QCD@LHC for beginners

Lesson 4.5

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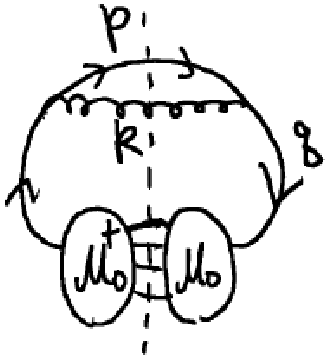
Outline

- Lesson 4
  - Around "IR divergence"
    - IR divergence in QCD/KLN theorem
    - Factorization
    - DGLAP Equation/PDF

GOAL : Understanding
(1) IR divergence structure.
(2) what is PDF.

IR divergence/DIGLAP Eq.

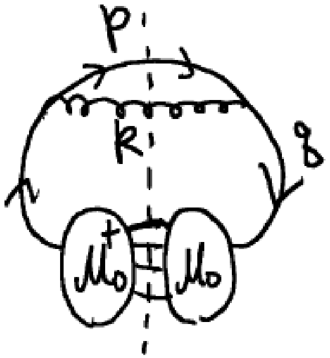


$$= g^2 Tr\left[\bar{u}(p)\not{\epsilon}^*(k)\frac{\sum u(q)\bar{u}(q)}{q^2}|\tilde{\mathcal{M}}_0|^2\frac{\sum u(q)\bar{u}(q)}{q^2}\not{\epsilon}(k)u(p)\right]$$

q=p+k

IR divergence/DIGLAP Eq.

$|M_0|^2$



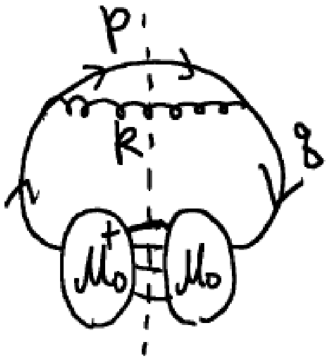
$$= g^2 Tr\left[\bar u(p)\not\epsilon^*(k)\frac{\sum u(q)\bar u(q)}{q^2}|\tilde{M}_0|^2\frac{\sum u(q)\bar u(q)}{q^2}\not\epsilon(k)u(p)\right]$$

$$= |M_0|^2\frac{g^2}{(q^2)^2}\frac{1}{2}\sum_\lambda Tr[\not p\not\epsilon_\lambda(k)\not q\not\epsilon_\lambda(k)]$$

$q=p+k$

IR divergence/DIGLAP Eq.

$$|M_0|^2$$



$$= g^2 Tr\left[\bar{u}(p)\not\epsilon^*(k)\frac{\sum u(q)\bar{u}(q)}{q^2}|\tilde{\mathcal{M}}_0|^2\frac{\sum u(q)\bar{u}(q)}{q^2}\not\epsilon(k)u(p)\right]$$

$$= |M_0|^2\frac{g^2}{(q^2)^2}\frac{1}{2}\sum_\lambda Tr[\not p\not\epsilon_\lambda(k)\not q\not\epsilon_\lambda(k)] \equiv R_I$$

$q=p+k$

$$\sum_\lambda \varepsilon^\mu_\lambda(k)\varepsilon^\nu_\lambda(k) = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n}$$

$$R_I = 2g_s^2\frac{k_T}{x(1-x)}(P(x) + (1-x)\epsilon_{IR})$$

$$P(x) = \frac{1+x^2}{1-x}$$

IR divergence/DIGLAP Eq.

Factorization: Coll. Approx.

Matrix Element

$$\left|\mathcal{M}^{(d)}_{N+1}\right|^2 = \left|\mathcal{M}^{(4)}_{N}\left(q \to \sum_{i=1}^{N} q_i\right)\right|^2 \frac{16\pi}{s\mu^{2\varepsilon_{IR}}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x) \frac{1}{k_T^2}\left(\frac{1-x}{x}\right)$$

$$q^\mu = p_1^\mu + p_2^\mu - k^\mu,$$

Phase Space

$$d\Phi^{(d)}_{N+1} = d\Phi^{(4)}_{N}\left(q \to \sum_{i=1}^{N} q_i\right) \times \frac{1}{16\pi^2\Gamma(1+\varepsilon_{IR})}\left(\frac{k_T^2}{4\pi x^2}\right)^{\varepsilon_{IR}} \frac{1}{1-x} dx\, dk_T^2$$

IR divergence/DIGLAP Eq.

Factorization: Coll. Approx.

Splitting functions:

$$P_{qq}^{(0)}(z) = C_F\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right] ,$$

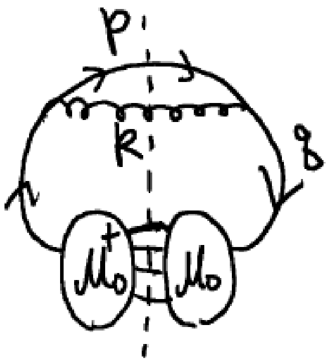
$$P_{qg}^{(0)}(z) = T_F[z^2+(1-z)^2] ,$$

$$P_{gq}^{(0)}(z) = C_F\frac{1+(1-z)^2}{z} ,$$

$$P_{gg}^{(0)}(z) = 2C_A\left[\frac{z}{(1-z)_+}-\frac{1-z}{z}+z(1-z)\right]+2\pi\beta_0\delta(1-z) ,$$

IR divergence/DIGLAP Eq.

Factorization: Coll. Approx.   Cross Section



$$= \frac{1}{(2p_1^0)(2p_2^0)v_{rel}} \int_{\Omega_{full}} d\Phi^{(d)}_{N+1} \left| \mathcal{M}^{(d)}_{N+1} \right|^2 ,$$

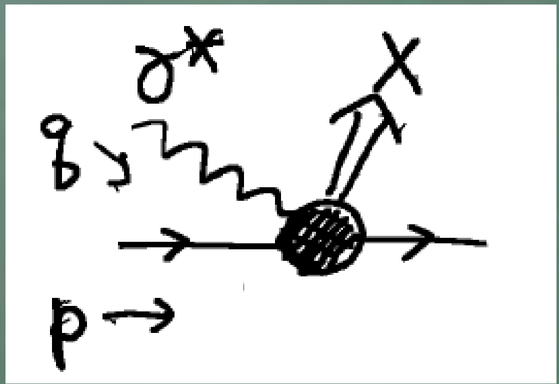
$$= \left(\frac{s}{4\pi\mu^2} \right)^{\varepsilon_{IR}} \frac{B(\varepsilon_{IR}, \varepsilon_{IR})}{2\Gamma(1+\varepsilon_{IR})} f_c \frac{\alpha_s}{2\pi} \int_0^1 dx \sigma_0(xs) \mathrm{P}(x) \left(\frac{1-x}{x} \right)^{2\varepsilon_{IR}}$$

$$\frac{e^{-\gamma\epsilon} P(x)\, \mathrm{B}(\epsilon, \epsilon) \left(\frac{s}{\mu^2} \right)^{\epsilon}}{2\,\Gamma(\epsilon+1)}$$

$$\frac{P(x)}{\epsilon} + P(x) \log\left(\frac{s}{\mu^2} \right) + O(\epsilon^1)$$

IR divergence/DIGLAP Eq.

Deep Inelastic Cross Section:



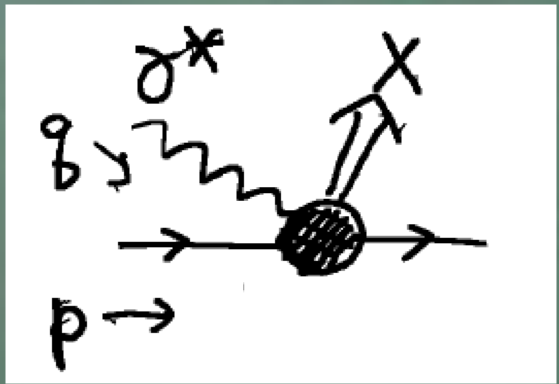
$= \sigma_{DIS}(p^2, Q^2 = -q^2)$

$$\sigma_{DIS}(p^2, Q^2) = 1 + ag^2 \ln\left(\frac{Q^2}{p^2}\right) + \cdots$$
$$= 1 + ag^2\left(\ln\left(\frac{\mu_F^2}{p^2}\right) + \ln\left(\frac{Q^2}{\mu_F^2}\right)\right) + \cdots$$
$$= \left[1 + ag^2 \ln\left(\frac{\mu_F^2}{p^2}\right) + \cdots\right]\left[1 + ag^2 \ln\left(\frac{Q^2}{\mu_F^2}\right) + \cdots\right]$$

PDF   Hard Part

IR divergence/DIGLAP Eq.

Deep Inelastic Cross Section:



$= \sigma_{DIS}(p^2, Q^2 = -q^2)$

Cross section must be scale independent: $\frac{d\sigma_{DIS}}{d \ln \mu_F^2} = 0$

$$\sigma_{DIS}(p, Q^2) = \int dx f(x, \mu_F^2) \sigma_H(x \cdot s, \mu_F^2)$$

$$\frac{d\sigma_{DIS}}{d \ln \mu_F^2} = \int dx \frac{df(x, \mu_F^2)}{d \ln \mu_F^2} \sigma_H(x \cdot s, \mu_F^2) + \int dx f(x, \mu_F^2) \frac{d\sigma_H(x \cdot s, \mu_F^2)}{d \ln \mu_F^2}$$

IR divergence/DIGLAP Eq.

Deep Inelastic Cross Section:

$$\frac{d\sigma_{DIS}}{d\ln\mu_F^2} = \int dx \frac{df(x,\mu_F^2)}{d\ln\mu_F^2}\sigma_H(x\cdot s,\mu_F^2) + \int dx f(x,\mu_F^2)\frac{d\sigma_H(x\cdot s,\mu_F^2)}{d\ln\mu_F^2}$$

$$\sigma_H \propto \frac{\alpha_s}{2\pi}\left(\frac{P(x)}{\epsilon} + P(x)\ln\left(\frac{Q^2}{\mu_F^2}\right) + O(\epsilon)\right)$$

$$\frac{d\sigma_H(x\cdot s,\mu_F^2)}{d\ln\mu_F^2} = -\frac{\alpha_s}{2\pi}P(x)$$

IR divergence/DIGLAP Eq.

Deep Inelastic Cross Section:

$$\frac{d\sigma_{DIS}}{d\ln\mu_F^2} = \int dx \frac{df(x,\mu_F^2)}{d\ln\mu_F^2}\sigma_H(x\cdot s,\mu_F^2) + \int dx f(x,\mu_F^2)\frac{d\sigma_H(x\cdot s,\mu_F^2)}{d\ln\mu_F^2}$$

$$\sigma_H \propto \frac{\alpha_s}{2\pi}\left(\frac{P(x)}{\epsilon} + P(x)\ln\left(\frac{Q\,2}{\mu_F^2}\right) + O(\epsilon)\right)$$

$$\frac{d\sigma_{DIS}}{d\ln\mu_F^2} = 0$$

$$\frac{d\sigma_H(x\cdot s,\mu_F^2)}{d\ln\mu_F^2} = -\frac{\alpha_s}{2\pi}P(x)$$

$$\frac{df(x,\mu_F^2)}{d\ln\mu_F^2} = \frac{\alpha_s(\mu_F)}{2\pi}\int\frac{dz}{z}P(z)f(\frac{x}{z},\mu_F^2)$$

IR divergence/DGLAP Equation

DGLAP Equation

Splitting function ← pQCD

$$\frac{dD(x, Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y} P_+(x/y) D(y, Q^2)$$

$$D(x, Q^2) = \Pi(Q^2, Q_s^2) D(x, Q_s^2) + \frac{\alpha}{2\pi}\int_{Q_s^2}^{Q^2} \frac{dK^2}{K^2} \Pi(Q^2, K^2) \int_x^{1-\epsilon} \frac{dy}{y} P(y) D(x/y, K^2)$$

$$\Pi(Q^2, Q'^2) = \exp\left(-\frac{\alpha}{2\pi}\int_{Q'^2}^{Q^2} \frac{dK^2}{K^2} \int_0^{1-\epsilon} dx P(x)\right)$$

Sudakov Factor

non-branch provability

IR divergence/DGLAP Equation

Initial parton distribution @ Low Q^2

PS(MC)
or
PDF(analy.)

Q2 evolution

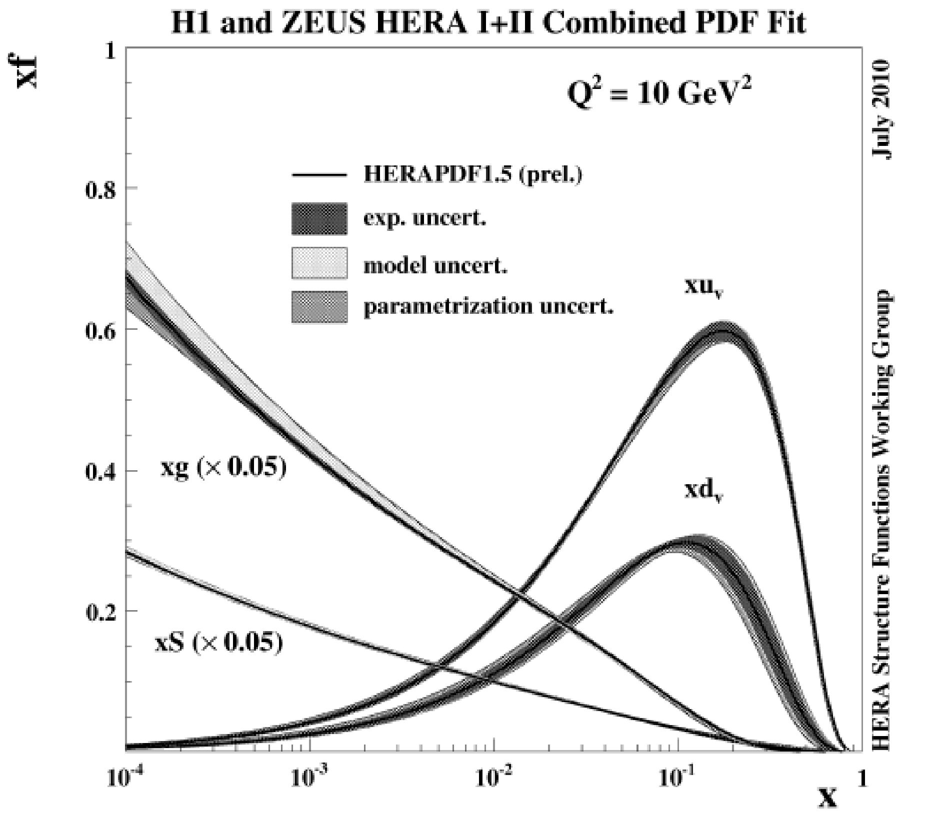
DGLAP Eq.

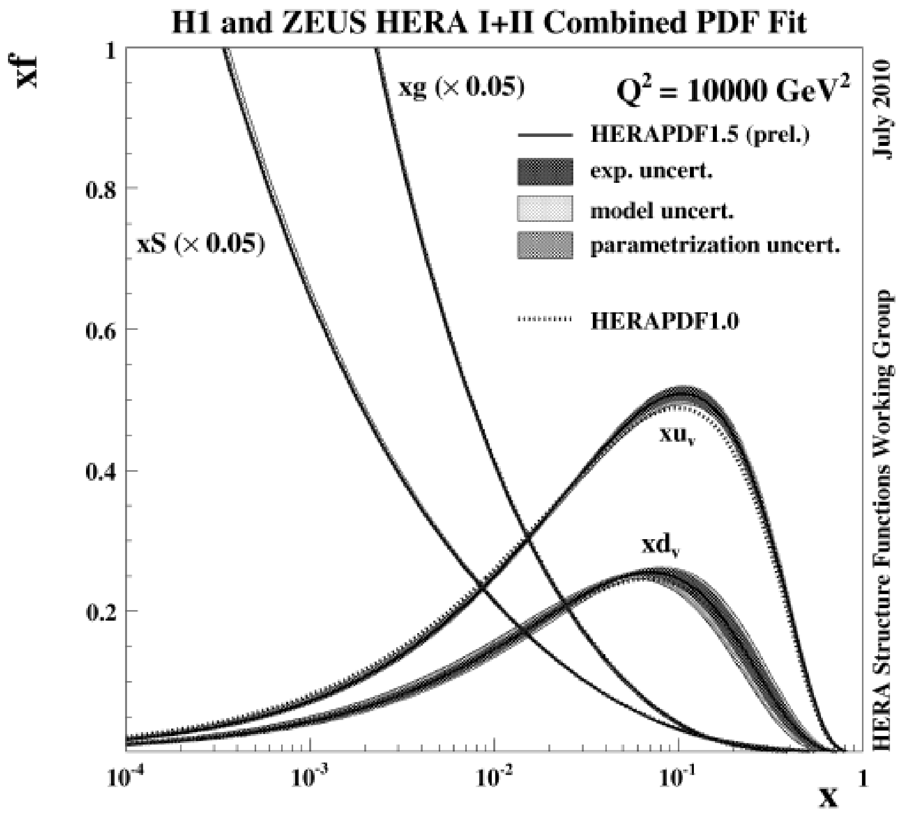
fitting

Ex. data

Parton distribution @ High Q^2

IR divergence/DGLAP Equation





IR divergence/DGLAP Equation

- PDF:
  - Initial distribution: data fitting
  - Q^2 evolution: Analytic solution of DGLAP Eq.
  - No kinematical information

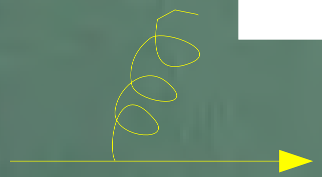
- PS:
  - Initial distribution: from PDF
  - Q^2 evolution: MC method to solve DGLAP Eq.
  - generate Pt distribution

IR divergence/DGLAP Equation

DGLAP Eq. (Integral-differential equation)

$$\frac{d\,D(x,Q^2)}{d\log(Q^2)} = \frac{\alpha}{2\pi}\int_0^x \frac{dy}{y} P(x/y) D(y,Q^2)$$

q→q+g : $P(x) = \left(\frac{1+x^2}{1-x}\right) \to \theta(1-\epsilon-x) - \delta(1-x)\int_0^{1-\epsilon} dy\, P(y)$



$$\frac{d\,D}{d\,K^2}K^2 + \frac{\alpha}{2\pi}c(\epsilon)D = \frac{\alpha}{2\pi}P\otimes D$$

$$c(\epsilon) = \int_0^{1-\epsilon} dz\, p(z)$$

IR divergence/DGLAP Equation

$$\frac{d\,\Pi}{d\,K^2}K^2+\frac{\alpha}{2\pi}c\,\Pi=0$$

Sudakov form factor

↓

non-branching prob.

Inhomogeneous Eq.

$$\frac{d\,D}{d\,K^2}K^2+\frac{\alpha}{2\pi}c\,D=\frac{\alpha}{2\pi}P\otimes D$$

$$D(x,K^2)=\hat{D}(x,K^2)\,\Pi(K^2,Q^2)$$

↓

$$\frac{d\,\hat{D}}{d\,K^2}K^2=\frac{\alpha}{2\pi}P\otimes\hat{D}$$

$$\hat{D}=\frac{\alpha}{2\pi}\int P\otimes\hat{D}\,\frac{d\,K^2}{K^2}+c$$

IR divergence/DGLAP Equation

Integral equation

$$D(x,Q^2)=\Pi(Q^2,Q_s^2)\,D(x,Q_s^2)+\frac{\alpha}{2\pi}P\otimes D(x,K^2)\frac{dK^2}{K^2}$$

$$\Pi(Q_1^2,Q_0^2)=-\exp\left(\int_{Q_0^2}^{Q_1^2}\frac{\alpha}{2\pi}\frac{dK^2}{K^2}\int_0^{1-\epsilon}dx\,P(x)\right)$$

Method of successive substitution

Equation $\phi(x)=f(x)+\lambda\int_a^x K(x,y)\phi(y)\,dy$

0th approx. $\phi_0(x)=f(x)$

1st $\phi_1(x)=f(x)+\lambda\int_a^x K(x,y)\phi_0(y)\,dy$

2nd $\phi_2(x)=f(x)+\lambda\int_a^x K(x,y)\phi_1(y)\,dy$

IR divergence/DGLAP Equation

Method of successive substitution

No emission

1 parton emission

$$\phi_2 = f(x) + \lambda \int_a^x K(x, y)\phi(y)dy$$
$$+ \lambda^2 \int_{y_1}^x dy \int_a^{y_1} dy_1 K(x, y_1)\phi(y_1) K(y_{1,} y)\phi(y)$$

2 parton emission

IR divergence/DGLAP Equation

Method of successive substitution

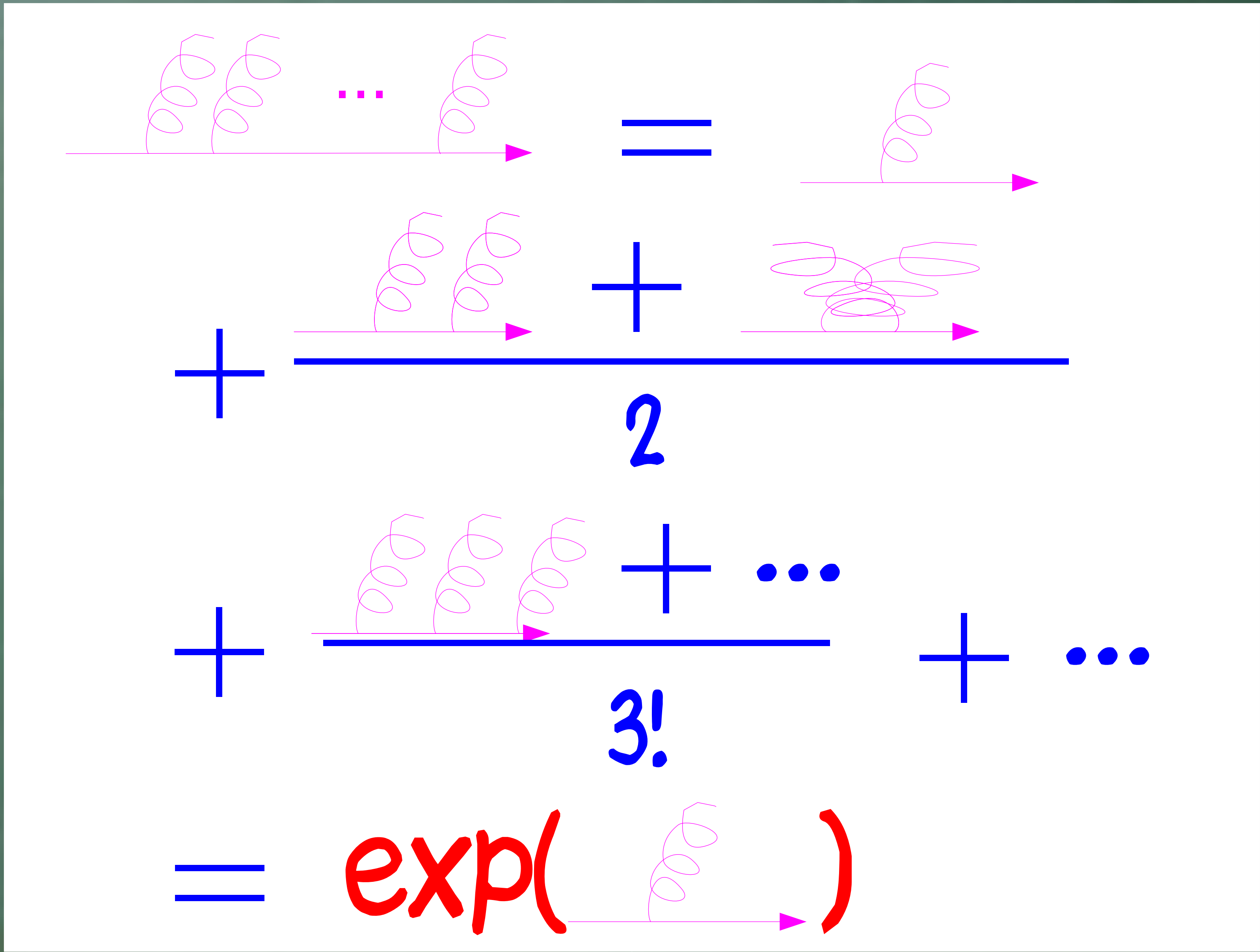
$$\phi(x)=f(x)+\lambda\int_a^x K(x,y)\phi(y)dy$$

$$\phi_2=f(x)+\lambda\int_a^x K(x,y)\phi(y)dy$$
$$+\lambda^2\int_{y_1}^x dy\int_a^{y_1} dy_1 K(x,y_1)\phi(y_1)K(y_{1,}y)\phi(y)$$

$$\phi_n(x)=f(x)+\sum_{l=1}^n \lambda^l\int_a^x K_l(x,y)f(y)dy$$

$$K_l(x,y)=\int\cdots\int dy_1\cdots dy\, K(x,y_1)\cdots K(y_{l-1},y)$$

IR divergence/DGLAP Equation



IR divergence/DGLAP Equation