QCD@LHC for beginners

Lesson 2

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Introduction/ Lagrangian

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

Exercise:

(1) Show covariant derivative D_μ is covariant under infinitesimal gauge transformation.

$$q \to \delta_g q = Uq$$
$$U = \exp\left(i\chi^a(x)t^a\right)$$

$$U(D_\mu)U^{-1} = \delta_g D_\mu$$

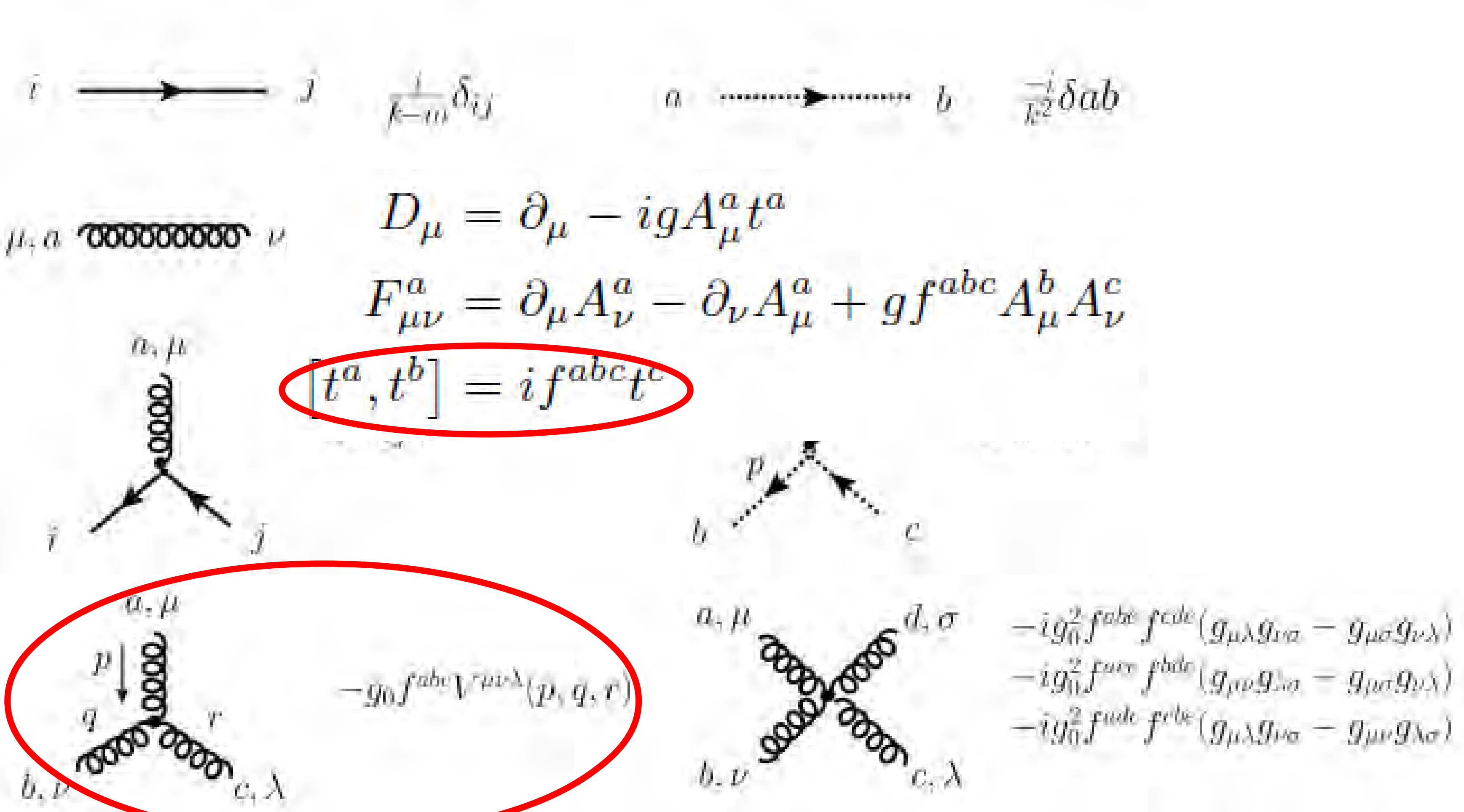
$$A^a_\mu t^a \to \delta_g A^a_\mu t^a = \frac{1}{g}\partial_\mu \chi^a(x)t^a - i[A^a_\mu t^a, \chi^b(x)t^b]$$

(2) Show kinetic term include gluon self-couplings

$$-\frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu}$$

Errata

Renormalization/Feynman Rule



$\frac{i}{k-m}\delta_{ij}$

$\frac{-i}{k^2}\delta ab$

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

$$[t^a, t^b] = if^{abc}t^c$$

$-g_0 f^{abc} V^{\mu\nu\lambda}(p, q, r)$

$-ig_0^2 f^{abe} f^{cde}(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda})$
$-ig_0^2 f^{ace} f^{bde}(g_{\mu\nu}g_{\lambda\sigma} - g_{\mu\sigma}g_{\nu\lambda})$
$-ig_0^2 f^{ade} f^{cbe}(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\nu}g_{\lambda\sigma})$

$$V_{\mu\alpha\beta}(p, -k-p, k) = (2k+p)_\mu g_{\alpha\beta} - (k-p)_\alpha g_{\beta\mu} - (k+2p)_\beta g_{\mu\alpha}$$

Outline

- Lesson 2
  - Renormalization
    - Toy example
    - quark self-energy
    - gluon vacuum-polarization
    - gluon vertex (3-point)
    - MS/$\overline{\text{MS}}$ Scheme

→ Divergent terms

GOAL: derivation of β-function

Renormalization/Toy example

Classical electro-dynamics

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x

Electrostatic potential: $$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

r

$$\mathrm{Log}\left[x + \sqrt{r^2 + x^2}\right]$$

$x \to \pm\infty$

$\mathrm{Log}(x+|x|)$

$\pm\infty$

Charge density: λ (C/m)

Renormalization/Toy example

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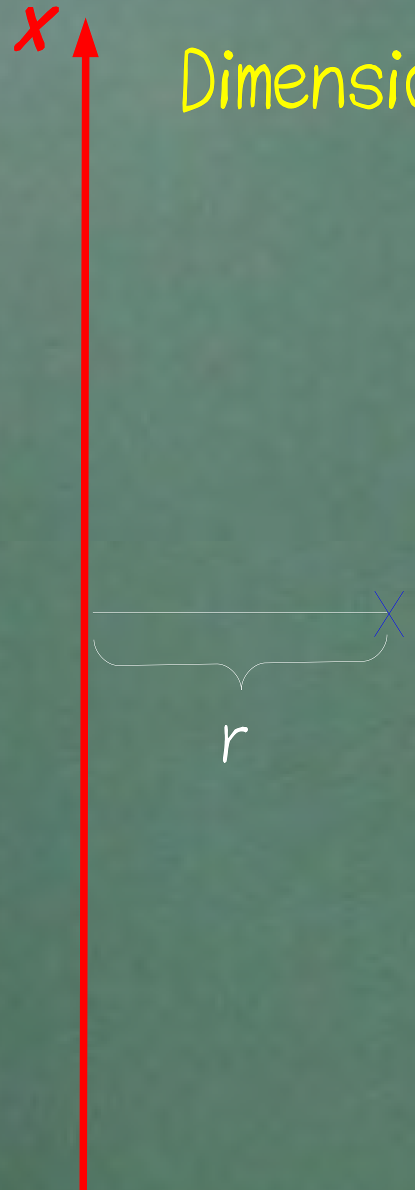

Dimensional Regularization

$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

space dimension: 1 → n = 1 - 2 ε

$$\frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}}$$

Renormalization/Toy example

Dimensional Regularization

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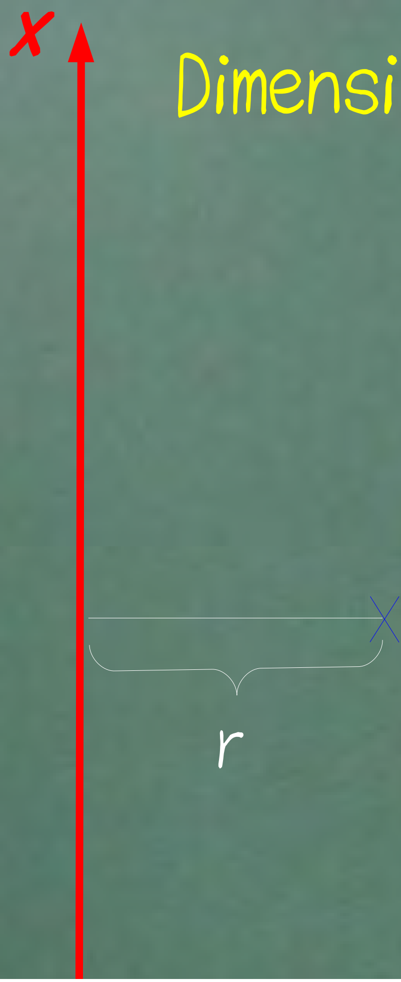

$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

space dimension: 1 → n = 1 - 2 ε

$$\frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}}$$

$$-\frac{r^{-1+n}\ \mathrm{Gamma}\left[\frac{1}{2}-\frac{n}{2}\right]\ \mathrm{Gamma}\left[\frac{n}{2}\right]}{2\sqrt{\pi}}$$

Renormalization/Toy example

Dimensional Regularization

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x

$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

space dimension: 1 → n = 1 - 2 ε

$$\frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}}$$

$$\mathrm{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$$

$$\mathrm{B}(x,y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

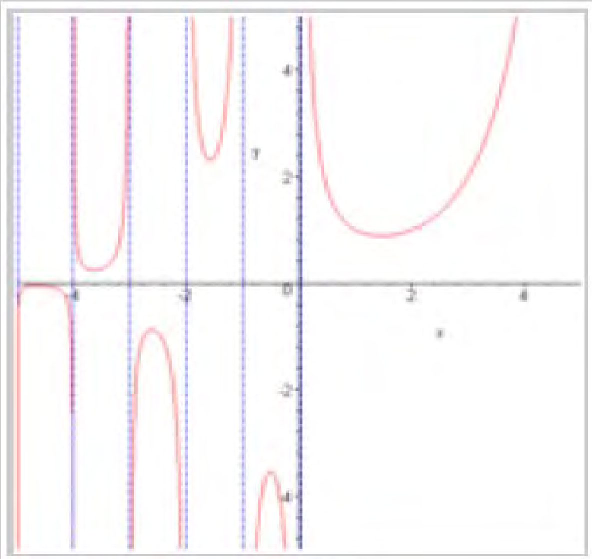
$$x = r\tan\left(\frac{\pi}{2}\theta\right)$$

$$\frac{r^{-1+n}\,\mathrm{Gamma}\left[\frac{1}{2}-\frac{n}{2}\right]\,\mathrm{Gamma}\left[\frac{n}{2}\right]}{2\sqrt{\pi}}$$

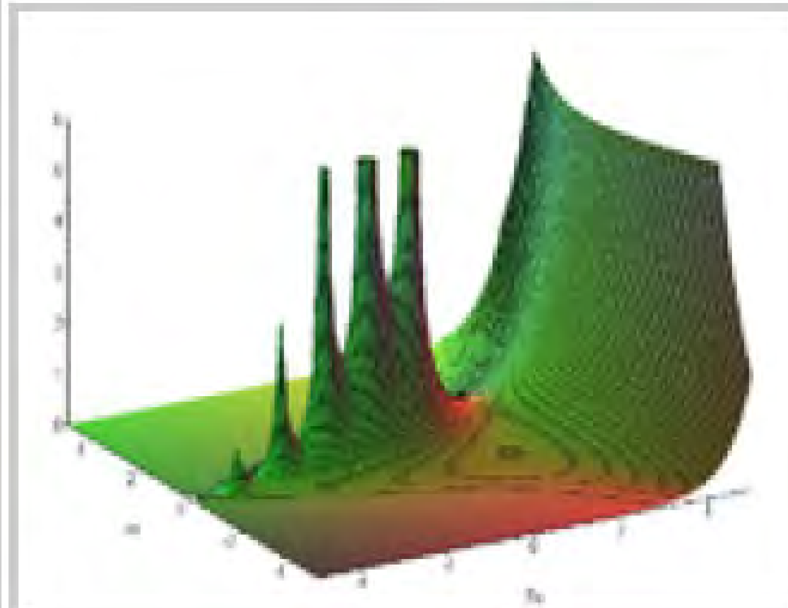
Renormalization/Γ function

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt.$$

$$\Gamma(n) = (n-1)!$$



y=Γ(x)



$$\Gamma(\epsilon) \to \frac{1}{\epsilon} - \gamma + \frac{1}{12}\left(6\gamma^2 + \pi^2\right)\epsilon + O\left(\epsilon^2\right)$$

Renormalization/Toy example

Dimensional Regularization

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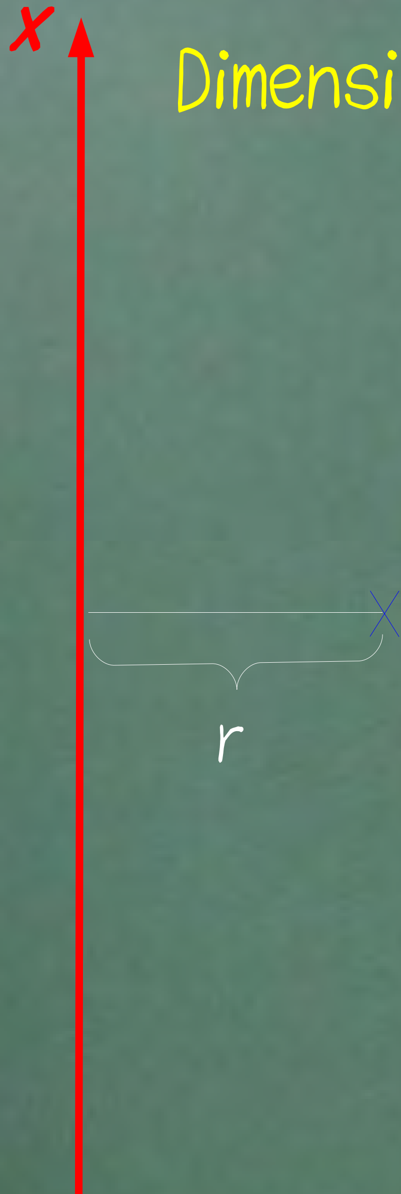


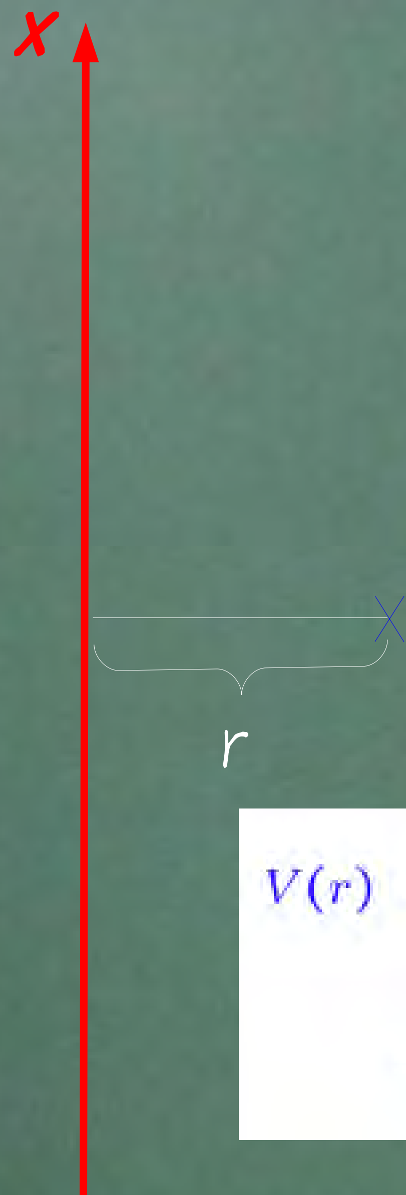
$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

space dimension: $1 \rightarrow n = 1 - 2\varepsilon$

$$\frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}}$$

$$\frac{2\,\pi^{n/2}}{\mathrm{Gamma}\left[\frac{n}{2}\right]}$$

Renormalization/Toy example



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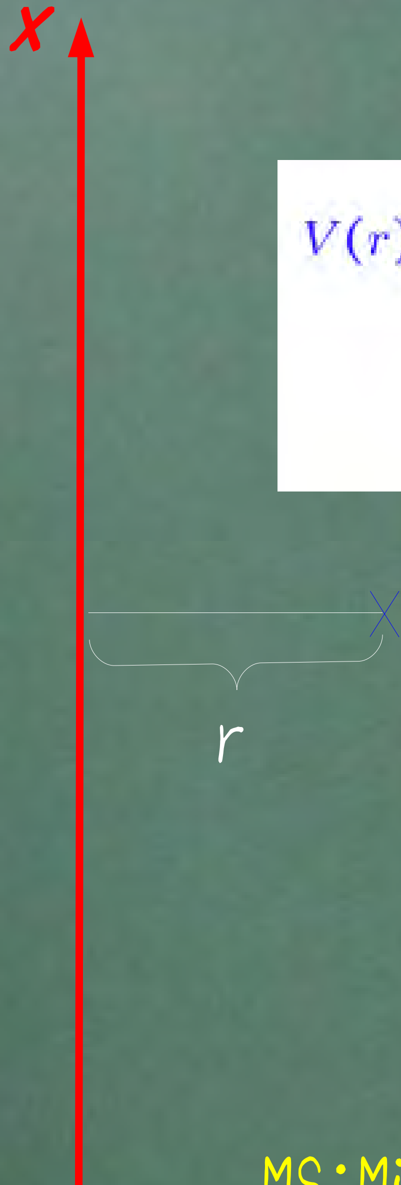
$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{dx}{\sqrt{x^2+r^2}}$$

space dimension: $1 \rightarrow n = 1 - 2\varepsilon$

$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}} = \frac{\lambda}{4\pi\epsilon_0}\frac{\Gamma\left(\frac{1-n}{2}\right)}{\left(\frac{r}{\mu}\sqrt{\pi}\right)^{1-n}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{\epsilon}+\ln\frac{e^{-\gamma_E}}{\pi}+\ln\frac{\mu^2}{r^2}+O(\epsilon)\right] \qquad n = 1-2\epsilon$$

Renormalization/Toy example



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$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}} = \frac{-\lambda}{4\pi\epsilon_0}\frac{\Gamma(\frac{1-n}{2})}{\left(\frac{r}{\mu}\sqrt{\pi}\right)^{1-n}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{\epsilon} + \ln\frac{e^{-\gamma_E}}{\pi} + \ln\frac{\mu^2}{r^2} + O(\epsilon)\right] \qquad n = 1-2\epsilon$$

Subtract

$$V_{MS}(r) = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\frac{e^{-\gamma_E}}{\sqrt{\pi}} + \ln\frac{\mu^2}{r^2}\right]$$

$$V_{\overline{MS}}(r) = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{\mu^2}{r^2}$$

MS: Minimum Subtraction

Renormalization/Toy example

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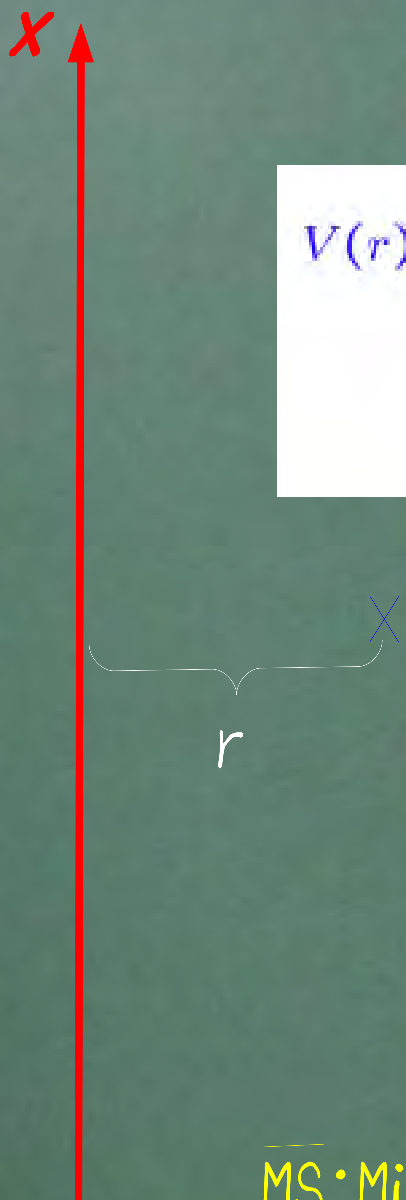

$$V(r) = \frac{\lambda}{4\pi\epsilon_0}\int d\Omega[n]\frac{x^{n-1}}{\mu^{n-1}}\frac{dx}{\sqrt{x^2+r^2}} = \frac{\lambda}{4\pi\epsilon_0}\frac{\Gamma(\frac{1-n}{2})}{\left(\frac{r}{\mu}\sqrt{\pi}\right)^{1-n}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{\epsilon} + \ln\frac{e^{-\gamma_E}}{\pi} + \ln\frac{\mu^2}{r^2} + O(\epsilon)\right] \qquad n = 1-2\epsilon$$

Subtract

$$V_{MS}(r) = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\frac{e^{-\gamma_E}}{\pi} + \ln\frac{\mu^2}{r^2}\right]$$

$$V_{\overline{MS}}(r) = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{\mu^2}{r^2}$$

$\overline{MS}$: Minimum Subtraction (bar)

Renormalization/Toy example

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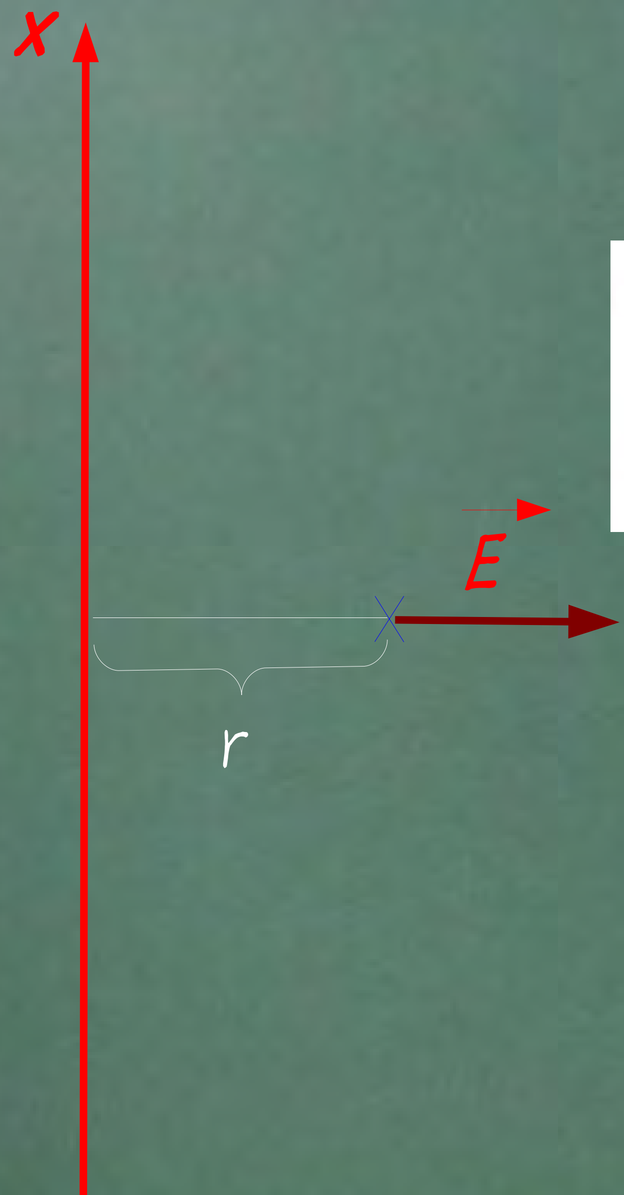

$$V_{MS}(r) = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\frac{e^{-\gamma_E}}{\sqrt{\pi}} + \ln\frac{\mu^2}{r^2}\right]$$

$$V_{\overline{MS}}(r) = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{\mu^2}{r^2}$$

potential ⇒ electric field

Scheme independent!!

Renormalization/Toy example

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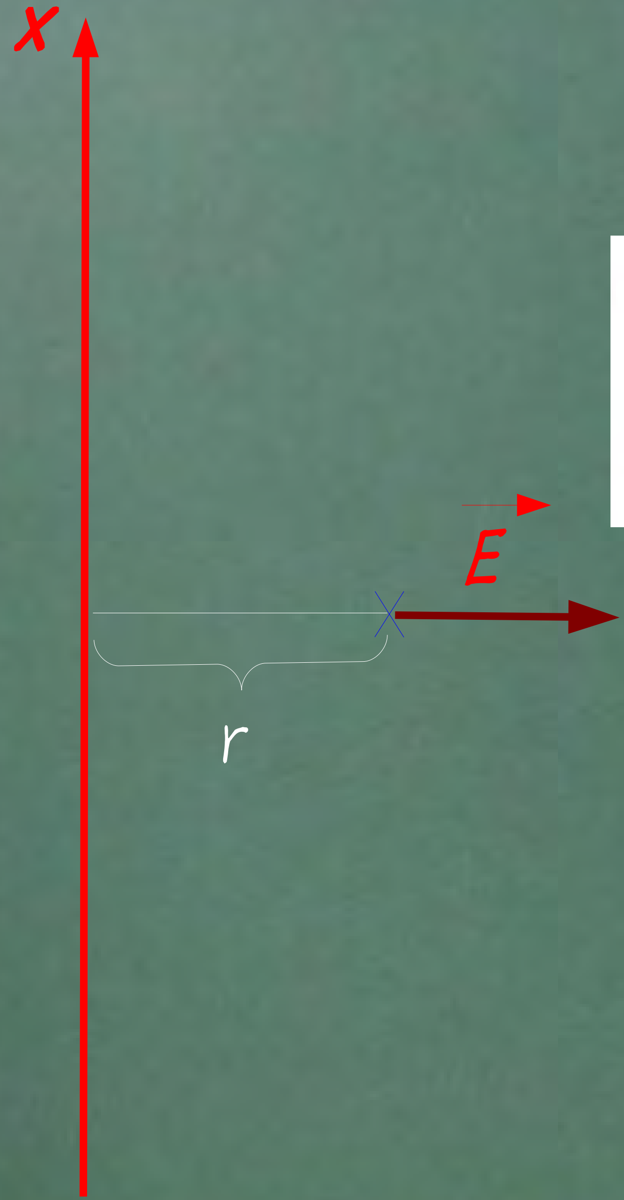


$$V_{MS}(r) = \frac{\lambda}{4\pi\epsilon_0}\left[\ln\frac{e^{-\gamma_E}}{\sqrt{\pi}} + \ln\frac{\mu^2}{r^2}\right]$$

$$V_{\overline{MS}}(r) = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{\mu^2}{r^2}$$

potential ⇒ electric field

$$E = -\frac{\partial V}{\partial r} = \frac{1}{2\pi\epsilon_0}\frac{1}{r}$$

Scheme independent!!

Renormalization/Feynman Rule

Lagrangian (bare)

$$\mathcal{L} = \sum_{k=1}^{n_f} \bar{q}_{0k}(i\not{D} - \cancel{m_{0k}})q_{0k} - \frac{1}{4}F^a_{0\mu\nu}F_0^{a\mu\nu} - \frac{1}{2a_0}(\partial_\mu A_0^{a\mu})^2 - \bar{c}_0^a \partial^\mu D_\mu c_0^a$$

(massless theory)

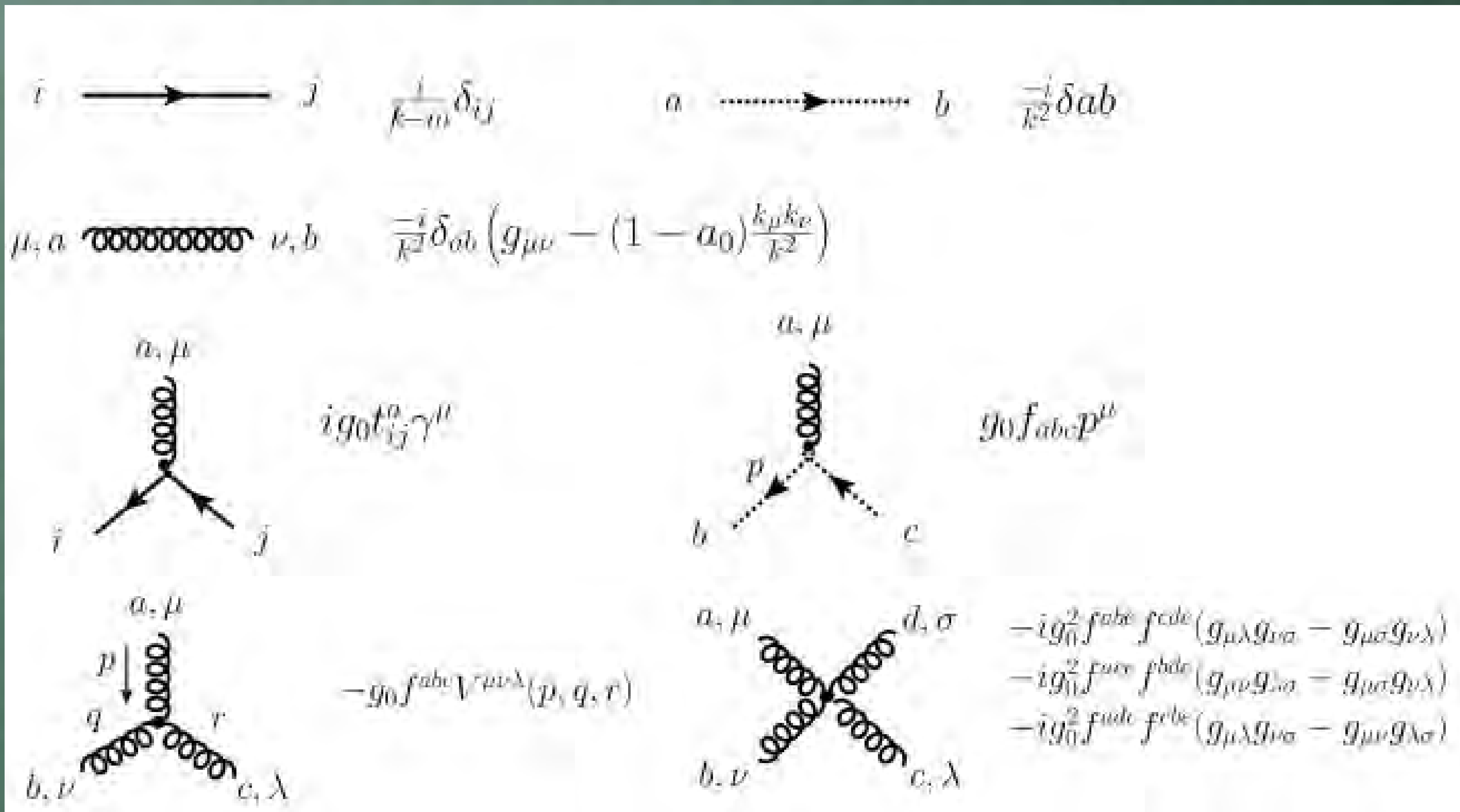
$$q_{0k} = Z_q^{1/2} q_k, \quad A^a_{0\mu} = Z_A^{1/2} A^a_\mu, \quad c_0^a = Z_c^{1/2} c^a, \quad a_0 = Z_A\, a, \quad g_0 = Z_\alpha^{1/2} g$$

renormalized

renormalization constant

bare

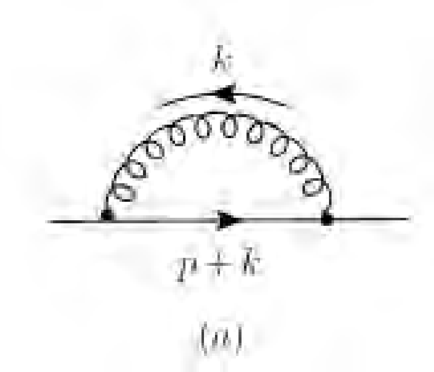
Renormalization/Feynman Rule



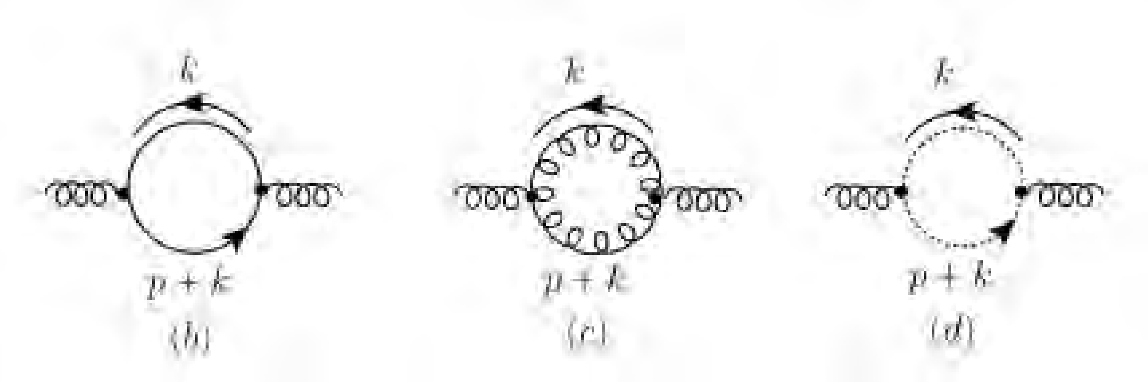
$$V_{\mu\alpha\beta}(p,-k-p,k)=(2k+p)_\mu g_{\alpha\beta}-(k-p)_\alpha g_{\beta\mu}-(k+2p)_\beta g_{\mu\alpha}$$

Renormalization/Counter terms

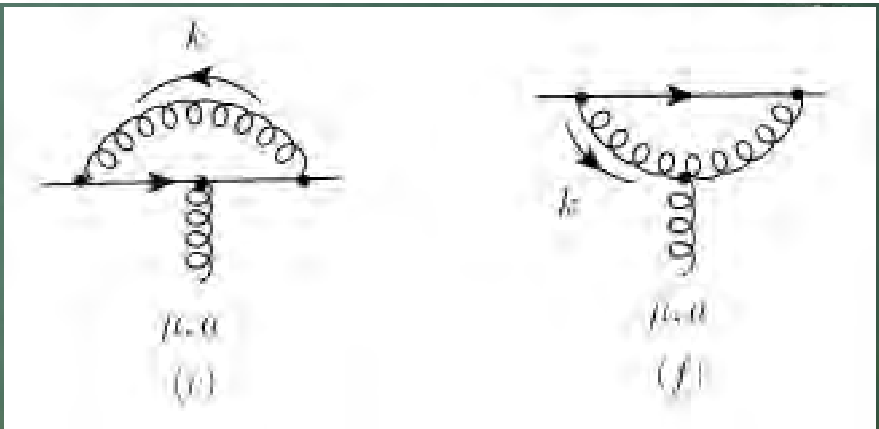
quark self-energy



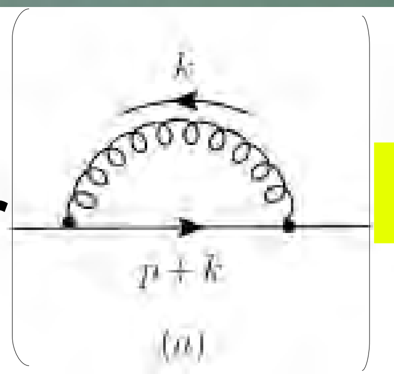
gluon
vacuum-polarization



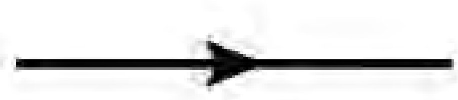
quark gluon vertex



Renormalization/Counter terms

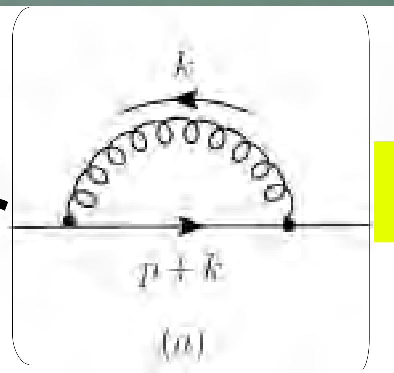
Tr 

$$\Sigma_V(p^2) = \frac{ig_0^2}{-p^2} C_F \int \frac{d^d k}{(2\pi)^d} \frac{N}{D_1 D_2}$$

i  j $\qquad \frac{i}{\not{k}-m}\delta_{ij}$

μ, a  ν, b $\qquad \frac{-i}{k^2}\delta_{ab}\left(g_{\mu\nu} - (1-a_0)\frac{k_\mu k_\nu}{k^2}\right)$

Renormalization/Counter terms

Tr 

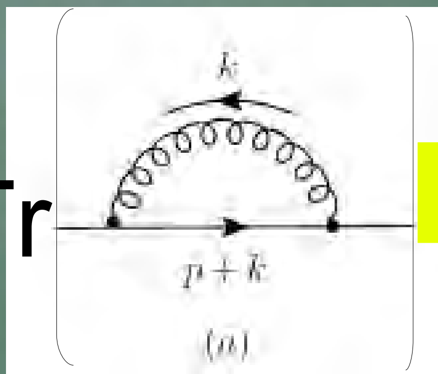
$$\Sigma_V(p^2) = \frac{ig_0^2}{-p^2} C_F \int \frac{d^d k}{(2\pi)^d} \frac{N}{D_1 D_2}$$

$$D_1 = -(p+k)^2 \qquad D_2 = -k^2$$

$$N = \frac{1}{4} \mathrm{Tr}\left[\not{p}\gamma^\mu(\not{p}+\not{k})\gamma^\nu\right]\left(g_{\mu\nu} - \xi\frac{k_\mu k_\nu}{k^2}\right)$$

$$C_F = t^a t^a = \frac{4}{3}$$

Renormalization/Counter terms

Tr 

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$$C_F = t^a t^a = \frac{4}{3}$$

Exercise!

Renormalization/Counter terms

$$N = -(d-2)(p^2 + p\cdot k) + \frac{\xi}{D_2}[k^2 p\cdot k + 2(p\cdot k)^2 - p^2 k^2]$$

$$= -\frac{1}{2}[(d-2)(-2+1+D_2-D_1) + \frac{\xi}{D_2}(-D_2(1+D_2-D_1) + (1+D_2-D_1)^2 + D_2)$$

$$(-p^2)^{-\epsilon}$$

$$\int d^d k \frac{k^{\mu_1} k^{\mu_2} \ldots}{(-k^2 - i0)^n} = 0$$

$$\Sigma_V(p^2) = \frac{g_0^2 (-p^2)^{-\epsilon}}{(4\pi)^{d/2}} C_F \frac{1}{2}[(d-2-\xi)G(1,1) + \xi G(1,2)]$$

$$G(n_1, n_2) = -i\pi^{-d/2}(-p^2)^{n_1+n_2-d/2} \int \frac{d^d k}{D_1^{n_1} D_2^{n_2}}$$

$$= \frac{\Gamma(-d/2+n_1+n_2)\Gamma(d/2-n_1)\Gamma(d/2-n_2)}{\Gamma(n_1)\Gamma(n_2)\Gamma(d-n_1-n_2)}$$

Renormalization/Counter terms

$$\Sigma_V(p^2) = -\frac{g_0^2}{(4\pi)^{d/2}} C_F \frac{1}{2} [(d-2-\xi)G(1,1) + \xi G(1,2)]$$

$\xi = 1 - a_0$

$$G(1,2) = -(d-3)G(1,1)$$

$$\Sigma_V(p^2) = -\frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} C_F \frac{d-2}{2} a_0 G(1,1)$$

Renormalization/Counter terms

$$\Sigma_V(p^2) = -\frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} C_F \frac{d-2}{2} a_0 G(1,1)$$

$$iS_F(p) = \frac{i}{\not p} + \frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p} + \frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p} + \cdots = \frac{i}{\not p[1-\Sigma_V(p^2)]}$$

μ : renormalization scale

$O(\alpha_s)$

$$\frac{g_0^2}{(4\pi)^{d/2}} \equiv \mu^{2\epsilon} \frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma_E \epsilon}$$

$$\frac{i}{\not p}\left[1 + \Sigma_V(p^2) + \mathcal{O}(\alpha_s^2)\right] = \frac{i}{\not p}\left[1 - \frac{\alpha_s(\mu)}{4\pi} C_F \left(\frac{\mu^2}{-p^2}\right)^{\epsilon} e^{\gamma_E \epsilon} a(1-\epsilon) G(1,1) + \mathcal{O}(\alpha_s^2)\right]$$

Renormalization/Counter terms

$$\Sigma_V(p^2) = -\frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} C_F \frac{d-2}{2} a_0 G(1,1)$$

$\overline{\text{MS}}$ scheme

$$iS_F(p) = \frac{i}{\not p} + \frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p} + \frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p}(-i\not p\Sigma_V)\frac{i}{\not p} + \cdots = \frac{i}{\not p\,[1-\Sigma_V(p^2)]}$$

$$e^{\gamma\epsilon}\Gamma(\epsilon) \to \frac{1}{\epsilon} + \frac{\pi^2\epsilon}{12} + \mathcal{O}(\epsilon^2)$$

$O(\alpha_s)$

$$\frac{g_0^2}{(4\pi)^{d/2}} \equiv \mu^{2\epsilon}\frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma_E \epsilon}$$

$$\frac{i}{\not p}\left[1 + \Sigma_V(p^2) + \mathcal{O}(\alpha_s^2)\right] = \frac{i}{\not p}\left[1 - \frac{\alpha_s(\mu)}{4\pi} C_F \left(\frac{\mu^2}{-p^2}\right)^{\epsilon} e^{\gamma_E \epsilon} a(1-\epsilon) G(1,1) + \mathcal{O}(\alpha_s^2)\right]$$

Renormalization/Counter terms

$$G(1,1) = \frac{1}{\epsilon} - \gamma_E + 2 + \mathcal{O}(\epsilon)$$

$$iS_F(p) = \frac{i}{\not p}\left[1 - \frac{\alpha_s(\mu)}{4\pi\epsilon} C_F a \left\{1 - \epsilon\left(\ln(-p^2/\mu^2) - 1\right)\right\}\right]$$

$$q_{0k} = Z_q^{1/2} q_k$$

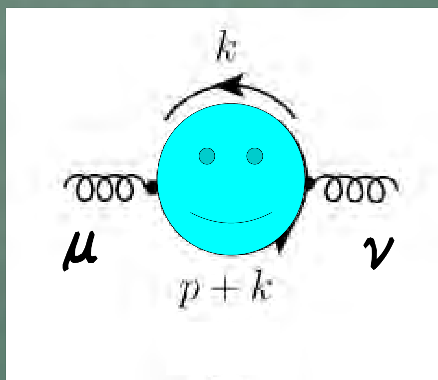
$$\bar{q}_0 S_F q_0 = Z_q^{1/2} \bar{q} S_F^r Z_q^{1/2} q$$
$$= \bar{q}\left(Z_q S_F^r\right) q$$

$\overline{\text{MS}}$ scheme

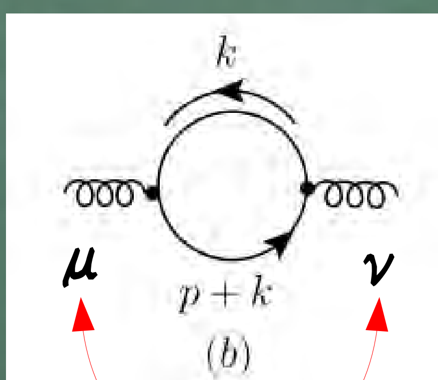
$$Z_q = 1 - a\frac{\alpha_s}{4\pi\epsilon} C_F + \mathcal{O}(\alpha_s^2)$$

Renormalization/Counter terms

gluon
vacuum-polarization



$$i\Pi_{\mu\nu}(p) = i(p^2 g_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$$



$g_{\mu\nu}$

$$\mathrm{Tr}(t^a t^b) = T_F \delta^{ab} = \delta^{ab}/2$$

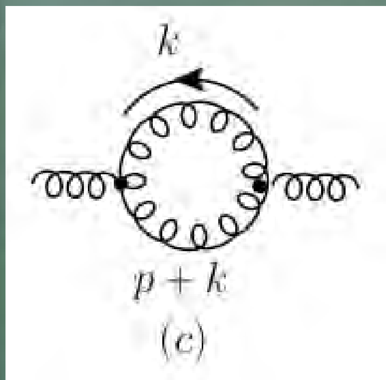
$$\Pi_q(p^2) = -i\frac{T_F\ n_f}{(d-1)(-p^2)} \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left[i g_0 \gamma_\mu \frac{i}{\not{p} + \not{k}} i g_0 \gamma^\mu \frac{i}{\not{k}} \right]$$

$$= -T_F n_f \frac{g_0^2 (-p^2)^{-\epsilon}}{(4\pi)^{d/2}} 2 \frac{d-2}{d-1} G(1,1)$$

$a_0 = 1$

Feynman gauge

Renormalization/Counter terms



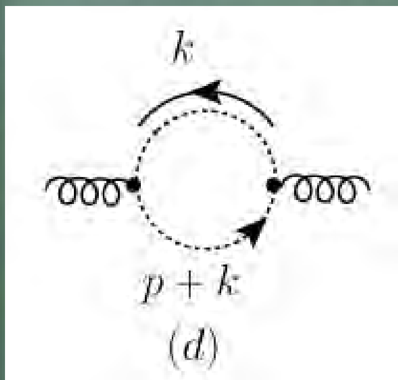
$$\Pi_g(p^2) = i\frac{C_A}{2}\frac{g_0^2}{(d-1)(-p^2)}\int\frac{d^dk}{(2\pi)^d}\frac{N}{D_1D_2}$$

$$N = V_{\mu\alpha\beta}(p,-k-p,k)V^{\mu\beta\alpha}(-p,-k,k+p)$$

$$C_A = 3$$

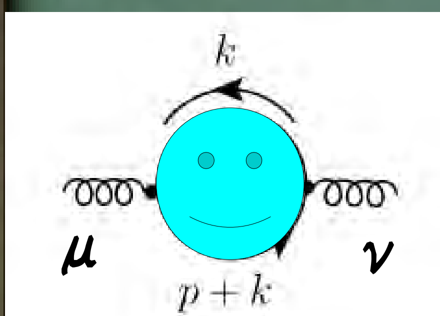
$$\Pi_g(p^2) = \frac{3}{2}C_A\frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}}G(1,1)$$

Renormalization/Counter terms



$$\Pi_{gh}(p^2) = -iC_A \frac{g_0^2}{(d-1)(-p^2)} \int \frac{d^d k}{(2\pi)^d} \frac{k(p+k)}{D_1 D_2}$$

$$= \frac{1}{2} C_A \frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} G(1,1) \frac{1}{d-1}$$



$$\Pi(p^2) = \Pi_q(p^2) + \Pi_g(p^2) + \Pi_{gh}(p^2)$$

$$= \frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} \frac{G(1,1)}{2(d-1)} \left[4(2-d)T_F n_f + C_A \left\{ 3d - 2 \right\} \right]$$

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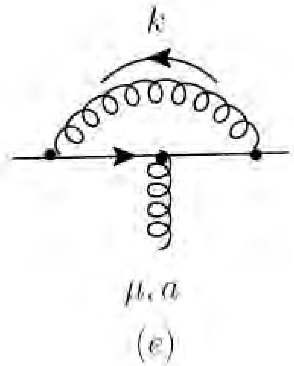
$$\Pi(p^2) = \Pi_q(p^2) + \Pi_g(p^2) + \Pi_{gh}(p^2)$$
$$= \frac{g_0^2(-p^2)^{-\epsilon}}{(4\pi)^{d/2}} \frac{G(1,1)}{2(d-1)} \left[4(2-d)T_F n_f + C_A \left\{3d-2\right\}\right]$$

$$-iD\mu\nu = \frac{-i}{p^2(1-\Pi(p^2))}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D_{\mu\nu}(p^2) = Z_A D^r_{\mu\nu}(p^2)$$

$$Z_A = 1 - \frac{\alpha_s}{4\pi\epsilon}\left[\frac{1}{2}\left(a - \frac{13}{3}\right)C_A + \frac{4}{3}T_F n_f\right] + \mathcal{O}(\alpha_s^2)$$

Renormalization/Counter terms



$$ig_0\Lambda_e^\alpha = \left(C_F - \frac{C_A}{2}\right)\int \frac{d^dk}{(2\pi)^d} ig_0\gamma^\mu \frac{i}{\not k} ig_0\gamma^\alpha \frac{i}{\not k} ig_0\gamma^\nu \frac{-i}{k^2}\left(g_{\mu\nu} - \xi\frac{k_\mu k_\nu}{k^2}\right)$$

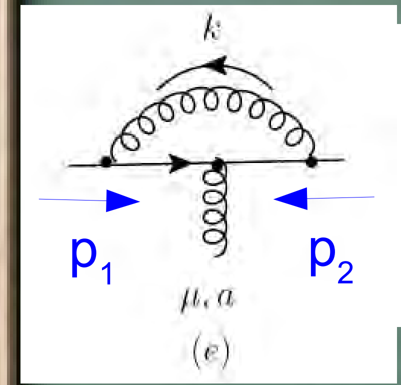
$$= g_0^3\left(C_F - \frac{C_A}{2}\right)\int \frac{d^dk}{(2\pi)^d}\frac{\gamma_\mu \not k\gamma^\alpha \not k\gamma^\mu - \xi k^2\gamma^\alpha}{(k^2)^3},$$

$$\Lambda_e^\alpha = -ig_0^2\left(C_F - \frac{C_A}{2}\right)a_0\gamma^\alpha\int\frac{d^dk}{(2\pi)^d}\frac{1}{(-k^2+i0)^2}$$

$$i\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{(k_E^2)^2}\bigg|_{\mathrm{UV}} = \frac{i\Omega_4}{(2\pi)^4}\int_\lambda^\infty dk_E k_E^{-1-2\epsilon} = \frac{i\lambda^{-2\epsilon}}{(4\pi)^2\epsilon} = \frac{i}{(4\pi)^2}\frac{1}{\epsilon}$$

$$\Lambda_e^\alpha = \left(C_F - \frac{C_A}{2}\right)a\frac{\alpha_s(\mu)}{4\pi\epsilon}\gamma^\alpha$$

Renormalization/Counter terms



$$ig_0\Lambda_e^\alpha = \left(C_F - \frac{C_A}{2}\right)\int \frac{d^dk}{(2\pi)^d} ig_0\gamma^\mu \frac{i}{\not k} ig_0\gamma^\alpha \frac{i}{\not k} ig_0\gamma^\nu \frac{-i}{k^2}\left(g_{\mu\nu} - \xi\frac{k_\mu k_\nu}{k^2}\right)$$

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$p_1=p_2=0$

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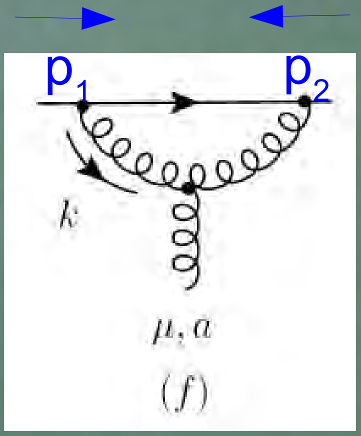


$p_1=p_2=0$

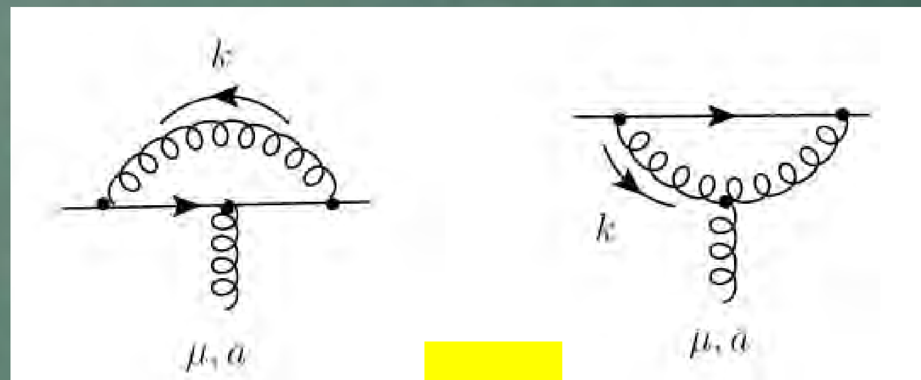
$$\Lambda_f^\alpha = i\frac{C_A}{2}g_0^2\int\frac{d^dk}{(2\pi)^d}\frac{\gamma_\mu \not{k}\gamma_\nu}{(k^2)^3}V^{\alpha\nu\mu}(0,-k,k)$$

$$= -i\frac{3}{2}C_A g_0^2\gamma^\alpha\int\frac{d^dk}{(2\pi)^d}\frac{1}{(-k^2+i0)^2}$$

$$i\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{(k_E^2)^2}\bigg|_{\mathrm{UV}} = \frac{i\Omega_4}{(2\pi)^4}\int_\lambda^\infty dk_E k_E^{-1-2\epsilon} = \frac{i\lambda^{-2\epsilon}}{(4\pi)^2\epsilon} = \frac{i}{(4\pi)^2}\frac{1}{\epsilon}$$

$$\Lambda_f^\alpha = \frac{3}{2}C_A\frac{\alpha_s}{4\pi\epsilon}\gamma^\alpha$$

Renormalization/Counter terms



$$Z_\Gamma = 1 + \left(C_F a + C_A \frac{a+3}{4} \right) \frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

$$Z_\Gamma = Z_q^{-1} Z_A^{-1/2} Z_\alpha^{1/2}$$

$$Z_\alpha = 1 - \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f \right) \frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

Renormalization/Counter terms

$$Z_\alpha = 1 - \left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right)\frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

$$\frac{g_0^2}{(4\pi)^{d/2}} \equiv \mu^{2\epsilon}\frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma_E \epsilon}$$

Now we are ready to discuss running α_s
and asymptotic free!

Renormalization/Counter terms

$$\ldots 1 - \left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right)\frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

Next Lesson!

$$\frac{g_0^2}{(4\pi)^{d/2}} \equiv \ldots e^{\gamma_E \epsilon}$$

Now we are ready to discuss running α_s and asymptotic free!