Chapter 5
Theories and Lagrangians III: The Standard Model

The previous two chapters were devoted to introducing the basic ingredients necessary in building up a physical description of elementary particles: the fermion matter fields and the gauge fields responsible for the interactions. The time has come to combine these elements into a description of the physics of elementary particles. The result will be the standard model.

In the next sections we are going to summarize the basic features of the standard model, also called the Glashow–Weinberg–Salam theory [1–3]. Our presentation here, however, will leave one important problem open: how particle masses in the standard model can be made compatible with gauge invariance. The missing ingredient to solve this problem, spontaneous symmetry breaking, will have to wait until Chap. 7. The presentation will remain mostly qualitative. The details of the construction of the standard model and a full study of its consequences for the phenomenology of elementary particles can be found in many textbooks (for example [4–8]).

5.1 Fundamental Interactions

Most of the phenomena we witness in our daily life can be explained in terms of two fundamental forces: gravity and electromagnetism. They are the only relevant interactions in a very wide range of phenomena that goes from the dynamics of galaxies to atomic and solid state physics.

These two interactions, however, do not suffice to give an account of all subnuclear physics. Gravity is indeed too weak to be of any relevance at the atomic level. The laws of electromagnetism, on the other hand, offer no explanation as to how a large number of positively charged protons can be confined in nuclei with a size of the order of 10^{-15} m. QED does not provide either any mechanism that could explain nuclear processes such as beta decay. These phenomena require invoking two nuclear interactions: a "strong" one responsible for binding protons and neutrons together in the atomic nuclei, and a "weak" one that, without producing bound states, accounts for nuclear disintegrations.

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To understand how the relevant interaction can be identified in subnuclear processes we need to recall some basic ideas from quantum mechanics. Take a system in a quantum state of energy E, $|\psi_E\rangle$. Let us assume that this state decays as a consequence of the interaction Hamiltonian H_{int}. Then, the lifetime τ of the state is equal to the inverse of its width Γ that, in turn, can be computed using Fermi's golden rule

$$\Gamma = 2\pi \sum_f \rho_f(E)|\langle f|H_{\text{int}}|\psi_E\rangle|^2. \tag{5.1}$$

Here the sum is over final states and $\rho_f(E)$ is the density of such states with energy E. The key point is that, generically, the overlap $\langle f|H_{\text{int}}|\psi_E\rangle$ is proportional to a power of the coupling constant (i.e., the charge) of the interaction involved in the process. Thus, the bottom line is that the lifetime of a quantum state is, roughly speaking, inversely proportional to the strength of the interaction responsible for its decay.

In high energy physics, this provides a good guiding principle to identify the interaction behind a decay process: the hierarchy in the strength of the three interactions should be reflected in a hierarchy of the characteristic times of the processes they mediate. This is indeed what happens. Strong interaction decays are characterized by very short lifetimes of the order

$$\tau_{\text{strong}} \sim 10^{-23}\,\text{s}. \tag{5.2}$$

Next in the hierarchy come electromagnetic processes, for which

$$\tau_{\text{em}} \sim 10^{-16}\,\text{s}. \tag{5.3}$$

Finally, the weak interaction is behind processes with typical times substantially longer than the ones above

$$\tau_{\text{weak}} \sim 10^{-8} - 10^{-6}\,\text{s}, \tag{5.4}$$

with some decays, such as the neutron β-decay, reaching characteristic times of the order of minutes.

Electromagnetic processes are described quantum mechanically using quantum electrodynamics (see Chap. 4). As for the strong and weak interactions, before entering into the details of their quantum field theory description we need to learn some basic facts about their phenomenology.

Strong Interaction

Let us begin with the strong interaction. The class of subatomic particles that feel the strong force, collectively denoted as *hadrons*, are classified in two types depending on their spin: *baryons* with half-integer spin (e.g. the proton and the neutron) and *mesons* with integer spin (e.g. the pions).

Approximate symmetries are a very useful tool in the study of physical processes mediated by the strong interaction. The best-known example is the isospin symmetry familiar from nuclear physics. Indeed, strong interactions do not seem to distinguish very much between protons and neutrons although both the weak and electromagnetic interactions do. This is shown by the similar energy levels of the so-called mirror nuclei, those related by replacing one or more protons by neutrons such as ^{11}B and ^{11}C. The slight differences in the spectrum of these nuclei can be explained by the small mass split between the proton and the neutron and by their different values for the electric charge.

That the strong interaction alone cannot tell apart neutrons from protons is codified in mathematical terms in a global $\mathrm{SU}(2)_I$ isospin symmetry that rotates these two particles into one another. Protons and neutrons form a doublet with isospin $I = \frac{1}{2}$ and third components $I_3(p) = \frac{1}{2}$ and $I_3(n) = -\frac{1}{2}$. The scheme is extended to other particles, such as the three pions $\pi^0,\ \pi^{\pm}$, that form an isospin triple ($I = 1$) where $I_3(\pi^{\pm}) = \pm 1,\ I_3(\pi^0) = 0$.

All this notwithstanding, isospin remains only an approximate symmetry of the strong force even after switching off both the electromagnetic and the weak interactions. This follows from the small but nonvanishing difference between the masses of the particles within an isospin multiplet. Isospin is nevertheless useful because the mass splitting is much smaller than the particle masses themselves and the symmetry breaking effects are small.

Besides isospin, the strong interaction preserves other quantum numbers, such as strangeness S. Adding this quantum number to isospin it is possible to extend $\mathrm{SU}(2)_I$ to the flavor $\mathrm{SU}(3)_f$ global symmetry. Strongly interacting particles are then classified in irreducible representations of this group: singlets, octets and decuplets but, interestingly, not triplets [the fundamental and antifundamental representations of $\mathrm{SU}(3)_f$]. To illustrate this we see that the isodoublet formed by the proton and the neutron is embedded into a $\mathrm{SU}(3)_f$ octet that also includes an isotriplet ($\Sigma^{\pm},\ \Sigma^0$) with $S = -1$ and the isodoublet ($\Xi^-,\ \Xi^0$) with $S = -2$. We can get an idea of the accuracy of this approximate symmetry by noticing that

$$m(p, n) \approx 930\,\mathrm{MeV}, \quad m(\Sigma) \approx 1190\,\mathrm{MeV}, \quad m(\Xi) \approx 1320\,\mathrm{MeV}. \tag{5.5}$$

The mass split between the states with different strangeness in the octet is about 30% of the average mass, much larger than the 0.1–0.7% mass split within each isospin multiplet. Similarly, the addition of the lowest-lying strange mesons to the pion isotriplet results in the $\mathrm{SU}(3)_f$ octet and singlet shown in Fig. 5.1. Very soon we will learn how these approximate symmetries reflect the inner structure of the hadrons.

An important relation is the Gell-Mann–Nishijima formula giving the electric charge of a strong-interacting particle in terms of its third isospin component and strangeness

$$Q = I_3 + \frac{B + S}{2}, \tag{5.6}$$

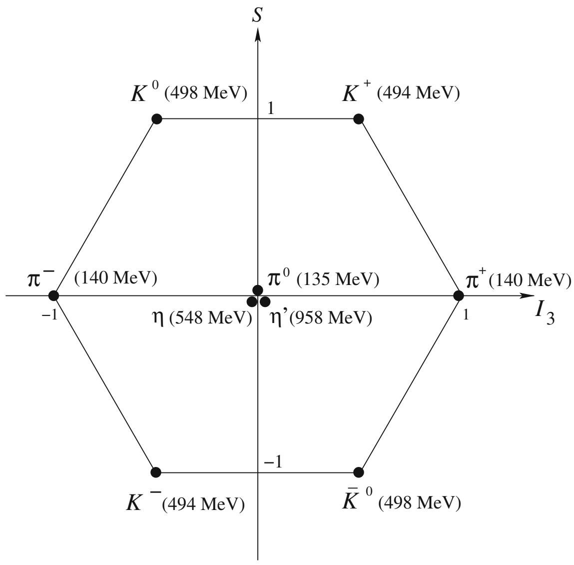

Fig. 5.1 The lowest-lying pseudoscalar mesons. The masses of the particles are indicated in parenthesis

where B is the baryon number, that takes the values $B = +1$ for baryons, $B = -1$ for antibaryons and $B = 0$ for mesons. The combination $Y = \frac{1}{2}(B + S)$ defines the strong hypercharge that is conserved in strong interaction processes.

Weak Interaction

After gravity, the weak interaction is the most universal force in Nature since every known matter particle takes part in it. This includes all hadrons as well as a number of nonhadronic particles called *leptons*. Although the weak interactions do not produce bound states, it is behind very important physical processes such as neutron beta decay

$$n \longrightarrow p + e^- + \overline{\nu}_e, \tag{5.7}$$

responsible for the radioactive disintegration of nuclei.

Neutron beta decay is an example of a process mediated by a so-called weak charged current: the hadronic (n, p) and leptonic $(e^-, \overline{\nu}_e)$ pairs contain particles whose electric charges differ in one unit. Another example of this kind of processes is provided by muon decay

$$\mu^- \longrightarrow e^- + \overline{\nu}_e + \nu_\mu. \tag{5.8}$$

Here the two pairs formed by the leptons of the same flavor, $(e^-, \overline{\nu}_e)$ and (μ^-, ν_μ), are composed of particles of different charge. Weak processes can also proceed through weak neutral currents in which the hadrons or the same-flavor leptons do not change their electric charge. One example is electron-neutrino scattering

$$e^- + \nu_\mu \longrightarrow e^- + \nu_\mu, \tag{5.9}$$

where the particles in each of the two same-flavor lepton pairs have the same electric charge.

One of the most distinctive features of the weak interaction is that it violates what once were cherished discrete symmetries. In the dominant decay channel of the negatively-charged pion into a muon and a muonic neutrino

$$\pi^- \longrightarrow \overline{\nu}_\mu + \mu^-, \tag{5.10}$$

it is experimentally observed that the muon is always emitted with positive helicity (i.e., it is right-handed). Since parity reverses the helicity of the particle, this result indicates that parity is violated by the weak interaction. Moreover, this violation is maximal because *all* muons emitted in the π^- decay are right-handed. This shows that any field-theoretical description of the weak interaction must necessarily be chiral, that is, the weak interaction coupling of the fermions should depend on their helicities. This feature singles out weak interaction among the fundamental forces in that it is the only one that distinguishes left from right. Why this is the case remains a mystery.

Charge conjugation, denoted by C, is a discrete operation that interchanges particles with their antiparticles. The properties of this discrete symmetry will be studied in detail in Chap. 11. Here we only need to know that the decay of the positively-charged pion is obtained by charge-conjugating (5.10)

$$\pi^+ \longrightarrow \nu_\mu + \mu^+. \tag{5.11}$$

An important property of the operation C is that it changes particles by antiparticles but does not modify the helicity of the fermions. This means that if charge conjugation is a symmetry of the weak interaction, the decay of the π^+ has to proceed by emission of a right-handed antimuon. Experimentally, however, it is observed that the antimuon emitted by the decaying pion is *always* left-handed! This shows that weak interactions not only violate parity but also charge conjugation and that this violation is also maximal.

This is not the end of the story. Not only P and C are violated by the weak interaction, but also their combination CP. How this happens is however more subtle (see Sect. 11.5).

5.2 Leptons and Quarks

One of the glaring features of the host of particles produced in high energy collisions is that there is only a small number of them that do not feel the strong nuclear force. The list is made of the following six leptons

e^-	electron	$q_e = -1$	ν_e	electron neutrino	$q_{\nu_e} = 0$
μ^-	muon	$q_\mu = -1$	ν_μ	muon neutrino	$q_{\nu_\mu} = 0$
τ^-	tau	$q_\tau = -1$	ν_τ	tau neutrino	$q_{\nu_\tau} = 0$

and their corresponding antiparticles. The rest of the over one hundred particles and resonances listed in the Review of Particle Physics [9], partake in physical processes mediated by the strong interaction.

Unlike the case of the leptons, the large number of hadronic particles strongly hints to them being composites of more fundamental objects. This idea is supported by the experimental evidence showing that hadrons are "extended" and have an internal structure. This is best seen in deep inelastic scattering where a hadron (typically a proton) is made to collide with a lepton (an electron, muon or neutrino). These processes are called inelastic because the hadron, as the result of the collision, is smashed into a bunch of hadrons. For example,

$$e^- + p \longrightarrow e^- + \text{hadrons}.$$

The incoming particles interact either electromagnetically or through the weak interaction. In either case the interchanged quanta probe the hadron with a resolution given by the inverse of the transferred momenta. The data obtained in these experiments is consistent with the interaction of the probe quanta with pointlike objects inside the hadron. In Sect. 5.3 we will see how the study of these processes provides plenty of useful information about the physical properties of the strong interaction. For the time being it suffices to know that they show that hadrons are made of pointlike objects.

In fact, the spectrum of hadrons can be reproduced by assuming that they are composed of particles with spin $\frac{1}{2}$ and fractional charge, called *quarks*. By simple addition of angular momentum we realize that the distinction between mesons and baryons comes out naturally. The first are bound states of a quark and an antiquark, whereas the second are composed of three quarks. All known hadrons can thus be explained as bound states of six different quarks. The quark types, called *flavors*, are conventionally denoted by the following names

u	up	$q_u = \frac{2}{3}$	d	down	$q_c = -\frac{1}{3}$
c	charm	$q_c = \frac{2}{3}$	s	strange	$q_s = -\frac{1}{3}$
t	top	$q_t = \frac{2}{3}$	b	bottom	$q_t = -\frac{1}{3}$

As a matter of fact, the top quark is too short-lived to give rise to bound states. Nevertheless it can be produced in the high energy collisions of protons, where its existence was verified in 1995 through the observation of its decay channels. One of the most remarkable properties of quarks is that, unlike leptons, they have fractional electric charge. Notice that, however, the charge of the bound state of a quark and an antiquark or of three quarks always results in a state with integer charge.

Many features of the hadronic spectrum can be predicted using the nonrelativistic quark model, where the quarks are taken to be nonrelativistic particles. In this model, the hadron wave function is constructed in terms of the wave functions of the constituent quarks. Thus, some quantum numbers of the hadrons can be obtained by doing "spectroscopy", in a similar fashion as it is done in atomic physics.

To see how this works we consider as an example the lightest hadrons composed only by the u and d quarks. We begin with the mesons for which we have four independent states in flavor space: $|u\overline{u}\rangle$, $|u\overline{d}\rangle$, $|d\overline{u}\rangle$ and $|d\overline{d}\rangle$. They have to be identified with the four lowest lying mesons, the pions $\pi^{\pm}$, π^0 and the η meson. To identify who is who in this case, we begin by looking at the electric charge. This allow us to identify the flavor wave function of the charged pions as

$$|\pi^+\rangle = |u\overline{d}\rangle, \quad |\pi^-\rangle = |d\overline{u}\rangle. \tag{5.12}$$

The wave function of the two neutral mesons π^0 and η, on the other hand, should be orthogonal combinations of the chargeless states $|u\overline{u}\rangle$ and $|d\overline{d}\rangle$. To identify them we need to invoke another quantum number that distinguishes betweeen the two particles. This is isospin. The neutral pion belongs, together with $\pi^{\pm}$, to a isospin triplet with $I_3(\pi^0) = 0$, whereas the η is an isospin singlet.

We have to assign then isospin quantum numbers of the u and d quarks. They are grouped together into an isodoublet transforming under isospin in the fundamental representation of SU(2), that is, $I_3(u) = \frac{1}{2}$ and $I_3(d) = -\frac{1}{2}$. With this choice we see that the flavor wave functions shown in (5.12) have the required isospin, $I_3(\pi^{\pm}) = \pm 1$. As for the third member of the $I = 1$ triplet, we have to decompose the product of two fundamental representations of SU(2) into irreducible representations. Using the rules familiar from the angular momentum algebra in quantum mechanics, we find the wave function of the neutral pion to be

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\overline{u}\rangle - |d\overline{d}\rangle\right). \tag{5.13}$$

Since the pions have zero spin, the total (flavor + spin) wave function is the tensor product of (5.12) and (5.13) with the spin wave function

$$|s = 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right). \tag{5.14}$$

Having studied the mesons we proceed to the baryons, starting with the proton and the neutron. By just looking at the electric charge of these particles we see that their quark composition has to be uud and udd respectively. However, the obvious choice for the proton and neutron wave functions, $|uud\rangle$ and $|udd\rangle$, are not good candidates. The reason is that these states are eigenstate of the third component of the isospin I_3 but not of the total isospin I^2. Indeed, for the case of the proton the states with well defined total isospin are[1]

$$\begin{aligned} |uud\rangle_S &= \frac{1}{\sqrt{6}}\left(|uud\rangle + |udu\rangle - 2|duu\rangle\right), \\ |uud\rangle_A &= \frac{1}{\sqrt{2}}\left(|uud\rangle - |udu\rangle\right). \end{aligned} \tag{5.15}$$

[1] Here we have to remember that the isospin operators acting on the Hilbert space of three particles have the form $I_i = I_i^{(1)} \otimes \mathbf{1} \otimes \mathbf{1} + \mathbf{1} \otimes I_i^{(2)} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{1} \otimes I_i^{(3)}$, where $I_i^{(a)}$ is the isospin operator acting on the Hilbert space of the a-th particle.

Both states have $I = \frac{1}{2},\ I_3 = \frac{1}{2}$. The subscripts indicate that the states are symmetric and antisymmetric with respect to the interchange of the two last states. The proton is in fact a linear combination of these two states. To find the precise one we need to take into account that the total wave function, including the spin degrees of freedom, has to be antisymmetric under the interchange of any two quarks. Taking this into account we have

$$
\begin{aligned}
|p\uparrow\rangle &= \frac{1}{\sqrt{2}}\left(|uud\rangle_S \otimes |\Uparrow\rangle_A + |uud\rangle_A \otimes |\Uparrow\rangle_S\right), \\
|p\downarrow\rangle &= \frac{1}{\sqrt{2}}\left(|uud\rangle_S \otimes |\Downarrow\rangle_A + |uud\rangle_A \otimes |\Downarrow\rangle_S\right).
\end{aligned}
\tag{5.16}
$$

The spin states $|\Uparrow\rangle_{A,S},\ |\Downarrow\rangle_{A,S}$ are eigenstates of the total spin (with $s = \frac{1}{2}$) and its third component ($s_z = \pm\frac{1}{2}$), the subscripts indicating again that the wave functions are symmetric and antisymmetric in the last two states. For example, for the spin-up states we have

$$
\begin{aligned}
|\Uparrow\rangle_S &= \frac{1}{\sqrt{6}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right), \\
|\Uparrow\rangle_A &= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right).
\end{aligned}
\tag{5.17}
$$

A similar analysis can be carried out for the neutron, whose flavor wave function is written in terms of the states $|ddu\rangle_{S,A}$ which have $I = \frac{1}{2},\ I_3 = -\frac{1}{2}$.

Protons and neutrons are not the only hadrons made out of u and d quarks. By simple counting we see that there are $2^3 = 8$ possible baryon states. Keeping in mind that quarks transform in the fundamental representation of the $\mathrm{SU}(2)_I$ isospin group, these states are classified by the irreducible representations contained in the product representation

$$
\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4} \oplus \mathbf{2}_S \oplus \mathbf{2}_A. \tag{5.18}
$$

The subscript in the last two terms indicates that these irreducible representations act on the spaces spanned by $\{|uud\rangle_S, |ddu\rangle_S\}$ and $\{|uud\rangle_A, |ddu\rangle_A\}$ respectively. The states transforming under the **4** are identified with the four Δ resonances: $\Delta^{++}\,(uuu),\ \Delta^{+}\,(uud),\ \Delta^{0}\,(udd)$ and $\Delta^{-}\,(ddd)$. They form an isoquadruplet with $I = \frac{3}{2}$. Notice that although Δ^{+} and Δ^{0} have the same quark composition as the proton and the neutron respectively, they differ in the spin, which is $S = \frac{3}{2}$ for the delta resonances. Their wave functions in flavor and spin spaces can be obtained along the lines showed above for the proton and the neutron.

The hadron spectroscopy described so far can be extended to include hadrons with nonvanishing strangeness. In the context of the quark model these are particles which contain a net number of s quarks. This quark has strangeness $S = -1$ and is an isospin singlet. The $\mathrm{SU}(2)_I$ isospin group is extended to flavor $\mathrm{SU}(3)_f$, where the three quarks u, d and s form a triplet that transforms in the fundamental representation **3**, with antiquarks transforming in the complex conjugate representation $\overline{\mathbf{3}}$. With this we can explain the hadron classification discussed in Sect. 5.1: the group theory identity

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \tag{5.19}$$

means that the lightest mesons (including those with nonvahinish strangeness) come in octets and singlets, whereas baryons are classified in decuplets, octets and singlets according to

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}. \tag{5.20}$$

The quark model gives a rationale for the existence of the approximate flavor symmetries of the strong interaction. The nine pseudoscalar mesons shown in Fig. 5.1 are the states on the right-hand side of the decomposition (5.19). The group theory analysis shows that the quark composition of the kaons is

$$|K^+\rangle = |u\overline{s}\rangle, \quad |K^0\rangle = |d\overline{s}\rangle. \tag{5.21}$$

In addition to the kaons, the multiplet also contains two more particles, η and η', with $I = 0$ and $S = 0$. The identification of the flavor wave function of these states requires a bit of extra work.

On purely group theoretical grounds, there are two possible ways to construct a state with vanishing isospin out of a quark and an antiquark triplet, namely

$$\begin{aligned} |\eta_1\rangle &= \frac{1}{\sqrt{3}}\left(|u\overline{u}\rangle + |d\overline{d}\rangle + |s\overline{s}\rangle\right), \\ |\eta_8\rangle &= \frac{1}{\sqrt{6}}\left(|u\overline{u}\rangle + |d\overline{d}\rangle - 2|s\overline{s}\rangle\right). \end{aligned} \tag{5.22}$$

With the subscript on the left-hand side we have indicated that the states come respectively from the singlet and the octet of $\mathrm{SU}(3)_f$. However, the identification of (5.22) with observed particles has to be done with care. Were $\mathrm{SU}(3)_f$ an exact symmetry of the strong interactions, $|\eta_1\rangle$ and $|\eta_8\rangle$ would be eigenstates of the Hamiltonian of the strong force. But we know that $\mathrm{SU}(3)_f$ is only an approximate symmetry and therefore time evolution mixes these two states. In fact, there are two particles, η and η', with the correct quantum numbers that are a mixture of the states (5.22)

$$\begin{aligned} |\eta\rangle &= \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_1\rangle, \\ |\eta'\rangle &= \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_1\rangle. \end{aligned} \tag{5.23}$$

The pseudoscalar mixing angle θ_P is experimentally found to be $\theta_P \simeq -17°$.

We have encountered a general phenomenon called *mixing*. This happens whenever the propagation eigenstates (i.e., states with a well-defined mass) do not coincide with other quantum number eigenstates (in this case the flavor quantum number), and it is at the origin of many interesting phenomena in particle physics.

Looking back at the lowest lying mesons shown in Fig. 5.1, we immediately notice the rather small mass difference between the three pions. In the context of the quark model this experimental fact can be interpreted as indicating that the masses of the u and d quarks should be very similar. Using the same argument, the mass difference between pions and kaons hints to a larger mass for the strange quark, $m_s > m_u \simeq m_d$. This conclusion, however, has to be taken with a grain of salt: as quarks are confined inside the hadrons talking about their masses is a very delicate issue that we will elaborate upon in Chap. 10 (see Sect. 10.2).

5.3 Quantum Chromodynamics

The failure to detect isolated quarks indicates that some physical mechanism should be responsible for their confinement inside hadrons. This property of the quark interaction contrasts very much with the picture of the quark–quark interaction that emerges from the deep inelastic scattering experiments already discussed in the previous section. One of the surprising conclusions following from the study of these collisions is that the data extracted is compatible with the quarks inside the hadrons behaving as nearly free particles. More precisely, the results can be reproduced assuming that while the lepton interacts with the nucleon constituents, to a very good approximation these constituents can be considered as not interacting with each other.

This means that a successful theory of the strong interaction should account for these two curious features of the quark interaction force: it should grow at large distances in order to prevent quarks from being "ionized" out of the hadrons, while at the same time it should be negligible when the quarks are within a distance well below the nucleon radius, i.e., approximately 10^{-15} m.

The very implementation of the quark model leads to the realization that quarks have an extra quantum number beyond flavor and spin. This is most easily seen in the case of the Δ^{++}. As we discussed above, this resonance is made out of three u quarks and has total spin $s = \frac{3}{2}$. Then, its wave function with $s_z = \frac{3}{2}$ has to be

$$|\Delta^{++};\ s_z = \tfrac{3}{2}\rangle = |uuu\rangle \otimes |\uparrow\uparrow\uparrow\rangle \equiv |u\uparrow, u\uparrow, u\uparrow\rangle. \tag{5.24}$$

As it stands, the wave function is symmetric under the interchange of any of the identical three quarks. This is indeed a problem, the quarks are fermions and therefore their total wave function has to be completely antisymmetric. One way to avoid the problem is if each quark has an extra index taking three values, u_i with $i = 1, 2, 3$. Then, the wave function

$$|\Delta^{++};\ s_z = \tfrac{3}{2}\rangle = \frac{1}{\sqrt{3!}}\varepsilon_{ijk}|u_i\uparrow, u_j\uparrow, u_k\uparrow\rangle. \tag{5.25}$$

is antisymmetric under the interchange of any of the constituent quarks, as required by their fermionic statistics. This new quantum number is called *color*. The conclusion we have reached is that each quark flavor comes in tree different states labeled by this new index.

The color quantum number is the key to the formulation of a theory of strong interaction able to account for the phenomenology. This theory is called Quantum Chromodynamics (QCD) and is a nonabelian gauge theory based on the gauge group SU(3). This group acts on the color index of the quark spinor field as

$$Q_i^f \longrightarrow U(g)_{ij} Q_j^f, \quad \text{with} \quad g \in \text{SU}(3), \tag{5.26}$$

where $f = 1, \ldots, 6$ runs over the six quark flavors and $U(g)$ is an element of the fundamental representation of the gauge group. The Lagrangian of the theory can be constructed using what we learned in Sect. 4.4

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{f=1}^{6} \overline{Q}^f \left(i \not{D} - m_f \right) Q^f. \tag{5.27}$$

To keep the notation simple we have omitted the color indices. The nonabelian gauge field strength $F^a_{\mu\nu}$ (with $a = 1, \ldots, 8$) and the covariant derivative D_μ are given in terms of the SU(3) gauge field A^a_μ by (4.52) and (4.46) respectively. In the latter case the generators T^a_R are the Gell–Mann matrices listed in Eq. (B.16).

The QCD Lagrangian (5.27) leads to a theory where the interactions between quarks have the features required to explain both quark confinement and the deep inelastic scattering experiments. Unfortunately, at this point we cannot be more explicit. We still have to learn how to quantize an interacting field theory such as QCD. The most we can say now is that quantum effects result in an effective force between quarks that grows at large distances, whereas it tends to zero at short distances. The clarification of this statement will have to wait until Chap. 8.

From the point of view of the quark model it seems rather arbitrary that hadrons result form bound states of either a quark and an antiquark or of three quarks. Why not, say, having hadrons made of two quarks? QCD offers an explanation of this fact. What happens is that hadrons are colorless objects, i.e., they transform as singlets under SU(3). Then, since quarks (resp. antiquarks) transform under the fundamental $\mathbf{3}_c$ (resp. antifundamental $\overline{\mathbf{3}}_c$) of SU(3), it is impossible to produce a colorless object out of two quarks

$$\mathbf{3}_c \otimes \mathbf{3}_c = \mathbf{6}_c \oplus \overline{\mathbf{3}}_c. \tag{5.28}$$

Here, to avoid confusion with the notation of previous sections, we have introduced a subscript to indicate that we are referring to irreducible representations of color SU(3). On the other hand, using the identities

$$\begin{aligned} \mathbf{3}_c \otimes \overline{\mathbf{3}}_c &= \mathbf{8}_c \oplus \mathbf{1}_c, \\ \mathbf{3}_c \otimes \mathbf{3}_c \otimes \mathbf{3}_c &= \mathbf{10}_c \oplus \mathbf{8}_c \oplus \mathbf{8}_c \oplus \mathbf{1}_c, \end{aligned} \tag{5.29}$$

we find that there is no problem in constructing colorless mesons and baryons. One example is the Δ^{++} wave function shown in equation (5.25). Notice that on purely group theoretical grounds there are ways other than (5.29) of producing color singlets. For example, the product of four fundamental and one antifundamental representations of SU(3) contains several singlets. These exotic baryons, however, have not been observed experimentally to date.

QCD includes, besides the six quarks, eight gauge fields mediating the strong interaction, one for each generator of SU(3). These intermediate vector bosons are the *gluons*. It is rather counterintuitive that a short-ranged force such as the strong interaction is mediated by massless particles. However, we have to recall that the strong nuclear force that we referred to in Sect. 5.1 is a force between colorless hadrons. The nuclear force between nucleons emerges as a residual interaction very much in the same fashion as the van der Waals force does in molecular physics between electrically neutral atoms, where the Coulomb force produces a residual potential falling off as r^{-6}. The problem is that in the case of QCD the complexity of the theory makes it very difficult to give a concrete form to this general idea. In spite of recent progresses [10], there is still no precise understanding of how nuclear effective potentials emerge from the gluon-mediated QCD interaction between quarks.

The approximate symmetries of the strong interaction are (approximate) global symmetries of the QCD Lagrangian. Focusing on the two lightest quark flavors, u and d, the fermionic part of this Lagrangian can be written as

$$\begin{aligned}\mathscr{L} &= (\overline{u}, \overline{d}) \begin{pmatrix} i\not{D} - \frac{m_u+m_d}{2} & 0 \\ 0 & i\not{D} - \frac{m_u+m_d}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \\ &\quad - \frac{m_u - m_d}{2} (\overline{u}, \overline{d}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}. \end{aligned} \tag{5.30}$$

In the limit when $m_u \simeq m_d$, the second term can be ignored and the Lagrangian is approximately invariant under the global $\mathrm{SU}(2)_I$ isospin transformations

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow M \begin{pmatrix} u \\ d \end{pmatrix}, \tag{5.31}$$

where M is a SU(2) matrix. Acting on the flavor wave function of the nucleons and the pions this gives the usual isospin transformation. In a similar fashion, $\mathrm{SU}(3)_f$ can be seen to emerge from the approximate global symmetry of the QCD Lagrangian in the limit in which the mass differences between the masses of the *u*, *d* and *s* are neglected. As in (5.31), these transformation acts linearly on the quark triplet.

5.4 The Electroweak Theory

At low energies weak processes such as those described in Sect. 5.1 can be phenomenologically described by interaction terms of the form

$$\mathscr{L}_{\text{int}} = \frac{G_F}{\sqrt{2}} J^\mu(x) J_\mu(x)^\dagger. \tag{5.32}$$

The dimensionful coupling constant G_F, called the Fermi constant, has the value

$$G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}. \tag{5.33}$$

The current in the Lagrangian (5.32) is split into hadronic and leptonic contributions, $J_\mu(x) = J_\mu^{(h)}(x) + J_\mu^{(\ell)}(x)$. As hadrons are composite objects, the hadronic current has to be expressed in terms of form factors. The leptonic current, on the other hand, is written in terms of the lepton fields as

$$J_\mu^{(\ell)} = \overline{\nu}_e(x)\gamma_\mu(1-\gamma_5)e(x) + \cdots \tag{5.34}$$

The dots stand for other fields. Currents like (5.34) are known as charged currents because the two fields forming it have electric charges that differ by one unit. We also have contributions to the Lagrangian coming from neutral currents made of like-charge leptons. It is important that all the terms appearing in the lepton current have the so-called V–A form including the chirality-sensitive factor $1-\gamma_5$. This is imposed by the fact that weak interactions maximally violate parity.

The interaction Lagrangian (5.32) describes weak interaction processes very successfully at low energies. However, for various reasons the theory runs into trouble when the energy gets close to the characteristic energy scale $1/\sqrt{G_F}$.

A way to deal with these problems is to give up the "contact" interaction (5.32) in favor of an intermediate boson, in analogy with QED or QCD. The only problem is that the intermediate boson now has to be massive if we want to recover the effective current–current interaction at low energies. This we can illustrate with a simple toy model of a massive abelian gauge field coupled to a real current J_μ

$$\mathscr{L}_{\text{int}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu + gJ_\mu A^\mu, \tag{5.35}$$

with g a dimensionless coupling constant. At energies below the mass m, the kinetic term of the gauge field is subleading with respect to the mass term. Solving the equations of motion for A_μ in this limit, and substituting the result in (5.35), we arrive at the low energy "contact" interaction

$$\mathscr{L}_{\text{int}} = \frac{g^2}{2m^2}J_\mu J^\mu. \tag{5.36}$$

Extrapolation of this result to the weak interaction leads to the conclusion that both charged and neutral weak currents are mediated by *massive* gauge bosons.[2]

The construction of a theory of weak interactions based on the interchange of vector bosons leads in fact to the unification of the weak and electromagnetic interaction based on a gauge theory with gauge group $\text{SU}(2) \times \text{U}(1)_Y$. There are four generators: two charged and one neutral bosons, responsible respectively for charged

[2] At the end of this chapter we will see that this is itself not free of problems. How these are overcome will be explained in Chap. 10.

and neutral weak currents, and the photon. As group generators we use $\{T^{\pm}, T^3, Y\}$. The first three are the ladder generators (B.10) of the SU(2) factor, called the weak isospin. In addition, the so-called weak hypercharge is the generator of the $U(1)_Y$ factor where the subscript is intended to avoid confusion with the electromagnetic U(1) gauge group. It is important to keep in mind that, despite their similar names, the weak isospin and hypercharge are radically different from the strong interaction namesakes introduced in Sect. 5.1. This notwithstanding, the value of the weak hypercharge of the different fields will be fixed in such a way that the analog of the Gell-Mann–Nishijima relation is satisfied

$$Q = T^3 + Y. \tag{5.37}$$

Once the gauge group is chosen, we exhibit the vector bosons of the theory. For this we introduce the Lie algebra valued gauge fields

$$\mathbf{W}_\mu = W_\mu^+ T^- + W_\mu^- T^+ + W_\mu^3 T^3, \quad \mathbf{B}_\mu = B_\mu Y. \tag{5.38}$$

Using the Gell-Mann–Nishijima formula (5.37) and the commutation relations of the SU(2) algebra shown in Eq. (B.10), we have

$$[Q, T^{\pm}] = \pm T^{\pm}, \quad [Q, T^3] = [Q, Y] = 0. \tag{5.39}$$

This means that the gauge fields $W_\mu^{\pm}$ are electrically charged, while W_μ^3 and B_μ are neutral fields.

We still have to identify the electromagnetic U(1) factor in the gauge group. Since the photon has no electric charge, the Maxwell gauge field A_μ must be a combination of the two neutral gauge bosons, W_μ^3 and B_μ. We define a new pair of neutral gauge fields (A_μ, Z_μ) by

$$\begin{aligned} A_\mu &= B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w, \\ Z_\mu &= -B_\mu \sin\theta_w + W_\mu^3 \cos\theta_w, \end{aligned} \tag{5.40}$$

where the transformation is parametrized by an angle θ_w called the weak mixing angle. The form of the linear combination is not arbitrary: it is the most general one guaranteeing that the new gauge fields A_μ and Z_μ have canonical kinetic terms in the action. The field A_μ is now identified with the electromagnetic potential. In short, what we have done is to parametrize our ignorance of how QED is embedded in the electroweak gauge theory by introducing the weak mixing angle. Its value will have to be determined experimentally. The fact that it is nonzero indicates that the weak and electromagnetic interactions are mixed.

This concludes our study of gauge bosons. Next we fix the representation of the matter fields, i.e. how matter fields transform under the gauge group. Here the experiment is our guiding principle. For example, we know that charged weak currents couple left-handed leptons to their corresponding left-handed neutrinos. Since these interactions are mediated by the charged gauge fields $\mathbf{W}_\mu^{\pm} = W_\mu^{\pm} T^{\mp}$, we are led to include both fields in a SU(2) doublet

Table 5.1 Transformation properties of the lepton fields under the electroweak gauge group $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$

Leptons i (generation)	1	2	3	T^3	Y
$\mathbf{L}^i$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$
ℓ_R^i	e_R^-	μ_R^-	τ_R^-	0	-1

In the last two columns on the right the values of the weak isospin and the hypercharge are shown for the different fields

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L. \tag{5.41}$$

In addition, we also know that the right-handed component of the electron does not take part in interactions mediated by weak charged currents. This indicates that they should be taken to be singlets under the SU(2) factor.

The Gell-Mann–Nishijima formula can be used now to fix the weak hypercharge of the leptons, i.e. their transformations under the $\mathrm{U}(1)_Y$ factor of the gauge group. Using that the left-handed isodoublets (5.41) transform in the fundamental ($s = \frac{1}{2}$) representation of SU(2) where $T^3 = \frac{1}{2}\sigma_3$, we have

$$Y(\nu_\ell) = -\frac{1}{2}, \quad Y(\ell) = -\frac{1}{2}, \tag{5.42}$$

where ℓ denotes e^-, μ^- or τ^-. For right-handed leptons, being singlets under SU(2), we have $T^3 = 0$ and therefore

$$Y(\ell_R) = -1. \tag{5.43}$$

We summarize the results in Table 5.1. We have introduced the compact notation $\mathbf{L}^i$ and ℓ^i to denote respectively the left-handed isodoublets and right-handed singlets.[3]

In all this discussion we have ignored the possibility of having a right-handed component for the neutrino. Being a SU(2) singlet and having zero charge, this particle would have also vanishing hypercharge. Thus, such a particle would be a singlet under all gauge groups of the standard model. This is called a sterile neutrino. It would only interact gravitationally or via some yet unknown interaction making their detection extremely difficult.

In the case of quarks we proceed along similar lines. We look first at the charged weak current that couples protons with neutrons. Taking into account the quark content of these particles, we see how this current in fact couples the u and d quark,

[3] It should be stressed that the quantum numbers appearing in Tables 5.1 and 5.2 summarize a great deal of experimental data resulting from decades of work.

Table 5.2 Transformation properties of the quarks in the electroweak sector of the standard model

Quarks i (generation)	1	2	3	T^3	Y
$\mathbf{Q}^i$	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$
U_R^i	u_R	c_R	t_R	0	$\frac{2}{3}$
D_R^i	d_R	s_R	b_R	0	$-\frac{1}{3}$

suggesting that they form an isodoublet. This structure is repeated for the three quark generations[4]

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L. \tag{5.44}$$

As with leptons, the right-handed quark components are singlet under SU(2). The hypercharges of the different quarks are shown in Table 5.2, where the notation $\mathbf{Q}^i$, U_L^i and D_L^i is respectively introduced to denote the left-handed doublets and right-handed SU(2) singlets.

The next task is to determine the couplings of the different matter fields (leptons and quarks) to the intermediate vector bosons. From Eq. (4.46), the covariant derivative acting on the matter fields in a representation R of the gauge field is of the form

$$\begin{aligned} D_\mu &= \partial_\mu - ig\mathbf{W}_\mu - ig'\mathbf{B}_\mu \\ &= \partial_\mu - igW_\mu^+ T_R^- - igW_\mu^- T_R^+ - igW_\mu^3 T_R^3 - ig' B_\mu Y_R, \end{aligned} \tag{5.45}$$

It is important to notice that we have introduced two distinct coupling constants g and g' associated with the two factors of the gauge group, SU(2) and $\mathrm{U}(1)_Y$. The reason is that gauge transformations do not mix the gauge field $\mathbf{W}_\mu$ with $\mathbf{B}_\mu$ and therefore gauge invariance does not require the coupling constants to be related. Applying Eq. (5.40), we express D_μ in terms of the gauge fields A_μ and Z_μ

$$\begin{aligned} D_\mu = \partial_\mu - igW_\mu^+ T_R^- - igW_\mu^- T_R^+ - iA_\mu(g\sin\theta_w T_R^3 + g'\cos\theta_w Y_R) \\ - iZ_\mu(gT_R^3\cos\theta_w - g' Y_R \sin\theta_w). \end{aligned} \tag{5.46}$$

We have identified A_μ with the electromagnetic gauge field. Thus, the third term in the covariant derivative gives the coupling of the matter field to electromagnetism

4 A word of warning is in order here. Although denoted by the same letter, the fields in the quark doublets are not necessarily the same ones that appear as the hadron constituents in the quark model. The two are related by a linear combination. This for the time being cryptic remark will find clarification in Chap. 10 (see page 197).

and, as a consequence, it should be of the form $-ieQA_\mu$, with Q the charge operator. With this in mind and using once more the Gell-Mann–Nishijima relation (5.37) we conclude that the electric charge e is related to the coupling constants g and g' by

$$e = g\sin\theta_w = g'\cos\theta_w. \tag{5.47}$$

This equation gives the physical interpretation of the weak mixing angle. It measures the ratio between the two independent coupling constants in the electroweak sector of the standard model

$$\tan\theta_w = \frac{g'}{g}. \tag{5.48}$$

Precise calculations with the standard model require writing a Lagrangian from where to start a quantization of the theory. A first part contains the dynamics of gauge fields and can be constructed using what we learned in Chap. 4 about nonabelian gauge fields

$$\begin{aligned}\mathscr{L}_{\text{gauge}} = &-\frac{1}{2}W^{+}_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{ig}{2}\cos\theta_w W^{+}_{\mu}W^{-}_{\nu}Z^{\mu\nu} \\ &+ \frac{ie}{2}W^{+}_{\mu}W^{-}_{\nu}F^{\mu\nu} - \frac{g^2}{2}\Big[(W^{+}_{\mu}W^{+\mu})(W^{-}_{\mu}W^{-\mu}) - (W^{+}_{\mu}W^{-\mu})^2\Big]\end{aligned} \tag{5.49}$$

where we have introduced the notation

$$\begin{aligned}W^{\pm}_{\mu\nu} &= \partial_\mu W^{\pm}_{\nu} - \partial_\nu W^{\pm}_{\mu} \mp ie\left(W^{\pm}_{\mu}A_\nu - W^{\pm}_{\nu}A_\mu\right) \mp ig\cos\theta_w\left(W^{\pm}_{\mu}Z_\nu - W^{\pm}_{\nu}Z_\mu\right) \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu,\end{aligned} \tag{5.50}$$

while $F_{\mu\nu}$ is the familiar field strength of the Maxwell field A_μ. The gauge part of the Lagrangian is a bit cumbersome because we have chosen to write it in terms of the fields A_μ and Z_μ. The $SU(2) \times U(1)_Y$ gauge symmetry is not obvious in this expression, but it has the advantage of making the invariance under the gauge transformations of electromagnetism manifest. We have also eliminated the coupling constant g' in favor of g and the weak mixing angle θ_w. Moreover, whenever the combination $g\sin\theta_w$ appeared we further used (5.47) and wrote the electric charge e.

For the matter fields we can write the following gauge invariant Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{matter}} = \sum_{i=1}^{3}\Big(& i\overline{\mathbf{L}}^j \not{D}\mathbf{L}^j + i\overline{\ell}^j_R\not{D}\ell^j_R \\ &+ i\overline{\mathbf{Q}}^j\not{D}\mathbf{Q}^j + i\overline{U}^j_R\not{D}U^j_R + i\overline{D}^j_R\not{D}D^j_R\Big).\end{aligned} \tag{5.51}$$

The covariant derivatives appearing in this Lagrangian can be written explicitly from (5.46) taking into account the representation for the different matter fields.

A glimpse to the Lagrangians $\mathscr{L}_{\text{gauge}}$ and $\mathscr{L}_{\text{matter}}$ and to the covariant derivative (5.46) shows the coupling between the standard model particles. As right-handed fields are singlets under SU(2), the $W^{\pm}$ boson only couples to the left-handed doublets. Using the expression of the $T^{\pm}$ generators in the fundamental representation of SU(2), we find that the terms in the standard model Lagrangian coupling the $W^{\pm}$ boson to the leptons take the form

$$gW^{+}_{\mu}\overline{\nu}_{\ell}\gamma^{\mu}\ell_{L}, \quad gW^{-}_{\mu}\overline{\ell}_{L}\gamma^{\mu}\nu_{\ell}. \tag{5.52}$$

Notice that the strength of these couplings is given by g.

From the covariant derivative (5.46), we see that the Z^0 couples to a combination of the two generators of the Cartan subalgebra of $\mathrm{SU}(2)\times\mathrm{U}(1)_Y$, namely T^3 and Y. Since they can be simultaneously diagonalized, this gauge boson couples to fermions of the same kind. In the case of the leptons the couplings are

$$\frac{g}{2\cos\theta_w}Z_{\mu}\overline{\nu}_{\ell}\gamma^{\mu}\nu_{\ell}, \quad \frac{g}{\cos\theta_w}\left(-\frac{1}{2}+\sin^2\theta_w\right)Z_{\mu}\overline{\ell}_{L}\gamma^{\mu}\ell_{L}, \tag{5.53}$$

and

$$\frac{g\sin^2\theta_w}{\cos\theta_w}Z_{\mu}\overline{\ell}_{R}\gamma^{\mu}\ell_{R}. \tag{5.54}$$

Unlike the $W^{\pm}$, the Z^0 boson couples to the right-handed components through the hypercharge.

The analysis can be repeated for quarks. The result is that once again right-handed quarks only couple to the Z^0, while the $W^{\pm}$ couple the upper and lower components of the left-handed doublets. Both left- and right-handed quarks, being charged, couple also to the electromagnetic field A_{μ}. The derivation of the form of these terms as well as the corresponding couplings is left as an exercise. Finally, the couplings between the gauge bosons can be read from the gauge Lagrangian (5.49).

5.5 Closing Remarks: Particle Masses in the Standard Model

The alert reader surely has noticed that in our discussion of the electroweak theory we have been conspicuously silent about particle masses. That particles such as the electron or the muon have nonvanishing masses is a well known experimental fact. Moreover, we have seen that phenomena such as beta decay cannot be explained by the model unless a mass is assumed for the intermediate vector bosons.

At the time when the standard model was developed in the 1960s, QED was the archetype of a successful quantum field theory: physical processes could be accurately computed at arbitrary high energies in terms of a small number of experimentally fixed parameters. From this point of view, there were fundamental obstacles to giving masses to the fields in the electroweak theory described in the previous section.

Adding explicit mass terms for fermions and gauge bosons to the Lagrangian breaks gauge invariance: in the case of the fermions a Dirac mass term mixes fields transforming in different representations of $SU(2) \times U(1)_Y$, whereas a term $\mathrm{Tr}(A_\mu A^\mu)$ is obviously not invariant under the gauge transformation of the vector field (4.42).

Giving up gauge invariance means destroying the possibility of building a theory of electroweak interactions valid to all energies. In more precise terms, gauge invariance is crucial for the *renormalizability* of the theory, a property whose physical relevance will be discussed in Chap. 8. Moreover, gauge invariance restricts the ways the standard model fields couple among themselves.

Note that the conflict between fermion masses and gauge invariance does not appear in the pure QCD sector. The reason is that the action of SU(3) is vector-like, i.e., the same for left- and right-handed quarks. Therefore a Dirac mass term is gauge invariant, and it can be included in the Lagrangian (5.27) without endangering desirable properties of the theory.

There is nevertheless a way of constructing massive intermediate gauge bosons and fermions in a manner compatible with $SU(2) \times U(1)_Y$ gauge invariance. It consists of generating the mass terms at low energies rather than putting them by hand in the Lagrangian, so gauge invariance is not broken, just hidden. This is achieved through the implementation of the Brout-Englert-Higgs mechanism to be presented in Chap. 7.

To conclude we must say that, as a matter of fact, there is nothing fundamentally wrong with adding explicit mass terms to the standard model Lagrangian, so long as we are only interested in describing the physics at energies *below* the mass scales appearing in the Lagrangian. We will elaborate on this statement in Chaps. 10 and 12, once we learn more about the quantum properties of interacting field theories.

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