupling f of (7.41) times an A parameter with the dimension of mass, cf. (7.44). Similarly, we have scaled the coefficient of the $SU(2)_L \times U(1)_Y$-invariant Higgs bilinear term by the supersymmetry invariant Higgsino mass μ. This ensures that the soft supersymmetry breaking parameter B (cf. 7.44) also has the dimension of mass. Note further the absence of any linear term in the Higgs fields, which would have been a $\mathcal{C}$-type term, cf. (7.43). If we allow all the new parameters, introduced in (9.3), to be complex, we would be dealing with some one hundred and twenty four [9.5] unknown real constants of which nineteen were already in the SM and one hundred and five are new. Fortunately, many processes are sensitive only to a small subset of these parameters, at least at the tree level[2]. In fact, in practical calculations in the MSSM (e.g. those for supersymmetry searches at colliders) several simplifying assumptions are usually made in order to drastically reduce the number of these additional parameters to only a handful. The final set of parameters is determined by the specific assumptions made. Different assumptions (usually motivated by different scenarios of supersymmetry breaking) result in different versions of the *Constrained Minimal Supersymmetric Standard Model* (**CMSSM**). Let us remark that, though well motivated, these assumptions do need to be tested in experiments and such tests form an important part of supersymmetry phenomenology at colliders. Of course, once again R_p conservation has been assumed in (9.3). The introduction of R_p violation in the soft supersymmetry breaking part of $\mathcal{L}$, without R_p nonconserving supersymmetric terms present in the superpotential $\mathcal{W}$ of (8.33), generally makes the scalar potential unbounded from below. We shall consider the latter kind of terms in Ch.14, when dealing with extensions of the MSSM.

Yet another issue confronting us is that of phases. As mentioned earlier, many of the new parameters in the part $\mathcal{L}_{SOFT}$ of (9.3) can, in general, be complex in a CP noninvariant theory. Two of these can be chosen to be real by appropriate phase rotations of the fields appearing in $\mathcal{L}_{SOFT}$ without compromising the form of $\mathcal{L}_{SUSY}$ in (7.41). However, many different nontrivial (i.e. in principle measurable) phases remain in the MSSM in addition to the single CP violating phase of the CKM matrix of the SM. On the other hand, some of these phases are subject to strong phenomenological constraints [9.6, 9.7] which come from the lack of observation of any additional, beyond-Standard-Model CP violation in low energy experiments so far. For example, if the phases in the gaugino/higgsino sector are large, effective cancellation mechanisms need to be devised [9.6] to meet those constraints. The

[2]For instance, negative search results from LEP, cf. Ch.15, already imply that both $|M_2|$ and $|\mu|$ must exceed M_W. Herein lies the origin of the μ problem about which we shall have more to say in §13.4 and §14.2.

simplest way to satisfy the experimental bounds on new sources of CP violation is to assume [9.8] that the phases of all soft supersymmetry breaking parameters are small. In fact, most analyses of CP conserving processes in softly broken supersymmetric scenarios have been performed under this assumption. Our phenomenological discussions will be mostly based on such a framework, but the mass matrices and couplings, given in this chapter, allow for the possibility of CP-violating phases.

Once supersymmetry and (at a lower energy scale) EW symmetry get broken, different sparticles of the same electric charge can mix. The sparticles, listed in Table 8.2, then no longer remain eigenstates of mass. Left squarks (sleptons) mix with right squarks (sleptons); there can be generation mixing as well. The EW gauginos and higgsinos mix too, as mentioned in Ch.8. The mixing patterns and mass values of sparticle mass eigenstates depend crucially on the manner of supersymmetry breaking. These masses and mixing angles, in turn, determine the experimental signals of supersymmetry. This is true both for sparticle production as well as decay analyses and for low energy signatures caused by the exchange of virtual sparticles in loops. We therefore need to study all nontrivial restrictions on sparticle mass matrices implied by low energy physics constraints, mainly from the absence [9.8] of FCNC processes in nature. These constraints also play a crucial role in relating softly broken supersymmetry to some higher scale physics which causes the transmission of supersymmetry breaking to the MSSM fields in the observable sector. The mass values of matter sfermions as well as of nonmatter fermions (i.e. gauginos and higgsinos) are controlled by the explicitly supersymmetry breaking soft operators, introduced at this higher scale. One then needs to consider the subsequent modification of these via renormalization group evolution down to electroweak energies. This scale dependence of the mass spectrum of sparticles will be discussed in Ch.11 whereas here we concentrate on the extra masses and mixing angles of the MSSM at laboratory energies. Let us note meanwhile that there is no really satisfactory theory of soft supersymmetry breaking terms at this point; only speculative models exist. Thus low energy constraints are the only phenomenological pointers to them that we have at present and these merit careful attention.

The next section contains a discussion of the masses of higgsinos and electroweak gauginos as well as of the two cases of mixing among them: one for charged ones and another for neutral ones. In subsequent sections we shall consider the general mass matrices for sleptons and squarks incorporating various supersymmetric and nonsupersymmetric mass terms. We shall also address the different cases of mixing among them and what effects these have on their interaction vertices.

9.2 Electroweak Gauginos and Higgsinos

We concentrate here on the spin half supersymmetric partners of the electroweak gauge and Higgs bosons: the electroweak gauginos and higgsinos. While gaugino mass terms are part of the soft supersymmetry breaking $\mathcal{L}_{SOFT}$ of (9.3), the spontaneous symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ forces the gaugino fields $\tilde{\lambda}^{\pm}$ of (8.27) to mix with the higgsino fields $\tilde{h}_i^{\pm}$ of (8.25), leading to physical mass eigenstate charginos $\tilde{\chi}^{\pm}_{1,2}$. This fact, already mentioned in §8.2, will receive our attention first. Similar mixings exist in the sector of neutral EW gauginos and higgsinos and will be discussed later. The soft supersymmetry breaking gaugino mass parameters $M_a (a = 1, 2)$ and the supersymmetry preserving higgsino mass parameter μ of (8.33), plus the ratio $\tan\beta$ of Higgs VEVs, cf. (8.24), are the only parameters of the model that are relevant to our present discussion. In case M_a $(a = 1, 2)$ and μ are complex, then M_2 can be chosen to be real and positive without loss of generality. In this situation two additional parameters enter the game, viz. Φ_μ and Φ_M – the relative phases between M_2 and μ and between M_2 and M_1 respectively. However, in some (though not all) of our discussions below we shall assume these phases to be zero.

The chargino mass matrix

Starting from (5.55), we can isolate in the Lagrangian density the matter-gaugino-Higgs coupling terms that generate chargino masses. They can be written generically in **two component notation** as

$$-\sqrt{2} g_2 (T^a)_{ij} \lambda^a \xi_j \phi_i^\star + \text{h.c.}$$

Here λ^a stands for a gaugino field, while ξ and ϕ stand for the fermionic and bosonic components of a Higgs chiral superfield respectively; T^a is a gauge group generator acting in the representation space of ξ and ϕ typified by indices i, j. Here ξ_i are two component spinorial fields in the $(\frac{1}{2}, 0)$ representation (cf. Ch.3) while their barred versions are the corresponding conjugate fields in the $(0, \frac{1}{2})$ representation. Once the fields $h^0_{1,2}$ of (8.20) acquire VEVs $v_{1,2}$ on the spontaneous breakdown of the EW symmetry, the above expression generates a sum of mixed gaugino and higgsino mass terms. We further need to add to the above the supersymmetry breaking gaugino mass terms from (9.3) and the supersymmetric bilinear higgsino mixing terms contained in the $\mu H_1{\cdot}H_2$ part of the superpotential (8.33). Thus the mass terms of the nonmatter charged fermions can finally be written as

$$\mathcal{L}^c_{MASS} = -\frac{g_2}{\sqrt{2}}(v_1 \lambda^+ \tilde{h}^2_1 + v_2 \lambda^- \tilde{h}^1_2 + \text{h.c.}) - (M_2 \lambda^+ \lambda^- + \mu \tilde{h}^2_1 \tilde{h}^1_2 + \text{h.c.})\ . \tag{9.4}$$

In (9.4) $\tilde{h}^2_1$ and $\tilde{h}^1_2$ are two component spinorial higgsino fields in the $(\frac{1}{2}, 0)$ representation carrying $Y = -1$, $Q = -1$ and $Y = 1$, $Q = 1$ respectively, cf. (8.25). Moreover, the two component charged gaugino fields $\lambda^\pm$ are defined as $(\sqrt{2})^{-1}(\lambda_1 \mp i\lambda_2)$. The mass term of (9.4) can now be rewritten in terms of a 2×2 matrix $\mathbf{X}$ as follows. Define two column vectors $\psi^\pm$, each consisting of one gaugino field component and one higgsino field component, as

$$\psi^+ \equiv \begin{pmatrix} \lambda^+ \\ \tilde{h}^1_2 \end{pmatrix},\ (\psi^+)^T \equiv (\lambda^+\ \tilde{h}^1_2)\ , \tag{9.5a}$$

$$\psi^- \equiv \begin{pmatrix} \lambda^- \\ \tilde{h}^2_1 \end{pmatrix},\ (\psi^-)^T \equiv (\lambda^-\ \tilde{h}^2_1)\ . \tag{9.5b}$$

Let us denote the components of $\psi^\pm$ by $\psi^\pm_m$ with $m = 1, 2$, i.e. $\psi^+_1 = \lambda^+$ etc. Now we can make use of (8.22) and (8.24) to rewrite (9.4) as

$$-\mathcal{L}^c_{MASS} = (\psi^-)^T \mathbf{X} \psi^+ + \text{h.c.}\ , \tag{9.6}$$

with

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}. \tag{9.7}$$

One can find unitary matrices $\mathcal{U}$ and $\mathcal{V}$ such that

$$\mathcal{U}^\star \mathbf{X} \mathcal{V}^{-1} = \mathbf{M}_c^D, \tag{9.8}$$

where $\mathbf{M}_c^D$ is a diagonal matrix with real nonnegative entries $\tilde{M}_1$ and $\tilde{M}_2$. The two component **chargino** mass eigenstate fields can then be identified as

$$\chi_k^+ = \mathcal{V}_{km}\psi_m^+ , \tag{9.9a}$$

$$\chi_k^- = \mathcal{U}_{km}\psi_m^- , \tag{9.9b}$$

with $k = 1, 2$. These two component $\chi^\pm$ fields enable one to recast (9.6) as

$$-\mathcal{L}^c_{MASS} = \chi_k^-(\mathbf{M}_D^c)_{km}\chi_m^+ + \text{h.c.} \tag{9.10}$$

We are now in a position to define four component Dirac chargino fields

$$\tilde{\chi}_1^+ \equiv \begin{pmatrix} \tilde{\chi}_1^+ \\ \overline{\chi_1^-}^T \end{pmatrix}, \tag{9.11a}$$

$$\tilde{\chi}_2^+ \equiv \begin{pmatrix} \tilde{\chi}_2^+ \\ \overline{\chi_2^-}^T \end{pmatrix}. \tag{9.11b}$$

By using (3.28a), the mass term (9.10) can be rewritten in terms of these Dirac chargino fields as

$$-\mathcal{L}^c_{MASS} = \widetilde{M}_1\overline{\tilde{\chi}_1^+}\tilde{\chi}_1^+ + \widetilde{M}_2\overline{\tilde{\chi}_2^+}\tilde{\chi}_2^+. \tag{9.12}$$

By convention, $\tilde{\chi}_1^+$ is chosen to be lighter than $\tilde{\chi}_2^+$, i.e. $\widetilde{M}_1 < \widetilde{M}_2$. $\widetilde{M}_{1,2}$ are actually the positive square roots of the eigenvalues of the matrix $\mathbf{X}^\dagger\mathbf{X}$. From (9.8) we see that

$$(\mathbf{M}_c^D)^2 = \mathcal{V}\mathbf{X}^\dagger\mathbf{X}\mathcal{V}^{-1} = \mathcal{U}^\star\mathbf{X}\mathbf{X}^\dagger(\mathcal{U}^\star)^{-1}, \tag{9.13}$$

i.e. $\mathcal{U}, \mathcal{V}$ are the unitary matrices which diagonalize the hermitian matrices $\mathbf{X}\mathbf{X}^\dagger$ and $\mathbf{X}^\dagger\mathbf{X}$ respectively. For such 2×2 matrices, the eigenvalues and mixing matrices are easy to write down analytically. The squared masses are given by

$$\begin{aligned}\widetilde{M}^2_{2,1} = \frac{1}{2}\Bigg[&|M_2^2| + |\mu^2| + 2M_W^2 \pm \Big\{(|M_2^2| - |\mu^2|)^2 \\ &+ 4M_W^4\cos^2 2\beta + 4M_W^2(|M_2^2| + |\mu^2| + 2\Re e(M_2\mu)\sin 2\beta)\Big\}^{1/2}\Bigg].\end{aligned} \tag{9.14}$$

If the phases of M_2 and μ are ignored, all the entries of $\mathbf{X}$ become real. We work in the convention where M_2 is positive, but μ can have either sign (N.B. $\tan\beta$ is always positive, cf. §10.2). Then the mixing matrices can be written as

$$\mathcal{U} = \mathbf{O}_u , \tag{9.15a}$$

$$\mathcal{V} = \begin{cases} \mathbf{O}_v \text{ for det } \mathbf{X} > 0 , \\ \sigma_3\mathbf{O}_v \text{ for det } \mathbf{X} < 0 , \end{cases} \tag{9.15b}$$

$$\mathbf{O}_{v,u} = \begin{pmatrix} \cos\phi_{v,u} & \sin\phi_{v,u} \\ -\sin\phi_{v,u} & \cos\phi_{v,u} \end{pmatrix}, \tag{9.15c}$$

where

$$\tan 2\phi_u = \frac{2\sqrt{2}M_W(\mu\sin\beta + M_2\cos\beta)}{M_2^2 - \mu^2 - 2M_W^2\cos 2\beta} , \tag{9.16a}$$

$$\tan 2\phi_v = \frac{2\sqrt{2}M_W(\mu\cos\beta + M_2\sin\beta)}{M_2^2 - \mu^2 + 2M_W^2\cos 2\beta} . \tag{9.16b}$$

The corresponding expressions for complex $\mathbf{X}$ can be found in Ref. [9.9]. Eqs. (9.16) are invariant under the change $\phi \to \phi + \pi/2$. However, these solutions are not equivalent. One has to check whether (9.8) holds in order to decide which of the four solutions of (9.16) is the correct one.

It is convenient at this stage to relate the starting two component charged gaugino[3] and higgsino fields to the four component weak interaction eigenstate ones of (8.25) and (8.27):

$$\tilde\lambda^+ = \begin{pmatrix} \lambda^+ \\ \overline{\lambda^-}^T \end{pmatrix}, \tag{9.17a}$$

$$\tilde h^+ = \begin{pmatrix} \tilde h_2^1 \\ \overline{\tilde h_1^2}^T \end{pmatrix}. \tag{9.17b}$$

The relations between these and the four component mass eigenstate chargino fields $\tilde\chi^\pm$ are:

$$P_L\tilde\lambda^+ = \mathcal{V}^\star_{k1}P_L\tilde\chi_k^+ , \tag{9.18a}$$

$$P_R\tilde\lambda^+ = \mathcal{U}_{k1}P_R\tilde\chi_k^+ , \tag{9.18b}$$

$$P_L\tilde h^+ = \mathcal{V}^\star_{k2}P_L\tilde\chi_k^+ , \tag{9.18c}$$

$$P_R\tilde h^+ = \mathcal{U}_{k2}P_R\tilde\chi_k^+ . \tag{9.18d}$$

Using these equations, we can also derive similar relations for the charge conjugate and adjoint spinors,

$$P_R(\tilde\lambda^+)^C = \mathcal{V}_{k1}P_R(\tilde\chi_k^+)^C, \tag{9.19a}$$

$$P_L(\tilde h^+)^C = \mathcal{U}^*_{k2}P_L(\tilde\chi_k^+)^C, \tag{9.19b}$$

$$\overline{\tilde\lambda^+}P_L = \mathcal{U}^\star_{k1}\overline{\tilde\chi_k^+}P_L , \tag{9.19c}$$

$$\overline{\tilde h^+}P_R = \mathcal{V}_{k2}\overline{\tilde\chi_k^+}P_R . \tag{9.19d}$$

Eqs. (9.18,19) and similar relations will prove useful later in deriving the interaction vertices involving various particles/sparticles and charginos.

[3]We should emphasize that our convention on gaugino field components is different from that of Haber and Kane [9.10]. However, our Feynman rules are the same as theirs except that β is the complement of their θ_v. Our $\mathcal{V}$ and $\mathcal{U}$ matrices are the same as the V and U respectively of Gunion and Haber [9.10].

The neutralino mass matrix

Let us now take up the issue of mass eigenstates for neutral non matter fermions. Again, the corresponding mass terms receive contributions from V_{SOFT}, from the superpotential as well as from the matter-gauge-Higgs couplings with the neutral Higgs fields replaced by their VEVs. Retaining only terms relevant to the neutral sector, the mass term in two component notation reads

$$\begin{aligned}\mathcal{L}^n_{MASS} &= -\frac{g_2}{2}\lambda_3\left(v_1\tilde{h}^1_1 - v_2\tilde{h}^2_2\right) + \frac{g_Y}{2}\lambda_0\left(v_1\tilde{h}^1_1 - v_2\tilde{h}^2_2\right) + \mu\tilde{h}^1_1\tilde{h}^2_2 \\ &\quad -\frac{1}{2}M_2\lambda_3\lambda_3 - \frac{1}{2}M_1\lambda_0\lambda_0 + \text{h.c.}\end{aligned} \tag{9.20}$$

In (9.20) we have extended the notation of (9.4) for two component EW charged gaugino and higgsino fields to the corresponding neutral ones. In general, the three mass parameters M_1, M_2 and μ, which determine the neutral nonmatter fermionic mass matrix and the mixing contained therein, are completely arbitrary. However, in simple grand unified theories M_1 and M_2 are related to each other. Such theories predict that $M_1 = M_2$ at the high scale where the gauge couplings are presumed to unify. The gaugino mass M_α will be shown in Ch.11 to evolve (at one loop) with the momentum scale in a way identical to that of the square of the corresponding gauge coupling strength g_α, the subscript α referring to one of the factors of the SM gauge group. The unification condition then implies

$$M_1(M_Z) = \frac{5}{3}\tan^2\theta_W M_2(M_Z) \simeq \frac{1}{2}M_2(M_Z)\ , \tag{9.21}$$

θ_W being the Weinberg angle. As explained more clearly in Ch.11, the factor 5/3 appears in (9.21) from the difference between the normalization of generators in a simple unifying gauge group and that of the electroweak hypercharge generator in the SM.

Define a row vector $(\psi^0)^T$ with two gaugino field components and two higgsino field components:

$$(\psi^0)^T \equiv (\lambda_0\ \lambda_3\ \tilde{h}^1_1\ \tilde{h}^2_2)\ . \tag{9.22}$$

Eq. (9.20) can then be recast as

$$\mathcal{L}^n_{MASS} = -\frac{1}{2}\left(\psi^0\right)^T \mathcal{M}^n\psi^0 + \text{h.c.} \tag{9.23}$$

In (9.23) the 4×4 mass matrix $\mathcal{M}^n$ is given by

$$\mathcal{M}^n = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \tag{9.24}$$

where $s_W \equiv \sin\theta_W, c_W \equiv \cos\theta_W, s_\beta \equiv \sin\beta,\ c_\beta \equiv \cos\beta$ in the notation of Ch.8. Let us denote the components of ψ^0 in (9.22) as ψ^0_n, with $n = 1,2,3,4$, i.e. $\psi^0_1 = \lambda_0$ etc. Now we can define two component neutralino mass eigenstate fields χ^0_l by

$$\chi^0_l = Z_{ln}\psi^0_n\ , \tag{9.25}$$

where $l = 1, 2, 3, 4$ and $\mathbf{Z}$ is a 4×4 unitary matrix, as defined by Gunion and Haber [9.10], satisfying

$$\mathbf{Z}^* \mathcal{M}^n \mathbf{Z}^{-1} = \mathbf{M}_n^D \ , \tag{9.26}$$

$\mathbf{M}_n^D$ being a diagonal matrix with only nonnegative entries. The latter can be computed from

$$(\mathbf{M}_n^D)^2 = \mathbf{Z} \mathcal{M}^{n^\dagger} \mathcal{M}^n \mathbf{Z}^{-1} \ . \tag{9.27}$$

Sometimes, for simplicity of calculation, the possible phases in the entries of $\mathcal{M}^n$ are ignored. Now the rows of $\mathbf{Z}$ can be either purely real or purely imaginary. A common practice in the literature is to choose a real, orthogonal $\mathbf{Z}$. In this case, however, the eigenvalues of M_n^D can sometimes be negative. *Then those neutralino mass eigenstates, which correspond to such negative mass eigenvalues, need to be redefined with chiral rotations so as to make the latter positive.* It is difficult to keep track of this during calculations, since one has to introduce an explicit $i\gamma_5$ factor whenever a neutralino corresponding to a negative eigenvalue of $\mathcal{M}^n$ appears at a vertex. So we shall not make such an assumption. There is one point to be noted, though. In many applications, it is sufficient to keep the sign of the neutralino mass in the neutralino propagator and in neutralino spin sums without any modification of Feynman rules.

As with charginos, the masses and mixing angles of the neutralinos are completely determined in terms of a few parameters; here these are $M_{1,2}$, μ and $\tan\beta$. We can choose to introduce four component Majorana spinorial fields $\tilde{\chi}_l^0$:

$$\tilde{\chi}_l^0 = \begin{pmatrix} \chi_l^0 \\ \overline{\chi}_l^{0T} \end{pmatrix} . \tag{9.28}$$

Now the mass term of (9.23) takes a simple four component Majorana form, namely

$$\mathcal{L}_{MASS}^n = -\frac{1}{2} \sum_l \widetilde{M}_l^n \overline{\tilde{\chi}_l^0} \tilde{\chi}_l^0 \ , \tag{9.29}$$

where $\widetilde{M}_l^n \equiv M_{\tilde{\chi}_l^0}$ are the nonnegative diagonal elements of $\mathbf{M}_n^D$. The eigenvalues $\widetilde{M}_\ell^n$ and the matrix $\mathbf{Z}$ *can most easily be obtained numerically.* If all entries of $\mathcal{M}^n$ are real, an analytical calculation of the former is possible [9.11]. However, the expressions are quite cumbrous and will not be given here. The neutralino eigenstates are labeled[4] in the mass order $M_{\tilde{\chi}_1^0} < M_{\tilde{\chi}_2^0} < M_{\tilde{\chi}_3^0} < M_{\tilde{\chi}_4^0}$ by convention. In most phenomenological discussions of the MSSM (unless there is a lighter gravitino or a violation of R-parity), the lightest neutralino $\tilde{\chi}_1^0$ is assumed to be the Lightest Supersymmetric Particle (LSP).

It is instructive to relate the mass eigenstate neutralino fields $\tilde{\chi}_l^0$ to four component gaugino and higgsino fields which are weak interaction eigenstates. Let us consider the latter first. They are the Majorana spinors

$$\tilde{\lambda}_3 = \begin{pmatrix} \lambda_3, \\ \bar{\lambda}_3^T \end{pmatrix} , \tag{9.30a}$$

[4]**Caution**: the subscripts 1,2,3 in $\tilde{\chi}_{1,2,3}^0$ do not have any specific association with the subscripts of the gaugino mass parameters $M_{1,2,3}$.

$$\tilde{\lambda}_0 = \begin{pmatrix} \lambda_0 \\ \bar{\lambda}_0^T \end{pmatrix}, \tag{9.30b}$$

$$\tilde{h}_1^0 = \begin{pmatrix} \tilde{h}_1^1 \\ \overline{\tilde{h}_1^1}^T \end{pmatrix}, \tag{9.30c}$$

$$\tilde{h}_2^0 = \begin{pmatrix} \tilde{h}_2^2 \\ \overline{\tilde{h}_2^2}^T \end{pmatrix}. \tag{9.30d}$$

Then the desired relations can be given as follows:

$$P_L\tilde{\lambda}_0 = P_L Z_{l1}^* \tilde{\chi}_l^0 \ , \tag{9.31a}$$

$$P_R\tilde{\lambda}_0 = P_R Z_{l1} \tilde{\chi}_l^0 \ , \tag{9.31b}$$

$$P_L\tilde{\lambda}_3 = P_L Z_{l2}^* \tilde{\chi}_l^0 \ , \tag{9.31c}$$

$$P_R\tilde{\lambda}_3 = P_R Z_{l2} \tilde{\chi}_l^0 \ , \tag{9.31d}$$

$$P_L\tilde{h}_s^0 = P_L Z_{l,s+2}^* \tilde{\chi}_l^0 \ , \tag{9.31e}$$

$$P_R\tilde{h}_s^0 = P_R Z_{l,s+2} \tilde{\chi}_l^0 \ . \tag{9.31f}$$

Note that the index l in (9.31) spans the values $1, 2, 3, 4$, while the index s covers $1, 2$ only. Similar relations can be written for $\overline{\tilde{\lambda}^0} P_L$ etc. using (9.31).

We can study and comment on the nature of the chargino and neutralino sectors in some limiting cases. If $|\mu| \gg |M_{1,2}| \gg M_Z$, the two lightest neutralinos $\tilde{\chi}_{1,2}^0$ are gaugino dominated. **If (9.21) is assumed**, it follows that $\tilde{\chi}_1^0$ is mostly the $U(1)_Y$ gaugino ("bino") field $\tilde{\lambda}_0$ and $\tilde{\chi}_2^0$ is largely the neutral $SU(2)_L$ gaugino ("wino") $\tilde{\lambda}^3$. The two higher mass neutralinos $\tilde{\chi}_{3,4}^0$ are then predominantly higgsinos. Similarly, the lighter chargino $\tilde{\chi}_1^\pm$ is more or less the charged "wino" and the heavier chargino is largely the charged higgsino. Furthermore, the magnitude of the μ parameter and the masses of the chargino and neutralino masses are roughly related by $M_{\tilde{\chi}_1^\pm} \simeq M_{\tilde{\chi}_2^0} \simeq 2M_{\tilde{\chi}_1^0}$ and $|\mu| \simeq M_{\tilde{\chi}_3^0} \simeq M_{\tilde{\chi}_4^0} \simeq M_{\tilde{\chi}_2^\pm} \gg M_{\tilde{\chi}_1^\pm}$. In the opposite limit $|\mu| \ll |M_{1,2}|$, the lighter neutralinos and the lighter chargino are mostly higgsinos with masses close to $|\mu|$, whereas the heavier chargino is predominantly the charged "wino". Finally, when $|\mu| \simeq |M_2|$ or $|M_1|$, strong kinds of mixing occur between gauginos and higgsinos in the formation of physical nonmatter fermions; in general, the masses are no longer related in any simple way. If $|\mu, M_2| \gg M_Z$ and (9.21) is assumed, then the approximate relation $M_{\tilde{\chi}_2^0} \simeq M_{\tilde{\chi}_1^\pm}$ holds [9.12] irrespective of the ordering and the relative magnitudes of $|\mu|$ and $|M_{1,2}|$. All these statements are insensitive to variations in $\tan\beta$ within the usually covered range (cf. Chs. 10 and 11).

9.3 Chargino and Neutralino Interactions with Gauge Bosons

Chargino-Neutralino-$W^\pm$ interactions

These receive contributions from two sources: (1) the analog of the fourth term in the RHS of (5.55) corresponding to the $SU(2)_L$ gauge group and (2) the analog of the first term, for the two Higgs superfields, for the $SU(2)_L \times U(1)$ gauge group. It is clear that only the gauge field part of the covariant derivative will contribute to the interaction. With I, J as gauge group representation indices and subscript s $(= 1, 2)$ distinguishing the two higgsino two component spinors, the latter reads,

$$-\overline{\tilde{h}_{sI}}\, \bar{\sigma}^\mu \left(\frac{g_Y}{2} Y_{h_s} \delta_{IJ} B_\mu + \frac{g_2}{2}(\tau^a)_{IJ} W^a_\mu\right) \tilde{h}_{sJ} \ .$$

The resulting charged weak boson terms in the Lagrangian density, expressed in the four component notation and in the weak basis after using (3.28c,d), read:

$$\mathcal{L}_{\tilde{\chi}^\pm_k \tilde{\chi}^0_l W^\pm} = g_2 W^-_\mu \left[\overline{\tilde{\lambda}}_3 \gamma^\mu \tilde{\lambda}^+ - \frac{1}{\sqrt{2}}\left(\overline{\tilde{h}^0_2}\gamma^\mu P_L \tilde{h}^+ - \overline{\tilde{h}^0_1}\gamma^\mu P_R \tilde{h}^+\right)\right] + \text{h.c.} \qquad (9.32)$$

One can rewrite the interaction (9.32) in terms of chargino and neutralino fields by using the chargino and neutralino mixing matrices using (9.17,18) and (9.30,31). The final expression is

$$\mathcal{L}_{\tilde{\chi}^\pm_k \tilde{\chi}^0_l W^\pm} = g_2 W^-_\mu \overline{\tilde{\chi}^0_l} \gamma^\mu \left(C^L_{lk} P_L + C^R_{lk} P_R\right) \tilde{\chi}^+_k + \text{h.c.} \ , \qquad (9.33)$$

where the couplings C^L_{lk} and C^R_{lk} are given by

$$C^L_{lk} = -\frac{1}{\sqrt{2}} Z_{l4} \mathcal{V}^*_{k2} + Z_{l2} \mathcal{V}^*_{k1} \ , \qquad (9.34a)$$

$$C^R_{lk} = \frac{1}{\sqrt{2}} Z^*_{l3} \mathcal{U}_{k2} + Z^*_{l2} \mathcal{U}_{k1} \ . \qquad (9.34b)$$

In (9.33) and (9.34) the subscript k takes values $1, 2$ while l goes from 1 to 4. The generic vertex corresponding to (9.33) is shown in Fig. 9.3. Note that an arrow has been put on the Majorana fermion line also in accordance with the convention in Appendix D of the first paper of Ref. [9.10].

Fig. 9.3 is included in Appendix A

Neutralino-Neutralino-Z and Chargino-Chargino-(Z, γ) interactions

In the four component basis of (9.17) and (9.30), after using the Majorana identities (3.29c,d) and the definitions (8.7), we can write in analogy with the previous case

$$\begin{aligned}\mathcal{L}_{Z(\gamma)\tilde{\chi}\tilde{\chi}} &= \frac{g_2}{c_W} Z_\mu \left(-c_W^2 \overline{\tilde{\lambda}^+} \gamma^\mu \tilde{\lambda}^+ - \frac{1}{2} \cos 2\theta_W \ \overline{\tilde{h}^+} \gamma^\mu \tilde{h}^+ \right) \\ &+ \frac{g_2}{4c_W} Z_\mu \left(\overline{\tilde{h}_1^0} \gamma^\mu \gamma_5 \tilde{h}_1^0 - \overline{\tilde{h}_2^0} \gamma^\mu \gamma_5 \tilde{h}_2^0 \right) \\ &- e A_\mu \left(\overline{\tilde{\lambda}^+} \gamma^\mu \tilde{\lambda}^+ + \overline{\tilde{h}^+} \gamma^\mu \tilde{h}^+ \right) . \end{aligned} \tag{9.35}$$

The second line of (9.35), when rewritten in terms of the mass eigenstates $\tilde{\chi}_\ell^0$, yields the $Z\tilde{\chi}_l^0\tilde{\chi}_n^0$ interaction. The use of (9.31) leads to

$$\mathcal{L}_{Z\tilde{\chi}_l^0\tilde{\chi}_n^0} = \frac{g_2}{2c_W} Z_\mu \overline{\tilde{\chi}_l^0} \gamma^\mu \left(N_{ln}^L P_L + N_{ln}^R P_R \right) \tilde{\chi}_n^0 \ . \tag{9.36}$$

Here the couplings $N_{ln}^{L,R}$ are given by

$$N_{ln}^L = -\frac{1}{2} Z_{l3} Z_{n3}^* + \frac{1}{2} Z_{l4} Z_{n4}^* \ , \tag{9.37a}$$

$$N_{ln}^R = -\left(N_{ln}^L \right)^* . \tag{9.37b}$$

Referring back to (9.27), note that, under the assumption of a real $\mathcal{M}^n$, the $Z\tilde{\chi}_l^0\tilde{\chi}_{l'}^0$ interaction will always involve a pure vector (axial vector) coupling, for a negative (positive) value of $\cos[2\mathrm{Arg}(Z_{ln}Z_{l'n})]$. In this situation the cosine is just a signature factor.

Turning to charginos, the last term in the RHS of (9.35) gives the $\gamma\tilde{\chi}_k^+\tilde{\chi}_k^-$ interaction as

$$\begin{aligned}\mathcal{L}_{\gamma\tilde{\chi}_k^-\tilde{\chi}_k^+} &= -e A_\mu \Big[\overline{\tilde{\chi}_m^+} \gamma^\mu \left(\mathcal{V}_{m1} \mathcal{V}_{k1}^* + \mathcal{V}_{m2} \mathcal{V}_{k2}^* \right) P_L \\ &\quad + \overline{\tilde{\chi}_m^+} \gamma^\mu \left(\mathcal{U}_{m1}^* \mathcal{U}_{k1} + \mathcal{U}_{l2}^* \mathcal{U}_{k2} \right) P_R \Big] \tilde{\chi}_k^+ \\ &= -e A_\mu \overline{\tilde{\chi}_k^+} \gamma^\mu \tilde{\chi}_k^+ \ , \end{aligned} \tag{9.38}$$

where we have used $\mathcal{U}^\dagger \mathcal{U} = \mathcal{V}^\dagger \mathcal{V} = \mathbb{1}$. Finally, the $Z\tilde{\chi}_m^-\tilde{\chi}_k^+$ interaction follows from the first RHS term of (9.35). Rewritten in terms of the mixing angles, it reads

$$\mathcal{L}_{Z\tilde{\chi}_m^-\tilde{\chi}_k^+} = \frac{g_2}{c_W} Z_\mu \overline{\tilde{\chi}_m^+} \gamma^\mu \left(O_{mk}^L P_L + O_{mk}^R P_R \right) \tilde{\chi}_k^+ \ , \tag{9.39}$$

with the couplings $O_{mk}^{L,R}$ given by

$$O_{mk}^L = -\mathcal{V}_{m1} \mathcal{V}_{k1}^* - \frac{1}{2} \mathcal{V}_{m2} \mathcal{V}_{k2}^* + \delta_{mk} s_W^2 \ , \tag{9.40a}$$

$$O_{mk}^R = -\mathcal{U}_{m1}^* \mathcal{U}_{k1} - \frac{1}{2} \mathcal{U}_{m2}^* \mathcal{U}_{k2} + \delta_{mk} s_W^2 \ . \tag{9.40b}$$

The unitarity properties of the $\mathcal{V}, \mathcal{U}$ matrices have again been used in deriving (9.39). The vertices corresponding to (9.36), (9.38) and (9.39) are given in Fig. 9.4. Those corresponding to (9.36) have an additonal factor of 2 in the Feynman rules [9.10] which appears due to $\tilde{\chi}_l^0$ being Majorana fermions. Once again, we have put arrows [9.10] on lines corresponding to the latter.

Fig. 9.4 is included in Appendix A

9.4 Masses and Mixing Patterns of Sfermions

Slepton mass terms

There are three sources of slepton mass terms in the Lagrangian density: 1) explicit mass terms as well as trilinear A-terms from the soft part of the scalar potential V_{SOFT}, cf. (9.3), 2) the contribution to the scalar potential by the F-terms of (8.34), which arise out of the superpotential $\mathcal{W}$ of (8.33) and 3) the contribution to the scalar potential from the D-terms given by (8.35). The F- and D-contributions, as well as those from the trilinear terms in V_{SOFT}, materialize after the neutral Higgs fields acquire nonvanishing VEVs on the spontaneous breakdown of the $SU(2)_L \times U(1)_Y$ symmetry. On the other hand, each sfermion mass term in V_{SOFT} is invariant under $SU(2)_L \times U(1)_Y$ transformations. If all sfermions are heavier than the electroweak gauge bosons, as indicated by present null search experiments, their large masses could be due to these terms. The pieces in the sfermion mass terms due to trilinear scalar couplings, as well as the terms which orginate from the higgsino mass term in the superpotential $\mathcal{W}$, mix the left and right sleptons $\tilde{e}_{iR}$ and $\tilde{e}_{jL}$. Depending upon the nature of V_{SOFT}, there can also be generation mixing for charged sleptons. However, we will show later that, under some simple assumptions about V_{SOFT}, one can often neglect some of the generation mixing in the slepton sector, once one has imposed the restrictions implied by strong experimental limits that exist on the nonconservation of lepton flavor.

The relevant terms in V (and hence in $-\mathcal{L}$), which contribute to slepton masses, can be written, using (9.3),(8.33)-(8.35) and (8.36), as

$$V^{\tilde{\ell}} = V^{\tilde{\ell}}_{SOFT} + V^{\tilde{\ell}}_F + V^{\tilde{\ell}}_D \ . \tag{9.41}$$

The different terms in the RHS of (9.41) can be shown, with repeated indices summed, as follows:

$$V^{\tilde{\ell}}_{SOFT} = \tilde{\ell}^*_{iL}(\mathcal{M}^2_{\tilde{\ell}})_{ij}\tilde{\ell}_{jL} + \tilde{e}^\star_{iR}(\mathcal{M}^2_{\tilde{e}})_{ij}\tilde{e}_{jR} + \left[h_1 \cdot \tilde{\ell}_{iL}(f^e A^e)_{ij}\tilde{e}^\star_{jR} + \text{h.c.}\right], \tag{9.42a}$$

$$\begin{aligned} V^{\tilde{\ell}}_F &= \left|\mu^\star h_2^- - \tilde{\nu}^\star_i f^{e\star}_{ij}\tilde{e}_{jR}\right|^2 + \left|\mu^\star h_2^{0\star} - \tilde{e}^\star_{iL} f^{e\star}_{ij}\tilde{e}_{jR}\right|^2 \\ &\quad + \sum_i \left|f^e_{ji} h_1 \cdot \tilde{\ell}_{jL}\right|^2 + f^e_{ij} f^{e*}_{ij'}\tilde{e}^\star_{jR}\tilde{e}_{j'R}\left(|h^0_1|^2 + h^+_1 h^-_1\right), \end{aligned} \tag{9.42b}$$

$$\begin{aligned} V^{\tilde{\ell}}_D &= \frac{1}{4} g^2_Y(|h_1|^2 - |h_2|^2)\sum_i \left(|\tilde{\ell}_{iL}|^2 - 2|\tilde{e}_{iR}|^2\right) \\ &\quad + \frac{1}{4} g^2_2 \left(h^\dagger_1 \vec{\tau} h_1 + h^\dagger_2 \vec{\tau} h_2\right) \tilde{\ell}^\dagger_{iL}\vec{\tau}\tilde{\ell}_{iL} \ . \end{aligned} \tag{9.42c}$$

When the neutral Higgs fields acquire vacuum expectation values, as per (8.21), (9.42) lead to the following mass terms in the Lagrangian density.

$$\begin{aligned} -\mathcal{L}^{\tilde{\ell}}_m &= \tilde{\nu}^*_i \left(\mathcal{M}^2_{\tilde{\ell}} + M^2_Z \cos 2\beta\ (1/2)\mathbb{1}\right)_{ij} \tilde{\nu}_j \\ &\quad + \tilde{e}^*_{iL}\left[\mathcal{M}^2_{\tilde{\ell}} - M^2_Z \cos 2\beta\ (1/2 - \sin^2\theta_W)\mathbb{1} + m^2_{e_i}\mathbb{1}\right]_{ij} \tilde{e}_{jL} \\ &\quad + \tilde{e}^*_{iR}\left(\mathcal{M}^2_{\tilde{e}} - M^2_Z \cos 2\beta \sin^2\theta_W \mathbb{1} + m^2_{e_i}\mathbb{1}\right)_{ij}\tilde{e}_{jR} \\ &\quad - \left[\tilde{e}^*_{iL}(m_{e_i} A^{e*}_{ij} + m_{e_i}\delta_{ij}\mu\tan\beta)\tilde{e}_{jR} + \text{h.c.}\right]. \end{aligned} \tag{9.43}$$

In writing the above mass term, we have absorbed the electroweak couplings and VEVs $v_{1,2}$ in M_Z, β and θ_W; m_{e_i} stands for the mass of the charged lepton e_i (cf. 8.23) and $\mathbb{1}_{ij} = \delta_{ij}$. Moreover, we have used (8.23a) for f^e_{ij}. The choice of the signs in front of the $f^e A^e$ etc. terms in (9.3) was made in accordance with the convention established in §7.7 and determines the sign of the A^e-term in (9.43). The opposite signs of the terms proportional to $M_Z^2 \cos 2\beta \sin^2\theta_W$ in the second and third lines of (9.43) are noteworthy. The coefficient of this term is essentially decided by the electric charge of the slepton field. The chiral superfield $\bar{E}_i$ contains $\tilde{e}^*_{iR}$ and hence carries the electric charge of the positron, unlike L_i containing $\tilde{e}_{iL}$ with the opposite charge. Furthermore, the term containing $M_Z^2 \cos 2\beta$ is proportional to T^L_{3f} and hence changes sign between the left selectron and the sneutrino. Clearly, the states $\tilde{e}_{iL}$ $\tilde{e}^*_{iR}$ and $\tilde{\nu}_i$, which appear in (9.43), are the interaction eigenstates; the corresponding mass eigenstates will be linear combinations of these. In principle, both lepton flavor mixing[5] as well as L-R mixing are now possible.

Squark mass terms

The supersymmetric and nonsupersymmetric mass terms for squark fields can be written in a manner analogous to that for slepton ones with the correspondence $\tilde{\ell}_L \to \tilde{q}_L$, $\tilde{\nu}_i \to \tilde{u}_{iL}$, $\tilde{e}_{iL,R} \to \tilde{d}_{iL,R}$. Just the additional singlet fields $\tilde{u}_{iR}$, that are present, need to be included. Moreover, the nontrivial CKM mixing, present in the quark sector, needs to be taken into account. Expressions similar to those appearing in (9.42) can be written for the squark scalar potential. In the following we first write the relevant part of the squark scalar potential which will contribute to squark masses as

$$V^{\tilde{q}} = V^{\tilde{q}}_{SOFT} + V^{\tilde{q}}_F + V^{\tilde{q}}_D \,, \tag{9.44}$$

without any specific assumptions about the supersymmetry breaking parametric matrices A^d, A^u. We then have

$$\begin{aligned} V^{\tilde{q}}_{SOFT} &= \tilde{q}^\dagger_{iL}(\mathcal{M}^2_{\tilde{q}})_{ij}\tilde{q}_{jL} + \tilde{d}^\dagger_{iR}(\mathcal{M}^2_{\tilde{d}})_{ij}\tilde{d}_{jR} + \tilde{u}^\dagger_{iR}(\mathcal{M}^2_{\tilde{u}})_{ij}\tilde{u}_{jR} \\ &\quad + \Big[h_1\cdot\tilde{q}_{iL}(f^dA^d)_{ij}\tilde{d}^\star_{jR} + \tilde{q}_{iL}\cdot h_2(f^uA^u)_{ij}\tilde{u}^\star_{jR} + \text{h.c.}\Big]\,, && (9.45\text{a}) \\ V^{\tilde{q}}_F &= \Big|\mu^\star h_2^- - \tilde{u}^\dagger_{iL}f^{d\star}_{ij}\tilde{d}_{jR}\Big|^2 + \Big|\mu^\star h_2^{0\star} - \tilde{d}^\dagger_{iL}f^{d\star}_{ij}\tilde{d}_{jR}\Big|^2 + \sum_i \big|f^d_{ji}h_1\cdot\tilde{q}_{jL}\big|^2 \\ &\quad + \Big|-\mu^\star h_1^+ + \tilde{d}^\dagger_{iL}f^u_{ij}\tilde{u}_{jR}\Big|^2 + \Big|-\mu^\star h_1^{0\star} + \tilde{u}^\dagger_{iL}f^{u\star}_{ij}\tilde{u}_{jR}\Big|^2 + \sum_i \big|f^{u\star}_{ji}h_2\cdot\tilde{q}_{jL}\big|^2 \\ &\quad + \sum_i \Big|f^{d\star}_{ij}h_1^{0\star}\tilde{d}_{jR} - f^{u\star}_{ij}h_2^-\tilde{u}_{jR}\Big|^2 + \sum_i \Big|f^{d\star}_{ij}h_1^+\tilde{d}_{jR} - f^{u\star}_{ij}h_2^{0\star}\tilde{u}_{jR}\Big|^2\,, && (9.45\text{b}) \\ V^{\tilde{q}}_D &= \frac{1}{4}g_Y^2\left(|h_1|^2 - |h_2|^2\right)\Big[-\frac{1}{3}\tilde{q}^\dagger_{iL}\tilde{q}_{iL} + \sum_i 2\Big(Q_u|\tilde{u}_{iR}|^2 + Q_d|\tilde{d}_{iR}|^2\Big)\Big] \\ &+ \frac{1}{4}g_2^2\big(h_1^\dagger\vec{\tau}h_1 + h_2^\dagger\vec{\tau}h_2\big)\cdot\tilde{q}^\dagger_{iL}\vec{\tau}\tilde{q}_{iL}\,, && (9.45\text{c}) \end{aligned}$$

[5]There are currently some scenarios, going beyond the MSSM, which anticipate a large $\tilde{\nu}_\mu$-$\tilde{\nu}_\tau$ mixing in analogy with what is observed in the ν_μ-ν_τ sector by the super-Kamiokande experiment.

where $Q_{u,d}$ are the electric charges of the u, d-type squarks in units of the positron charge. From (9.49) the squark mass terms in the Lagrangian density can be written as

$$\begin{aligned}
-\mathcal{L}_m^{\tilde{q}} \;=\;& \tilde{u}_{iL}^* \left[\mathcal{M}_{\tilde{q}}^2 + M_Z^2 \cos 2\beta\ (1/2 - Q_u \sin^2\theta_W)\mathbb{1} + (\mathbf{m_u}\mathbf{m_u}^\dagger)\right]_{ij} \tilde{u}_{jL} \\
&+ \tilde{d}_{iL}^* \left[\mathcal{M}_{\tilde{q}}^2 - M_Z^2 \cos 2\beta\ (1/2 + Q_d \sin^2\theta_W)\mathbb{1} + (\mathbf{m_d}\mathbf{m_d}^\dagger)\right]_{ij} \tilde{d}_{jL} \\
&+ \tilde{u}_{iR}^* \left[\mathcal{M}_{\tilde{u}}^2 + Q_u M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1} + (\mathbf{m_u}^\dagger \mathbf{m_u})\right]_{ij} \tilde{u}_{jR} \\
&+ \tilde{d}_{iR}^* \left[\mathcal{M}_{\tilde{d}}^2 + Q_d M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1} + (\mathbf{m}_d^\dagger \mathbf{m}_d)\right]_{ij} \tilde{d}_{jR} \\
&- \tilde{u}_{iL}^* \left[(\mathbf{m}_u A^{u^*})_{ij} + \mu(\mathbf{m}_u)_{ij} \cot\beta\right] \tilde{u}_{jR} + \text{h.c.} \\
&- \tilde{d}_{iL}^* \left[(\mathbf{m}_d A^{d^*})_{ij} + \mu(\mathbf{m}_d)_{ij} \tan\beta\right] \tilde{d}_{jR} + \text{h.c.}
\end{aligned} \quad (9.46)$$

In (9.50) $\mathbf{m}_u$ and $\mathbf{m}_d$ are the up and down type quark mass matrices respectively in generation space (cf. 8.11). One may note that , just as with sleptons, the squarks are massive even in the limit of unbroken $SU(2)_L \times U(1)_Y$ symmetry. Once more, there is a relative negative sign between the mass terms for left squarks and right squarks for the pieces proportional to the charge Q_u and Q_d. The mixing between the left and right squark fields, given in the last two RHS terms, is caused by the trilinear A-terms as well as by the higgsino mass contribution to the F-terms. Because of extant mixing in the quark sector, both L-R mixing and generation mixing are nontrivial and complicated for squarks.

Sfermion mixing: some generalities

Let us define a six component vector

$$\tilde{\mathbf{f}} = \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \quad (9.47)$$

where $\tilde{f}_L, \tilde{f}_R$ are each a three component column vector in generation space with components $\tilde{f}_{iL}, \tilde{f}_{iR}, \tilde{f}$ being the superpartner of any matter fermion field f, quark or lepton. Thus $\tilde{f}$ can be $\tilde{\nu}, \tilde{e}, \tilde{u}, \tilde{d}$ except that we put $\tilde{\nu}_R = 0$. The general squared mass matrix for such sfermions can then be written as a 2×2 Hermitian matrix of 3×3 blocks in the space spanned by the vector of (9.47):

$$\mathcal{M}_{\tilde{\mathbf{f}}}^2 = \begin{pmatrix} \mathcal{M}_{\tilde{f}_{LL}}^2 & \mathcal{M}_{\tilde{f}_{LR}}^2 \\ \mathcal{M}_{\tilde{f}_{LR}}^{2\dagger} & \mathcal{M}_{\tilde{f}_{RR}} \end{pmatrix}. \quad (9.48)$$

In (9.48) $\mathcal{M}_{\tilde{f}_{LL}}^2$ and $\mathcal{M}_{\tilde{f}_{RR}}^2$ are hermitian in generation space. Now all the sfermion mass terms of (9.43) and (9.46) can be collected under

$$-\mathcal{L}_{SFERMION\ MASS} = \sum_{\tilde{\mathbf{f}}} \tilde{\mathbf{f}}^\dagger \mathcal{M}_{\tilde{\mathbf{f}}}^2 \tilde{\mathbf{f}}\ . \quad (9.49)$$

Specifically, for sneutrinos, charged sleptons, u-squarks and d-squarks, we can respectively write from (9.43) and (9.46) the 6×6 squared mass matrices in terms of 3×3 submatrix blocks as

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} \mathcal{M}^2_{\tilde{\ell}} + M_Z^2 T_{3L}^{\tilde{\nu}} \cos 2\beta \; \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix} , \tag{9.50a}$$

$$\mathcal{M}^2_{\tilde{\mathbf{e}}} =$$
$$\begin{pmatrix} \mathcal{M}^2_{\tilde{\ell}} + M_Z^2 (T_{3L}^{\tilde{e}} - Q_e \sin^2\theta_W) \cos 2\beta \; \mathbb{1} + \mathbf{m}_e \mathbf{m}_e^\dagger & -\mathbf{m}_e (A^{e^*} + \mu \tan\beta) \\ -(A^{e^T} + \mu^\star \tan\beta) \mathbf{m}_e^\dagger & \mathcal{M}^2_{\tilde{e}} + Q_e M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1} + \mathbf{m}_e^\dagger \mathbf{m}_e \end{pmatrix} , \tag{9.50b}$$

$$\mathcal{M}^2_{\tilde{\mathbf{u}}} =$$
$$\begin{pmatrix} \mathcal{M}^2_{\tilde{q}} + M_Z^2 (T_{3L}^{\tilde{u}} - Q_u \sin^2\theta_W) \cos 2\beta \; \mathbb{1} + \mathbf{m}_u \mathbf{m}_u^\dagger & -\mathbf{m}_u (A^{u^\star} + \mu \cot\beta) \\ -(A^{u^T} + \mu^\star \cot\beta) \mathbf{m}_u^\dagger & \mathcal{M}^2_{\tilde{u}} + Q_u M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1} + \mathbf{m}_u^\dagger \mathbf{m}_u \end{pmatrix} , \tag{9.50c}$$

$$\mathcal{M}^2_{\tilde{\mathbf{d}}} =$$
$$\begin{pmatrix} \mathcal{M}^2_{\tilde{q}} + M_Z^2 (T_{3L}^{\tilde{d}} - Q_d \sin^2\theta_W) \cos 2\beta \; \mathbb{1} + \mathbf{m}_d \mathbf{m}_d^\dagger & -\mathbf{m}_d (A^{d^*} + \mu \tan\beta) \\ -(A^{d^T} + \mu^\star \tan\beta) \mathbf{m}_d^\dagger & \mathcal{M}^2_{\tilde{d}} + Q_d M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1} + \mathbf{m}_d^\dagger \mathbf{m}_d \end{pmatrix} . \tag{9.50d}$$

In (9.50) $T_{3L}^{\tilde{f}}$ is the third component of the weak isospin of $\tilde{f}_L$, Q_f the electromagnetic charge of f and $\mathbf{m}_f$ the mass matrix (cf. 8.11 and 8.12) for f in generation space, with $(m_e)_{ij}$ being of course $m_{e_i}\delta_{ij}$. However, $\mathcal{M}^2_{\tilde{\mathbf{f}}}$ involves not only $\mathbf{m}_f$ but also the soft supersymmetry breaking squared mass matrices $\mathcal{M}^2$ both for the $SU(2)_L$ doublet left sfermions and for the $SU(2)_L$ singlet right sfermions plus the matrix A^f in generation space and finally the supersymmetric higgsino mass parameter μ. Note that A^f is in general a complex 3×3 matrix and μ can be complex too. Observe furthermore that the D-term contributions are diagonal in generation space. The offdiagonal LR mixing terms are proportional to fermion masses and hence appreciable only for the third generation. Otherwise, generation mixing is really controlled by the soft supersymmetry breaking terms.

Referring back to (9.47), we can define mass eigenstate sfermions through the six component column vector $\tilde{\mathbf{f}}^m$ which is unitarily transformed from $\tilde{\mathbf{f}}$:

$$\tilde{\mathbf{f}}^m = \mathbf{W}^{\tilde{f}\dagger} \tilde{\mathbf{f}} \; . \tag{9.51}$$

The 6×6 unitary matrices $\mathbf{W}^{\tilde{f}}$ then diagonalize the squared mass matrices $\mathcal{M}^2_{\tilde{\mathbf{f}}} \; \forall \tilde{f}$:

$$\mathbf{M}^{2(D)}_{\tilde{\mathbf{f}}} = \mathbf{W}^{\tilde{f}\dagger} \mathcal{M}^2_{\tilde{\mathbf{f}}} \mathbf{W}^{\tilde{f}} . \tag{9.52}$$

Let us introduce the indices s, t running from 1 to 6 while we keep the generation indices as i, j running from 1 to 3. We make a convention to order the sfermions by mass, $\tilde{f}_1^m$ being the lightest and $\tilde{f}_6^m$ the heaviest among sfermions of a given charge. Eq. (9.51) can then be rewritten as

$$\tilde{f}_s^m = W_{ts}^{\tilde{f}\star} \tilde{f}_t = W_{is}^{\tilde{f}\star} \tilde{f}_{iL} + W_{i+3\ s}^{\tilde{f}\star} \tilde{f}_{iR} \; , \tag{9.53}$$

the generation index i being summed on repetition. The second step of (9.53) shows the decomposition of a mass eigenstate sfermion field into left and right chiral interaction eigenstate sfermions. The latter can be written, by inverting (9.53), as

$$\tilde{f}_{iL} = W^{\tilde{f}}_{is}\tilde{f}^m_s, \qquad (9.54a)$$

$$\tilde{f}_{iR} = W^{\tilde{f}}_{i+3\ s}\tilde{f}^m_s. \qquad (9.54b)$$

Two limiting cases of the above most general sfermion mass mixing are also quite transparent.

(a) No L-R mixing

In this case $\mathcal{M}_{\tilde{f}LR}$ vanishes and (9.48) reduces to

$$\mathcal{M}^2_{\tilde{\mathbf{f}}} = \begin{pmatrix} \mathcal{M}^2_{\tilde{f}LL} & 0 \\ 0 & \mathcal{M}^2_{\tilde{f}RR} \end{pmatrix}. \qquad (9.55)$$

Now the 6×6 unitary matrix $\mathbf{W}^{\tilde{f}}$ has the chiral block diagonal form

$$\mathbf{W}^{\tilde{f}} = \begin{pmatrix} \mathbf{U}^{\tilde{f}_L} & 0 \\ 0 & \mathbf{U}^{\tilde{f}_R} \end{pmatrix}, \qquad (9.56)$$

where $\mathbf{U}^{\tilde{f}_L}$ and $\mathbf{U}^{\tilde{f}_R}$ are unitary submatrices for the distinct left and right sfermion sectors. In terms of explicit generation indices i, j $(= 1, 2, 3)$ we can write

$$W^{\tilde{f}}_{i\ j+3} = W^{\tilde{f}}_{i+3\ j} = 0\ , \qquad (9.57a)$$

$$W^{\tilde{f}}_{ij} = U^{\tilde{f}_L}_{ij}, \qquad (9.57b)$$

$$W^{\tilde{f}}_{i+3\ j+3} = U^{\tilde{f}_R}_{ij}. \qquad (9.57c)$$

The 3×3 unitary matrices $\mathbf{U}^{\tilde{f}_L}$ and $\mathbf{U}^{\tilde{f}_R}$ in generation space, appearing in (9.56) and (9.57), are sfermionic generalizations of the flavor rotation matrices $\mathbf{U}^{f_L}, \mathbf{U}^{f_R}$ for a chiral fermion f that we introduced for $f = u, d$ in Ch.8 to put the quark mass matrices $\mathbf{m}_u, \mathbf{m}_d$ into diagonal form via biunitary transformations. The chiral block submatrices of (9.50), for $f = \tilde{\nu}, \tilde{e}, \tilde{u}, \tilde{d}$, now have the following respective expressions after diagonalization.

$$\mathbf{M}^{2(D)}_{\tilde{\nu}} = \mathbf{U}^{\tilde{\nu}\dagger}(\mathcal{M}^2_{\tilde{\ell}} - M_Z^2\cos 2\beta\ T^{\tilde{\nu}}_{3L}\mathbb{1})\mathbf{U}^{\tilde{\nu}}, \qquad (9.58a)$$

$$\mathbf{M}^{2(D)}_{\tilde{e}LL} = \mathbf{U}^{\tilde{e}^\dagger_L}\left[\mathcal{M}^2_{\tilde{\ell}} + M_Z^2\cos 2\beta(T^{\tilde{e}}_{3L} - \sin^2\theta_W)\mathbb{1} + \mathbf{m}^{2(D)}_e\right]\mathbf{U}^{\tilde{e}_L}, \qquad (9.58b)$$

$$\mathbf{M}^{2(D)}_{\tilde{e}RR} = \mathbf{U}^{\tilde{e}^\dagger_R}\left[\mathcal{M}^2_{\tilde{e}} + Q_e M_Z^2\cos 2\beta\sin^2\theta_W\mathbb{1} + \mathbf{m}^{2(D)}_e\right]\mathbf{U}^{\tilde{e}_R}, \qquad (9.58c)$$

$$\mathbf{M}^{2}_{\tilde{u}LL} = \mathbf{U}^{\tilde{u}^\dagger_L}\left[\mathcal{M}^2_{\tilde{u}} + M_Z^2\cos 2\beta(T^{\tilde{u}}_{3L} - Q_u\sin^2\theta_W)\mathbb{1} + \mathbf{m}^\dagger_u\mathbf{m}_u\right]\mathbf{U}^{\tilde{u}_L}, \qquad (9.58d)$$

$$\mathbf{M}^{2(D)}_{\tilde{u}RR} = \mathbf{U}^{\tilde{u}^\dagger_R}\left[\mathcal{M}^2_{\tilde{u}} + Q_u M_Z^2\cos 2\beta\sin^2\theta_W\mathbb{1} + \mathbf{m}^\dagger_u\mathbf{m}_u\right]\mathbf{U}^{\tilde{u}_R}, \qquad (9.58e)$$

$$\mathbf{M}^{2(D)}_{\tilde{d}LL} = \mathbf{U}^{\tilde{d}^\dagger_L}\left[\mathcal{M}^2_{\tilde{d}} + M_Z^2\cos 2\beta(T^{\tilde{d}}_{3L} - Q_d\sin^2\theta_W)\mathbb{1} + \mathbf{m}^\dagger_d\mathbf{m}_d\right]\mathbf{U}^{\tilde{d}_L}, \qquad (9.58f)$$

$$\mathbf{M}^{2(D)}_{\tilde{d}RR} = \mathbf{U}^{\tilde{d}^\dagger_R}\left[\mathcal{M}^2_{\tilde{d}} + Q_{\tilde{d}} M_Z^2\cos 2\beta\sin^2\theta_W\mathbb{1} + \mathbf{m}^\dagger_d\mathbf{m}_d\right]\mathbf{U}^{\tilde{d}_R}. \qquad (9.58g)$$

Note that mass eigenstate sfermions will now be ordered by mass within $s = 1, 2, 3$ for left sfermions and within $s = 4, 5, 6$ for right sfermions, i.e. now we have

$$\text{mass}(f_1^m) < \text{mass}(f_2^m) < \text{mass}(f_3^m), \quad \text{(left sfermions)} , \tag{9.59a}$$

$$\text{mass}(f_4^m) < \text{mass}(f_5^m) < \text{mass}(f_6^m), \quad \text{(right sfermions)} , \tag{9.59b}$$

without any definite ordering between the two groups. Thus a program, made to diagonalize the original 6×6 matrix, will not automatically return a block diagonal mixing matrix as in (9.56) since the program will insist on all mass eigenstate sfermions being ordered according to their masses. The latter can be obtained just by interchanging certain rows and columns of $\mathbf{W}^{\tilde{f}}$ without affecting physics.

(b) No flavor mixing

In this limit the 6×6 mixing matrix only couples the two sfermionic states labelled by the indices i and $i + 3$, i.e. the left and the right states of a given flavor. For a real mass matrix, one has

$$W^{\tilde{f}}_{ii} = W^{\tilde{f}}_{i+3\ i+3} = \cos\theta_{\tilde{f}_i} , \tag{9.60a}$$

$$W^{\tilde{f}}_{i\ i+3} = -W^{\tilde{f}}_{i+3\ i} = -\sin\theta_{\tilde{f}_i} . \tag{9.60b}$$

Thus, for instance, mass eigenstate charged sleptons will now be described by

$$\tilde{\mathbf{f}}^m = \begin{pmatrix} \tilde{e}_1 \\ \tilde{\mu}_1 \\ \tilde{\tau}_1 \\ \tilde{e}_2 \\ \tilde{\mu}_2 \\ \tilde{\tau}_2 \end{pmatrix} , \tag{9.61}$$

i.e. the mass ordering is enforced between f_i^m and f_{i+3}^m and not between different flavor states.

Before closing this section, we want to comment specifically on the squared mass matrices of staus, sbottoms and stops. These third generation sleptons and squarks are somewhat special. It is reasonable to take them to be decoupled from other sleptons and squarks i.e. assume no flavor mixing for them. On the other hand, they do involve substantial L-R mixing on account of the nonnegligible masses of their fermion partners. Indeed, they physically manifest themselves as the mass eigenstates $\tilde{\tau}_{1,2}, \tilde{b}_{1,2}$ and $\tilde{t}_{1,2}$. In this picture the corresponding squared mass matrices can be written approximately in 2×2 form

$$\mathcal{M}^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{\tilde{\ell}_3} - (1/2 - \sin^2\theta_W) M_Z^2 \cos 2\beta + m_\tau^2 & -m_\tau(A^{\tau\star} + \mu\tan\beta) \\ -m_\tau(A^\tau + \mu^\star \tan\beta) & m^2_{\tilde{\tau}} - M_Z^2 \cos 2\beta \sin^2\theta_W + m_\tau^2 \end{pmatrix} , \tag{9.62a}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} m^2_{\tilde{q}_3} - (1/2 - 1/3 \sin^2\theta_W) M_Z^2 \cos 2\beta + m_b^2 & -m_b(A^{b\star} + \mu\tan\beta) \\ -m_b(A^b + \mu^\star \tan\beta) & m^2_{\tilde{b}} - 1/3 M_Z^2 \cos 2\beta \sin^2\theta_W + m_b^2 \end{pmatrix} , \tag{9.62b}$$

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{q}_3} + (1/2 - 2/3\sin^2\theta_W)M_Z^2\cos 2\beta + m_t^2 & -m_t(A^{t\star} + \mu\cot\beta) \\ -m_t(A^t + \mu^\star\cot\beta) & m^2_{\tilde{t}} + 2/3M_Z^2\cos 2\beta\sin^2\theta_W + m_t^2 \end{pmatrix}. \tag{9.62c}$$

The off-diagonal L-R mixing term is particularly large in the stop case, being proportional to the mass of the top quark. This can in principle make $\tilde{t}_1$ the lightest sfermion.

9.5 The Flavor Problem in Supersymmetry

Many discussions in previous sections have hinted that there is a generic flavor problem [9.8] in supersymmetric theories. The origin of the problem is in the occurrence of sizable flavor dependence in sfermion mass matrices. The latter naturally leads to large induced FCNC amplitudes which are, however, unobserved by experiment. The lack of observation of the decay $\mu \to e\gamma$ puts some constraints on the lepton-slepton sector. Though processes like $D^0 \leftrightarrow \bar{D}^0$ and $B^0 \leftrightarrow \bar{B}^0$ transitions as well as $b \to s\gamma$ decay yield constraints on the quark-squark sector, the most stringent restrictions here come from what is already known about K^0-$\bar{K}^0$ mixing. Let us elaborate on this last statement by following the treatment of Hagelin et al [9.8]. At the one loop level the box diagram of Fig. 9.5 can induce an operator such as $\bar{d}_L\gamma_\mu s_L\ \bar{s}_L\gamma^\mu d_L$

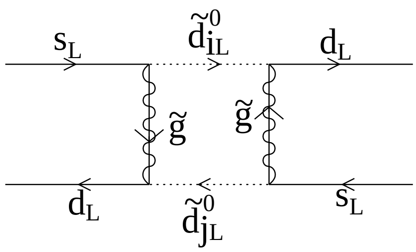

Fig. 9.5 One loop squark induced K^0-$\bar{K}^0$ mixing

into the effective Lagrangian density contributing to the said mixing. From the product of two squark propagators and four elements of the matrix $\mathbf{U}^{\tilde{d}_L}$ of (9.56) in this diagram, the transition amplitude for $\bar{s}_L d_L \to \bar{d}_L s_L$ picks up a factor[6]

$$\sum_i \frac{U^{\tilde{d}_L}_{di} U^{\tilde{d}^\dagger_L}_{is}}{k^2 - m^2_{\tilde{d}_i} + i\epsilon} \sum_j \frac{U^{\tilde{d}_L}_{dj} U^{\tilde{d}^\dagger_L}_{js}}{k^2 - m^2_{\tilde{d}_j} + i\epsilon},$$

k being the loop momentum. We have set all external momenta to zero because $m_K^2 \ll m^2_{\tilde{q}}$. Since the unitarity of $\mathbf{U}^{\tilde{d}_L}$ makes this factor vanish in case $m^2_{\tilde{d}_i}$ is the same for $\tilde{d}_i = \tilde{d}, \tilde{s}, \tilde{b}$, it can be rewritten as

$$\frac{1}{(k^2 - m^2_{\tilde{d}} + i\epsilon)^4} \left| \sum_i U^{\tilde{d}_L}_{si} U^{\tilde{d}^\dagger_L}_{id} \Delta m^2_{\tilde{d}_i} \right|^2 + \mathcal{O}\left(\left[k^2 - m^2_{\tilde{d}}\right]^{-5}\right),$$

[6]We work in an interaction basis where the down quark mass matrix is diagonal.

where $m_{\tilde{d}}^2$ is an average mass squared for charge $-1/3$ squarks and $m_{\tilde{d}_i}^2 = m_{\tilde{d}}^2 + \Delta m_{\tilde{d}_i}^2$. The aforementioned transition amplitude has the dimensionality of an inverse mass squared. So after inserting the product of the two gluino propagators and four powers of the QCD coupling strength g_s and performing the loop integration, one is left with an amplitude proportional to

$$\frac{g_s^4}{\tilde{m}^6}\left|\sum_i U_{di}^{\tilde{d}_L} U_{is}^{\tilde{d}_L^\dagger} \Delta m_{\tilde{d}_i}^2\right|^2,$$

where $\tilde{m} = \max\,(m_{\tilde{q}}, M_{\tilde{g}})$, i.e. the larger of the squark and gluino masses. With $|\Delta m_{\tilde{d}_i}| \sim m_{\tilde{d}_i} = \mathcal{O}(10^2)$ GeV, this yields a contribution which is three orders of magnitude larger than that from the SM. The latter obtains through the replacement of the gluino lines by $W^\pm$ lines and of the squark lines by u_i-quark ones in Fig. 9.5 and reproduces the observed value of the K_L-K_S mass difference [9.13] rather well.

The above discussion raises an important question : how can such undesirable amplitudes be suppressed in supersymmetric theories? The structure of the expression in the above paragraph implies that there are basically three ways in which a suppression of the desired nature can be achieved. One may also consider various combinations of these options. We shall describe these three possibilities one by one. Note that we keep our focus on the quark-squark sector here. Analogous arguments do apply to the lepton-slepton sector, though with a certain simplification; flavor mixing among leptons can be neglected – at least in the limit of vanishing neutrino masses. Thus constraints, from the yet unobserved $\mu \to e\gamma$ decay and muon conversion to electron in atoms, can also be taken care of.

The first choice is to make the prefactor in the said expression small, i.e. to take [9.14] the masses of sfermions of the first two generations to be very large, in the multi-TeV range. Of course, the naturalness argument, discussed in Ch.1, requires one to keep third generation sfermion and Higgs boson masses at or below the TeV scale. However, the smallness of first and second generation Yukawa couplings allows the choice of quite large masses for the corresponding sfermions without destabilizing the hierarchy[7]. This is a "brute force" solution of the flavor problem, since all loop corrections involving internal first or second generation sfermions and external fermion or gauge boson legs are then suppressed, including in particular those corrections that give rise to FCNC transitions. The prevention of unacceptably large loop corrections from the hypercharge $U(1)_Y$ D-terms to Higgs masses requires the condition $\sum_i Y_i m_{\tilde{f}_i}^2 \lesssim \mathcal{O}(1)$ TeV^2. Another problem arises in any attempt to implement such a spectrum at a high energy scale: two loop contributions to the renormalization group equations due to $SU(3)_C$ interactions tend to drive the squared stop masses to negative values [9.15], leading to color and/or charge symmetry breaking. On the positive side, this kind of model also easily satisfies constraints on flavor conserving CP violating amplitudes. In particular, those from the yet unobserved electric dipole moments of the neutron and electron are respected even though all soft supersymmetry breaking parameters have CP violating phases of $\mathcal{O}(1)$. This kind of "inverted hierarchy" model sometimes goes under the name *more minimal* or *Effective Supersymmetry*, since first and second generation sfermions essentially decouple from physics at energies that will be accessible in the foreseeable future at collider experiments.

[7]If $\tan\beta$ is small, the $\tilde{b}_R, \tilde{\tau}$ and h masses can also be made large.

The second strategy [9.16] is to assume a (presumably dynamically generated) **alignment** between the fermion and sfermion mass matrices so that both can be made diagonal in the same basis. In fact, in that case the mixing matrix, appearing in the expression for the box graph of Fig. 9.5, is diagonal. The expression then vanishes and the problem is solved. This is actually only a partial solution, since, owing to nontrivial CKM mixing, $\mathcal{M}^2_{\tilde{q}}$ cannot commute simultaneously with the u and d quark mass matrices (except when $\mathcal{M}^2_{\tilde{q}}$ is proportional to the unit matrix; this case will be treated below). As already stated (see also Table 9.1 below), by far the most stringent constraints come from the kaon sector. Models of alignment hence usually assume that $\mathcal{M}^2_{\tilde{q}}$ is aligned with the d quark mass matrix. Since CKM mixing angles are in fact quite small, an approximate alignment with the u quark mass matrix then also obtains. However, generically one would expect nonnegligible D^0-$\bar{D}^0$ mixing in this class of models.

The third option is to assume a high degree of mass degeneracy, or **universality of masses**, among sfermions with given $SU(2)_L \times U(1)_Y$ quantum numbers (including electromagnetic charge) but occurring in different generations. In this scenario the K^0-$\bar{K}^0$ mixing expression is suppressed because the $\Delta m^2_{\tilde{d}_i}$ are very small. Large flavor mixing is possible in this option if on-shell sparticles can be produced,[8] but FCNC amplitudes, involving only SM particles as external legs, are suppressed by a super-GIM mechanism. In practice, it suffices to assume a near mass degeneracy between sfermions of the first and second generations; experimental flavor mixing constraints on the third generation are weak, mostly because the SM contribution to B^0-$\bar{B}^0$ mixing is quite large, and has a sizable theoretical uncertainty. Indeed, with substantial L-R mixing, one may expect $\tilde{\tau}_1$, $\tilde{b}_1$ and $\tilde{t}_1$ to be significantly lighter than the corresponding mass degenerate charge -1, charge $-1/3$ and charge $2/3$ sfermions of the first two generations, respectively. Note that FCNC constraints do not lead to any relations between, say, $\mathcal{M}^2_{\tilde{u}}$, $\mathcal{M}^2_{\tilde{d}}$ and $\mathcal{M}^2_{\tilde{q}}$. As will be shown in more detail in Chs.12 and 13, specific models with high scale supersymmetry breaking nevertheless do usually imply a high degree of degeneracy between first and second generation squarks with different $SU(2)_L \times U(1)_Y$ quantum numbers. On the other hand, in such models exact universality only holds at a high scale. Quantum corrections will typically lead to deviations from universality at the weak scale. We shall see later that many such models, while still compatible with the present constraints, therefore predict significant new contributions to certain FCNC processes. In the remaining sections of this chapter we shall hence present Feynman rules for sfermion interactions allowing for a completely general mixing between all six sfermions of a given electric charge.

Before coming to the Feynman rules, mentioned above, however, we would like to give a more quantitative discussion of the bounds on flavor violation in the sfermion sector. This can most easily be done using the *mass insertion* method [9.18]. In this approach one works in a basis where the mass matrix of quarks of a given charge as well as the corresponding quark-squark-neutral gaugino couplings are diagonal in flavor space. As a result, different bases need to be used for problems involving external d-type or external u-type quarks. Flavor violation is then described by flavor nondiagonal entries $(\Delta^{\tilde{f}}_{ij})_{AB}$ of the sfermion squared mass matrices in that basis, where i and j are generation indices and $A, B \in \{L, R\}$

[8]The effects of such large mixing may be observable as slepton oscillations [9.17] in pp and $\ell^+\ell^-$ colliders.

labels the four 3×3 blocks in (9.48). These off-diagonal entries are treated as two point interactions in the perturbation expansion, leading to nondiagonal propagators with explicit flavor offdiagonal mass insertions. The experimental constraints can most conveniently be

quantity	$x = 0.3$	$x = 1.0$	measurable
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{12})^2_{LL}\right\|}$	1.9×10^{-2}	4.0×10^{-2}	
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{12})^2_{LR}\right\|}$	7.9×10^{-3}	4.4×10^{-3}	Δm_K
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{12})_{LL}(\delta^{\tilde{d}}_{12})_{RR}\right\|}$	2.5×10^{-3}	2.5×10^{-3}	
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{13})^2_{LL}\right\|}$	4.6×10^{-2}	9.8×10^{-2}	
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{13})^2_{LR}\right\|}$	5.6×10^{-2}	3.3×10^{-2}	Δm_B
$\sqrt{\left\|\Re e(\delta^{\tilde{d}}_{13})_{LL}(\delta^{\tilde{d}}_{13})_{RR}\right\|}$	1.6×10^{-2}	1.8×10^{-2}	
$\sqrt{\left\|\Re e(\delta^{\tilde{u}}_{12})^2_{LL}\right\|}$	4.7×10^{-2}	1.0×10^{-1}	
$\sqrt{\left\|\Re e(\delta^{\tilde{u}}_{12})^2_{LR}\right\|}$	6.3×10^{-2}	3.1×10^{-2}	Δm_D
$\sqrt{\left\|\Re e(\delta^{\tilde{u}}_{12})_{LL}(\delta^{\tilde{u}}_{12})_{rr}\right\|}$	1.6×10^{-2}	1.7×10^{-2}	
$\Im m(\delta^{\tilde{d}}_{12})_{LL}$	1.0×10^{-1}	4.8×10^{-1}	ϵ'_K/ϵ_K
$\Im m(\delta^{\tilde{d}}_{12})_{LR}$	1.1×10^{-5}	2.0×10^{-5}	ϵ'_K/ϵ_K
$(\delta^{\tilde{d}}_{23})_{LL}$	4.4	8.2	$\mathrm{BR}(b \to s\gamma)$
$(\delta^{\tilde{d}}_{23})_{LR}$	1.3×10^{-2}	1.6×10^{-2}	$\mathrm{BR}(b \to s\gamma)$
$(\delta^{\tilde{\ell}}_{12})_{LL}$	4.1×10^{-3}	7.7×10^{-3}	$\mathrm{BR}(\mu \to e\gamma)$
$(\delta^{\tilde{\ell}}_{12})_{LR}$	1.4×10^{-6}	1.7×10^{-6}	$\mathrm{BR}(\mu \to e\gamma)$
$(\delta^{\tilde{\ell}}_{13})_{LL}$	15	29	$\mathrm{BR}(\tau \to e\gamma)$
$(\delta^{\tilde{\ell}}_{13})_{LR}$	8.9×10^{-2}	1.1×10^{-1}	$\mathrm{BR}(\tau \to e\gamma)$
$(\delta^{\tilde{\ell}}_{23})_{LL}$	2.8	5.3	$\mathrm{BR}(\tau \to \mu\gamma)$
$(\delta^{\tilde{\ell}}_{23})_{LR}$	1.7×10^{-2}	2.0×10^{-2}	$\mathrm{BR}(\tau \to \mu\gamma)$

Table 9.1. Experimental upper bounds [9.18] on flavor violation in the soft supersymmetry breaking terms of sfermions.

expressed as bounds on the dimensionless quantities $(\delta^{\tilde{f}}_{ij})_{AB}$. In the simplest case, $(\delta^{\tilde{f}}_{ij})_{AB} = (\Delta^{\tilde{f}}_{ij})_{AB}/(\overline{m^2}^{\tilde{f}}_{ij})_{AB}$, where the "average" sfermion squared mass is given by $(\overline{m^2}^{\tilde{f}}_{ij})_{AB} = \sqrt{(\mathcal{M}^2_{ii})_{AA}(\mathcal{M}^2_{jj})_{BB}}$. Moreover, this formalism also allows the inclusion of higher order contributions. Thus, for instance, the second order contribution to $(\delta^{\tilde{f}}_{ij})_{RR}$ is given by $(\Delta^{\tilde{f}}_{ik})_{RL}(\Delta^{\tilde{f}}_{kj})_{LR}/\left((\overline{m^2}^{\tilde{f}}_{ik})_{RL}(\overline{m^2}^{\tilde{f}}_{kj})_{LR}\right)$.

The experimental bounds on the various off-diagonal entries $(\delta^{\tilde{f}}_{ij})_{AB}$ are summarized in Table 9.1, which has been extracted from Ref.[9.18]. It has been assumed here that each supersymmetric contribution separately satisfies the overall constraint on the quantity indicated, i.e. "accidental" cancellations between different kinds of contributions have not been considered. For simplicity, moreover, $(\delta^{\tilde{f}}_{ij})_{LR}$ and $(\delta^{\tilde{f}}_{ij})_{RL}$ are taken to be equal, though the assumption could be avoided. The bounds on the slepton sector have been computed from loop diagrams involving a photino, rather than treating the two neutral electroweak gauginos separately. It is in this limit that the bounds on $(\delta^{\tilde{\ell}}_{ij})_{RR}$ are identical to those on $(\delta^{\tilde{\ell}}_{ij})_{LL}$ that have been listed in the table. An analogous statement holds for the bounds in the squark sector, which come from diagrams involving gluinos. All these bounds scale inversely with the relevant sfermion mass. The numerical values, given in Table 9.1, assume a common squark mass of 500 GeV and a common slepton mass of 100 GeV. Thus the bounds on $\delta^{\tilde{\ell}}$ scale like $m_{\tilde{\ell}}/(100\text{ GeV})$ while those on $\delta^{\tilde{q}}$ $(q = u, d)$ scale like $m_{\tilde{q}}/(5000\text{ GeV})$. Note that we only quote bounds from contributions involving flavor changing couplings to neutral gauginos (gluinos or neutralinos) $\tilde{\lambda}^0$; values are given for two values of the ratio $x \equiv (m_{\tilde{\lambda}^0}/\overline{m}_{\tilde{f}})^2$. Each entry in the last column in this table indicates the physical measurable from which the corresponding bound has been derived. Note moreover that the bounds on $(\delta^{\tilde{f}}_{ij})_{RR}$ and $(\delta^{\tilde{f}}_{ij})_{RL}$ are equal to those on the corresponding $(\delta^{\tilde{f}}_{ij})_{LL}$ and $(\delta^{\tilde{f}}_{ij})_{RL}$ respectively. As mentioned earlier, the most severe constraints exist on the mixing between first and second generation charge $-1/3$ squarks. The constraint on the mixing between charge $2/3$ squarks of the first two generations is considerably milder. Furthermore, $\mathcal{O}(1)$ mixing between second and third generation squarks is allowed in the LL or RR sector. The constraint on mixing between left and right sfermions is often much more stringent than that on LL and RR mixing. The reason is that the relevant effective fermionic operators leading to radiative decays treated in the last eight rows of Table 9.5, break chiral symmetry, i.e. cause couplings between left chiral and right chiral fermions, facilitated by the transitions between the corresponding sfermions.

9.6 Interactions of Sfermions with Gauge Bosons

A sfermion participates in an MSSM gauge interaction in two ways: (1) as a member of a sfermion pair and (2) along with another fermion. We shall take up (2) in the next Section. Here we consider (1) and enumerate the different possibilities below.

Slepton-slepton-electroweak gauge boson interactions

In this category come cubic/quartic vertices involving a *pair* of sleptons and one/two EW gauge boson(s). The two sleptons could be various combinations of charged and neutral ones while the gauge boson(s) would be correspondingly neutral and/or charged. These interactions were covered earlier in (8.44b–d) and (8.47c–h), but we now describe physical vertices with mass eigenstate sleptons and general mixing as described at the end of §9.4. We can collect all such vertices in three groups.

(1) The first group (Fig. 9.6) consists of vertices which involve either only one (two) Z boson(s) interacting with a sneutrino pair or only one (two) photon(s). These vertices have the feature that the mixing matrices $\mathbf{W}^{\tilde{f}}$ cancel out. The crucial observation here is that $\mathbf{W}^{\tilde{f}\dagger}\mathbf{W}^{\tilde{f}} = \mathbb{1}$.

(2) The second group (Fig. 9.7), comprising either a W^+W^- pair interacting with two sleptons or a Z interacting with a charged slepton pair, shows a nontrivial dependence on the mixing matrices $\mathbf{W}^{\tilde{f}}$ only in the presence of left right mixing. Without such mixing, i.e. if the 6×6 slepton mixing matrix has the form (9.56), flavor mixing would again drop out, owing to the unitarity of the $\mathbf{U}$ matrices. On the other hand, if L-R mixing is present, a nontrivial dependence on the mixing angle emerges even in the absence of flavor mixing. The relevant Feynman rules for this case can be derived easily from the general rules listed in Fig. 9.7, using (9.60). Notice that we have replaced $\mathbf{W}^{\tilde{\nu}}$ by $\mathbf{U}^{\tilde{\nu}}$ since in the MSSM no righthanded (s)neutrinos exist at the weak scale.

(3) All the remaining vertices, which are in general affected by generation mixing even in the absence of L-R mixing, make up the third group (Fig. 9.8). In this case a nontrivial dependence on the mixing angle will survive in both simplified scenarios discussed in §9.4, i.e. (9.57) and (9.60). For the convenience of the reader we give both the $W^+\tilde{l}\tilde{\nu}^*$ and $W^-\tilde{\nu}\tilde{l}^*$ vertices, which are related to each other by complex conjugation.

Figs. 9.6, 9.7 and 9.8 are included in Appendix A

Squark-squark-gauge boson interactions

The simplest set of vertices in this category are those that involve only squarks and gluons in SQCD. The cubic $\tilde{q}\tilde{\bar{q}}g$ and the quartic $\tilde{q}\tilde{\bar{q}}gg$ vertices have already been fully discussed in §5.5 and §8.4. Nothing needs to be added to those discussions, since the mixing matrices will cancel out in these vertices.

Turning to cubic and quartic vertices of physical mass eigenstate squarks with electroweak gauge bosons, we can again collect them in three groups as in the case of sleptons. However, the first group – which is free from *any* mixing – now has only pure photon vertices, cf. Fig. 9.9. The second group (Fig. 9.10), involving a W^+W^- pair or one (two) neutral gauge boson(s), at least one being the Z, shows a nontrivial mixing dependence only in the presence of L-R mixing. Only the third group, containing a single W either by itself or in association with a neutral gauge boson interacting with a squark pair, has the complication of both types of mixing, i.e. generation as well as left right. These vertices are given in Fig. 9.11.

As with sleptons, it is straightforward to derive the corresponding Feynman rules if flavor or L-R mixing can be ignored, using (9.57) and (9.60), respectively.

Figs. 9.9, 9.10 and 9.11 are included in Appendix A

This brings us to the end of the discussion of sfermion-gauge boson vertices.

9.7 Fermion-sfermion-gaugino/higgsino interactions

Fermion-sfermion-chargino interactions

Let us first discuss the fermion-sfermion-chargino vertices in the "current" basis in which generically $\tilde{f}_{u_iL}, \tilde{f}_{d_iL}$ are the left sfermions of the up, down type, $\tilde{f}_{u_iR}, \tilde{f}_{d_iR}$ are the corresponding right sfermions and f_{u_i}, f_{d_i} the corresponding fermions following the notation introduced in §8.4. Our starting points are (1) the gaugino-sfermion-fermion interactions, as given by expressions analogous to (5.55) and (2) the higgsino-sfermion-fermion couplings arising from the superpotential (8.33). In the two component spinor notation used in previous sections, the relevant part of the Lagrangian density reads

$$\begin{aligned}
\mathcal{L}_{f\tilde{f}'^*\tilde{\chi}^\pm} &= -g_2\left(\lambda^-\xi^1_{Q_i}\tilde{d}^*_{iL} + \lambda^+\xi^2_{Q_i}\tilde{u}^*_{iL} + \lambda^-\xi^1_{L_i}\tilde{e}^*_{iL} + \lambda^+\xi^2_{L_i}\tilde{\nu}^*_{iL}\right) \\
&+ \frac{g_2(\mathbf{m}^*_u)_{ij}}{\sqrt{2}M_W\sin\beta}\left(\tilde{h}^1_2\xi^2_{Q_i}\tilde{u}^*_{jR} + \tilde{h}^1_2\xi_{\bar{U}_j}\tilde{d}_{iL}\right) \\
&+ \frac{g_2(\mathbf{m}^*_d)_{ij}}{\sqrt{2}M_W\cos\beta}\left(\tilde{h}^2_1\xi^1_{Q_i}\tilde{d}^*_{jR} + \tilde{h}^2_1\xi_{\bar{D}_j}\tilde{u}_{iL}\right) \\
&+ \frac{g_2(\mathbf{m}^*_e)_{ij}}{\sqrt{2}M_W\cos\beta}\left(\tilde{h}^2_1\xi^1_{L_i}\tilde{e}^*_{jR} + \tilde{h}^2_1\xi_{\bar{E}_j}\tilde{\nu}_{iL}\right) + \text{h.c.}
\end{aligned} \tag{9.63}$$

We have written out the squark and slepton terms separately. The first term in the RHS of (9.62) describe the gaugino-fermion-sfermion couplings, while the last three terms correspond to higgsino-fermion-sfermion interactions. The latter are proportional to fermion mass matrices and vanish in the limit of massless fermions. In this expression $\xi^{1(2)}_{Q_i}$ and $\xi^{1(2)}_{L_i}$ are the two component spinors representing the $T_{3L} = 1/2$ $(-1/2)$ fermionic components of a chiral $SU(2)_L$ doublet superfield such as Q_i or L_i of (8.15)-(8.17). Furthermore, $\xi_{\bar{D}_j}$ and $\xi_{\bar{E}_j}$ are the fermionic components of $SU(2)_L$ singlet superfields. The four component Dirac spinor fields corresponding to the various matter fermions are constructed out of $\xi_{U_i}, \xi_{D_i}, \xi_{L_i}, \xi_{\bar{U}_i}, \xi_{\bar{D}_i}, \xi_{\bar{E}_i}$ (where *e.g.* $\xi_{U_i} \equiv \xi^1_{Q_i}$ and so on) as described in (3.20) of §3.2. For example, for the up type quarks

$$u_i = \begin{pmatrix} \xi_{U_i} \\ \bar{\xi}^T_{\bar{U}_i} \end{pmatrix}. \tag{9.64}$$

Recall that each of the singlet superfields $\bar{E}_i, \bar{D}_i$ and $\bar{U}_i$ contains the left chiral component of the antifermion field. Let us define Dirac fields f_{u_i,d_i} for general up, down type matter fermions (covering both quarks and leptons) in analogy with the u_i of (9.64). In terms of these generic up, down fermions and sfermions and the four component wino and higgsino eigenstates defined in (9.17), we can rewrite (9.63) as

$$
\begin{aligned}
\mathcal{L}_{f\tilde{f}'^{*}\tilde{\chi}^{\pm}} &= -g_2 \left[\bar{f}_{u_i} P_R \tilde{\lambda}^+ \tilde{f}_{d_i L} + \bar{f}_{d_i} P_R (\tilde{\lambda}^+)^C \tilde{f}_{u_i L} \right] \\
&+ \frac{g_2 (\mathbf{m}_{f_d})_{ij}}{\sqrt{2} M_W \cos\beta} \left[\bar{f}_{u_i} P_R \tilde{h}^+ \tilde{f}_{d_j R} + \overline{(\tilde{h}^+)^C} P_R f_{d_j} \tilde{f}^*_{u_i L} \right] \\
&+ \frac{g_2 (\mathbf{m}_{f_u})_{ij}}{\sqrt{2} M_W \sin\beta} \left[\bar{f}_{d_i} P_R (\tilde{h}^+)^C \tilde{f}_{u_j R} + \overline{\tilde{h}^+} P_R f_{u_j} \tilde{f}^*_{d_i L} \right] + \text{h.c.} \qquad (9.65)
\end{aligned}
$$

Of course, a sum over all fermions f_{u_i}, f_{d_i} covering quarks and leptons (and corresponding sfermions) is implied. On utilizing (9.18) and (9.19), this Lagrangian can be recast in terms of the chargino mass eigenstates $\tilde{\chi}^{\pm}_k$, $k = 1, 2$, as

$$
\begin{aligned}
\mathcal{L}_{f\tilde{f}'^{*}\tilde{\chi}^{\pm}} &= -g_2 \left[\mathcal{U}_{k1} \bar{f}_{u_i} P_R \tilde{\chi}^+_k \tilde{f}_{d_i L} + \mathcal{V}_{k1} \bar{f}_{d_i} P_R (\tilde{\chi}^+_k)^C \tilde{f}_{u_i L} \right] \\
&+ \frac{g_2 (\mathbf{m}_{f_d})_{ij}}{\sqrt{2} M_W \cos\beta} \mathcal{U}_{k2} \left[\bar{f}_{u_i} P_R \tilde{\chi}^+_k \tilde{f}_{d_j R} + \tilde{f}^*_{u_i L} \overline{(\tilde{\chi}^+_k)^C} P_R f_{d_j} \right] \\
&+ \frac{g_2 (\mathbf{m}_{f_u})_{ij}}{\sqrt{2} M_W \sin\beta} \mathcal{V}_{k2} \left[\bar{f}_{d_i} P_R (\tilde{\chi}^+_k)^C \tilde{f}_{u_j R} + \tilde{f}^*_{d_i L} \overline{\tilde{\chi}^+_k} P_R f_{u_j} \right] + \text{h.c.} \qquad (9.66)
\end{aligned}
$$

In the supersymmetric limit the lepton-slepton-chargino vertices can be read off from this expression, using $(\mathbf{m}_e)_{ij} = m_{e_i} \delta_{ij}$, modulo $\tilde{e}_L$-$\tilde{e}_R$ mixing in the slepton sector. However, the existence of the soft supersymmetric breaking terms can change that. In case of the quark-squark-chargino interaction, there is also the additional complication of generation mixing which is present even in the supersymmetric limit. A further point to note in (9.66) is the occurrence of $(\tilde{\chi}^+_k)^C$. The appearance of charge conjugated fermion fields is generic in supersymmetric theories and gives rise to the explicit presence of the charge conjugation matrix C in Feynman rules. The basic reason for the necessity of introducing these ugly C-factors in Feynman rules is the following. In contrast with charged fermions in the SM, charginos do not carry a "fermion number" like lepton or baryon number. The same field can thus couple to $\bar{u}\tilde{d}$ and to $d\bar{\tilde{u}}$. If the first vertex is written in terms of an incoming (positive) chargino field, the second vertex has to be written in terms of the outgoing charge conjugate of that chargino field (or vice versa).

We are now in a position to write down the interaction terms of (9.66) explicitly for the quark/squark and lepton/slepton sectors in terms of mass diagonal matter fermion and sfermion fields. We use quark flavor rotation matrices $\mathbf{U}^{u_{L,R}}$ and $\mathbf{U}^{d_{L,R}}$, introduced in Ch. 8, as well as the sfermion rotation matrices $\mathbf{U}^{\tilde{\nu}}$, $\mathbf{W}^{\tilde{e}}$, $\mathbf{W}^{\tilde{u}}$ and $\mathbf{W}^{\tilde{d}}$ of §9.4. We employ $i, j, k = 1, 2, 3$ as indices in generation space, while $s = 1, \ldots, 6$ labels charged slepton or squark mass eigenstates. The physical quark masses are denoted by m_{d_i} and m_{u_i}. Finally, quark/squark fields are taken to be row or column vectors in color space. The quark/squark part of (9.66) then reads (for simplicity we omit the superscript m denoting mass eigenstates)

$$
\begin{aligned}
\mathcal{L}_{q\tilde{q}'\chi^{\pm}} &= \bar{u}_i C^L_{isk} P_R \tilde{d}_s \tilde{\chi}^+_k + \bar{d}_i D^L_{isk} P_R \tilde{u}_s (\tilde{\chi}^+_k)^C \\
&+ \tilde{u}^\dagger_s \overline{(\tilde{\chi}^+_k)^C} E^R_{isk} P_R d_i + \tilde{d}^\dagger_s \overline{\tilde{\chi}^+_k} F^R_{isk} P_R u_i + \text{h.c.}\ , \qquad (9.67)
\end{aligned}
$$

with

$$C^L_{isk} = -g_2\mathcal{U}_{k1}\sum_{j=1}^{3} U^{u_L *}_{ji} W^{\tilde{d}}_{js} + \frac{g_2\mathcal{U}_{k2}}{\sqrt{2}M_W\cos\beta}\sum_{j,n=1}^{3} V^{q_L}_{in} m_{d_n} U^{d_R *}_{jn} W^{\tilde{d}}_{j+3\ s} , \tag{9.68a}$$

$$D^L_{isk} = -g_2\mathcal{V}_{k1}\sum_{j=1}^{3} U^{d_L *}_{ji} W^{\tilde{u}}_{js} + \frac{g_2\mathcal{V}_{k2}}{\sqrt{2}M_W\sin\beta}\sum_{j,n=1}^{3} V^{q_L *}_{ni} m_{u_n} U^{u_R *}_{jn} W^{\tilde{u}}_{j+3\ s} , \tag{9.68b}$$

$$E^R_{isk} = \frac{g_2\mathcal{U}_{k2} m_{d_i}}{\sqrt{2}M_W\cos\beta}\sum_{j=1}^{3} U^{d_L}_{ji} W^{\tilde{u}*}_{js} , \tag{9.68c}$$

$$F^R_{isk} = \frac{g_2\mathcal{V}_{k2} m_{u_i}}{\sqrt{2}M_W\sin\beta}\sum_{j=1}^{3} U^{u_L}_{ji} W^{\tilde{d}*}_{js} . \tag{9.68d}$$

The corresponding vertices are given in Fig. 9.12. It may be noted that left (right) fermions connect to the left (right) components of the sfermions through the gaugino components of the charginos, which are described by $\mathcal{U}_{\ell 1}$ and $\mathcal{V}_{\ell 1}$. In contrast, the terms coming from Yukawa couplings, which are proportional to a quark mass, couple a left (right) fermion to the right (left) component of the corresponding sfermion. If squarks and quarks could be aligned exactly (see §9.5), all combinations of quark and squark mixing matrices appearing in (9.68) would reduce either to the unit matrix (in the right handed sector) or to the standard KM matrix $\mathbf{V}^{q_L}$ (in the left handed sector); however, as discussed earlier, alignment cannot be exact in the u and d sectors simultaneously. Note finally that, as per the convention of Appendix D of Haber and Kane [9.10], a charge conjugation matrix $\underset{\sim}{C}$ to the right operates on the transposed $\bar{u}$-spinor $\bar{u}^T$ or $\bar{v}$-spinor $\bar{v}^T$ while a $\underset{\sim}{C}^{-1}$ to the left requires a transposed v-spinor v^T or u-spinor u^T to left multiply it.

Fig. 9.12 is included in Appendix A

We turn next to the lepton/slepton part of (9.63). It reads

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^\pm} = \bar{\nu}_i c^L_{isk}\tilde{e}_s P_R\tilde{\chi}^+_k + d^L_{ijk}\bar{e}_i P_R(\tilde{\chi}^+_k)^C\tilde{\nu}_j + e^R_{ijk}\overline{(\tilde{\chi}^+_k)^C} P_R e_i\tilde{\nu}^*_j + \text{h.c.} , \tag{9.69}$$

with

$$c^L_{isk} = -g_2\mathcal{U}_{k1}W^{\tilde{e}}_{is} + \frac{g_2 m_{e_i}}{\sqrt{2}M_W\cos\beta}\mathcal{U}_{k2}W^{\tilde{e}}_{i+3\ s} , \tag{9.70a}$$

$$d^L_{ijk} = -g_2 U^{\tilde{\nu}}_{ij}\mathcal{V}_{k1} , \tag{9.70b}$$

$$e^R_{ijk} = \frac{g_2 m_{e_i}}{\sqrt{2}M_W\cos\beta}\mathcal{U}_{k2}U^{\tilde{\nu}*}_{ij} . \tag{9.70c}$$

The corresponding vertices are drawn in Fig. 9.13; they can be obtained from those of Fig. 9.12 with the replacements $u \to \nu$, $d \to e$, $\mathbf{V}^{q_L} \to 1\!\!1$, $\mathbf{U}^{u_L}$, $\mathbf{U}^{d_L}$, $\mathbf{U}^{u_R}$, $\mathbf{U}^{d_R} \to 1\!\!1$, $\mathbf{W}^{\tilde{u}} \to \mathbf{U}^{\tilde{\nu}}$, $\mathbf{W}^{\tilde{d}} \to \mathbf{W}^{\tilde{e}}$, and $m_{u_k} \to 0$.

Fig. 9.13 is included in Appendix A

Fermion-sfermion-neutralino interactions

The neutralino-fermion-sfermion interaction can be written down in a similar fashion. This time we need to isolate the $a = 3$ term from (5.55) for the $SU(2)_L$ gauge group and the $U(1)_Y$ analog of the terms in (5.36) and express them in terms of the four component matter fermions as well as the four component gauginos and higgsinos in the weak interaction basis, defined in (9.30):

$$
\begin{aligned}
\mathcal{L}_{f\tilde{f}\tilde{\chi}^0} &= -\sqrt{2}g_2\tilde{f}_{iL}\sum_{f=e,\nu,u,d}\bar{f}_iP_R\left[T_3^f\tilde{\lambda}_3+\tan\theta_W(Q_f-T_3^f)\tilde{\lambda}_0\right] \\
&+\sqrt{2}g_2\tan\theta_W\ Q_f\tilde{f}^*_{iR}\overline{\tilde{\lambda}_0}P_Rf_i-\frac{g_2}{\sqrt{2}M_W\cos\beta}(\mathbf{m}_d^*)_{ij}\left[\ \overline{\tilde{h}_1^0}P_L\tilde{d}^\dagger_{jR}d_i+\bar{d}_jP_L\tilde{h}_1^0\tilde{d}_{iL}\right] \\
&-\frac{g_2}{\sqrt{2}M_W\sin\beta}(\mathbf{m}_u^*)_{ij}\left[\ \overline{\tilde{h}_2^0}P_L\tilde{u}^\dagger_{jR}u_i+\bar{u}_jP_L\tilde{h}_2^0\tilde{u}_{iL}\right] \\
&-\frac{g_2}{\sqrt{2}M_W\cos\beta}(\mathbf{m}_e^*)_{ij}\left[\ \overline{\tilde{h}_1^0}P_Le_i\tilde{e}^*_{jR}+\bar{e}_jP_L\tilde{h}_1^0\tilde{e}_{iL}\right]+\text{h.c.} \qquad (9.71)
\end{aligned}
$$

In (9.71) T_{3L}^f and Q_f are respectively the third component of weak isospin and the electromagnetic charge of fermion type f and i, j are generation indices as before. In terms of the neutralino mass eigenstates $\tilde{\chi}_l^0$, (9.71) becomes

$$
\begin{aligned}
\mathcal{L}_{f\tilde{f}\tilde{\chi}^0} &= \sum_{f=u,d,e,\nu}\overline{\tilde{\chi}_l^0}\left(G_l^{f_L}\tilde{f}^*_{iL}P_L+G_l^{f_R}\tilde{f}^*_{iR}P_R\right)f_i \\
&- \frac{g_2}{\sqrt{2}M_W\sin\beta}\left[(\mathbf{m}_u^*)_{ij}Z^*_{l4}\tilde{u}^\dagger_{jR}\overline{\tilde{\chi}_l^0}P_Lu_i+(\mathbf{m}_u)_{ij}Z_{l4}\tilde{u}^\dagger_{iL}\overline{\tilde{\chi}_l^0}P_Ru_j\right] \\
&- \frac{g_2}{\sqrt{2}M_W\cos\beta}\left[(\mathbf{m}_d^*)_{ij}Z^*_{l3}\tilde{d}^\dagger_{jR}\overline{\tilde{\chi}_l^0}P_Ld_i+(\mathbf{m}_d)_{ij}Z_{l3}\tilde{d}^\dagger_{iL}\overline{\tilde{\chi}_l^0}P_Rd_j\right] \\
&- \frac{g_2}{\sqrt{2}M_W\cos\beta}\left[(\mathbf{m}_e^*)_{ij}Z^*_{l3}\tilde{e}^*_{jR}\overline{\tilde{\chi}_l^0}P_Le_i+(\mathbf{m}_e)_{ij}Z_{l3}\tilde{e}^\star_{iL}\overline{\tilde{\chi}_l^0}P_Re_j\right]+\text{h.c.}\ , \qquad (9.72)
\end{aligned}
$$

where we have used (9.31). The coupling strengths $G_l^{f_L}$ and $G_l^{f_R}$ in (9.71) can be written as

$$G_l^{f_L} = -\sqrt{2}g_2\left[T_{3L}^fZ^*_{l2}+\tan\theta_W(Q_f-T_{3L}^f)Z^*_{l1}\right], \qquad (9.73a)$$

$$G_l^{f_R} = \sqrt{2}g_2\tan\theta_WQ_fZ_{l1}\ . \qquad (9.73b)$$

Once more, we can rewrite the interactions of (9.72) in terms of mass diagonal quark and lepton fields by performing flavor rotations in generation space with indices i, j. Similarly, the squark and slepton interaction eigenstates appearing in (9.72) can be related to the corresponding mass eigenstates through (9.54). The quark and squark fields are also three component row or column vectors in color space. Altogether the relevant interaction terms for the quark/squark sector can be written as (we again suppress the superscript m indicating mass eigenstates):

$$\mathcal{L}_{q\tilde{q}'\tilde{\chi}^0} = \overline{\tilde{\chi}_l^0}\Big[(G^{u_L}_{isl}P_L + G^{u_R}_{isl}P_R)\tilde{u}_s^\dagger u_i + (G^{d_L}_{isl}P_L + G^{d_R}_{isl}P_R)\tilde{d}_s^\dagger d_i\Big] + \text{h.c.} \tag{9.74}$$

In (9.74) we have defined the couplings

$$G^{u_L}_{isl} = G^{u_L}_l \sum_{j=1}^{3} W^{\tilde{u}*}_{js} U^{u_L}_{ji} - \frac{g_2}{\sqrt{2}M_W \sin\beta} m_{u_i} Z^*_{l4} \sum_{j=1}^{3} W^{\tilde{u}*}_{j+3\ s} U^{u_R}_{ji} \ , \tag{9.75a}$$

$$G^{u_R}_{isl} = G^{u_R}_l \sum_{j=1}^{3} W^{\tilde{u}*}_{j+3\ s} U^{u_R}_{ji} - \frac{g_2}{\sqrt{2}M_W \sin\beta} m_{u_i} Z_{l4} \sum_{j=1}^{3} W^{\tilde{u}*}_{js} U^{u_L}_{ji} \ , \tag{9.75b}$$

$$G^{d_L}_{isl} = G^{d_L}_l \sum_{j=1}^{3} W^{\tilde{d}*}_{js} U^{d_L}_{ji} - \frac{g_2}{\sqrt{2}M_W \cos\beta} m_{d_i} Z^*_{l3} \sum_{j=1}^{3} W^{\tilde{d}*}_{j+3\ s} U^{d_R}_{ji} \ , \tag{9.75c}$$

$$G^{d_R}_{isl} = G^{d_R}_l \sum_{j=1}^{3} W^{\tilde{d}*}_{j+3\ s} U^{d_R}_{ji} - \frac{g_2}{\sqrt{2}M_W \cos\beta} m_{d_i} Z_{l3} \sum_{j=1}^{3} W^{\tilde{d}*}_{js} U^{d_L}_{ji} \ , \tag{9.75d}$$

where the coefficients $G^{q_L}_l$ and $G^{q_R}_l$ are as in eqs.(9.73a) and (9.73b), respectively. Feynman rules for the vertices of (9.75) are given in Fig. 9.14. An arrow has been put on the neutralino line in conformity with the convention in Appendix D of the first paper of Ref. [9.10].

Fig. 9.14 is included in Appendix A

Let us remark once again that, in the limit of massless fermions, the higgsinos will decouple from the matter fermion/sfermion sector. Note also that the couplings of neutral higgsinos to quark mass eigenstates are proportional to the mass of that eigenstate. This is in contrast to the couplings of the charged higgsinos, where heavy quark masses contribute to the coupling of light quarks. However, due to the smallness of the KM elements mixing the third generation with the first two, in practice one can still often neglect the Yukawa contributions to chargino and neutralino couplings to first and second generation fermions. In the alignment option of §9.5 the products of flavor rotation matrices can be put equal to unity in either the up or down quark sector (but not for both simultaneously, as we noted earlier). On the other hand, if squarks of all three generations are degenerate and LR mixing can be ignored, all products of rotation matrices appearing in (9.75) collapse to Kronecker-δs, where either $i = s$ or $i + 3 = s$.

Let us now turn our attention to the lepton/slepton sector. The interaction terms with neutralinos can be written as

$$\mathcal{L}_{\ell\tilde{\ell}'\tilde{\chi}^0} = \overline{\tilde{\chi}_l^0}\Big[G^{\nu}_{ijl}\tilde{\nu}_j^* P_L \nu_i + (G^{e_L}_{isl}P_L + G^{e_R}_{isl}P_R)\tilde{e}_s^* e_i\Big] + \text{h.c.} \tag{9.76}$$

In (9.76) we have introduced the couplings

$$G^{\nu}_{ijl} = G^{\nu}_{l} U^{\tilde{\nu}*}_{ij}, \tag{9.77a}$$

$$G^{e_L}_{isl} = G^{e_L}_{l} W^{\tilde{e}*}_{is} - \frac{g_2}{\sqrt{2} M_W \cos\beta} m_{e_i} Z^*_{l3} W^{\tilde{e}*}_{i+3\ s}, \tag{9.77b}$$

$$G^{e_R}_{isl} = G^{e_R}_{l} W^{\tilde{e}*}_{i+3\ s} - \frac{g_2}{\sqrt{2} M_W \cos\beta} m_{e_i} Z_{l3} W^{\tilde{e}*}_{is}, \tag{9.77c}$$

The vertex Feynman rules appear in Fig. 9.15. In the alignment option, or if sleptons are mass degenerate, the slepton flavor rotation matrices $\mathbf{U}^{\tilde{\nu}}$ and $\mathbf{W}^{\tilde{e}}$ can be put equal to the identity matrix, if $\tilde{e}_L$-$\tilde{e}_R$ mixing is negligible. LR mixing can, as usual, be included in these options by using (9.60). Once again an arrow has been put [9.10] on the neutralino line.

Fig. 9.15 is included in Appendix A

Quark-squark-gluino interactions

These are now different from the pure SQCD case, cf. (5.60) and Fig. 5.2. However, with the armory of quark and squark flavor rotation matrices that have been developed already, we can write the relevant interaction terms in a straightforward way as follows[9].

$$\mathcal{L}_{q\tilde{q}'^{\star}\tilde{g}} = -\sqrt{2} g_s \sum_{q=u,d} \bar{q}_i \left[U^{q_L *}_{ji} W^{\tilde{q}}_{js} P_R - U^{q_R *}_{ji} W^{\tilde{q}}_{j+3\ s} P_L \right] T^a \tilde{g}^a \tilde{q}_s + \text{h.c.} \tag{9.78}$$

We have again suppressed the superscript m denoting mass eigenstates, and have written (s)quark fields as vectors in color space. The corresponding Feynman rules are given in Fig. 9.16; we have used them already in §9.5, in the basis where $\mathbf{U}^{d_L} = \mathbf{U}^{d_R} = \mathbb{1}$.

Fig. 9.16 is included in Appendix A

Eqs. (9.68), (9.75) and (9.78) are in a general basis of the quark and squark interaction eigenstates. Not all the rotation matrices appearing in these equations are separately physical quantities. Despite the occurrence of the matrices $\mathbf{U}^{u_R}$ and $\mathbf{U}^{d_R}$ in some of these equations, one can only measure the products of quark and squark mixing matrices which appear in these couplings. Note that exactly one factor in these products is always the hermitian conjugate of a rotation matrix. This shows that only any *misalignment* between righthanded quarks and "righthanded" ($SU(2)$ singlet) squarks is measurable. That can also be seen by defining $U^{q_R *}_{ij} q_{iR}$ and $U^{q_R *}_{ij} \tilde{q}_{iR}$ as new "interaction eigenstates". This redefinition does not modify any of the gauge interactions in the MSSM Lagrangian. The righthanded quark mixing matrices would then disappear from (9.68), (9.75) and (9.78); more exactly, they would be absorbed in the squark rotation matrices $\mathbf{W}^{\tilde{q}}$, which are not invariant under this redefinition of the $\tilde{q}_R$ "interaction eigenstates".[10] Indeed, practical calculations are usually performed in this basis,

[9] The s subscript of g, referring to the strong coupling, should not be confused with the squark mass eigenstate label s.

[10] Of course, *products* of rotation matrices that appear in couplings of mass eigenstates are invariant under redefinitions of current eigenstates.

because the relevant couplings are simpler than in a general basis. One can even go one step further and chose the $SU(2)_L$ doublet (s)quark interaction eigenstates in such a way that either the up or the down quark mass matrix (but not both!) becomes diagonal. The only quark rotation matrix appearing in the quark squark chargino/neutralino/gluino couplings is then the KM matrix. Of course, such a procedure will yet again modify the squark rotation matrices. In these bases our interactions are modified as follows: $\mathbf{U}^{u_R}, \mathbf{U}^{d_R} \rightarrow \mathbb{1}$, and either $\mathbf{U}^{d_L} \rightarrow \mathbb{1}, \mathbf{U}^{u_L} \rightarrow (\mathbf{V}^{q_L})^\dagger$ (in the basis where $\mathbf{m}_d$ is diagonal), or $\mathbf{U}^{u_L} \rightarrow \mathbb{1}, \mathbf{U}^{d_L} \rightarrow \mathbf{V}^{q_L}$ (in the basis where $\mathbf{m}_u$ is diagonal).

Flavor mixing in the fermion-sfermion-bosino couplings is of much greater phenomenological importance than the "super-CKM mixing" introduced in §8.4. The latter appears in the coupling of W bosons to squarks and sleptons; the only process of current interest where these couplings play a role is slepton production at hadron colliders, which is however difficult to detect anyway (see §15.3). In contrast, the couplings listed in this section not only determine the constraints on flavor mixing described in §9.5; they also largely determine how sparticles decay. For example, the "flavor" of a squark is usually defined through the quark to which this squark decays. However, in the presence of significant flavor mixing this definition may not be unique: several different quarks might couple to the same squark mass eigenstate. The relative branching ratios into different quark flavors may even depend on the -ino that is produced in that decay. For example, different combinations of mixing matrices appear in squark to neutralino plus quark decays, described by the Lagrangian (9.74), than in squark to gluino plus quark decays described by (9.78). Conversely, these couplings determine which (combinations of) flavors are produced in the decays of gluinos, charginos and neutralinos. For example, (9.76) and (9.77) show that the observation of decays of the type $\tilde{\chi}_l^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell'^-$, with $l > 1$ and $\ell \neq \ell'$, would be an unambiguous sign for slepton flavor mixing.

This completes our discussion of vertices with gauginos/higgsinos interacting with a fermion-sfermion combination.

9.8 Quartic Sfermion Vertices

The final nongauge and nonHiggs interaction that needs to be discussed is the interaction of four sfermions. These vertices appear e.g. in one loop corrections to sfermion pair production processes, and in two loop corrections to reactions without external superparticles. In (8.49) we gave the relevant part of the Lagrangian in the absence of $\tilde{f}_L$-$\tilde{f}_R$ mixing. In that case sfermion mixing matrices only appeared in the F-term (Yukawa) contributions, and in the part of the $SU(2)_L$ D-term that couples $\tilde{u}_L$ to $\tilde{d}_L$ squarks, and $\tilde{\nu}$ to $\tilde{e}_L$ sleptons. However, since $\tilde{f}_L$ and $\tilde{f}_R$ have different gauge quantum numbers, nonvanishing $\tilde{f}_L$-$\tilde{f}_R$ mixing means that sfermion mixing in general affects almost all terms in the quartic interaction Lagrangian. This is true even for the $SU(3)_C$ D-terms, since the $\tilde{q}_L$ squarks reside in left chiral superfields that transform as triplets under $SU(3)_C$, while the $\tilde{\bar{q}}_R^*$ reside in antitriplet left chiral superfields: The $SU(3)_C$ D-term contributions from the two therefore differ by a relative sign, as shown in (5.60). The only exception is the term involving four sneutrinos, since the MSSM assumes the absence of $SU(2)_L$ singlet sneutrinos with weak scale masses.

The relevant part of the Lagrangian can now be written as

$$-\mathcal{L}_{\tilde{f}^4} = \sum_{\tilde{f}_1,\tilde{f}_2,\tilde{f}_3,\tilde{f}_4} Y[\tilde{f}_1,\tilde{f}_2,\tilde{f}_3,\tilde{f}_4]\tilde{f}_1^*\tilde{f}_2\tilde{f}_3^*\tilde{f}_4\ , \tag{9.79}$$

where the Y are constant (field independent) coefficients. In (9.79) the indices $\tilde{f}_i$ of Y have been written in the form of arguments, rather than as superscripts or subscripts, in order to avoid an excessive proliferation of subscripts. The sum in (9.79) runs over sfermion type ($\tilde{u}, \tilde{d}, \tilde{e}$ and $\tilde{\nu}$), mass eigenstate labels, and color indices.

The Y coefficients of (9.79) are given explicitly by

$$\begin{aligned}
Y[\tilde{u}_s^{a*},\tilde{u}_t^a,\tilde{u}_u^{b*},\tilde{u}_v^b] &= \frac{g_2^2}{2M_W^2\sin^2\beta}\sum_{i,j,k,l,m,n=1}^{3} W_{is}^{\tilde{u}*}U_{ik}^{u_L}m_{u_k}U_{jk}^{u_R*}W_{j+3\ t}^{\tilde{u}}W_{lv}^{\tilde{u}}U_{lm}^{u_L*}m_{u_m}U_{nm}^{u_R}W_{n+3\ u}^{\tilde{u}*}\\
&+ \frac{g_s^2}{4}\left[\delta_{sv}\delta_{tu}-\frac{1}{3}\delta_{st}\delta_{uv}-4\sum_{i,j=1}^{3}W_{is}^{\tilde{u}*}W_{j+3\ u}^{\tilde{u}*}\left(W_{iv}^{\tilde{u}}W_{j+3\ t}^{\tilde{u}}-\frac{1}{3}W_{it}^{\tilde{u}}W_{j+3\ v}^{\tilde{u}}\right)\right]\\
&+ \frac{g_2^2}{8}\left(1+\frac{\tan^2\theta_W}{9}\right)\sum_{i,j=1}^{3}W_{is}^{\tilde{u}*}W_{it}^{\tilde{u}}W_{ju}^{\tilde{u}*}W_{jv}^{\tilde{u}}\\
&+ \frac{g_2^2\tan^2\theta_W}{9}\sum_{i,j=1}^{3}\left(2W_{i+3\ s}^{\tilde{u}*}W_{i+3\ t}^{\tilde{u}}-W_{is}^{\tilde{u}^*}W_{it}^{\tilde{u}}\right)W_{j+3\ u}^{\tilde{u}*}W_{j+3\ v}^{\tilde{u}}\ ,
\end{aligned}\tag{9.80a}$$

$$\begin{aligned}
Y[\tilde{d}_s^{a*},\tilde{d}_t^a,\tilde{d}_u^{b*},\tilde{d}_v^b] &= \frac{g_2^2}{2M_W^2\cos^2\beta}\sum_{i,j,k,l,m,n=1}^{3} W_{is}^{\tilde{d}*}U_{ik}^{d_L}m_{d_k}U_{jk}^{d_R*}W_{j+3\ t}^{\tilde{d}}W_{lv}^{\tilde{d}}U_{lm}^{d_L*}m_{d_m}U_{nm}^{d_R}W_{n+3\ u}^{\tilde{d}*}\\
&+ \frac{g_s^2}{4}\left[\delta_{sv}\delta_{tu}-\frac{1}{3}\delta_{st}\delta_{uv}-4\sum_{i,j=1}^{3}W_{is}^{\tilde{d}*}W_{j+3\ u}^{\tilde{d}*}\left(W_{iv}^{\tilde{d}}W_{j+3\ t}^{\tilde{d}}-\frac{1}{3}W_{it}^{\tilde{d}}W_{j+3\ v}^{\tilde{d}}\right)\right]\\
&+ \frac{g_2^2}{8}\left(1+\frac{\tan^2\theta_W}{9}\right)\sum_{i,j=1}^{3}W_{is}^{\tilde{d}*}W_{it}^{\tilde{d}}W_{ju}^{\tilde{d}*}W_{jv}^{\tilde{d}}\\
&+ \frac{g_2^2\tan^2\theta_W}{36}\sum_{i,j=1}^{3}\left(W_{i+3\ s}^{\tilde{d}*}W_{i+3\ t}^{\tilde{d}}-W_{is}^{\tilde{d}^*}W_{it}^{\tilde{d}}\right)W_{j+3\ u}^{\tilde{d}*}W_{j+3\ v}^{\tilde{d}}\ ,
\end{aligned}\tag{9.80b}$$

$$\begin{aligned}
Y[\tilde{u}_s^{a*},\tilde{u}_t^a,\tilde{d}_u^{b*},\tilde{d}_v^b] &= -\frac{g_s^2}{6}\left[\delta_{st}\delta_{uv}-2\sum_{i,j=1}^{3}\left(W_{is}^{\tilde{u}*}W_{it}^{\tilde{u}}W_{j+3\ u}^{\tilde{d}^*}W_{j+3\ v}^{\tilde{d}}+W_{i+3\ s}^{\tilde{u}*}W_{i+3\ t}^{\tilde{u}}W_{ju}^{\tilde{d}*}W_{jv}^{\tilde{d}}\right)\right]\\
&- \frac{g_2^2}{4}\left(1-\frac{\tan^2\theta_W}{9}\right)\sum_{i,j=1}^{3}W_{is}^{\tilde{u}*}W_{it}^{\tilde{u}}W_{ju}^{\tilde{d}*}W_{jv}^{\tilde{d}}\\
&+ \frac{g_2^2\tan^2\theta_W}{18}\sum_{i,j=1}^{3}\Big[W_{is}^{\tilde{u}*}W_{it}^{\tilde{u}}W_{j+3\ u}^{\tilde{d}*}W_{j+3\ v}^{\tilde{d}}\\
&\qquad +2W_{i+3\ s}^{\tilde{u}^*}W_{i+3\ t}^{\tilde{u}}\left(W_{ju}^{\tilde{d}*}W_{jv}^{\tilde{d}}+2W_{j+3\ u}^{\tilde{d}*}W_{j+3\ v}^{\tilde{d}}\right)\Big]\ ,
\end{aligned}\tag{9.80c}$$

$$
\begin{aligned}
Y[\tilde{u}_s^{a*}, \tilde{d}_t^{a}, \tilde{d}_u^{b*}, \tilde{u}_v^{b}] &= \frac{g_2^2}{2M_W^2 \sin^2\beta} \sum_{i,j,k,l,m,n=1}^{3} W_{iu}^{\tilde{d}*} U_{ik}^{u_L} m_{u_k} U_{jk}^{u_R *} W_{j+3\ v}^{\tilde{u}} W_{lt}^{\tilde{d}} U_{lm}^{u_L *} m_{u_m} U_{nm}^{u_R} W_{n+3\ s}^{\tilde{u}*} \\
&+ \frac{g_2^2}{2M_W^2 \cos^2\beta} \sum_{i,j,k,l,m,n=1}^{3} W_{is}^{\tilde{u}*} U_{ik}^{d_L} m_{d_k} U_{jk}^{d_R *} W_{j+3\ t}^{\tilde{d}} W_{lv}^{\tilde{u}} U_{lm}^{d_L *} m_{d_m} U_{nm}^{d_R} W_{n+3\ u}^{\tilde{d}*} \\
&+ \frac{g_s^2}{2} \left[\delta_{sv}\delta_{tu} - 2 \sum_{i,j=1}^{3} \left(W_{is}^{\tilde{u}*} W_{iv}^{\tilde{u}} W_{j+3\ u}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} + W_{i+3\ s}^{\tilde{u}*} W_{i+3\ v}^{\tilde{u}} W_{ju}^{\tilde{d}*} W_{jt}^{\tilde{d}} \right) \right] \\
&+ \frac{g_2^2}{2} \sum_{i,j=1}^{3} W_{is}^{\tilde{u}*} W_{it}^{\tilde{d}} W_{ju}^{\tilde{d}*} W_{jv}^{\tilde{u}} \,, \qquad (9.80\text{d})
\end{aligned}
$$

$$
\begin{aligned}
Y[\tilde{u}_s^{a*}, \tilde{u}_t^{a}, \tilde{e}_u^{*}, \tilde{e}_v] &= -\frac{g_2^2}{4} \left(1 + \frac{\tan^2\theta_W}{3} \right) \sum_{i,j=1}^{3} W_{is}^{\tilde{u}*} W_{it}^{\tilde{u}} W_{ju}^{\tilde{e}*} W_{jv}^{\tilde{e}} \\
&+ \frac{g_2^2 \tan^2\theta_W}{6} \sum_{i,j=1}^{3} \left[W_{is}^{\tilde{u}*} W_{it}^{\tilde{u}} W_{j+3\ u}^{\tilde{e}*} W_{j+3\ v}^{\tilde{e}} \right. \\
&\qquad \left. + 2 W_{i+3\ s}^{\tilde{u}^*} W_{i+3\ t}^{\tilde{u}} \left(W_{ju}^{\tilde{e}*} W_{jv}^{\tilde{e}} - 2 W_{j+3\ u}^{\tilde{e}*} W_{j+3\ v}^{\tilde{e}} \right) \right] \,, \qquad (9.80\text{e})
\end{aligned}
$$

$$
\begin{aligned}
Y[\tilde{d}_s^{a*}, \tilde{d}_t^{a}, \tilde{e}_u^{*}, \tilde{e}_v] &= \frac{g_2^2}{2M_W^2 \cos^2\beta} \sum_{i,j,k,l=1}^{3} \left(W_{is}^{\tilde{d}*} U_{ik}^{d_L} m_{d_k} U_{jk}^{d_R *} W_{j+3\ t}^{\tilde{d}} W_{l+3\ u}^{\tilde{e}*} m_{e_l} W_{lv}^{\tilde{e}} \right. \\
&\qquad \left. + W_{i+3\ s}^{\tilde{d}*} U_{ik}^{d_R} m_{d_k} U_{jk}^{d_L *} W_{jt}^{\tilde{d}} W_{lu}^{\tilde{e}*} m_{e_l} W_{l+3\ v}^{\tilde{e}} \right) \\
&+ \frac{g_2^2}{4} \left(1 - \frac{\tan^2\theta_W}{3} \right) \sum_{i,j=1}^{3} W_{is}^{\tilde{d}*} W_{it}^{\tilde{d}} W_{ju}^{\tilde{e}*} W_{jv}^{\tilde{e}} \\
&+ \frac{g_2^2 \tan^2\theta_W}{6} \sum_{i,j=1}^{3} \left[W_{is}^{\tilde{d}*} W_{it}^{\tilde{d}} W_{j+3\ u}^{\tilde{e}*} W_{j+3\ v}^{\tilde{e}} \right. \\
&\qquad \left. - W_{i+3\ s}^{\tilde{d}^*} W_{i+3\ t}^{\tilde{d}} \left(W_{ju}^{\tilde{e}*} W_{jv}^{\tilde{e}} - 2 W_{j+3\ u}^{\tilde{e}*} W_{j+3\ v}^{\tilde{e}} \right) \right] \,, \qquad (9.80\text{f})
\end{aligned}
$$

$$
Y[\tilde{u}_s^{a*}, \tilde{u}_t^{a}, \tilde{\nu}_i^{*}, \tilde{\nu}_i] = \frac{g_2^2}{4} \sum_{j=1}^{3} \left[W_{js}^{\tilde{u}*} W_{jt}^{\tilde{u}} \left(1 - \frac{\tan^2\theta_W}{3} \right) + \frac{4\tan^2\theta_W}{3} W_{j+3\ s}^{\tilde{u}^*} W_{j+3\ t}^{\tilde{u}} \right] \,, \qquad (9.80\text{g})
$$

$$
Y[\tilde{d}_s^{a*}, \tilde{d}_t^{a}, \tilde{\nu}_i^{*}, \tilde{\nu}_i] = -\frac{g_2^2}{4} \sum_{j=1}^{3} \left[W_{js}^{\tilde{d}*} W_{jt}^{\tilde{d}} \left(1 + \frac{\tan^2\theta_W}{3} \right) - \frac{2\tan^2\theta_W}{3} W_{j+3\ s}^{\tilde{d}^*} W_{j+3\ t}^{\tilde{d}} \right] \,, \qquad (9.80\text{h})
$$

$$
\begin{aligned}
Y[\tilde{u}_s^{a*}, \tilde{d}_t^{a}, \tilde{e}_u^{*}, \tilde{\nu}_i] &= \frac{g_2^2}{2M_W^2 \cos^2\beta} \sum_{j,k,l,m=1}^{3} W_{js}^{\tilde{u}*} U_{jk}^{d_L} m_{d_k} U_{lk}^{d_R *} W_{l+3\ t}^{\tilde{d}} U_{mi}^{\tilde{\nu}} m_{e_m} W_{m+3\ u}^{\tilde{e}*} \\
&+ \frac{g_2^2}{2} \sum_{j,k=1}^{3} W_{js}^{\tilde{u}*} W_{jt}^{\tilde{d}} U_{ki}^{\tilde{\nu}} W_{ku}^{\tilde{e}*} \,, \qquad (9.80\text{i})
\end{aligned}
$$

$$
Y[\tilde{d}_t^{a*}, \tilde{u}_s^{a}, \tilde{\nu}_i^{*}, \tilde{e}_u] = \left(Y[\tilde{u}_s^{a*}, \tilde{d}_t^{a}, \tilde{e}_u^{*}, \tilde{\nu}_i] \right)^* \,, \qquad (9.80\text{j})
$$

$$
\begin{aligned}
Y[\tilde{e}_s^*, \tilde{e}_t, \tilde{e}_u^*, \tilde{e}_v] &= \frac{g_2^2}{2M_W^2 \cos^2\beta} \sum_{i,j=1}^{3} W_{is}^{\tilde{e}*} m_{e_i} W_{i+3\ t}^{\tilde{e}} W_{j+3\ u}^{\tilde{e}*} m_{e_j} W_{jv}^{\tilde{e}} \\
&+ \frac{g_2^2}{8}\left(1+\tan^2\theta_W\right) \sum_{i,j=1}^{3} W_{is}^{\tilde{e}*} W_{it}^{\tilde{e}} W_{ju}^{\tilde{e}*} W_{jv}^{\tilde{e}} \\
&+ \frac{g_2^2 \tan^2\theta_W}{2} \sum_{i,j=1}^{3} \left(W_{i+3\ s}^{\tilde{e}*} W_{i+3\ t}^{\tilde{e}} - W_{is}^{\tilde{e}^*} W_{it}^{\tilde{e}}\right) W_{j+3\ u}^{\tilde{e}*} W_{j+3\ v}^{\tilde{e}}\ , \qquad (9.80\text{k}) \\
Y[\tilde{e}_s^*, \tilde{e}_t, \tilde{\nu}_i^*, \tilde{\nu}_j] &= \frac{g_2^2}{2M_W^2 \cos^2\beta} \sum_{k,l=1}^{3} U_{ki}^{\tilde{\nu}*} m_{e_k} W_{k+3\ t}^{\tilde{e}} U_{lj}^{\tilde{\nu}} m_{e_l} W_{l+3\ s}^{\tilde{e}*} \\
&+ \frac{g_2^2}{2} \sum_{k,l=1}^{3} W_{ks}^{\tilde{e}^*} U_{kj}^{\tilde{\nu}} W_{lt}^{\tilde{e}} U_{li}^{\tilde{\nu}*} \\
&- \delta_{ij} \frac{g_2^2}{4} \sum_{k=1}^{3} \left[\left(1-\tan^2\theta_W\right) W_{ks}^{\tilde{e}*} W_{kt}^{\tilde{e}} + 2\tan^2\theta_W W_{k+3\ s}^{\tilde{e}*} W_{k+3\ t}^{\tilde{e}}\right]\ , \qquad (9.80\text{l}) \\
Y[\tilde{\nu}_i^*, \tilde{\nu}_i, \tilde{\nu}_j^*, \tilde{\nu}_j] &= \frac{g_2^2}{8}\left(1+\tan^2\theta_W\right)\ . \qquad (9.80\text{m})
\end{aligned}
$$

In (9.80) we have used $s, t, u, v = 1, \ldots, 6$ to label sfermion mass eigenstates, $i, j, k, l, m, n = 1, 2, 3$ are generation (or sneutrino mass eigenstate) labels, and superscripts $a, b = 1, 2, 3$ are $SU(3)$-color labels. Note that there are two different color connections for $\tilde{u}^*\tilde{u}\tilde{d}^*\tilde{d}$ vertices, as shown in (9.80c,d). These two color connections are equivalent for interactions of four squarks of the same type, since they can be transformed into each other by simply exchanging mass eigenstate labels, which are summed in (9.79). When computing the Feynman rules from (9.79) and (9.80), care must be taken to symmetrize properly. The result is displayed in Fig. 9.17.

Fig. 9.17 is included in Appendix A

We conclude with a remark (cf. ftnt.9). The mixing matrices appearing in (9.80) are again not separately invariant under redefinitions of the "current" eigenstates. However, the *products* of mixing matrices appearing in these expressions are invariant under such redefinitions, since they describe couplings of physical particles (i.e. mass eigenstates).

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Chapter 10

HIGGS BOSONS IN THE MSSM

10.1 Higgs Potential in the MSSM

As discussed in Ch.1, low energy supersymmetry has been theoretically motivated to stabilize the mass and the VEV of the Standard Model Higgs boson with respect to higher scales. This makes the Higgs sector of a supersymmetric extension of the Standard Model especially interesting. We have already shown in Ch. 8 that the minimal supersymmetric model requires two Higgs doublets $h_{1,2}$ (with D as an SU(2) doublet index and $Y = -1, 1$ respectively):

$$h_1^D \equiv \begin{pmatrix} h_1^1 \\ h_1^2 \end{pmatrix} = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad h_2^D \equiv \begin{pmatrix} h_2^1 \\ h_2^2 \end{pmatrix} = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}. \tag{10.1}$$

We shall see in this chapter how these doublets lead to five physical Higgs particles $h, H, A, H^{\pm}$ and what one can say about their masses and couplings [10.1], [10.2]. A noteworthy feature, specific to this supersymmetric extension, is that all quartic self couplings of the Higgs fields get related to the gauge couplings of the electroweak theory. This is quite unlike in nonsupersymmetric theories where the former are a priori arbitrary. This restriction is the key to various mass bounds and relations [10.3] which exist for physical Higgs particles in the supersymmetric extension of the Standard Model. A second important feature is that the couplings of the neutral Higgs particles to quark mass eigenstates turn out to be flavor diagonal. This happens because up type quarks obtain their masses purely from the VEV $v_2/\sqrt{2}$ of h_2^0 while down type ones do so from the VEV $v_1/\sqrt{2}$ of h_1^0. In the language of Glashow and Weinberg [10.4] the Higgs sector of the MSSM is a special case of the 'type 2' two Higgs doublet model.

We have already given the MSSM superpotential and the soft explicit supersymmetry breaking terms in Chs. 8 and 9 respectively. The tree level scalar potential is

$$V = V_{SUSY} + V_{SOFT} \ , \tag{10.2}$$

where V_{SUSY} was defined in (8.36) and V_{SOFT} in (9.3). Recall that

$$F_k = -\partial \mathcal{W}_{\text{MSSM}}/\partial \Phi_k^\dagger \Big| , \ \mathcal{W}_{\text{MSSM}} = \mu H_1 \cdot H_2 - f_{ij}^\ell H_1 \cdot L_i \bar{E}_j - f_{ij}^d H_1 \cdot Q \bar{D}_j - f_{ij}^u Q \cdot H_2 \bar{U}_j \ . \tag{10.3}$$

As before, i, j are generation indices and, for any two SU(2)-doublet superfields A^D and B^E, $A{\cdot}B \equiv \epsilon_{DE}A^D B^E$. Moreover,

$$\vec{D}_H = -g_2 h_k^\dagger \frac{\vec{\tau}}{2} h_k \ , \tag{10.4a}$$

$$D_H^Y = -g_Y h_k^\dagger \frac{Y}{2} h_k \ , \tag{10.4b}$$

where we are now using the subscript H to refer exclusively to the Higgs sector and k is summed. Needless to say, both $\vec{D}$ and D^Y will have additional bilinear terms involving squarks as well as those with sleptons.

The tree level Higgs potential follows from (10.2) – (10.4) by inputting V_{SOFT} from (9.3) and utilizing the relation $\vec{\tau}_{AB}{\cdot}\vec{\tau}_{CD} = 2\delta_{AD}\delta_{BC} - \delta_{AB}\delta_{CD}$. Using the notation $h^\dagger h \equiv |h|^2$, it can be written as

$$V_H = \frac{1}{8}(g_Y^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \frac{g_2^2}{2}\ |h_1^\dagger h_2|^2 + |\mu|^2(|h_1|^2 + |h_2|^2) + V_{H,SOFT} \ , \tag{10.5a}$$

$$V_{H,SOFT} = m_1^2|h_1|^2 + m_2^2|h_2|^2 + (m_{12}^2 h_1{\cdot}h_2 + h.c.) \ , \tag{10.5b}$$

with coefficients m_1^2, m_2^2 and $m_{12}^2 \equiv B\mu$, cf.(9.3), having the dimension[1] of squared mass. In following the steps to (10.5), it may be noted that $h_1{\cdot}h_2 = \tilde{h}_1^\dagger h_2$ where $\tilde{h}_1 = i\tau_2 h_1^*$ is an SU(2) doublet with $Y = 1$. (10.5a) and (10.5b) can be rewritten as

$$V_H = \frac{1}{8}(g_Y^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \frac{g_2^2}{2}|h_1^\dagger h_2|^2 + m_{1h}^2|h_1|^2 + m_{2h}^2|h_2|^2 + (m_{12}^2 h_1{\cdot}h_2 + h.c.), \tag{10.6}$$

where

$$m_{1,2h}^2 = m_{1,2}^2 + |\mu|^2 \ . \tag{10.7}$$

The sign of the last RHS term in (10.6) has been chosen with care. It will be seen later that $m_{12}^2 = B\mu$ is expected to be positive.

10.2 Spontaneous Symmetry Breakdown and VEVs

A Higgs induced spontaneous symmetry breaking will take place if the minimum of V_H is attained at nonzero values of the Higgs fields:

$$\langle h_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{10.8}$$

In (10.6) one can[2] always absorb a relative phase between h_1 and h_2 by redefining one of them with an additional phase; this freedom enables us to define $v_{1,2}$ as **real and positive**

[1] We remind the reader that B is a soft supersymmetry breaking parameter with the dimension of mass, while μ is a supersymmetry invariant (higgsino mass) parameter.

[2] Any VEV for one charged Higgs field can be rotated to zero by an $SU(2)$ transformation and then the minimization condition means a vanishing VEV for the other charged Higgs. This is a consequence of inbuilt $U(1)_{\mathrm{em}}$ invariance which thus remains unbroken.

and also to treat m_1^2, m_2^2 and m_{12}^2 as real. Recall from §8.2 that these VEVs can be related to the W and Z masses by

$$M_W = \frac{g_2}{2}(v_1^2 + v_2^2)^{1/2}, \quad M_Z = \frac{(g_Y^2 + g_2^2)^{1/2}}{2}(v_1^2 + v_2^2)^{1/2}, \tag{10.9}$$

i.e.

$$(v_1^2 + v_2^2)^{1/2} = (\sqrt{2}G_F)^{-1} \simeq 246 \text{ GeV}. \tag{10.10}$$

Let us consider the parameter $\tan\beta$, as introduced in (8.24), namely

$$\tan\beta \equiv v_2/v_1. \tag{10.11}$$

Now, our phase freedom to define $v_{1,2}$ as positive restricts β to the range

$$0 \leq \beta \leq \pi/2 \ .$$

Though $\tan\beta$ will generally be left undetermined in this book, current theoretical widsom suggests [10.5] that the value of $\tan\beta$ is restricted to the range $1 \leq \tan\beta \lesssim 60$. The lower and upper bounds both stem from the desired requirement (cf. Ch.11) of radiatively induced electroweak symmetry breakdown by which one of the eigenvalues of the neutral Higgs mass squared matrix, evaluated at $v_1 = 0 = v_2$, is driven to be negative by the top Yukawa coupling via Renormalization Group Evolution. They come also from the requirement of all the couplings participating in the RGE equations remaining perturbative upto a high grand unifying scale like 2×10^{16} GeV. These issues, including additional experimental constraints on $\tan\beta$, will be discussed more thoroughly in Ch.11.

Near the minimum, characterized by the VEVs $\langle h_{1,2}^0 \rangle = v_{1,2}/\sqrt{2}$, $\langle h_1^- \rangle = 0 = \langle h_2^+ \rangle$, it is sufficient to explore the Higgs potential retaining only the neutral Higgs fields. This part of the Higgs potential can be written from (10.6) as

$$V_H^0 = \frac{1}{8}(g_Y^2 + g_2^2)^2(|h_1^0|^2 - |h_2^0|^2)^2 + m_{1h}^2|h_1^0|^2 + m_{2h}^2|h_2^0|^2 - m_{12}^2(h_1^0 h_2^0 + \text{h.c.}) \ , \tag{10.12}$$

where the negative sign before the last RHS term proportional to m_{12}^2 has arisen because $\epsilon_{12} = -1$. The quartic terms in (10.12) vanish along $|h_1^0| = |h_2^0|$. By further choosing $h_1^0 = \pm h_2^0$, we see that the fact that V_H^0 must be bounded from below requires that

$$m_{1h}^2 + m_{2h}^2 = m_1^2 + m_2^2 + 2|\mu|^2 > 2|m_{12}^2| \ . \tag{10.13}$$

Because of quantum corrections and renormalization group evolution (cf. Ch.11), m_{1h}^2, m_{2h}^2 and m_{12}^2 become running quantities — varying with the energy scale, cf. §6.9. However, (10.13) has to be valid at all scales. On the other hand, the quadratic part of V_H^0 can be written as

$$V_H^{0,quadr.} = (h_1^{0*} \ \ h_2^0) \begin{pmatrix} m_{1h}^2 & -m_{12}^2 \\ -m_{12}^2 & m_{2h}^2 \end{pmatrix} \begin{pmatrix} h_1^0 \\ h_2^{0*} \end{pmatrix} . \tag{10.14}$$

For the nonzero VEVs $v_{1,2}$ to develop, at least one of the eigenvalues of the mass squared matrix in (10.14) has to be negative. Since (10.13) requires the matrix to have a positive trace, one is led to the necessary condition for spontaneous symmetry breakdown that its determinant be negative, i.e.

$$m_{12}^4 > m_{1h}^2 m_{2h}^2 = (m_1^2 + |\mu|^2)(m_2^2 + |\mu|^2) \ . \tag{10.15}$$

(10.15) is valid only at and below the energy scale where the spontaneous breaking of electroweak symmetry becomes operative. Furthermore, (10.13) and (10.15) become mutually incompatible in the supersymmetry invariant limit when $m_{1h}^2 = m_{2h}^2 = \mu^2$. Hence there is **an intimate connection between the breaking of supersymmetry and that of electroweak symmetry in the MSSM**.

Let us return to (10.6) and explore V_H at its supposed minimum, i.e. at $h_{1,2} = \langle h_{1,2} \rangle$, as given by (10.8). Thus

$$V_H^{min} = \frac{1}{32}(g_Y^2 + g_2^2)(v_1^2 - v_2^2)^2 + \frac{1}{2} m_{1h}^2 v_1^2 + \frac{1}{2} m_{2h}^2 v_2^2 - m_{12}^2 v_1 v_2 \ . \tag{10.16}$$

The consistency conditions for the above mentioned minimum is the vanishing of $\partial V_H^{min}/\partial v_1$ and $\partial V_H^{min}/\partial v_2$. These respectively imply the relations

$$m_{1h}^2 = m_{12}^2 \frac{v_2}{v_1} - \frac{1}{8}(g_Y^2 + g_2^2)(v_1^2 - v_2^2) \ , \tag{10.17a}$$

$$m_{2h}^2 = m_{12}^2 \frac{v_1}{v_2} + \frac{1}{8}(g_Y^2 + g_2^2)(v_1^2 - v_2^2) \ . \tag{10.17b}$$

By using (10.7), (10.10) and (10.11) in (10.17), the latter can be recast into the following equations:

$$-2B\mu = -2m_{12}^2 = (m_1^2 - m_2^2)\tan 2\beta + M_Z^2 \sin 2\beta \ , \tag{10.18a}$$

$$|\mu|^2 = (\cos 2\beta)^{-1}(m_2^2 \sin^2\beta - m_1^2 \cos^2\beta) - \frac{1}{2} M_Z^2 \ . \tag{10.18b}$$

10.3 Higgs Masses at the Tree Level

Though we shall see in §10.6 that there are significant radiative corrections to Higgs masses in the MSSM, we first discuss their tree level values here. The mass squared matrix of the Higgs scalars can be obtained from the quadratic part of V_H, i.e. $V_H^{(2)} = \frac{1}{2} m_{lm}^2 \phi_l \phi_m$ with

$$m_{lm}^2 = \left\langle \frac{\partial^2 V_H}{\partial \phi_l \partial \phi_m} \right\rangle , \tag{10.19}$$

where ϕ_l is the generic notation for the real or imaginary part of any Higgs component field and the double derivative is evaluated at the minimum. The 8×8 Higgs mass squared matrix then breaks up diagonally into a set of 2×2 matrices.

Charged Goldstones and Higgs

The total charged Higgs mass term, obtained by using (10.8) in (10.6), is given by

$$V_{h^\pm}^{quadr} =$$
$$(h_1^+ \ \ h_2^+) \begin{pmatrix} m_{1h}^2 + \frac{1}{8}(g_Y^2+g_2^2)(v_1^2-v_2^2) + \frac{1}{4}g_2^2 v_2^2 & m_{12}^2 + \frac{1}{4}g_2^2 v_1 v_2 \\ m_{12}^2 + \frac{1}{4}g_2^2 v_1 v_2 & m_{2h}^2 - \frac{1}{8}(g_Y^2+g_2^2)(v_1^2-v_2^2) + \frac{1}{4}g_2^2 v_1^2 \end{pmatrix} \begin{pmatrix} h_1^- \\ h_2^- \end{pmatrix}$$
$$= \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g_2^2\right)(h_1^+ \ \ h_2^+) \begin{pmatrix} v_2^2 & v_1 v_2 \\ v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} h_1^- \\ h_2^- \end{pmatrix}, \tag{10.20}$$

where – in the last step – eqs. (10.17) have been used. The vanishing determinant and the nonvanishing trace of the matrix in the RHS of (10.20) imply massless as well as massive charged modes. The former are the Goldstone boson pair $G^\pm$ which combine with the massless $W^\pm$ to give them mass. The latter pertain to the physical charged Higgs particles $H^\pm$. Thus one has

$$m_{G^\pm}^2 = 0 \ , \tag{10.21a}$$

$$m_{H^\pm}^2 = \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g_2^2\right)(v_1^2 + v_2^2) \ . \tag{10.21b}$$

It follows from (10.20) and (10.11) that the corresponding mass diagonal fields are

$$H^\pm = \sin\beta \ h_1^\pm + \cos\beta \ h_2^\pm \ , \tag{10.22a}$$

$$G^\pm = -\cos\beta \ h_1^\pm + \sin\beta \ h_2^\pm \ , \tag{10.22b}$$

The couplings of $G^\pm$ in a general R-gauge are given in Ref. [10.1]. However, we formulate our discussions in the unitary gauge where $G^\pm$ are set equal to zero.

Neutral Goldstone and CP odd Higgs

Choosing $\phi_{\ell,m}$ in (10.19) to be $\Im m \ h_{1,2}^0$, we have the corresponding mass squared matrix:

$$m_{\Im m \ h^0}^2 =$$
$$\begin{pmatrix} m_{1h}^2 + \frac{1}{8}(g_Y^2+g_2^2)(v_1^2-v_2^2) & m_{12}^2 \\ m_{12}^2 & m_{2h}^2 - \frac{1}{8}(g_Y^2+g_2^2)(v_1^2-v_2^2) \end{pmatrix} = m_{12}^2 \begin{pmatrix} v_2/v_1 & 1 \\ 1 & v_1/v_2 \end{pmatrix}, \tag{10.23}$$

once again using (10.17). As before, the vanishing determinant and the nonvanishing trace imply a massless neutral Goldstone mode G (which combines with the massless Z) and a neutral scalar which is CP odd on being a linear combination of the imaginary components of the neutral Higgs fields. In fact, we have

$$m_{G^0}^2 = 0 \ , \tag{10.24a}$$

$$m_A^2 = \frac{m_{12}^2}{v_1 v_2}\left(v_1^2 + v_2^2\right) = \frac{2m_{12}^2}{\sin 2\beta} \ . \tag{10.24b}$$

N.B. since $\sin 2\beta$ is restricted to be positive, (10.24b) makes sense only if m_{12}^2 is positive – at least at electroweak energy scales. This is the explanation of the choice of the sign of the last RHS term in (10.6). The mass diagonal fields corresponding to (10.24) are

$$\frac{A}{\sqrt{2}} = \Im m\ h_1^0 \sin\beta + \Im m\ h_2^0 \cos\beta\ , \tag{10.25a}$$

$$\frac{G^0}{\sqrt{2}} = -\Im m\ h_1^0 \cos\beta + \Im m\ h_2^0 \sin\beta\ . \tag{10.25b}$$

In the physical basis, the CP odd neutral Higgs mass term in the Lagrangian density becomes

$$\frac{1}{2}(G^0 \quad A)\begin{pmatrix} 0 & \\ & m_A^2 \end{pmatrix}\begin{pmatrix} G^0 \\ A \end{pmatrix},$$

and the correct normalization of m_A^2 in (10.24b) can be checked from this. The couplings of G^0 in a general R-gauge can be found in Ref. [10.1], but again, in the U-gauge of ours, $G^0 = 0$.

Neutral CP even Higgs

Turning to the $\Re e\ h_{1,2}^0$ components, we find the corresponding mass squared matrix in an analogous way to be

$$\begin{aligned} m_{\Re e\ h^0}^2 &= \frac{1}{2}\begin{pmatrix} 2m_{1h}^2 + \frac{1}{4}(g_Y^2+g_2^2)(3v_1^2-v_2^2) & -2m_{12}^2 - \frac{1}{2}v_1v_2(g_Y^2+g_2^2) \\ -2m_{12}^2 - \frac{1}{2}v_1v_2(g_Y^2+g_2^2) & 2m_{2h}^2 + \frac{1}{4}(g_Y^2+g_2^2)(3v_2^2-v_1^2) \end{pmatrix} \\ &= \begin{pmatrix} m_A^2\sin^2\beta + M_Z^2\cos^2\beta & -(m_A^2+M_Z^2)\sin\beta\cos\beta \\ -(m_A^2+M_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}, \end{aligned} \tag{10.26}$$

where (10.10), (10.17) and (10.24b) have been used. The eigenvalues of the matrix in the RHS of (10.26), standing for the tree level physical squared masses of the two CP even Higgs scalars (H, h) of the MSSM, are

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \pm \{(m_A^2+M_Z^2)^2 - 4M_Z^2 m_A^2\cos^2 2\beta\}^{1/2}\right]. \tag{10.27}$$

In (10.27) we have defined H to be the heavier of the two, i.e. $m_h \leq m_H$. The corresponding mass diagonal fields are

$$\frac{1}{\sqrt{2}}H = (\Re e\ h_1^0 - \frac{v_1}{\sqrt{2}})\cos\alpha + (\Re e\ h_2^0 - \frac{1}{\sqrt{2}}v_2)\sin\alpha\ , \tag{10.28a}$$

$$\frac{1}{\sqrt{2}}h = -(\Re e\ h_1^0 - \frac{v_1}{\sqrt{2}})\sin\alpha + (\Re e\ h_2^0 - \frac{1}{\sqrt{2}}v_2)\cos\alpha\ . \tag{10.28b}$$

Referring back to the matrix of (10.26) as $\begin{pmatrix} A & B \\ B & C \end{pmatrix}$, the angle of rotation α in (10.28) is seen to obey the relations [10.1], [10.2]

$$\sin 2\alpha = \frac{2B}{\sqrt{(A-C)^2 + 4B^2}} = -\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}\sin 2\beta\ , \tag{10.29a}$$

$$\cos 2\alpha = \frac{A-C}{\sqrt{(A-C)^2+4B^2}} = -\frac{m_A^2-M_Z^2}{m_H^2-m_h^2}\cos 2\beta \ , \tag{10.29b}$$

$$\tan 2\alpha = \frac{m_h^2+m_H^2}{m_A^2-M_Z^2}\tan 2\beta = \frac{m_A^2+M_Z^2}{m_A^2-M_Z^2}\tan 2\beta \ . \tag{10.29c}$$

Since β is in the range $0 \le \beta \le \pi/2$, (10.29a) restricts α to the interval

$$-\pi/2 \le \alpha \le 0 \ .$$

A geometrical depiction is given in Fig. 10.1. Note that we always have

$$\sin(\beta-\alpha), \cos(\beta+\alpha) \ge 0 \ .$$

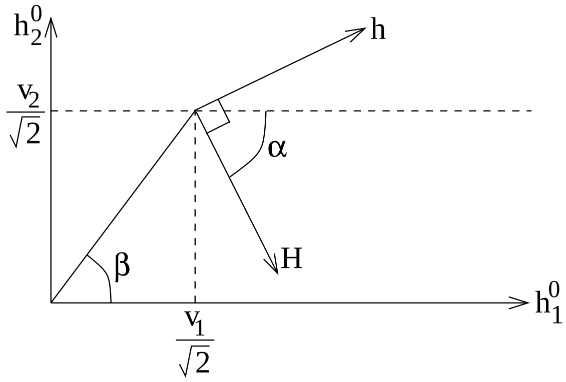

Fig.10.1. Geometrical depiction of physical CP even neutral Higgs states.

Relations and constraints

The Higgs mass spectrum is completely controlled by two new parameters which can be taken to be m_A and $\tan\beta$. These strongly influence the other masses, e.g. $m_h \to 0$ if $m_A \to 0$. The following tree level relations and constraints [10.1–10.3] emerge from (10.21b), (10.24b) and (10.27):

$$m_{H^\pm}^2 = m_A^2 + M_W^2 > \max\ (M_W^2, m_A^2) \ , \tag{10.30a}$$

$$m_h^2 + m_H^2 = m_A^2 + M_Z^2 \ , \tag{10.30b}$$

$$m_h < \min\ (m_A, M_Z)|\cos 2\beta| < \min\ (m_A, M_Z) \ , \tag{10.30c}$$

$$m_H > \max\ (m_A, M_Z) \ , \tag{10.30d}$$

$$\cos^2(\beta-\alpha) = \frac{m_h^2(M_Z^2-m_h^2)}{m_A^2(m_H^2-m_h^2)} \ . \tag{10.30e}$$

Thus the charged Higgs bosons $H^\pm$ are predicted to be heavier than the W. Of the CP even neutral ones, one light Higgs h is expected to be lighter than the Z and one heavier H is expected to exceed the Z in mass. The mass of the CP odd Higgs A is expected to be between those of the two CP even ones. The contents of (10.30c) and (10.30d) are illustrated in Fig. 10.2 below where $m_{h<}$ and $m_{H>}$ are the absolute (β-independent) upper and lower bounds on m_h and m_H respectively. For large $\tan\beta$ (i.e. $|\cos 2\beta| \to 1$) , m_h saturates $m_{h<}$ from below and m_H comes down to $m_{H>}$ from above. These are all **tree level predictions**; we discuss radiative effects on these mass bounds in §10.6.

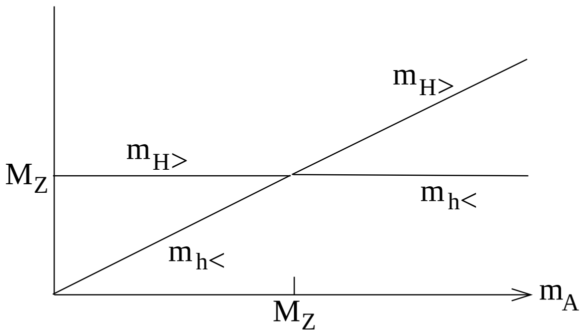

Fig.10.2. Tree level upper and lower mass bounds on h and H as a function of m_A.

10.4 Higgs-particle Vertices

The electroweak parameters of the Standard Model, together with $\tan\beta$ and α, completely determine the couplings of the physical Higgs particles to the Standard Model gauge bosons and fermions. We shall first discuss those and then come to Higgs self couplings. A discussion of Higgs couplings to sparticles is relegated to §10.5. In (8.32) we have already given the Higgs contribution to the supersymmetric part of the MSSM Lagrangian density. From this one can obtain all the Higgs couplings to fermions and gauge bosons in terms of the originally introduced but unphysical Higgs fields $h^0_{1,2}$ and $h^{\pm}_{1,2}$. The conversion to couplings with mass diagonal Higgs fields can be easily done through the transformations (10.22), (10.25) and (10.28). One should also put $G^{\pm} = 0 = G^0$ in the unitary gauge which we choose. For simplicity, we confine ourselves to one generation of up and down type fermions (masses m_u and m_d respectively): $f_L \equiv \begin{pmatrix} f_{uL} \\ f_{dL} \end{pmatrix}$, f_{uR}, f_{dR}, where f covers both quarks and leptons. Generation effects can be obtained by interpreting $m_{u,d}$ as 3×3 diagonal quark mass matrices and multiplying the charged Higgs coupling to fermions by the Cabibbo-Kobayashi-Maskawa matrix V.

The Higgs-fermion-antifermion Yukawa interactions can then be written as

$$\begin{aligned} \mathcal{L}_Y &= -\frac{g_2 m_d}{2M_W\cos\beta}\sum_f \bar{f}_d f_d (H\cos\alpha - h\sin\alpha) + \frac{ig_2 m_d\tan\beta}{2M_W}\sum_f \bar{f}_d\gamma_5 f_d A \\ &\quad -\frac{g_2 m_u}{2M_W\sin\beta}\sum_f \bar{f}_u f_u (H\sin\alpha + h\cos\alpha) + \frac{ig_2 m_u\cot\beta}{2M_W}\sum_f \bar{f}_u\gamma_5 f_u A \\ &\quad +\frac{g_2}{\sqrt{2}M_W}\sum_f \left[H^+\bar{f}_u(m_u\cot\beta\ P_L + m_d\tan\beta\ P_R)f_d + h.c.\right], \qquad (10.31) \end{aligned}$$

with f being summed over quarks and leptons. The corresponding vertex couplings (i times the coefficients of the interaction terms in $\mathcal{L}$) are given in Fig. 10.3. We do the same for the trilinear gauge-gauge-Higgs and Higgs-gauge-Higgs as well as the quartic gauge-gauge-Higgs-Higgs vertices instead of writing out the algebraic expressions in $\mathcal{L}$.

Fig. 10.3 is included in Appendix B

We can make the following comments on the couplings of Fig. 10.3.

- Tree level Higgs couplings to fermions are parity conserving and that of A to matter fermions involves a γ_5. That is why, in contrast with the 'scalars' h and H, A is sometimes called the 'pseudoscalar' Higgs. But, in the presence of CP violation, loop effects can mix the 'scalar' and 'pseudoscalar' Higgs bosons, especially since the MSSM admits additional sources of CP violation beyond the CKM phase.

- The parameters m_u and m_d refer to masses of up and down type quarks respectively for each generation.

- Bose statistics forbids the ZHH and Zhh trilinear couplings, while any ZhH coupling is forbidden by CP invariance. Since the latter is violable, a ZhH coupling could exist.

- The absence of any tree level $ZW^\pm H^\mp$ or $\gamma W^\pm H^\mp$ coupling is not surprising since neither can occur [10.1] in any model containing just $SU(2)_L$ doublet and singlet Higgs fields.

- The couplings for the vertices (W^+W^-h and W^+W^-H), (W^+HH^- and W^+hH^-), (ZHA and ZhA), (ZZH and ZZh) and (ZZh and ZhA) are pairwise complementary, i.e. if one is suppressed by the combination of mixing angles, the other is nearly full strength.

- For large $\tan\beta$ and moderate α, the neutral CP even Higgs couplings with the down type fermions get enhanced relative to those with up type ones. For the CP odd Higgs, this statement is true independent of α.

Turning to the self couplings of the Higgs bosons, we notice that they follow from the Higgs potential V_H of (10.5) on using the formulae for the physical Higgs fields, namely (10.22a), (10.25a) and (10.28). Following Ref. [10.1], one can introduce the convenient differential operators

$$
\begin{aligned}
D_H &\equiv (\sqrt{2})^{-1}[\cos\alpha(\partial/\partial h_1^0 + \partial/\partial h_1^{0*}) + \sin\alpha(\partial/\partial h_2^0 + \partial/\partial h_2^{0*})]\ , \\
D_h &\equiv (\sqrt{2})^{-1}[-\sin\alpha(\partial/\partial h_1^0 + \partial/\partial h_1^{0*}) + \cos\alpha(\partial/\partial h_2^0 + \partial/\partial h_2^{0*})]\ , \\
D_A &\equiv (\sqrt{2})^{-1}i[\sin\beta(\partial/\partial h_1^0 - \partial/\partial h_1^{0*}) + \cos\beta(\partial/\partial h_2^0 - \partial/\partial h_2^{0*})]\ , \\
D_{H^-} &\equiv \sin\beta\ \partial/\partial h_1^- + \cos\beta\ \partial/\partial h_2^-\ , \\
D_{H^+} &\equiv \sin\beta\ \partial/\partial h_1^+ + \cos\beta\ \partial/\partial h_2^+\ .
\end{aligned}
$$

Now the cubic and quartic vertex factors listed beside each vertex below (with legs, a, b, c spanning h, H, A and a, b, c, d spanning $h, H, A, H^{\pm}$) can be obtained respectively from $D_a D_b D_c V_H$ and $D_a D_b D_c D_d V_H$ evaluated at $\langle h_1^0 \rangle = v_1/\sqrt{2}, \langle h_1^- \rangle = 0 = \langle h_2^+ \rangle, \langle h_2^0 \rangle = v_2/\sqrt{2}$. We list these cubic and quartic self coupling vertices of the physical Higgs bosons in Fig. 10.4.

Fig. 10.4 is included in Appendix B

The **decoupling limit** in the Higgs sector of the MSSM is attained [10.5] by taking m_A to be very large : $m_A \to \infty$. (In practice, this usually obtains once m_A exceeds 250 GeV). From (10.27) and (10.30) we now have the results

$$m_h \to M_Z |\cos 2\beta|, \quad \cos^2 2\beta \to m_h^2/M_Z^2 \ , \tag{10.32a}$$

$$m_H^2 \to m_A^2 + M_Z^2 \sin^2 2\beta \ , \tag{10.32b}$$

$$|\cos(\beta - \alpha)| \to M_Z^2 |\sin 4\beta|/(2m_A^2) \ . \tag{10.32c}$$

In this limit we have $m_A \sim m_H \sim m_{H^\pm}$ and $\cos(\beta - \alpha) \simeq 0$, i.e. $\beta - \alpha \to \pi/2$ and $\sin\alpha \simeq -\cos\beta$ up to corrections $\mathcal{O}(M_Z^2/m_A^2)$. Thus the lightest Higgs particle h saturates[3] its upper mass bound $M_Z|\cos 2\beta|$ while the other Higgses all become uniformly heavy. Moreover, a perusal of the gauge couplings of the Higgs particles, all described above, shows that the vertices HW^+W, HZZ, ZAh, $W^\pm H^\mp h$, $ZW^\pm H^\mp h$ and $\gamma W^\pm H^\mp h$ are all proportional to $\cos(\beta - \alpha)$ while the vertices hZZ, ZAH, $W^\mp H^\pm H$, $ZW^\pm H^\mp H$ and $\gamma W^\pm H^\mp H$ are all proportional to $\sin(\beta - \alpha)$. Hence any vertex involving at least one vector boson and *exactly* one heavy Higgs particle (H, A or $H^\pm$) vanishes as $\cos(\beta - \alpha)$ when $m_A \to \infty$. Turning to matter fermions, the coupling strengths of the CP even neutral Higgs scalars to down type and up type fermions – *relative to those of the Standard Model Higgs* – are given below

$$hf_d\bar{f}_d\text{:} \quad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta - \alpha) - \tan\beta\cos(\beta - \alpha) \ , \tag{10.33a}$$

$$hf_u\bar{f}_u\text{:} \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta - \alpha) + \cot\beta\cos(\beta - \alpha) \ , \tag{10.33b}$$

$$Hf_d\bar{f}_d\text{:} \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta - \alpha) + \tan\beta\sin(\beta - \alpha) \ , \tag{10.33c}$$

$$Hf_u\bar{f}_u\text{:} \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta - \alpha) - \cot\beta\sin(\beta - \alpha) \ . \tag{10.33d}$$

Fig. 10.3 and (10.33) imply that, in the decoupling limit $|\beta - \alpha| \to \pi/2$, the couplings of the lightest Higgs scalar h to fermions and gauge boson pairs are identical to those of the Standard Model Higgs. Likewise, Fig. 10.4 shows that the self couplings hhh and $hhhh$ also reduce to their SM values in this limit. Thus, for a heavy A with $m_A \gg M_Z$, the effects of the extra scalars $H^\pm$, H and A in the MSSM decouple and the residual scalar h, while saturating its appear mass bound, looks just like the SM Higgs boson ϕ^0. The onset of decoupling is

[3]The reader is reminded that the present discussion is at the tree level.

controlled by (10.32c), and is depicted in Fig. 10.5 where the functions $\sin^2(\beta-\alpha)$ and $\cos^2(\beta-\alpha)$, i.e. the squared coupling strengths of h and H respectively to WW (cf. Fig. 10.3) relative to that of the SM Higgs, are plotted against m_A for two characteristic values of $\tan\beta$. Though the tree level results, mentioned above, change somewhat on account of radiative corrections (cf. §10.6), this last statement remains valid. The low energy effective scalar sector of the MSSM indeed becomes indistinguishable from that of the SM in the decoupling limit, except that, unlike in the latter, the mass of the lightest Higgs particle perforce remains bounded from above.

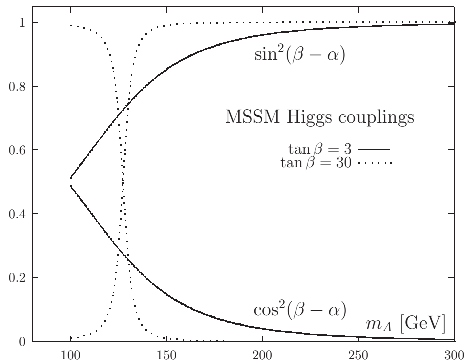

Fig.10.5. Squared coupling strengths of h and H to WW, relative to that of the SM Higgs, as functions of m_A, courtesy A. Djouadi.

10.5 Higgs-sparticle Vertices

Higgs couplings to neutralinos and charginos

The couplings of the Higgs bosons to the electroweak neutralinos and charginos originate from the gauge strength Yukawa couplings of gauginos to the scalar and fermionic component of a given chiral supermultiplet. In two component notation these are given by the last RHS term in (5.36) for abelian interactions, and by the fifth RHS term in (5.55) for nonabelian, presently $SU(2)$, interactions with the Higgs superfields H_1 and H_2 being the relevant chiral superfields. The corresponding terms in the interaction Lagrangian density can be rewritten in terms of the four component gaugino and higgsino fields of (9.17) and (9.30) by using the identities (3.28a,b) and (3.29a,b) to obtain the result

$$\begin{aligned}\mathcal{L}_{H\tilde{\chi}\tilde{\chi}} &= -\frac{g_2}{\sqrt{2}}\Big[h_1^0\Big(\overline{\tilde{h}_1^0}P_R\tilde{\lambda}_3+\sqrt{2}\,\overline{\tilde{\lambda}^+}P_R\tilde{h}^+\Big)+h_1^-\Big(\sqrt{2}\,\overline{\tilde{h}_1^0}P_R\tilde{\lambda}^+-\overline{\tilde{\lambda}_3}P_R\tilde{h}^+\Big)\\ &\quad +h_2^0\Big(\sqrt{2}\,\overline{\tilde{h}^+}P_R\tilde{\lambda}^+-\overline{\tilde{h}_2^0}P_R\tilde{\lambda}_3\Big)+h_2^+\Big(\overline{\tilde{h}^+}P_R\tilde{\lambda}_3+\sqrt{2}\,\overline{\tilde{\lambda}^+}P_R\tilde{h}_2^0\Big)\Big]\\ &\quad -\frac{g_Y}{\sqrt{2}}\Big(h_2^+\overline{\tilde{h}^+}P_R\tilde{\lambda}_0+h_2^0\overline{\tilde{h}_2^0}P_R\tilde{\lambda}_0-h_1^0\overline{\tilde{h}_1^0}P_R\tilde{\lambda}_0-h_1^-\overline{\tilde{\lambda}_0}P_R\tilde{h}^+\Big)+h.c. \qquad (10.34)\end{aligned}$$

Finally, we use (9.18), (9.19), (9.27), (10.22a), (10.25a) and (10.28) to express (10.34) in terms of chargino, neutralino and Higgs mass eigenstates:

$$\begin{aligned}
\mathcal{L}_{H\tilde{\chi}\tilde{\chi}} &= -g_2 \left(H\cos\alpha - h\sin\alpha\right)\left[\overline{\tilde{\chi}_k^+}\left(P_R Q_{km} + P_L Q_{mk}^*\right)\tilde{\chi}_m^+ + \frac{1}{2}\overline{\tilde{\chi}_n^0}\left(P_R Q''_{nl} + P_L Q''^*_{ln}\right)\tilde{\chi}_l^0\right] \\
&\quad - g_2\left(H\sin\alpha + h\cos\alpha\right)\left[\overline{\tilde{\chi}_k^+}\left(P_R S_{km} + P_L S_{mk}^*\right)\tilde{\chi}_m^+ - \frac{1}{2}\overline{\tilde{\chi}_n^0}\left(P_R S''_{nl} + P_L S''^*_{ln}\right)\tilde{\chi}_l^0\right] \\
&\quad - i g_2 A\left\{\overline{\tilde{\chi}_k^+}\left[P_R\left(Q_{km}\sin\beta + S_{km}\cos\beta\right) - P_L\left(Q_{mk}^*\sin\beta + S_{mk}^*\cos\beta\right)\right]\tilde{\chi}_m^+\right. \\
&\qquad \left. + \frac{1}{2}\overline{\tilde{\chi}_n^0}\left[P_R\left(Q''_{nl}\sin\beta - S''_{nl}\cos\beta\right) + P_L\left(S''^*_{ln}\cos\beta - Q''^*_{ln}\sin\beta\right)\right]\tilde{\chi}_l^0\right\} \\
&\quad - \left[g_2 H^- \overline{\tilde{\chi}_l^0}\left(P_R Q'^R_{lk} + P_L Q'^L_{lk}\right)\tilde{\chi}_k^+ + h.c.\right]. \qquad (10.35)
\end{aligned}$$

In (10.35) we have introduced the following quantities:

$$Q_{km} \equiv \frac{1}{\sqrt{2}}\mathcal{V}_{k1}\mathcal{U}_{m2}\ , \qquad (10.36a)$$

$$S_{km} \equiv \frac{1}{\sqrt{2}}\mathcal{V}_{k2}\mathcal{U}_{m1}\ , \qquad (10.36b)$$

$$Q'^R_{lk} \equiv \sin\beta\left[Z_{l3}\mathcal{U}_{k1} - \frac{1}{\sqrt{2}}\mathcal{U}_{k2}\left(Z_{l2} + \tan\theta_W Z_{l1}\right)\right], \qquad (10.36c)$$

$$Q'^L_{lk} \equiv \cos\beta\left[Z_{l4}^*\mathcal{V}_{k1}^* + \frac{1}{\sqrt{2}}\mathcal{V}_{k2}^*\left(Z_{l2}^* + \tan\theta_W Z_{l1}^*\right)\right], \qquad (10.36d)$$

$$Q''_{nl} \equiv \frac{1}{2}\left[Z_{n3}\left(Z_{l2} - \tan\theta_W Z_{l1}\right) + Z_{l3}\left(Z_{n2} - \tan\theta_W Z_{n1}\right)\right], \qquad (10.36e)$$

$$S''_{nl} \equiv \frac{1}{2}\left[Z_{n4}\left(Z_{l2} - \tan\theta_W Z_{l1}\right) + Z_{l4}\left(Z_{n2} - \tan\theta_W Z_{n1}\right)\right]. \qquad (10.36f)$$

We have closely followed the notation of Ref.[10.2] in defining the above quantities. The only difference is an overall factor of g_2 in the definition of Q'^R and Q'^L which have been put in order to conform with the convention used for the other coefficients in the interaction Lagrangian density (10.35). The corresponding Feynman rules are given in Fig. 10.6 in the Appendix, the only nontrivial feature being an extra factor of two in vertices involving two Majorana (neutralino) fermions.

Fig. 10.6 is included in Appendix B

Recall from Ch.9 that $\mathcal{U}_{k1}$, $\mathcal{V}_{k1}$, Z_{k1} and Z_{k2} label gaugino components, while $\mathcal{U}_{k2}$, $\mathcal{V}_{k2}$, Z_{k3} and Z_{k4} label higgsino components. Thus eqs. (10.36) clearly reflect the origin of the quantities defined from Higgs–higgsino–gaugino interactions. These couplings are *not* proportional to the masses of the corresponding charginos and neutralinos. In fact, as discussed in §9.2, gaugino–higgsino mixing in the chargino and neutralino sectors is often suppressed. The neutral Higgs bosons will then predominantly couple to two *different* charginos and neutralinos. However, chargino and neutralino final states can nonetheless play a prominent role in the decays of the heavy neutral Higgs bosons A and H if $m_A, m_H \leq 2m_t$ and $\tan\beta$ is not large. Conversely, final states containing the light neutral Higgs boson h can play an important role in the decays of the heavier neutralinos and charginos into lighter ones. On the other hand, the lower bound $m_{\tilde{\chi}_1^\pm} > 100$ GeV, which comes from chargino searches at LEP, implies that the decays $H^- \to \tilde{\chi}_k^- \tilde{\chi}_l^0$ can dominate only over the small region of parameter space where $m_{\tilde{\chi}_1^+} + m_{\tilde{\chi}_1^0} < m_{H^+} < m_t + m_b$. Indeed, LEP searches imply that $m_{\tilde{\chi}_1^+} + m_{\tilde{\chi}_1^0} \geq 140$ GeV, if the "gaugino mass unification condition", cf. (9.21), holds. Finally, note that the couplings of h, H and A would be scalar and pseudoscalar respectively, were all rotation matrices in the chargino and neutralino sector strictly real.

Sfermion Higgs couplings

The couplings between Higgs bosons and sfermions receive contributions from the supersymmetric F- and D-terms in the scalar potential, as well as from trilinear soft supersymmetry breaking terms. The same terms also contribute to sfermion mass matrices and have been collected in (9.42) and (9.45) for sleptons and squarks respectively. We use (10.22a), (10.25a) and (10.28) to move to the Higgs mass eigenstate basis. The quartic F- and D-terms then also give rise to trilinear interactions of a single Higgs particle with two sfermions, due to the VEVs of the neutral components of the Higgs fields. We first present the relevant pieces of the interaction Lagrangian density in the current basis for sfermions. This allows easier comparison with results in the literature. Moreover, as discussed in §9.5, in many realistic SUSY models intergeneration sfermion mixing can often be neglected, in which case the mass eigenstates are essentially equal to current eigenstates. For ease of presentation, we show the trilinear and quartic interactions of Higgs bosons with sleptons and squarks separately. For any angle ϕ, we use s_ϕ, c_ϕ, t_ϕ and $(ct)_\phi$ to mean $\sin\phi$, $\cos\phi$, $\tan\phi$ and $\cot\phi$ respectively, except that the corresponding symbols for θ_W are s_W, c_W, t_W and $(ct)_W$ respectively. The results for the relevant cubic and quartic interactions are

$$\begin{aligned}
\mathcal{L}_{H\tilde{\ell}\tilde{\ell}} &= \frac{g_2}{\sqrt{2}M_W} H^+ \Big\{ \tilde{\nu}_i^* \tilde{e}_{jR} \left[\mu \left(\mathbf{m_e}\right)_{ij} - \left(\mathbf{m_e} A^{e*}\right)_{ij} t_\beta \right] \\
&\qquad + \tilde{\nu}_i^* \tilde{e}_{jL} \left[\left(\mathbf{m_e} \mathbf{m_e}^\dagger\right)_{ij} t_\beta - M_W^2 \delta_{ij} s_{2\beta} \right] \Big\} \\
&+ \frac{g_2}{2M_W c_\beta} \tilde{e}_{iL}^* \tilde{e}_{jR} \Big[\left(\mathbf{m_e} A^{e*}\right)_{ij} \left(H c_\alpha - h s_\alpha - i A s_\beta\right) \\
&\qquad + \mu \left(\mathbf{m_e}\right)_{ij} \left(H s_\alpha + h c_\alpha + i A c_\beta\right) \Big] + h.c. \\
&+ \frac{g_2}{M_W c_\beta} \left[\tilde{e}_{iL}^* \tilde{e}_{jL} \left(\mathbf{m_e} \mathbf{m_e}^\dagger\right)_{ij} + \tilde{e}_{iR}^* \tilde{e}_{jR} \left(\mathbf{m_e}^\dagger \mathbf{m_e}\right)_{ij} \right] \left(h s_\alpha - H c_\alpha\right) \\
&+ \frac{g_2 M_W}{2} \left[h s_{(\alpha+\beta)} - H c_{(\alpha+\beta)} \right] \sum_i \Big[\left|\tilde{\nu}_i\right|^2 \left(1 + t_W^2\right) - \left|\tilde{e}_{iL}\right|^2 \left(1 - t_W^2\right) \\
&\qquad - 2 \left|\tilde{e}_{iR}\right|^2 t_W^2 \Big] .
\end{aligned} \tag{10.37}$$

$$
\begin{aligned}
\mathcal{L}_{H\tilde{q}\bar{\tilde{q}}} &= \frac{g_2}{\sqrt{2}M_W} H^+ \Big\{ \tilde{u}^\dagger_{iL}\tilde{d}_{jR}\left[\mu\left(\mathbf{m_d}\right)_{ij} - \left(\mathbf{m_d}A^{d*}\right)_{ij} t_\beta\right] \\
&\qquad + \tilde{u}^\dagger_{iR}\tilde{d}_{jL}\left[\mu^*\left(\mathbf{m_u^*}\right)_{ij} - \left(\mathbf{m_u^*}A^{u}\right)_{ij}(ct)_\beta\right] \\
&\qquad + \tilde{u}^\dagger_{iL}\tilde{d}_{jL}\left[\left(\mathbf{m_u}\mathbf{m_u^\dagger}\right)_{ij}(ct)_\beta + \left(\mathbf{m_d}\mathbf{m_d^\dagger}\right)_{ij} t_\beta - M_W^2\delta_{ij}s_{2\beta}\right] \\
&\qquad + \tilde{u}^\dagger_{iR}\tilde{d}_{jR}\left(\mathbf{m_u^\dagger}\mathbf{m_d}\right)_{ij}\left(t_\beta + (ct)_\beta\right)\Big\} \\
&+ \frac{g_2}{2M_W c_\beta}\Big[\tilde{d}^\dagger_{iL}\tilde{d}_{jR}\Big\{\left(\mathbf{m_d}A^{d*}\right)_{ij}\left(Hc_\alpha - hs_\alpha - iAs_\beta\right) \\
&\qquad\qquad + \mu\left(\mathbf{m_d}\right)_{ij}\left(Hs_\alpha + hc_\alpha + iAc_\beta\right)\Big\} + \text{h.c.}\Big] \\
&+ \frac{g_2}{2M_W s_\beta}\Big[\tilde{u}^\dagger_{iL}\tilde{u}_{jR}\Big\{\left(\mathbf{m_u}A^{u*}\right)_{ij}\left(Hs_\alpha + hc_\alpha - iAc_\beta\right) \\
&\qquad\qquad + \mu\left(\mathbf{m_u}\right)_{ij}\left(Hc_\alpha - hs_\alpha + iAs_\beta\right)\Big\} + \text{h.c.}\Big] \\
&+ \frac{g_2}{M_W c_\beta}\left(hs_\alpha - Hc_\alpha\right)\left[\tilde{d}^\dagger_{iL}\tilde{d}_{jL}\left(\mathbf{m_d}\mathbf{m_d^\dagger}\right)_{ij} + \tilde{d}^\dagger_{iR}\tilde{d}_{jR}\left(\mathbf{m_d^\dagger}\mathbf{m_d}\right)_{ij}\right] \\
&- \frac{g_2}{M_W s_\beta}\left(Hs_\alpha + hc_\alpha\right)\left[\tilde{u}^\dagger_{iL}\tilde{u}_{jL}\left(\mathbf{m_u}\mathbf{m_u^\dagger}\right)_{ij} + \tilde{u}^\dagger_{iR}\tilde{u}_{jR}\left(\mathbf{m_u^\dagger}\mathbf{m_u}\right)_{ij}\right] \\
&+ g_2(M_W/2)\sum_i\Big[\left|\tilde{u}_{iL}\right|^2\left(1 - t_W^2/3\right) - \left|\tilde{d}_{iL}\right|^2\left(1 + t_W^2/3\right) \\
&\qquad + (2t_W^2/3)\left(2\left|\tilde{u}_{iR}\right|^2 - \left|\tilde{d}_{iR}\right|^2\right)\Big]\left[hs_{(\alpha+\beta)} - Hc_{(\alpha+\beta)}\right]. \qquad (10.38)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{HH\tilde{\ell}\bar{\tilde{\ell}}} &= \frac{g_2^2}{2\sqrt{2}M_W^2} H^+\tilde{\nu}^*_i\tilde{e}_{jL}\Big\{\frac{s_\beta}{c_\beta^2}\left(\mathbf{m_e}\mathbf{m_e^\dagger}\right)_{ij}\left(Hc_\alpha - hs_\alpha + iAs_\beta\right) \\
&\qquad - M_W^2\delta_{ij}\left[Hs_{(\alpha+\beta)} + hc_{(\alpha+\beta)} - iAc_{2\beta}\right]\Big\} + h.c. \\
&- \frac{g_2^2}{4M_W^2 c_\beta^2}\left(H^2c_\alpha^2 + h^2s_\alpha^2 - Hhs_{2\alpha} + A^2s_\beta^2\right) \\
&\qquad \cdot\left[\tilde{e}^*_{iL}\tilde{e}_{jL}\left(\mathbf{m_e}\mathbf{m_e^\dagger}\right)_{ij} + \tilde{e}^*_{iR}\tilde{e}_{jR}\left(\mathbf{m_e^\dagger}\mathbf{m_e}\right)_{ij}\right] \\
&- \frac{g_2^2 t_\beta^2}{2M_W^2} H^+H^-\left[\tilde{\nu}^*_i\tilde{\nu}_j\left(\mathbf{m_e}\mathbf{m_e^\dagger}\right)_{ij} + \tilde{e}^*_{iR}\tilde{e}_{jR}\left(\mathbf{m_e^\dagger}\mathbf{m_e}\right)_{ij}\right] \\
&+ \frac{g_2^2}{8}\left[\left(h^2 - H^2\right)c_{2\alpha} + 2Hhs_{2\alpha} + A^2c_{2\beta}\right] \\
&\qquad \cdot\sum_i\left[\left|\tilde{\nu}_i\right|^2\left(1 + t_W^2\right) - \left|\tilde{e}_{iL}\right|^2\left(1 - t_W^2\right) - 2t_W^2\left|\tilde{e}_{iR}\right|^2\right] \\
&- \frac{g_2^2}{4}H^+H^-c_{2\beta}\sum_i\left[\left|\tilde{\nu}_i\right|^2\left(1 - t_W^2\right) - \left|\tilde{e}_{iL}\right|^2\left(1 + t_W^2\right) + 2t_W^2\left|\tilde{e}_{iR}\right|^2\right]. \qquad (10.39)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{HH\tilde{q}\tilde{q}} \;=\;& \frac{g_2^2}{2\sqrt{2}M_W^2} H^+ \Big\{ \tilde{u}_{iL}^\dagger \tilde{d}_{jL} \Big[\left(\mathbf{m_u}\mathbf{m_u}^\dagger\right)_{ij} \frac{c_\beta}{s_\beta^2} \left(H s_\alpha + h c_\alpha - i A c_\beta\right) \\
&\qquad + \left(\mathbf{m_d}\mathbf{m_d}^\dagger\right)_{ij} \frac{s_\beta}{c_\beta^2} \left(H c_\alpha - h s_\alpha + i A s_\beta\right) \\
&\qquad - M_W^2 \delta_{ij} \left(H s_{(\alpha+\beta)} + h c_{(\alpha+\beta)} - i A c_{2\beta}\right) \Big] \\
&\quad + \tilde{u}_{iR}^\dagger \tilde{d}_{jR} \left[H c_{(\alpha-\beta)} - h s_{(\alpha-\beta)}\right] \frac{2}{s_{2\beta}} \left(\mathbf{m_u}^\dagger \mathbf{m_d}\right)_{ij} \Big\} + h.c. \\
&- \frac{g_2^2}{4M_W^2 c_\beta^2} \left(H^2 c_\alpha^2 + h^2 s_\alpha^2 - H h s_{2\alpha} + A^2 s_\beta^2\right) \\
&\quad \cdot \left[\tilde{d}_{iL}^\dagger \tilde{d}_{jL} \left(\mathbf{m_d}\mathbf{m_d}^\dagger\right)_{ij} + \tilde{d}_{iR}^\dagger \tilde{d}_{jR} \left(\mathbf{m_d}^\dagger \mathbf{m_d}\right)_{ij} \right] \\
&- \frac{g_2^2}{4M_W^2 s_\beta^2} \left(H^2 s_\alpha^2 + h^2 c_\alpha^2 + H h s_{2\alpha} + A^2 c_\beta^2\right) \\
&\quad \cdot \left[\tilde{u}_{iL}^\dagger \tilde{u}_{jL} \left(\mathbf{m_u}\mathbf{m_u}^\dagger\right)_{ij} + \tilde{u}_{iR}^\dagger \tilde{u}_{jR} \left(\mathbf{m_u}^\dagger \mathbf{m_u}\right)_{ij} \right] \\
&- \frac{g_2^2}{2M_W^2} H^+ H^- \Big[\tilde{u}_{iL}^\dagger \tilde{u}_{jL} \left(\mathbf{m_d}\mathbf{m_d}^\dagger\right)_{ij} t_\beta^2 + \tilde{d}_{iL}^\dagger \tilde{d}_{jL} \left(\mathbf{m_u}\mathbf{m_u}^\dagger\right)_{ij} (ct)_\beta^2 \\
&\quad + \tilde{u}_{iR}^\dagger \tilde{u}_{jR} \left(\mathbf{m_u}^\dagger \mathbf{m_u}\right)_{ij} (ct)_\beta^2 + \tilde{d}_{iR}^\dagger \tilde{d}_{jR} \left(\mathbf{m_d}^\dagger \mathbf{m_d}\right)_{ij} t_\beta^2 \Big] \\
&+ \frac{g_2^2}{8} \left[\left(h^2 - H^2\right) c_{2\alpha} + 2 H h s_{2\alpha} + A^2 c_{2\beta} \right] \\
&\quad \cdot \sum_i \Big[\left|\tilde{u}_{iL}\right|^2 \left(1 - t_W^2/3\right) - \left|\tilde{d}_{iL}\right|^2 \left(1 + t_W^2/3\right) \\
&\qquad + (2t_W^2/3) \left(2\left|\tilde{u}_{iR}\right|^2 - \left|\tilde{d}_{iR}\right|^2 \right) \Big] \\
&- \frac{g_2^2}{4} H^+ H^- c_{2\beta} \sum_i \Big[\left|\tilde{u}_{iL}\right|^2 \left(1 + \frac{1}{3} t_W^2\right) - \left|\tilde{d}_{iL}\right|^2 \left(1 - t_W^2/3\right) \\
&\qquad - (2t_W^2/3) \left(2\left|\tilde{u}_{iR}\right|^2 - \left|\tilde{d}_{iR}\right|^2 \right) \Big] .
\end{aligned}
\tag{10.40}
$$

The following points about (10.37)–(10.40) are noteworthy:

- The hermitian conjugation in these equations acts only on terms to the left of the *h.c.* as written, terms to the right being already hermitian after summation over the generation indices i and j.
- The coupling of one Higgs boson to two sfermions is again *not* proportional to the sfermion mass. In the case of third generation sfermions the usually most important contributions to such a trilinear coupling are those proportional to μ or one of the

$A-$parameters, since the absolute values of these quantities can be significantly larger than M_W. Indeed, in principle, such couplings offer the only *direct* experimental access to the $A-$parameter. In practice, however, these couplings are difficult to measure since they involve three as yet undiscovered particles.

- The only significant contributions to the Higgs couplings to first and second generation sfermions are the pure gauge terms. In contrast, the quartic interactions of third generation sfermions are often dominated by contributions proportional to m_f^2.
- The F-term contributions to the couplings of $SU(2)_L$-doublet, "left chiral" sfermions are proportional to $\mathbf{m_f}\mathbf{m_f}^\dagger$, while those of $SU(2)_L$-singlet, "right chiral" sfermions are proportional to $\mathbf{m_f}^\dagger\mathbf{m_f}$. This is analogous to the LL and RR entries of the squared squark mass matrix listed in (9.46).
- The relative sign between the $SU(2)$ and $U(1)_Y$ D-term contributions to the quartic interactions differs for neutral and charged Higgs boson pairs. For example, the H^+H^- pair couples more strongly to $\tilde{e}_L$ pairs than to $\tilde{\nu}$ pairs, while the opposite is true for pairs of neutral Higgs bosons.

In the final step, (9.51) and (9.54) are to be utilized to convert the current eigenstate sfermion fields in (10.37)–(10.40) into mass eigenstate ones. We can use a notation similar to that in §9.8. The final result for Higgs-sfermion interactions can then be written as

$$\mathcal{L}_{H\tilde{f}} = \sum_{\phi,\tilde{f},\tilde{f}'} C[\phi,\tilde{f},\tilde{f}']\phi\tilde{f}^*\tilde{f}' + \sum_{\phi,\phi',\tilde{f},\tilde{f}'} D[\phi,\phi',\tilde{f},\tilde{f}']\phi\phi'\tilde{f}^*\tilde{f}', \tag{10.41}$$

where ϕ and ϕ' stand for any of the five physical Higgs fields of the MSSM, while $\tilde{f}$ and $\tilde{f}'$ are sfermion fields. Invariance under $SU(3)_C$ implies that $\tilde{f}'$ must be a squark if and only if $\tilde{f}$ is a squark; both squarks must then have the same color index, which therefore need not be displayed. Moreover, we shall again assume that the superpotential is written in a basis where the leptonic Yukawa couplings are flavor diagonal. The coefficients describing slepton-Higgs interactions can then be written down. First, we display the coefficients of the various cubic **Higgs-slepton-slepton** terms. They are

$$\begin{aligned} C[H^+,\tilde{\nu}_i,\tilde{e}_s] \;=\; & \frac{g_2}{\sqrt{2}M_W}\Big\{\sum_{k=1}^{3} U^{\tilde{\nu}*}_{ki}\Big[\mu m_{e_k} W^{\tilde{e}}_{k+3\ s} + \left(m^2_{e_k} t_\beta - M_W^2 s_{2\beta}\right) W^{\tilde{e}}_{ks}\Big] \\ & - \sum_{j,k=1}^{3} t_\beta \left(\mathbf{m_e} A^{e*}\right)_{kj} U^{\tilde{\nu}*}_{ki} W^{\tilde{e}}_{j+3\ s}\Big\}, \end{aligned} \tag{10.42a}$$

$$C[H^-,\tilde{e}_s,\tilde{\nu}_i] = \left(C[H^+,\tilde{\nu}_i,\tilde{e}_s]\right)^*, \tag{10.42b}$$

$$C[H,\tilde{\nu}_i,\tilde{\nu}_j] = -c_g[\tilde{\nu}] c_{\alpha+\beta}\delta_{ij}\ , \tag{10.42c}$$

$$C[h,\tilde{\nu}_i,\tilde{\nu}_j] = c_g[\tilde{\nu}] s_{\alpha+\beta}\delta_{ij}\ , \tag{10.42d}$$

$$C[H,\tilde{e}_s,\tilde{e}_t] = c_A[\tilde{e}_s,\tilde{e}_t]c_\alpha + c_\mu[\tilde{e}_s,\tilde{e}_t]s_\alpha - c_g[\tilde{e}_s,\tilde{e}_t]c_{\alpha+\beta}\ , \tag{10.42e}$$

$$C[h,\tilde{e}_s,\tilde{e}_t] = -c_A[\tilde{e}_s,\tilde{e}_t]s_\alpha + c_\mu[\tilde{e}_s,\tilde{e}_t]c_\alpha + c_g[\tilde{e}_s,\tilde{e}_t]s_{\alpha+\beta}\ , \tag{10.42f}$$

$$\begin{aligned} C[A,\tilde{e}_s,\tilde{e}_t] &= \frac{ig_2}{2M_W}\Big\{\sum_{i=1}^{3}\left(\mu m_{e_i} W^{\tilde{e}*}_{is} W^{\tilde{e}}_{i+3\ t} - \mu^* m_{e_i} W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{it}\right) \\ &\quad + t_\beta \sum_{i,j=1}^{3}\Big[\left(\mathbf{m_e}A^e\right)_{ij} W^{\tilde{e}*}_{j+3\ s} W^{\tilde{e}}_{it} - \left(\mathbf{m_e}A^{e*}\right)_{ij} W^{\tilde{e}*}_{is} W^{\tilde{e}}_{j+3\ t}\Big]\Big\}. \end{aligned} \tag{10.42g}$$

All coefficients $C[\phi,\tilde{\ell},\tilde{\ell}']$ not listed in eqs.(10.42) vanish. One could rewrite the matrix $\mathbf{m_e}A_e^*$ in terms of its eigenvalues and corresponding 3×3 rotation matrices. We have not done so since the eigenvalues of this matrix have no special physical meaning, in contrast to those of the SM matter fermion mass matrices. Moreover, we have introduced the quantities

$$\begin{aligned} c_A[\tilde{e}_s,\tilde{e}_t] &\equiv \frac{g_2}{M_W c_\beta}\Big\{-\sum_{i=1}^{3} m^2_{e_i}\left(W^{\tilde{e}*}_{is} W^{\tilde{e}}_{it} + W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{i+3\ t}\right) \\ &\quad + \frac{1}{2}\sum_{i,j=1}^{3}\Big[\left(\mathbf{m_e}A^{e\dagger}\right)_{ij} W^{\tilde{e}*}_{is} W^{\tilde{e}}_{j+3\ t} + \left(\mathbf{m_e}A^e\right)_{ij} W^{\tilde{e}*}_{j+3\ s} W^{\tilde{e}}_{it}\Big]\Big\}, \end{aligned} \tag{10.43a}$$

$$c_\mu[\tilde{e}_s,\tilde{e}_t] \equiv \frac{g_2}{2M_W c_\beta}\sum_{i=1}^{3} m_{e_i}\left(\mu W^{\tilde{e}*}_{is} W^{\tilde{e}}_{i+3\ t} + \mu^* W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{it}\right), \tag{10.43b}$$

$$c_g[\tilde{\nu}] \equiv \frac{g_2 M_W}{2}\left(1+t_W^2\right), \tag{10.43c}$$

$$c_g[\tilde{e}_s,\tilde{e}_t] \equiv \frac{g_2 M_W}{2}\sum_{i=1}^{3}\left[W^{\tilde{e}*}_{is} W^{\tilde{e}}_{it}\left(t_W^2 - 1\right) - 2t_W^2 W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{i+3\ t}\right]. \tag{10.43d}$$

The coefficients of the various quartic **Higgs-Higgs-slepton-slepton interactions** can also be displayed. They are

$$D[H^+,H^-,\tilde{\nu}_i,\tilde{\nu}_j] = \frac{g_2^2 c_{2\beta}}{4}\left(t_W^2 - 1\right)\delta_{ij} - \frac{g_2^2 t_\beta^2}{2M_W^2}\sum_{k=1}^{3} U^{\tilde{\nu}*}_{ki} U^{\tilde{\nu}}_{kj} m^2_{e_k}\ , \tag{10.44a}$$

$$\begin{aligned} D[H^+,H^-,\tilde{e}_s,\tilde{e}_t] &= \sum_{i=1}^{3}\Big\{\frac{g_2^2 c_{2\beta}}{4}\left[W^{\tilde{e}*}_{is} W^{\tilde{e}}_{it}\left(1+t_W^2\right) - 2t_W^2 W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{i+3\ t}\right] \\ &\quad - \frac{g_2^2 t_\beta^2}{2M_W^2} m^2_{e_i} W^{\tilde{e}*}_{i+3\ s} W^{\tilde{e}}_{j+3\ t}\Big\}, \end{aligned} \tag{10.44b}$$

$$D[H^+,H,\tilde{\nu}_i,\tilde{e}_s] = -d_g[\tilde{\nu}_i,\tilde{e}_s]s_{\alpha+\beta} + d_Y[\tilde{\nu}_i,\tilde{e}_s]c_\alpha\ , \tag{10.44c}$$

$$D[H^-,H,\tilde{e}_s,\tilde{\nu}_i] = \left(D[H^+,H,\tilde{\nu}_i,\tilde{e}_s]\right)^*, \tag{10.44d}$$

$$D[H^+, h, \tilde{\nu}_i, \tilde{e}_s] = -d_g[\tilde{\nu}_i, \tilde{e}_s]c_{\alpha+\beta} - d_Y[\tilde{\nu}_i, \tilde{e}_s]s_\alpha \ , \tag{10.44e}$$

$$D[H^-, h, \tilde{e}_s, \tilde{\nu}_i] = \left(D[H^+, h, \tilde{\nu}_i, \tilde{e}_s]\right)^* , \tag{10.44f}$$

$$D[H^+, A, \tilde{\nu}_i, \tilde{e}_s] = id_g[\tilde{\nu}_i, \tilde{e}_s]c_{2\beta} + id_Y[\tilde{\nu}_i, \tilde{e}_s]s_\beta \ , \tag{10.44g}$$

$$D[H^-, A, \tilde{e}_s, \tilde{\nu}_i] = \left(D[H^+, A, \tilde{\nu}_i, \tilde{e}_s]\right)^* , \tag{10.44h}$$

$$D[H, H, \tilde{\nu}_i, \tilde{\nu}_j] = -d_g[\tilde{\nu}]c_{2\alpha}\,\delta_{ij} \ , \tag{10.44i}$$

$$D[H, h, \tilde{\nu}_i, \tilde{\nu}_j] = 2d_g[\tilde{\nu}]s_{2\alpha}\,\delta_{ij} \ , \tag{10.44j}$$

$$D[h, h, \tilde{\nu}_i, \tilde{\nu}_j] = d_g[\tilde{\nu}]c_{2\alpha}\,\delta_{ij} \ , \tag{10.44k}$$

$$D[A, A, \tilde{\nu}_i, \tilde{\nu}_j] = d_g[\tilde{\nu}]c_{2\beta}\,\delta_{ij} \ , \tag{10.44l}$$

$$D[H, H, \tilde{e}_s, \tilde{e}_t] = -d_Y[\tilde{e}_s, \tilde{e}_t]c_\alpha^2 - d_g[\tilde{e}_s, \tilde{e}_t]c_{2\alpha} \ , \tag{10.44m}$$

$$D[H, h, \tilde{e}_s, \tilde{e}_t] = d_Y[\tilde{e}_s, \tilde{e}_t]s_{2\alpha} + 2d_g[\tilde{e}_s, \tilde{e}_t]s_{2\alpha} \ , \tag{10.44n}$$

$$D[h, h, \tilde{e}_s, \tilde{e}_t] = -d_Y[\tilde{e}_s, \tilde{e}_t]s_\alpha^2 + d_g[\tilde{e}_s, \tilde{e}_t]c_{2\alpha} \ , \tag{10.44o}$$

$$D[A, A, \tilde{e}_s, \tilde{e}_t] = -d_Y[\tilde{e}_s, \tilde{e}_t]s_\beta^2 + d_g[\tilde{e}_s, \tilde{e}_t]c_{2\beta} \ . \tag{10.44p}$$

We again list only the nonvanishing coefficients; for example, $D[H^-, H^+, \tilde{f}, \tilde{f}'] \equiv 0$, for $\tilde{f} \neq \tilde{f}'$. The Lagrangian density in (10.41) is nonetheless hermitian. Moreover, we have introduced the quantities

$$d_g[\tilde{\nu}] \equiv \frac{g_2^2}{8}\left(1 + t_W^2\right) , \tag{10.45a}$$

$$d_g[\tilde{\nu}_i, \tilde{e}_s] \equiv \frac{g_2^2}{2\sqrt{2}} \sum_{j=1}^{3} U_{ji}^{\tilde{\nu}*} W_{js}^{\tilde{e}} \ , \tag{10.45b}$$

$$d_g[\tilde{e}_s, \tilde{e}_t] \equiv -\frac{g_2^2}{8} \sum_{i=1}^{3} \left[2t_W^2 W_{i+3\ s}^{\tilde{e}*} W_{i+3\ t}^{\tilde{e}} + W_{is}^{\tilde{e}*} W_{it}^{\tilde{e}}\left(1 - t_W^2\right)\right] , \tag{10.45c}$$

$$d_Y[\tilde{\nu}_i, \tilde{e}_s] \equiv \frac{g_2^2 s_\beta}{2\sqrt{2}M_W^2 c_\beta^2} \sum_{j=1}^{3} m_{e_j}^2 U_{ji}^{\tilde{\nu}*} W_{js}^{\tilde{e}} \ , \tag{10.45d}$$

$$d_Y[\tilde{e}_s, \tilde{e}_t] \equiv \frac{g_2^2}{4M_W^2 c_\beta^2} \sum_{i=1}^{3} m_{e_i}^2 \left(W_{is}^{\tilde{e}*} W_{it}^{\tilde{e}} + W_{i+3\ s}^{\tilde{e}*} W_{i+3\ t}^{\tilde{e}}\right) . \tag{10.45e}$$

The analogous expressions for Higgs interactions with squarks are complicated by nontrivial quark flavor mixing. In addition to rotating the squarks into mass eigenstates using the matrices $\mathbf{W}^{\tilde{\mathbf{u}}}$ and $\mathbf{W}^{\tilde{\mathbf{d}}}$, we also need to diagonalize the quark mass matrices using (8.12). The coefficients of the various cubic **Higgs-squark-squark interaction** terms are given below. They are

$$
\begin{aligned}
C[H^+,\tilde{u}_s,\tilde{d}_t] \;=\; & \frac{g_2}{\sqrt{2}M_W}\sum_{i,j=1}^{3}\Big\{W^{\tilde{u}*}_{is}W^{\tilde{d}}_{j+3\;t}\Big[\sum_{k=1}^{3}\mu U^{d_L}_{ik}m_{d_k}U^{d_R*}_{jk}-\big(\mathbf{m_d}A^{d*}\big)_{ij}\,t_\beta\Big] \\
& +W^{\tilde{u}*}_{j+3\;s}W^{\tilde{d}}_{it}\Big[\sum_{k=1}^{3}\mu^* U^{u_L*}_{ik}m_{u_k}U^{u_R}_{jk}-\left(\mathbf{m_u^*}A^{u}\right)_{ij}(ct)_\beta\Big] \\
& +W^{\tilde{u}*}_{is}W^{\tilde{d}}_{jt}\Big[\sum_{k=1}^{3}\Big(U^{u_L}_{ik}m^2_{u_k}U^{u_L*}_{jk}(ct)_\beta+U^{d_L}_{ik}m^2_{d_k}U^{d_L*}_{jk}t_\beta\Big) \\
& \qquad -M_W^2\delta_{ij}s_{2\beta}\Big] \\
& +W^{\tilde{u}*}_{i+3\;s}W^{\tilde{d}}_{j+3\;t}\left(t_\beta+(ct)_\beta\right)\sum_{k,l=1}^{3}U^{u_R}_{ik}m_{u_k}V^{q_L}_{kl}m_{d_l}U^{d_R*}_{jl}\Big\},
\end{aligned}
\tag{10.46a}
$$

$$
C[H^-,\tilde{d}_t,\tilde{u}_s]=\Big(C[H^+,\tilde{u}_s,\tilde{d}_t]\Big)^*, \tag{10.46b}
$$

$$
C[H,\tilde{d}_s,\tilde{d}_t]=c_A[\tilde{d}_s,\tilde{d}_t]c_\alpha+c_\mu[\tilde{d}_s,\tilde{d}_t]s_\alpha-c_g[\tilde{d}_s,\tilde{d}_t]c_{\alpha+\beta}\;, \tag{10.46c}
$$

$$
C[h,\tilde{d}_s,\tilde{d}_t]=-c_A[\tilde{d}_s,\tilde{d}_t]s_\alpha+c_\mu[\tilde{d}_s,\tilde{d}_t]c_\alpha+c_g[\tilde{d}_s,\tilde{d}_t]s_{\alpha+\beta}\;, \tag{10.46d}
$$

$$
C[H,\tilde{u}_s,\tilde{u}_t]=c_A[\tilde{u}_s,\tilde{u}_t]s_\alpha+c_\mu[\tilde{u}_s,\tilde{u}_t]c_\alpha-c_g[\tilde{u}_s,\tilde{u}_t]c_{\alpha+\beta}\;, \tag{10.46e}
$$

$$
C[h,\tilde{u}_s,\tilde{u}_t]=c_A[\tilde{u}_s,\tilde{u}_t]c_\alpha-c_\mu[\tilde{u}_s,\tilde{u}_t]s_\alpha+c_g[\tilde{u}_s,\tilde{u}_t]s_{\alpha+\beta}\;, \tag{10.46f}
$$

$$
\begin{aligned}
C[A,\tilde{d}_s,\tilde{d}_t] \;=\; & \frac{ig_2}{2M_W}\sum_{i,j=1}^{3}\Big\{t_\beta\left[\left(\mathbf{m_d^*}A^{d}\right)_{ij}W^{\tilde{d}*}_{j+3\;s}W^{\tilde{d}}_{it}-\left(\mathbf{m_d}A^{d*}\right)_{ij}W^{\tilde{d}*}_{is}W^{\tilde{d}}_{j+3\;t}\right] \\
& +\sum_{k=1}^{3}m_{d_k}\left[\mu U^{d_L}_{ik}U^{d_R*}_{jk}W^{\tilde{d}*}_{is}W^{\tilde{d}}_{j+3\;t}-\mu^*U^{d_L*}_{ik}U^{d_R}_{jk}W^{\tilde{d}*}_{j+3\;s}W^{\tilde{d}}_{it}\right]\Big\},
\end{aligned}
\tag{10.46g}
$$

$$
\begin{aligned}
C[A,\tilde{u}_s,\tilde{u}_t] \;=\; & \frac{ig_2}{2M_W}\sum_{i,j=1}^{3}\Big\{(ct)_\beta\left[\left(\mathbf{m_u^*}A^{u}\right)_{ij}W^{\tilde{u}*}_{j+3\;s}W^{\tilde{u}}_{it}-\left(\mathbf{m_u}A^{u*}\right)_{ij}W^{\tilde{u}*}_{is}W^{\tilde{u}}_{j+3\;t}\right] \\
& +\sum_{k=1}^{3}m_{u_k}\left[\mu U^{u_L}_{ik}U^{u_R*}_{jk}W^{\tilde{u}*}_{is}W^{\tilde{u}}_{j+3\;t}-\mu^*U^{u_L*}_{ik}U^{u_R}_{jk}W^{\tilde{u}*}_{j+3\;s}W^{\tilde{u}}_{it}\right]\Big\}.
\end{aligned}
\tag{10.46h}
$$

The quantity $V^{q_L}_{k\ell}$ appearing in (10.46a) is an element of the Cabibbo-Kobayashi–Maskawa matrix of (8.40). Moreover, we have introduced the quantities

$$c_A[\tilde{d}_s, \tilde{d}_t] \equiv \frac{g_2}{M_W c_\beta} \sum_{i,j=1}^{3} \left\{ \frac{1}{2} \left[W_{is}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} \left(\mathbf{m_d} A^{d*} \right)_{ij} + W_{j+3\ s}^{\tilde{d}*} W_{it}^{\tilde{d}} \left(\mathbf{m_d^*} A^{d} \right)_{ij} \right] - \sum_{k=1}^{3} m_{d_k}^2 \left[U_{ik}^{d_L} U_{jk}^{d_L *} W_{is}^{\tilde{d}*} W_{jt}^{\tilde{d}} + U_{ik}^{d_R} U_{jk}^{d_R *} W_{i+3\ s}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} \right] \right\}, \tag{10.47a}$$

$$c_A[\tilde{u}_s, \tilde{u}_t] \equiv \frac{g_2}{M_W s_\beta} \sum_{i,j=1}^{3} \left\{ \frac{1}{2} \left[W_{is}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} \left(\mathbf{m_u} A^{u*} \right)_{ij} + W_{j+3\ s}^{\tilde{u}*} W_{it}^{\tilde{u}} \left(\mathbf{m_u^*} A^{u} \right)_{ij} \right] - \sum_{k=1}^{3} m_{u_k}^2 \left[U_{ik}^{u_L} U_{jk}^{u_L *} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{u}} + U_{ik}^{u_R} U_{jk}^{u_R *} W_{i+3\ s}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} \right] \right\}, \tag{10.47b}$$

$$c_\mu[\tilde{d}_s, \tilde{d}_t] \equiv \frac{g_2}{2 M_W c_\beta} \sum_{i,j,k=1}^{3} m_{d_k} \left[\mu U_{ik}^{d_L} U_{jk}^{d_R *} W_{is}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} + \mu^* U_{ik}^{d_L *} U_{jk}^{d_R} W_{j+3\ s}^{\tilde{d}*} W_{it}^{\tilde{d}} \right], \tag{10.47c}$$

$$c_\mu[\tilde{u}_s, \tilde{u}_t] \equiv \frac{g_2}{2 M_W s_\beta} \sum_{i,j,k=1}^{3} m_{u_k} \left[\mu U_{ik}^{u_L} U_{jk}^{u_R *} W_{is}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} + \mu^* U_{ik}^{u_L *} U_{jk}^{u_R} W_{j+3\ s}^{\tilde{u}*} W_{it}^{\tilde{u}} \right], \tag{10.47d}$$

$$c_g[\tilde{d}_s, \tilde{d}_t] \equiv -\frac{g_2 M_W}{2} \sum_{i=1}^{3} \left[W_{is}^{\tilde{d}*} W_{it}^{\tilde{d}} \left(1 + \frac{1}{3} t_W^2 \right) + \frac{2}{3} W_{i+3\ s}^{\tilde{d}*} W_{i+3\ t}^{\tilde{d}} t_W^2 \right], \tag{10.47e}$$

$$c_g[\tilde{u}_s, \tilde{u}_t] \equiv \frac{g_2 M_W}{2} \sum_{i=1}^{3} \left[W_{is}^{\tilde{u}*} W_{it}^{\tilde{u}} \left(1 - \frac{1}{3} t_W^2 \right) + \frac{4}{3} W_{i+3\ s}^{\tilde{u}*} W_{i+3\ t}^{\tilde{u}} t_W^2 \right]. \tag{10.47f}$$

Similarly the coefficients of the various quartic **Higgs-Higgs-squark-squark interaction** terms can be written down. They are

$$D[H^+, H^-, \tilde{d}_s, \tilde{d}_t] = -\frac{g_2^2}{2 M_W^2} \sum_{i,j,k=1}^{3} \left[U_{ik}^{d_R} m_{d_k}^2 U_{jk}^{d_R *} W_{i+3\ s}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} t_\beta^2 + U_{ik}^{u_L} m_{u_k}^2 U_{jk}^{u_L *} W_{is}^{\tilde{d}*} W_{jt}^{\tilde{d}} (ct)_\beta^2 \right] + \frac{g_2^2 c_{2\beta}}{4} \sum_{i=1}^{3} \left[W_{is}^{\tilde{d}*} W_{it}^{\tilde{d}} \left(1 - \frac{1}{3} t_W^2 \right) - \frac{2}{3} W_{i+3\ s}^{\tilde{d}*} W_{i+3\ t}^{\tilde{d}} t_W^2 \right], \tag{10.48a}$$

$$\begin{aligned} D[H^+, H^-, \tilde{u}_s, \tilde{u}_t] &= -\frac{g_2^2}{2M_W^2} \sum_{i,j,k=1}^{3} \left[U_{ik}^{u_R} m_{u_k}^2 U_{jk}^{u_R *} W_{i+3\ s}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} (ct)_\beta^2 \right. \\ & \qquad \left. + U_{ik}^{d_L} m_{d_k}^2 U_{jk}^{d_L *} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{u}} t_\beta^2 \right] \\ & - \frac{g_2^2 c_{2\beta}}{4} \sum_{i=1}^{3} \left[W_{is}^{\tilde{u}*} W_{it}^{\tilde{u}} \left(1 + \frac{1}{3} t_W^2 \right) \right. \\ & \qquad \left. - \frac{4}{3} W_{i+3\ s}^{\tilde{u}*} W_{i+3\ t}^{\tilde{u}} t_W^2 \right], \end{aligned} \tag{10.48b}$$

$$\begin{aligned} D[H^+, H, \tilde{u}_s, \tilde{d}_t] &= d_{Yu}[\tilde{u}_s, \tilde{d}_t] s_\alpha + d_{Yd}[\tilde{u}_s, \tilde{d}_t] c_\alpha \\ & + d_{Yud}[\tilde{u}_s, \tilde{d}_t] c_{\alpha-\beta} - d_g[\tilde{u}_s, \tilde{d}_t] s_{\alpha+\beta} \ , \end{aligned} \tag{10.48c}$$

$$D[H^-, H, \tilde{d}_t, \tilde{u}_s] = \left(D[H^+, H, \tilde{u}_s, \tilde{d}_t] \right)^*, \tag{10.48d}$$

$$\begin{aligned} D[H^+, h, \tilde{u}_s, \tilde{d}_t] &= d_{Yu}[\tilde{u}_s, \tilde{d}_t] c_\alpha - d_{Yd}[\tilde{u}_s, \tilde{d}_t] s_\alpha \\ & - d_{Yud}[\tilde{u}_s, \tilde{d}_t] s_{\alpha-\beta} - d_g[\tilde{u}_s, \tilde{d}_t] c_{\alpha+\beta} \ , \end{aligned} \tag{10.48e}$$

$$D[H^-, h, \tilde{d}_t, \tilde{u}_s] = \left(D[H^+, h, \tilde{u}_s, \tilde{d}_t] \right)^*, \tag{10.48f}$$

$$D[H^+, A, \tilde{u}_s, \tilde{d}_t] = -i d_{Yu}[\tilde{u}_s, \tilde{d}_t] c_\beta + i d_{Yd}[\tilde{u}_s, \tilde{d}_t] s_\beta + i d_g[\tilde{u}_s, \tilde{d}_t] c_{2\beta} \ , \tag{10.48g}$$

$$D[H^-, A, \tilde{d}_t, \tilde{u}_s] = \left(D[H^+, A, \tilde{u}_s, \tilde{d}_t] \right)^*, \tag{10.48h}$$

$$D[H, H, \tilde{d}_s, \tilde{d}_t] = -d_Y[\tilde{d}_s, \tilde{d}_t] c_\alpha^2 - d_g[\tilde{d}_s, \tilde{d}_t] c_{2\alpha} \ , \tag{10.48i}$$

$$D[H, h, \tilde{d}_s, \tilde{d}_t] = d_Y[\tilde{d}_s, \tilde{d}_t] s_{2\alpha} + 2 d_g[\tilde{d}_s, \tilde{d}_t] s_{2\alpha} \ , \tag{10.48j}$$

$$D[h, h, \tilde{d}_s, \tilde{d}_t] = -d_Y[\tilde{d}_s, \tilde{d}_t] s_\alpha^2 + d_g[\tilde{d}_s, \tilde{d}_t] c_{2\alpha} \ , \tag{10.48k}$$

$$D[A, A, \tilde{d}_s, \tilde{d}_t] = -d_Y[\tilde{d}_s, \tilde{d}_t] s_\beta^2 + d_g[\tilde{d}_s, \tilde{d}_t] c_{2\beta} \ , \tag{10.48l}$$

$$D[H, H, \tilde{u}_s, \tilde{u}_t] = -d_Y[\tilde{u}_s, \tilde{u}_t] s_\alpha^2 - d_g[\tilde{u}_s, \tilde{u}_t] c_{2\alpha} \ , \tag{10.48m}$$

$$D[H, h, \tilde{u}_s, \tilde{u}_t] = -d_Y[\tilde{u}_s, \tilde{u}_t] s_\alpha^2 + 2 d_g[\tilde{u}_s, \tilde{u}_t] s_{2\alpha} \ , \tag{10.48n}$$

$$D[h, h, \tilde{u}_s, \tilde{u}_t] = -d_Y[\tilde{u}_s, \tilde{u}_t] c_\alpha^2 + d_g[\tilde{u}_s, \tilde{u}_t] c_{2\alpha} \ , \tag{10.48o}$$

$$D[A, A, \tilde{u}_s, \tilde{u}_t] = -d_Y[\tilde{u}_s, \tilde{u}_t] c_\beta^2 + d_g[\tilde{u}_s, \tilde{u}_t] c_{2\beta} \ . \tag{10.48p}$$

In (10.48) we have introduced the quantities

$$d_{Yu}[\tilde{u}_s, \tilde{d}_t] \equiv \frac{g_2^2 c_\beta}{2\sqrt{2} M_W^2 s_\beta^2} \sum_{i,j,k=1}^{3} U_{ik}^{u_L} m_{u_k}^2 U_{jk}^{u_L *} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{d}} \ , \tag{10.49a}$$

$$d_{Yd}[\tilde{u}_s, \tilde{d}_t] \equiv \frac{g_2^2 s_\beta}{2\sqrt{2} M_W^2 c_\beta^2} \sum_{i,j,k=1}^{3} U_{ik}^{d_L} m_{d_k}^2 U_{jk}^{d_L *} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{d}} \ , \tag{10.49b}$$

$$d_{Yud}[\tilde{u}_s, \tilde{d}_t] \equiv \frac{g_2^2}{\sqrt{2} M_W^2 s_{2\beta}} \sum_{i,j,k,l=1}^{3} U_{ik}^{u_R} m_{u_k} V_{kl}^{q_L} m_{d_l} U_{jl}^{d_R *} W_{i+3\ s}^{\tilde{u}*} W_{j+3\ t}^{\tilde{d}} \ , \tag{10.49c}$$

$$d_Y[\tilde{d}_s, \tilde{d}_t] \equiv \frac{g_2^2}{4 M_W^2 c_\beta^2} \sum_{i,j,k=1}^{3} m_{d_k}^2 \left[U_{ik}^{d_L} U_{jk}^{d_L *} W_{is}^{\tilde{d}*} W_{jt}^{\tilde{d}} + U_{ik}^{d_R} U_{jk}^{d_R *} W_{i+3\ s}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} \right] , \tag{10.49d}$$

$$d_Y[\tilde{u}_s, \tilde{u}_t] \equiv \frac{g_2^2}{4 M_W^2 s_\beta^2} \sum_{i,j,k=1}^{3} m_{u_k}^2 \left[U_{ik}^{u_L} U_{jk}^{u_L *} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{u}} + U_{ik}^{u_R} U_{jk}^{u_R *} W_{i+3\ s}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} \right] , \tag{10.49e}$$

$$d_g[\tilde{u}_s, \tilde{d}_t] \equiv \frac{g_2^2}{2\sqrt{2}} \sum_{i=1}^{3} W_{is}^{\tilde{u}*} W_{jt}^{\tilde{d}} \ , \tag{10.49f}$$

$$d_g[\tilde{d}_s, \tilde{d}_t] \equiv -\frac{g_2^2}{8} \sum_{i=1}^{3} \left[W_{is}^{\tilde{d}*} W_{it}^{\tilde{d}} \left(1 + \frac{1}{3} t_W^2 \right) + \frac{2}{3} W_{i+3\ s}^{\tilde{d}*} W_{j+3\ t}^{\tilde{d}} \tan^2 \theta_W \right] , \tag{10.49g}$$

$$d_g[\tilde{u}_s, \tilde{u}_t] \equiv \frac{g_2^2}{8} \sum_{i=1}^{3} \left[W_{is}^{\tilde{u}*} W_{it}^{\tilde{u}} \left(1 - \frac{1}{3} t_W^2 \right) + \frac{4}{3} W_{i+3\ s}^{\tilde{u}*} W_{j+3\ t}^{\tilde{u}} t_W^2 \right] . \tag{10.49h}$$

The corresponding Feynman rules are shown in Fig. 10.7.

Fig. 10.7 is included in Appendix B

10.6 Radiative Effects on MSSM Higgs Particles

The properties of the Higgs particles in the MSSM and the relations among them, following naturally from supersymmetry, have been discussed in §10.3–§10.5 at the tree level. However, it is now known [10.6] that significant changes are induced radiatively in many of the expressions and relations, appearing in those sections, by quantum loop corrections. We shall discuss some of these effects at the one loop level, confining ourselves largely to the mass of the lightest Higgs h. That is where they are most spectacular and are of the greatest importance to experiment. Before going into the details, let us make three general points:

- One needs to be clear about the meaning of a physical Higgs mass when radiative effects are to be taken into account. The on-shell mass is defined as the square root of *that* value of q^2 for which the real part of the inverse scalar propagator $q^2 - m_{tree}^2 + \Sigma(q^2)$, $\Sigma(q^2)$ being the one loop self energy correction, vanishes. However, we will compute radiative corrections to static Higgs masses.

- Radiative corrections to the mass of h are dominated by loops involving the top quark t and its stop partners $\tilde{t}_{1,2}$, cf. Fig. 10.8. This domination occurs owing to the large Yukawa coupling that these states have with h. Contributions from loops mediated by other states are negligible by comparison and will be ignored.

- In the limit of exact supersymmetry, tree level Higgs masses are protected by the nonrenormalization theorem discussed in Ch.6. This explains why radiative corrections to those masses are controlled by M_s, the scale of soft supersymmetry breaking.

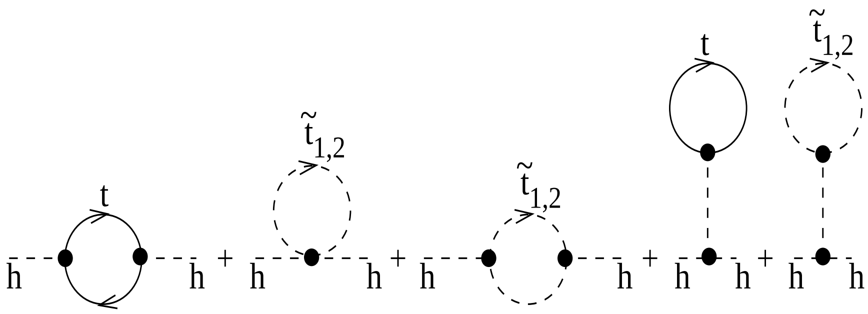

Fig.10.8. One loop self energy diagrams for h.

The radiatively corrected Higgs sector of the MSSM has been the subject of considerable study over several years. We do not go here into the initial and orginal works, but a detailed discussion with a historical perspective and a complete set of pertinent references may be found in the second paper cited in Ref. [10.1]. Three main tools have been employed in the literature: (1) direct diagrammatic calculations, (2) renormalization group methods and (3) effective potential techniques. Let us focus our attention on the correction to m_h as a sample case. The procedure in (1) is to adopt a straight computational approach by calculating the one loop self energy diagrams for h as given in Fig. 10.8.

In contrast, the methodology in (2) is that of Renormalization Group Evolution (RGE). For instance, when the sparticle mass spectrum (characterized by the scale M_s) is much heavier than the weak scale, i.e. $M_s \gg M_Z$, the quartic self coupling of h at the scale M_s is taken from (10.12) and (10.28b) to be

$$\frac{1}{32} \cos^2 2\alpha \, [g_Y^2(M_s) + g_2^2(M_s)]^2 \ .$$

It is then evolved to its value at the electroweak scale by means of the Standard Model RGE and used in the computation of the mass of h utilizing Standard Model expressions. However, we choose to present below an exposition of approach (3) – namely that of the effective potential – in calculating the correction to the tree level value of m_h. Though this method is numerically not as accurate as the diagrammatic one, it is pedagogically more interesting and gives a better theoretical insight into these loop induced radiative corrections. In addition, the inclusion of leading two loop corrections and the computation of corrections to static Higgs self couplings are more straightforward in this approach.

We start by considering the static approximation in which the effective action is approximated by the one loop effective potential. The actual calculation of the one loop effective potential can be found in standard text books [10.7–10.9]. The final expression reads

$$V_H^1(Q) = V_H^0(Q) + \Delta V_H^{(1)}(Q) \ , \tag{10.50a}$$

$$\Delta V_H^{(1)}(Q) = \frac{1}{64\pi^2} S\mathrm{Tr}\ \mathcal{M}^4(h) \left[\ln \frac{\mathcal{M}^2(h)}{Q^2} - \frac{3}{2} \right] . \tag{10.50b}$$

In (10.50), $V_H^0(Q)$ is the tree level Higgs potential with its couplings renormalized at some scale Q, $\mathcal{M}(h)$ is the field dependent mass matrix and the supertrace, cf. 5.10, covers all supermultiplet fields whose masses depend on the VEVs of the Higgs fields.

Corrections in the absence of $\tilde{t}_L$-$\tilde{t}_R$ mixing

We have earlier noted that the most important loop corrections to the Higgs potential V_H come from the top-stop sector of the theory. To keep the discussion simple, we will consider only these. First, we neglect any mixing between the $SU(2)_L$ doublet ($\tilde{t}_L$) and singlet ($\tilde{t}_R$) squarks and assume equal soft supersymmetry breaking squared masses $\tilde{m}^2$ for those two fields. The relevant field dependent masses then are

$$m_t^2(h) = f_t^2 |h_2^0|^2 , \tag{10.51a}$$

$$m_{\tilde{t}_1}^2(h) = m_{\tilde{t}_2}^2(h) = \tilde{m}^2 + f_t^2 |h_2^0|^2 , \tag{10.51b}$$

where f_t is the top Yukawa coupling strength, being equal to $m_t (2\sqrt{2} G_F)^{1/2} / \sin\beta$. We have neglected D-term contributions to the stop masses since they are proportional to electroweak gauge couplings. They are thus suppressed by at least one power of $M_W^2 m_t^{-2}$ compared to the pure Yukawa contribution.

Each fermion or boson contributes to the supertrace of (10.50b) with a multiplicative weight factor equal to the number of independent degrees of freedom associated with it. Let us recall that each Dirac fermion contains four degrees of freedom, while each complex scalar has two. In addition, we have to include a color factor of three. Altogether, we thus have from (10.50b) that

$$\Delta V_{H,t-\tilde{t}}^{(1)}(Q) = \frac{3}{16\pi^2} \left[(\tilde{m}^2 + f_t^2 |h_2^0|^2)^2 \left(\ln \frac{\tilde{m}^2 + f_t^2 |h_2^0|^2}{Q^2} - \frac{3}{2} \right) - (f_t^2 |h_2^0|^2)^2 \left(\ln \frac{f_t^2 |h_2^0|^2}{Q^2} - \frac{3}{2} \right) \right] , \tag{10.52}$$

where the overall factor of 3 in the numerator comes from color. As already advertised, the two terms in the RHS of (10.52) cancel exactly in the limit of unbroken supersymmetry.

In order to understand the physical significance of these corrections, we first have to redo the minimization of the Higgs potential. In §10.2 we had minimized the tree level expression which we now call $V_H^{(0)}$. Here we will do the same for $V_H^{(0)} + \Delta V_{H,t-\tilde{t}}^{(1)}$. As evident from (10.52), in the limit of vanishing $\tilde{t}_L$-$\tilde{t}_R$ mixing, corrections from the top-stop sector only involve the second Higgs doublet h_2. Therefore (10.17a) remains unchanged. On the other hand, (10.17b) now becomes

$$m_{2h}^2 = m_{12}^2 \cot\beta + \frac{M_Z^2}{2} \cos 2\beta - \frac{3 f_t^2}{16\pi^2} \left[f(m_{\tilde{t}}^2) - f(m_t^2) \right] , \tag{10.53}$$

where we have introduced the function

$$f(m^2) \equiv 2m^2 \left(\ln \frac{m^2}{Q^2} - 1 \right) \tag{10.54}$$

and m_t^2, $m_{\tilde{t}}^2$ are squared masses of the top and the stops respectively. We need not, for the moment, be bothered by the presence of the $\ln Q^2$ term since it can ultimately be absorbed in the renormalization of m_{2h}^2.

As in (10.19), we calculate the mass squared matrices now for the CP odd and CP even Higgs bosons by taking second derivatives of $V_H^{(0)} + \Delta V_{H,t-\tilde{t}}^{(1)}$ with respect to the imaginary and real parts of the neutral Higgs fields respectively. Once again, only the VEV v_2 (and not v_1) contributes to (10.53) as a consequence of our assumption of no $\tilde{t}_L$-$\tilde{t}_R$ mixing. Therefore, only the 2, 2 entries in the concerned matrices can possibly receive corrections from the top-stop sector. Moreover, since the VEVs v_1 and v_2 are real, the derivatives have to be taken at $\Im m\ h_1^0 = \Im m\ h_2^0 = 0$. It is then easy to see that the final result for the mass squared matrix of the CP odd states is the same as at the tree level, i.e. (10.23). The explicit correction to the 2,2 entry from the second derivative of (10.53) exactly cancels the correction to m_{2h}^2, given in (10.53). However, such is not the case for the 2,2 element of the mass squared matrix of the CP even Higgs scalars, (cf. 10.26). There we find the following finite and positive correction:

$$\Delta_{22}^{LL} = \frac{3 f_t^2 m_t^2}{4\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} \equiv \frac{\epsilon_h}{\sin^2\beta} , \tag{10.55}$$

where

$$\epsilon_h = \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} . \tag{10.56}$$

We have put the superscript LL on Δ_{22} to denote the fact that (10.56) is a leading logarithm (in the ratio $m_{\tilde{t}}/m_t$) expression.

The one loop correction to (10.26), in this scenario, reads

$$\delta m^2_{\Re e\ h^0} = \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{\epsilon_h}{\sin^2\beta} \end{pmatrix} , \tag{10.57}$$

so that (10.27), (10.29c) and (10.30b) extend respectively to

$$m_{h,H}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 + \frac{\epsilon_h}{\sin^2\beta} \pm \left\{ (m_A^2 + M_Z^2)^2 \sin^2 2\beta + \left[(M_Z^2 - m_A^2)\cos 2\beta + \frac{\epsilon_h}{\sin^2\beta} \right]^2 \right\}^{1/2} \right] , \tag{10.58a}$$

$$\tan 2\alpha = (m_A^2 + M_Z^2) \tan 2\beta \left(m_A^2 - M_Z^2 + \frac{\epsilon_h}{\sin^2\beta \cos 2\beta} \right)^{-1} , \tag{10.58b}$$

$$m_h^2 + m_H^2 = m_A^2 + M_Z^2 + \frac{\epsilon_h}{\sin^2\beta} . \tag{10.58c}$$

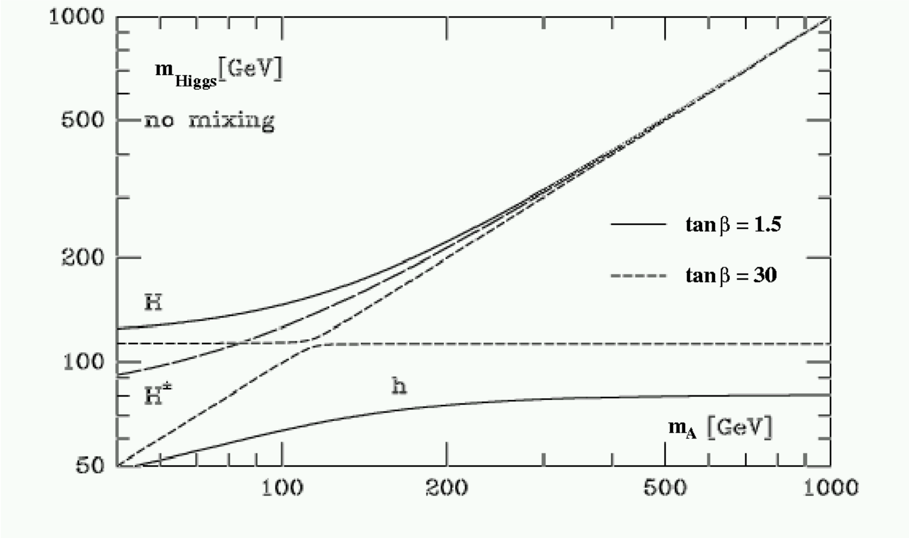

Fig.10.9. Other Higgs masses vs m_A for $\tan\beta = 1.5$ and 30 with $m_{\tilde{t}} \simeq 1$ TeV; adapted from Ref. [10.10].

In Fig. 10.9 the Higgs masses m_h, m_H and $m_{H^\pm}$, as given in (10.58a), are plotted [10.10] against m_A, the mass of the CP odd Higgs boson, for two rather extreme values of $\tan\beta$. For $\tan\beta > 1$, the mass eigenvalue of h increases monotonically with m_A, saturating to its maximum upper bound

$$m_h \ < \ (M_Z^2 \cos^2 2\beta + \epsilon_h)^{1/2} \tag{10.59}$$

for modest values of m_A, i.e. $m_A > 300$ GeV. For large $\tan\beta$ and $m_{\tilde{t}}$ taken to be $\mathcal{O}$(TeV), the RHS of (10.59) is ~ 110 GeV. We shall see later that the possibility of $\tilde{t}_L$–$\tilde{t}_R$ mixing can increase this upper bound[4]. At this level, the charged Higgs mass is still given by (10.30a) and is hence independent of $\tan\beta$, as shown in [10.10] Fig. 10.9. The tree level properties of the Higgs mass spectrum in the decoupling limit ($m_A \to \infty$) are still maintained. Now the $A, H, H^\pm$ Higgs particles remain nearly degenerate while the lightest h saturates its maximum mass value. The tree level mass orderings, $m_H > m_H^\pm > m_A$ remain valid for small $\tan\beta$. Otherwise, the larger $\tan\beta$ curves in Fig. 10.9 are fairly similar to the curves in Fig. 10.2, with M_Z replaced by $\{M_Z^2 + \epsilon_h^2\}^{1/2}$.

Eq. (10.56) represents the celebrated correction which has a quartic power dependence on the mass of the top quark. Note that it has only a logarithmic dependence on the stop mass squared $m_{\tilde{t}}^2$ which is characteristic of the square of the soft supersymmetry breaking scale M_s. This would seem to contradict our starting proposition that corrections to the masses of Higgs bosons should be proportional to supersymmetry breaking masses. This apparent contradiction is resolved by the fact that the shift in the tree level parameter m_{2h}^2 is indeed proportional to $m_{\tilde{t}}^2 - m_t^2$, c.f.(10.53). One would need to fine tune the parameters appearing in this equation if the tree level part were much smaller than the correction term. Furthermore, notice that the renormalization scale Q has disappeared from (10.56). This is to be expected since this equation describes the correction to a relation among *physical*

[4]Indeed, the final experimental lower bound on the mass of an SM-like Higgs boson of about 115 GeV from the completed runs at LEP indicates the need for some amount of $\tilde{t}_L$-$\tilde{t}_R$ mixing unless $\tilde{t}_{L,R}$-masses are much in excess of 1 TeV. This point will be discussed in more detail in §15.5.

quantities (masses of CP even Higgs bosons one hand and M_Z, m_A on the other). Indeed, it can be shown already at the level of the effective potential (10.50) that the explicit $\ln Q^2$ dependence of the one loop correction cancels against a similar dependence of the running quantities appearing in the tree level potential. In the simplified scenario, considered by us so far, it follows from (10.52) that the entire $\ln Q^2$ dependence collapses to

$$\frac{\partial \Delta V_H^{(1)}}{\partial \ln\ Q^2} = -\frac{3}{8\pi^2}\tilde{m}^2 \left(f_t^2 |h_2^0|^2 + \frac{1}{2}\tilde{m}^2 \right). \tag{10.60}$$

Thus the first term in the RHS of (10.60) exactly cancels the Q^2-dependence of $m_{2h}^2(Q)|h_2^0|^2$. The second term in the RHS of (10.60), a field independent constant, is of no immediate interest to particle physics, though it may contribute to the cosmological constant.

Corrections with $\tilde{t}_L$-$\tilde{t}_R$ mixing

Let us now introduce a nonzero $\tilde{t}_L$-$\tilde{t}_R$ mixing, described (c.f. 9.62c) by the off-diagonal matrix element[5] $-m_t(A^t + \mu \cot\beta)$ of the stop squared mass matrix. We will also allow the soft supersymmetry breaking $\tilde{t}_L$ and $\tilde{t}_R$ mass terms to differ. The eigenvalues of the field dependent $\tilde{t}$ squared mass matrix are then given by

$$m_{\tilde{t}_{1,2}}^2(h) = f_t^2|h_2^0|^2 + \frac{1}{2}\left[m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \pm \sqrt{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2) + 4 f_t^2 |A^t h_2^0 + \mu h_1^{0*}|^2}\ \right]. \tag{10.61}$$

Note that these eigenvalues depend on both neutral Higgs fields $h_{1,2}^0$. The corresponding one loop correction to the Higgs effective potential now becomes

$$\begin{aligned}\Delta V_{H,t-\tilde{t}}^{(1)}(Q) \;=\; & \frac{3}{32\pi^2}\Bigg[m_{\tilde{t}_1}^4(h)\left\{ \ln\frac{m_{\tilde{t}_1}^2(h)}{Q^2} - \frac{3}{2}\right\} + m_{\tilde{t}_2}^4(h)\left\{ \ln\frac{m_{\tilde{t}_2}^2(h)}{Q^2} - \frac{3}{2}\right\} \\ & -2 f_t^4|h_2^0|^4 \left\{ \ln\frac{f_t^2|h_2^0|^2}{Q^2} - \frac{3}{2}\right\}\Bigg].\end{aligned}$$

Both the minimization conditions $\partial V_H/\partial h_1^0 = 0,\ \partial V_H/\partial h_2^0 = 0$ are now affected by radiative corrections. Therefore, (10.17) change to

$$m_{1h}^2 = m_{12}^2 \tan\beta - \frac{1}{2}M_Z^2\cos 2\beta - \frac{3f_t^2}{32\pi^2}\frac{\mu(\mu + A^t\tan\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left[f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)\right], \tag{10.62a}$$

$$\begin{aligned} m_{2h}^2 \;=\; & m_{12}^2\cot\beta + \frac{1}{2}M_Z^2\cos 2\beta \\ & -\frac{3f_t^2}{32\pi^2}\left\{ f(m_{\tilde{t}_1}^2) + f(m_{\tilde{t}_2}^2) - 2f(m_t^2) + \frac{A^t(A^t + \mu\cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}[f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)]\right\}.\end{aligned} \tag{10.62b}$$

[5] We take A^t and μ to be real here.

Once again, the squared mass matrices for the neutral Higgs bosons can be computed from the second derivatives of the Higgs potential. The calculation for the CP odd case is greatly simplified by the observation that the first derivatives of any of the field dependent top (stop) masses with respect to the imaginary parts of h_1^0 and h_2^0 vanish in the (real) minimum of the Higgs potential. A straightforward calculation yields the result

$$m^2_{\Im\mathrm{m}\ h^0} = (m_{12}^2 + \Delta) \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix}, \tag{10.63}$$

with

$$\Delta = -\frac{3f_t^2}{32\pi^2} \frac{\mu A^t}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} [f(m^2_{\tilde{t}_1}) - f(m^2_{\tilde{t}_2})] \,. \tag{10.64}$$

As anticipated, this correction vanishes in the limit of no $\tilde{t}_L$-$\tilde{t}_R$ mixing ($\mu = A^t = 0$). The one loop corrected mass of the physical CP odd Higgs boson A thus becomes

$$m_A^2 = \frac{2(m_{12}^2 + \Delta)}{\sin 2\beta} \,. \tag{10.65}$$

The explicit $\ln Q^2$ dependence can again be shown to cancel in this equation, if m_{12}^2 and $\tan\beta$ are understood to be running parameters. However, this cancellation works exactly only in one loop order; beyond that, terms of order $[f_t^2 m_{\tilde{t}}^2 \ln(m_{\tilde{t}}^2/Q^2)]^2$ remain in (10.65). In the interest of perturbative stability, one should therefore choose a renormalization scale Q close to the stop mass, e.g. $Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. This is totally analogous to the choice made in perturbative QCD calculations (involving massless partons) of the renormalization scale to be close to the external momentum.

The generalization of (10.57), including $\tilde{t}_L$-$\tilde{t}_R$ mixing, now reads

$$\delta m^2_{\Re\mathrm{e}\ h^0} = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix}, \tag{10.66}$$

with

$$\Delta_{11} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \left[\frac{\mu(A^t + \mu\cot\beta)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \right]^2 \left(2 - \frac{m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \ln \frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} \right), \tag{10.67a}$$

$$\Delta_{12} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{\mu(A^t + \mu\cot\beta)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \ln \frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} + \frac{A^t}{\mu}\Delta_{11} \,, \tag{10.67b}$$

$$\Delta_{22} = \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \left[\ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \frac{A^t(A^t + \mu\cot\beta)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \ln \frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} \right] + \left(\frac{A^t}{\mu} \right)^2 \Delta_{11}. \tag{10.67c}$$

Again, each of eqs. (10.67) is independent of Q^2. Note also that the corrected value (10.65) of m_A^2 has to be used in the tree level squared mass matrix (10.26).

We have, so far, considered corrections only from the top-stop sector. If $\tan\beta$, the ratio of the Higgs VEVs, becomes very large, the bottom Yukawa coupling can be comparable to that of the top and make substantial additional corrections to (10.62), (10.65) and (10.66). These can be obtained from our expressions by the following three substitutions:- (1) interchange top (stop) masses and couplings with those of the bottom (sbottom); (2) interchange h_1^0 and h_2^0, i.e. $\tan\beta \leftrightarrow \cot\beta$; (3) interchange the shifts of m_{1h}^2 and m_{2h}^2, i.e. the leading logarithmic corrections from the bottom-sbottom sector only affect m_{1h}^2. Note that even if $\tan\beta$ is as high as m_t/m_b, the 'leading' logarithmic corrections from the b-$\tilde{b}$ sector to the squared mass matrix of the CP even Higgs bosons are suppressed by a factor $(m_b/m_t)^2$ as compared with those from the t-$\tilde{t}$ sector and thus can be safely neglected; however, the nonlogarithmic corrections from $\tilde{b}_L$-$\tilde{b}_R$ mixing can be significant in this case.

The question can be raised as to whether one can go beyond the one loop corrections from heavy quarks/squarks, presented above. Leading two loop corrections at $O(\alpha\alpha_s)$ to (10.67) can be incorporated with just a little more effort. This is done by treating the top mass in the overall m_t^4 factor as a scale dependent running quantity. In other words, m_t should be interpreted as the $\overline{\text{MS}}$ (or $\overline{\text{DR}}$) mass, *not* the pole mass. The two quantities are related by the boundary condition [10.11]

$$m_t(m_t) = m_t^{\text{pole}} \left(1 - \frac{4\alpha_S}{3\pi}\right) \tag{10.68}$$

plus higher order corrections. The scale dependence of m_t for scales $Q \leq m_{\tilde{t}}$ is the same as in the nonsupersymmetric SM:

$$m_t(Q) = m_t(m_t) \left[\frac{\alpha_S(Q)}{\alpha_S(m_t)}\right]^{12/23} . \tag{10.69}$$

The first (leading log) term in (10.67c) can be understood to have originated from the running of the SM Higgs self coupling from the scale[6]

$$M_s = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \tag{10.70}$$

to the scale m_t. Using this observation, the leading two loop corrections to this term can be easily incorporated by taking the factor m_t^4 at the intermediate scale $\sqrt{M_s m_t}$. All other terms in (10.67) can be absorbed in the boundary condition on the Higgs self coupling at the scale M_s; the m_t^4 factors in all such nonlogarithmic terms should therefore be taken at the high scale M_s.

By far, the most significant effect of the radiative corrections, discussed in this section, is that they relax the upper bound (10.30c) on the mass of the lighter CP even Higgs scalar h. We had already derived an upper bound $\sim$ 110 GeV in the absence of $\tilde{t}_L$-$\tilde{t}_R$ mixing, but here we give the more general result when such a mixing is present. For a given value of $\tan\beta$, m_h is still maximal when m_A is large (the "decoupling limit", as discussed earlier), but the bound is now given by

$$m_h^2 < M_Z^2 \cos^2 2\beta + \Delta_{11}\cos^2\beta + \Delta_{12}\sin 2\beta + \Delta_{22}\sin^2\beta\ , \tag{10.71}$$

with the Δ's given by (10.67). Numerically, the correction Δ_{22} is usually the most important one. The absolute upper bound is still reached for $\tan\beta \gg 1$ (i.e. $|\cos 2\beta| \to 1$) just as at

[6]We assume here for simplicity that supersymmetry breaking is characterized by this one mass scale.

the tree level. For equal $\tilde{t}_L$ and $\tilde{t}_R$ soft supersymmetry breaking mass terms, a simple yet accurate formula for this upper bound obtains in the limit $|m_t A^t| \ll m_{\tilde{t}}^2$:

$$m_h^2 \leq M_Z^2 + \frac{3G_F}{\sqrt{2}\pi^2} \Bigg[m_t^4(\sqrt{m_t M_s}) \ln(M_s^2/m_t^2) + (A^t)^2 M_s^{-2} m_t^4(M_s)(1 - \frac{1}{12}(A^t)^2 M_s^{-2}) \Bigg], \quad (10.72)$$

with M_s as given by (10.70). We have explained why the two m_t^4 factors in the RHS of (10.72) have to be taken at different scales. Taking $M_s = 1$ TeV and $m_t^{\text{pole}} = 175$ GeV from direct TEVATRON experiments [10.12], one finds $m_t(\sqrt{m_t M_s}) \simeq 157$ GeV and $m_t(M_s) \simeq 150$ GeV. Since the last RHS term in (10.72) is maximal at $A^t = \sqrt{6} M_s$, one then obtains an absolute upper bound on m_h which is a critical test of MSSM, namely

$$m_h < 132 \text{ GeV}. \quad (10.73)$$

Comparing with (10.59), we see that the effect of nonzero A^t, μ are quite significant and shifts the upper bound on the h-mass by more[7] than 20 GeV. Radiative corrections can therefore push m_h well beyond the reach of existing e^+e^- colliders. We finally mention that the treatment presented here has recently been extended by including corrections $\mathcal{O}(f_t^2 g^2)$ to the squared Higgs mass matrix and by allowing for large CP violating phases in the third generation squark sector [10.15]. These phases lead CP even and CP odd Higgs states to mix but do not alter the upper bound (10.73) on m_h. Later, in Ch.14, we shall discuss the generalization of (10.73) to cover extensions of the MSSM.

Concluding remarks

Before concluding this section, we want to make some brief general remarks on one loop radiative corrections to the charged Higgs mass and also to Higgs couplings in the MSSM. We have already shown that, in the absence of $\tilde{t}_L$-$\tilde{t}_R$ mixing (i.e. neglecting the effects of μ and A^t) – the charged Higgs mass is given by (10.30a) and is independent of $\tan\beta$. Even with $\tilde{t}_L$-$\tilde{t}_R$ mixing, the one loop corrections to $m_{H^\pm}^2$ remain small if the renormalization scale Q is chosen in a way such that perturbation theory is reliable. Explicit expressions for these corrections may be found in Ref. [10.6]. It is worth remarking here, though, that corrections from the top (stop)-bottom (sbottom) sector go to zero in the limit of a vanishing bottom Yukawa coupling. We further remind the reader that all D-term contributions to the squark mass matrices were neglected. The inclusion of such terms will introduce additional corrections of order $g_2^2 m_t^2/(8\pi^2)$ or $g_2^2 M_W^2/(8\pi^2)$. These corrections can be computed along

[7] We have presented here the results within the effective potential framework, implicitly working with $\overline{\text{MS}}$ renormalized parameters. A more recent analysis [10.13] shows that a diagrammatic calculation in the on-shell renormalization scheme, again including leading two loop corrections, almost exactly reproduces the result from the effective potential approach, once the difference between the two renormalization schemes has been taken into account. One should nonetheless assign a theoretical error of two to three GeV to the predicted value of m_h, due to higher order terms. An upward shift of such a magnitude has very recently been found [10.14] from two loop $\mathcal{O}(f_t^4)$ corrections.

the lines presented here. Though, strictly speaking, these modify [10.6] the relation (10.30a), they are numerically unimportant. Note also that a complete calculation of pure electroweak $O(g_2^2 M_W^2)$ corrections should include contributions from loops involving first and second generation sfermions as well as those from the gauge-Higgs-gaugino-higgsino sector. Turning to Higgs couplings, one loop radiative corrections, at the level discussed in this section, do not affect Higgs-gauge and Higgs-fermion couplings[8] directly. They only come in indirectly through a shift in the value of α, as indicated by (10.58b). Only in the case of Higgs self couplings are there some direct contributions [10.16]. For instance, ignoring $\tilde{t}_L$-$\tilde{t}_R$ mixing, the coefficients (denoted by $\lambda_{...}$) of $-ig_2 M_Z/(2\cos\theta_W)$ in the triple scalar Hhh and HAA vertices are changed from what appear in Fig. 10.4 to

$$\lambda_{Hhh} = 2\sin 2\alpha \sin(\alpha+\beta) - \cos 2\alpha \cos(\alpha+\beta) + 3\frac{\epsilon_h \sin\alpha}{M_Z^2 \sin^3\beta}\cos^2\beta \ , \tag{10.74a}$$

$$\lambda_{HAA} = -\cos 2\beta \cos(\alpha+\beta) + \frac{\epsilon_h \sin\alpha}{M_Z^2 \sin^3\beta}\cos^2\beta \ . \tag{10.74b}$$

Our final comment is on the static approximation. That may not work so well for Higgs bosons which are heavy, e.g. with masses comparable to those of the top/stop(s). Indeed, on-shell couplings of H and A then often develop imaginary dispersive parts from loops induced by the latter.

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