QCD@LHC for beginners

Lesson 1

Y.Kurihara
(KEK)

VSOP-18@Quy Nhon

H.Kawamura(KEK)

DATE		Tue 24 July	Fri. 27 July	Mon 30 July
Morning	08:20 – 08:30			
	08:30 – 09:30	Alvarez-Gaume 1	Kurihara 3	Kurihara 5
	09:30 – 09:40			
	09:40 – 10:40	Alvarez-Gaume 2	Kurihara 4	Kurihara 6
	10:40 – 11:00			
	11:00 – 12:00	Kurihara 1	Kim 5	Godbole 5
After noon	14:00 – 15:00	Kurihara 2	Kim 6	Godbole 6
	15:00 – 15:20			
	15:20 – 16:20	Discussions/ exercises	Mini-seminars	Osaka program
	16:20 – 16:30			
	16:30 – 17:30	Reserved for extra-session	Mini-seminars	Osaka program

Outline

- Lesson 1
  - Introduction
    - Why QCD?
    - Yan-Mills Lagrangian
    - Perturbative QCD
    - Feynman Rule
- Lesson 2
  - Renormalization
    - Toy example
    - quark self-energy
    - gluon vacuum-polarization
    - gluon vertex (3-point)
    - MS/$\overline{\text{MS}}$ Scheme

Divergent terms

Outline

- Lesson 3
  - Asymptotic Free
    - β-function and Renormalization Equation
    - α_s determination
    - Scale dependence/R-ratio
- Lesson 4
  - Around "IR divergence"
    - IR divergence in QCD/KLN theorem
    - Factorization
    - DGLAP Equation/PDF

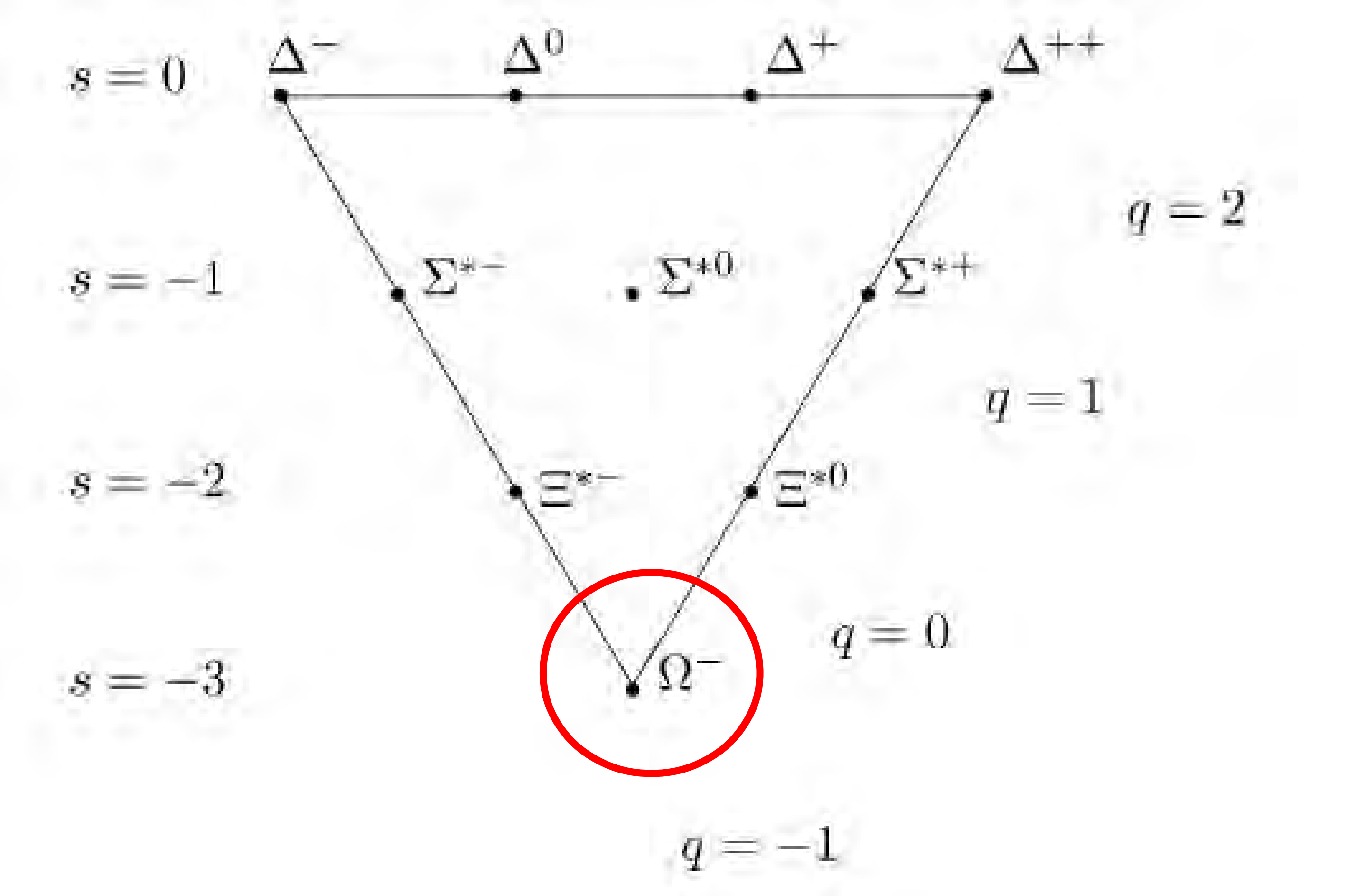
Outline

- Lesson 5
  - Examples
    - Drell-Yan
    - gluon emission
    - Higgs production/decay
- Lesson 6
  - Event Generator
    - structure
    - hard/soft parts and their connection
    - parton shower
  - Non perturbative effect

Introduction/Why QCD?

1954	Yan-Mills theory	C.N.Yan/R.Mills
1961	Eightfoldway/$SU(3)_f$	M.Gell-Mann/Y.Ne'eman
1964	Quark	M.Gell-Mann, G. Zweig
1965	Color $SU(3)_c$	Y.Nambu/M.Y.Han
1969	Parton model	R.Feynman
1973	Asymptotic free	D.Gross/F.Wilczek,
	(2004年 Nobel Prize)	D.Polizer
1977	Evolution Equation	G.Altarelli/G.Parisi,
		V.N.Gribov/L.N.Lipatov(1972),
		Yu.L.Dokshitzer

Introduction/ Why QCD?



Introduction/ Why QCD?

1954 Yan-Mills theory — C.N.Yan/R.Mills

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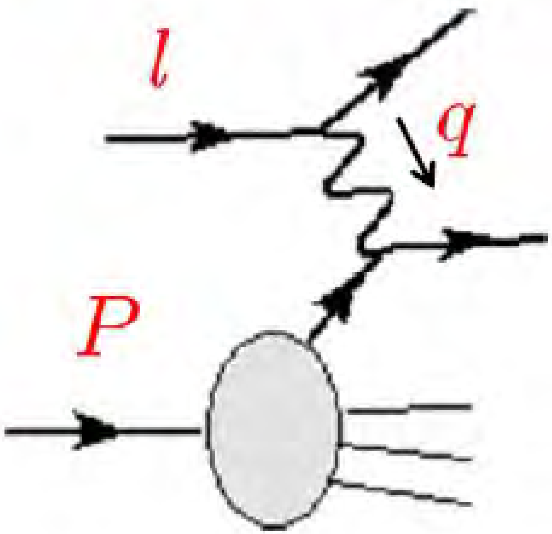
Introduction/ Why QCD?

Deep inelastic scattering



$Q^2 = -q^2$ momentum transfer squared

$x = \dfrac{Q^2}{2P \cdot q}$ Bjorken x : approximately equal to mom. fraction of the scattered quark in "Infinite Momentum Frame"

$y = \dfrac{P \cdot q}{P \cdot l}$ E_γ / E_l in proton's rest frame

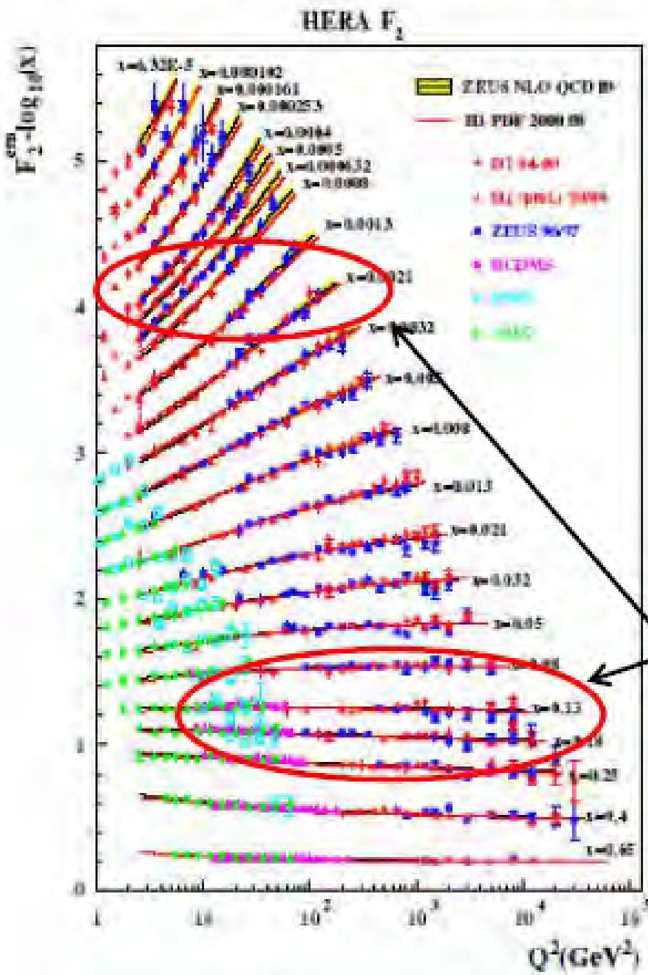
$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[\left\{ 1 + (1-y)^2 \right\} F_2(x, Q^2) - y^2 F_L(x, Q^2) \right]$$

Structure Functions

Introduction/ Why QCD?

DIS cont'd

HERA F2 data vs. NLO pQCD



H1+ZEUS, Moriond 2004

$$F_2(x, Q^2) = x \sum_a C_{2,a} \otimes f_a(x, Q^2)$$
$$\approx x \sum_a e_a^2 f_a(x, Q^2)$$

NLO (next-to-leading order)

$$C_{2,a}^{(1)}(z) \; + \; \left\{ P_{ab}^{(0)}(z), \; P_{ab}^{(1)}(z) \right\}$$

$x \sim 0.2$

dep. only on x → Bjorken scaling

$x \sim 0.8 - 2.5 \cdot 10^{-3}$

scaling violation

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1974	Discovery of J/ Ψ	S.Ting,B.Richter
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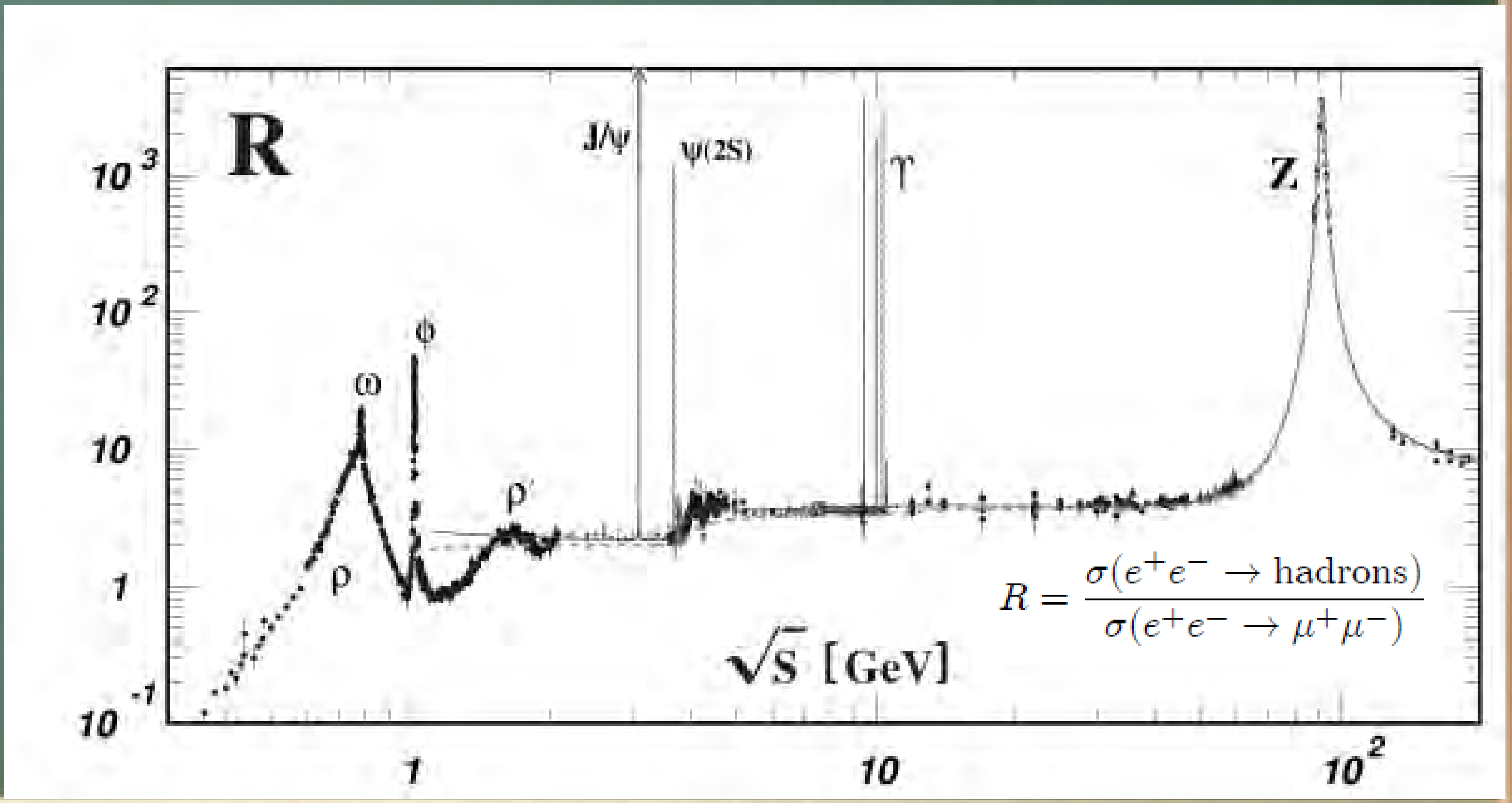
Introduction/ Why QCD?

- QCD: Color $SU_c(3)$ gauge theory (Yan-Mills theory)
- Experimental Evidences

Introduction/ Why QCD?

- QCD: Color $SU_c(3)$ gauge theory (Yan-Mills theory)
- Experimental Evidences
  - R-ratio

$$R \equiv R_0 = N_c \sum_{i=1}^{n_f} e_{q_i}^2$$



Introduction/ Why QCD ?

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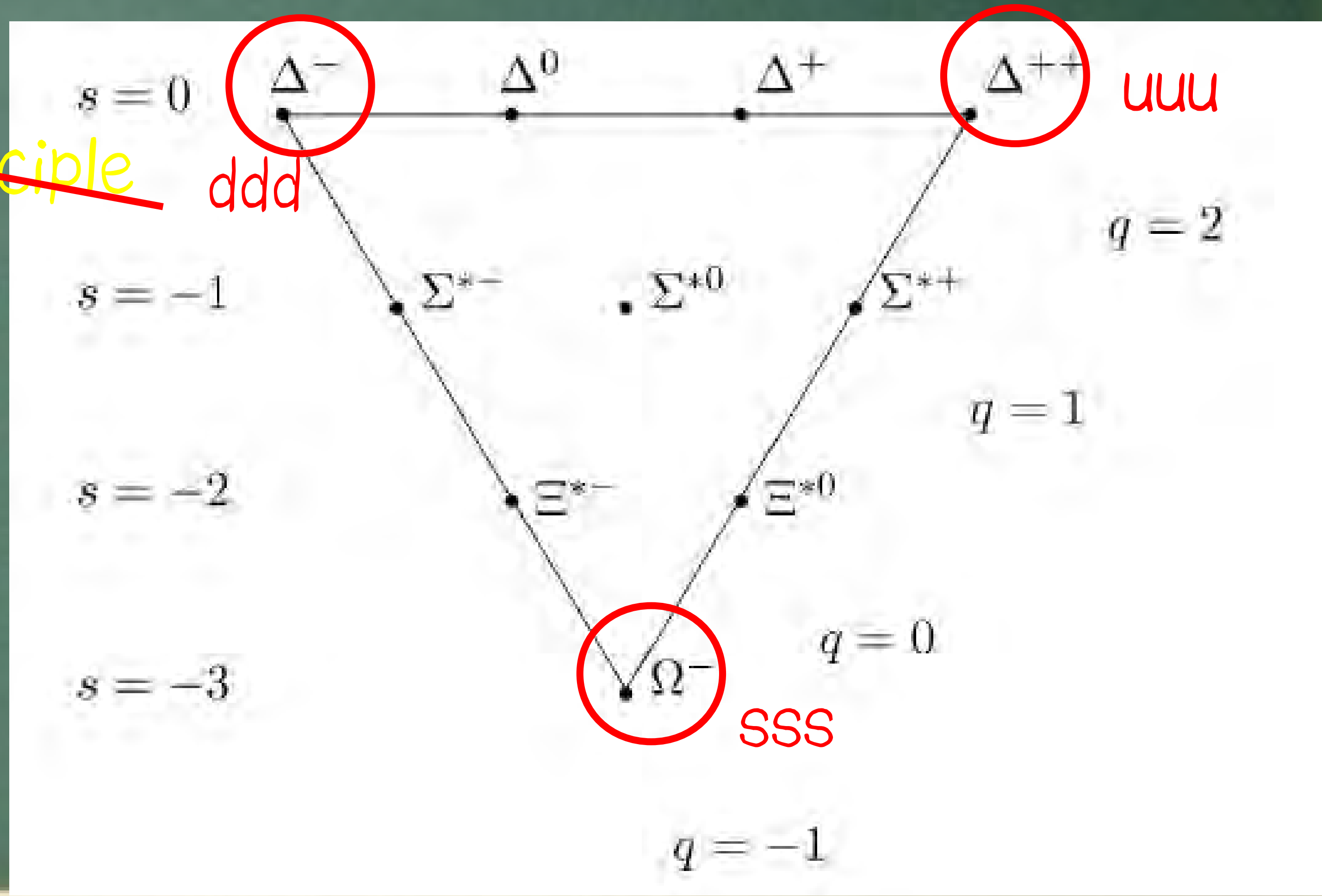
  - ~~Pauli Principle~~



Introduction/ Why QCD ?

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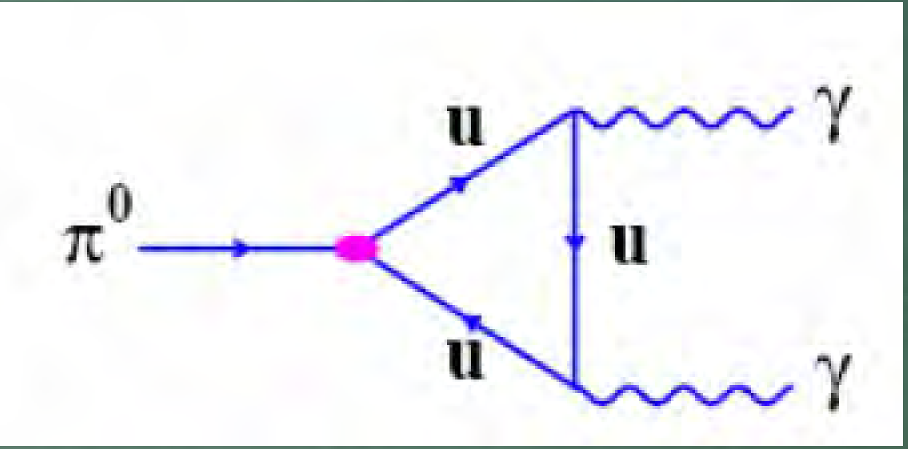
  - π^0 decay rate: $\pi^0 \to \gamma\,\gamma$



$$\Gamma(\pi^0 \to \gamma\gamma) \propto N^2_{colour}$$

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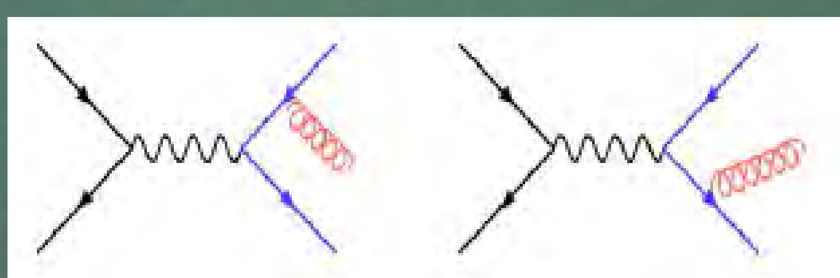
  - 3-jet event: gluon jet



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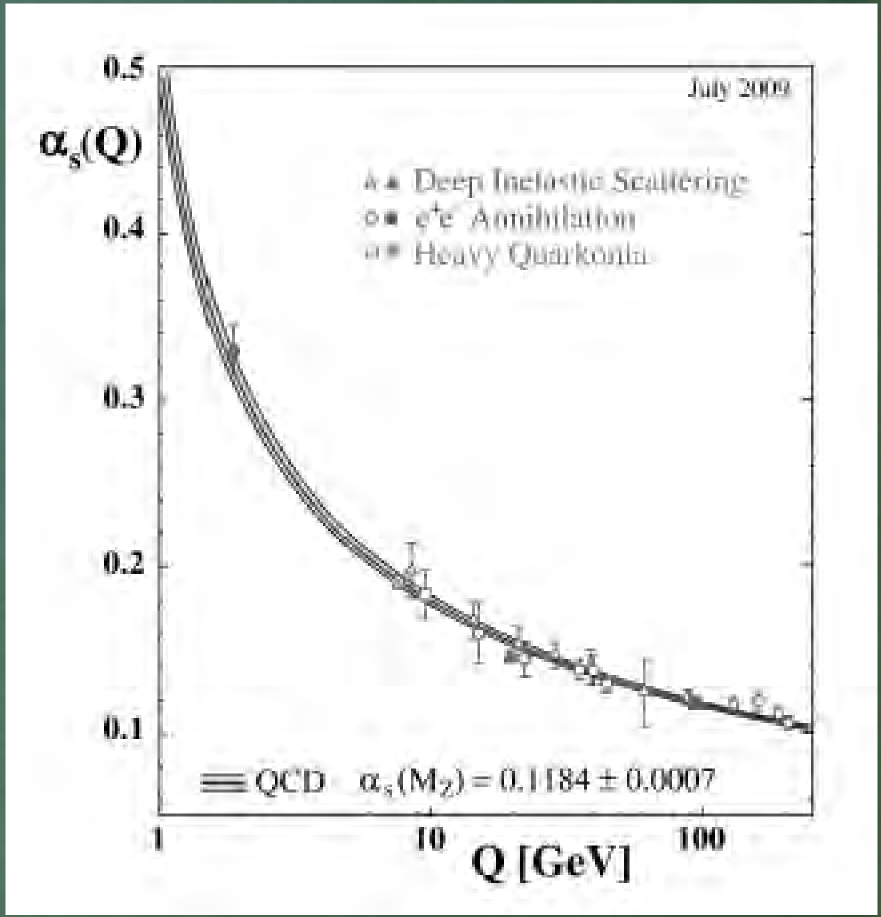
  - Running α_s



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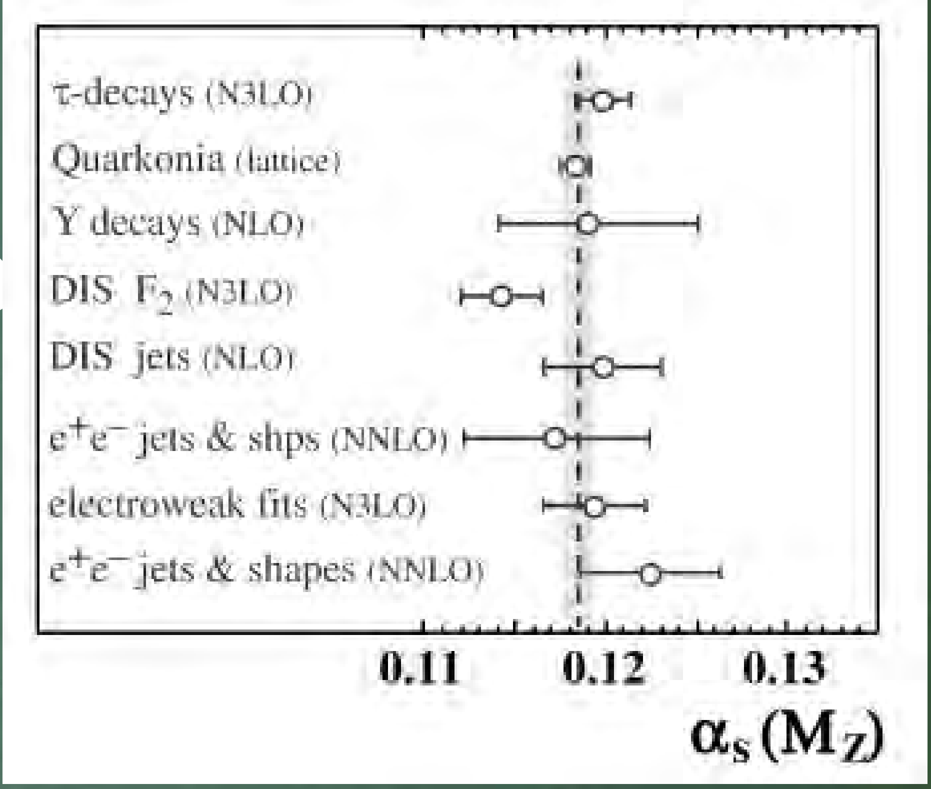


World Average '09 $\alpha_s(M_z) = 0.1184 \pm 0.0007$

Introduction/ Lagrangian

- QCD: Color $SU_c(3)$ gauge theory (Yan-Mills theory)

quark wave functions

$$\mathcal{L} = \sum_{k=1}^{n_f} \bar{q}_k (i\gamma^\mu D_\mu - m_k) q_k - \frac{1}{4} F^{a,\mu\nu} F^a_{\mu\nu}$$

$$D_\mu = \partial_\mu - i g A^a_\mu t^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$[t^a, t^b] = i f^{abc} t^c$$

covariant derivative

gluon field

structure constant

Field strength

SU(3) generator

Introduction/ Lagrangian

- QCD: Color $SU_c(3)$ gauge theory (Yan-Mills theory)

$$\mathcal{L} = \sum_{k=1}^{n_f} \bar{q}_k (i\gamma^\mu D_\mu - m_k) q_k - \frac{1}{4} F^{a,\mu\nu} F^a_{\mu\nu}$$

$$-\frac{1}{\alpha} (\partial_\mu A^{a,\mu})^2 - \bar{c}^a \partial^\mu D_\mu c^a$$

gauge fixing term

ghost term

Introduction/ Lagrangian

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

Exercise:

(1) Show covariant derivative D_μ is covariant under infinitesimal gauge transformation.

$$q \to \delta_g q = U q$$
$$U = \exp\left(i\chi^a(x)t^a\right)$$

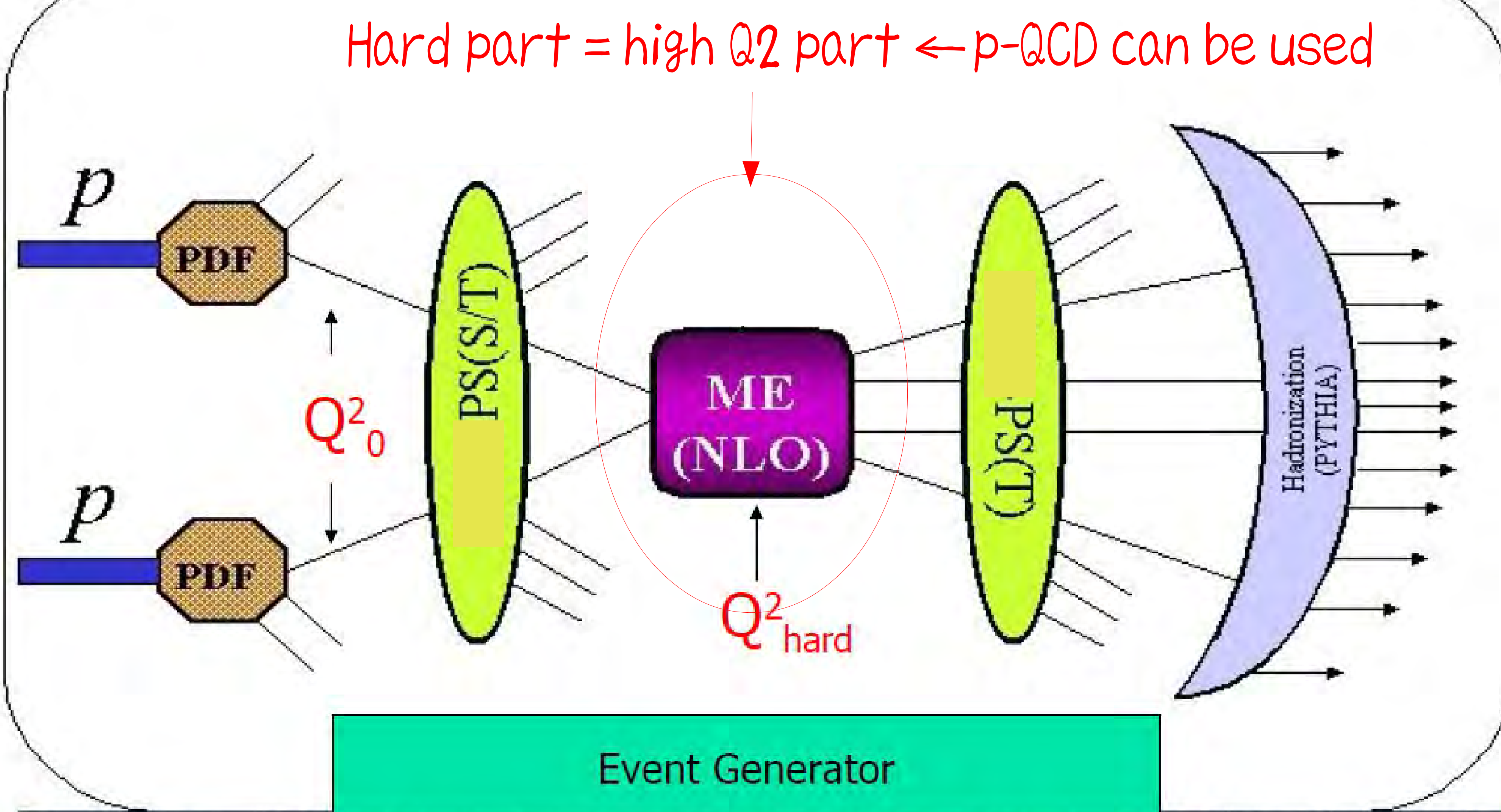
$$U(D_\mu)U^{-1} = \delta_g D_\mu$$

$$A^a_{\mu\nu} t^a \to \delta_g A^a_{\mu\nu} t^a = \frac{1}{g}\partial_\mu - i[A^a_\mu t^a, \chi^b(x)t^b]$$

(2) Show kinetic term include gluon self-couplings

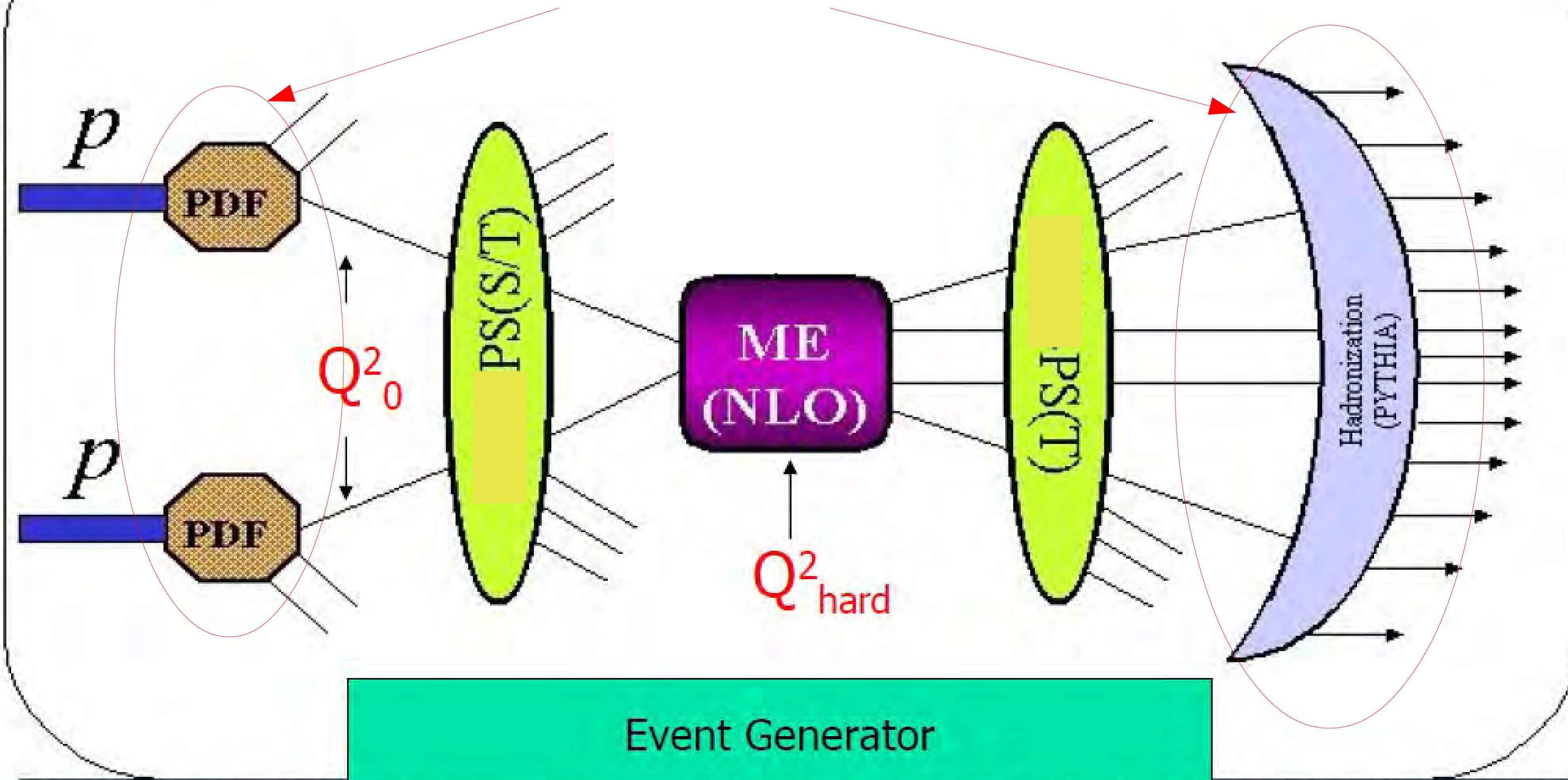
$$-\frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu}$$

Introduction/p-QCD



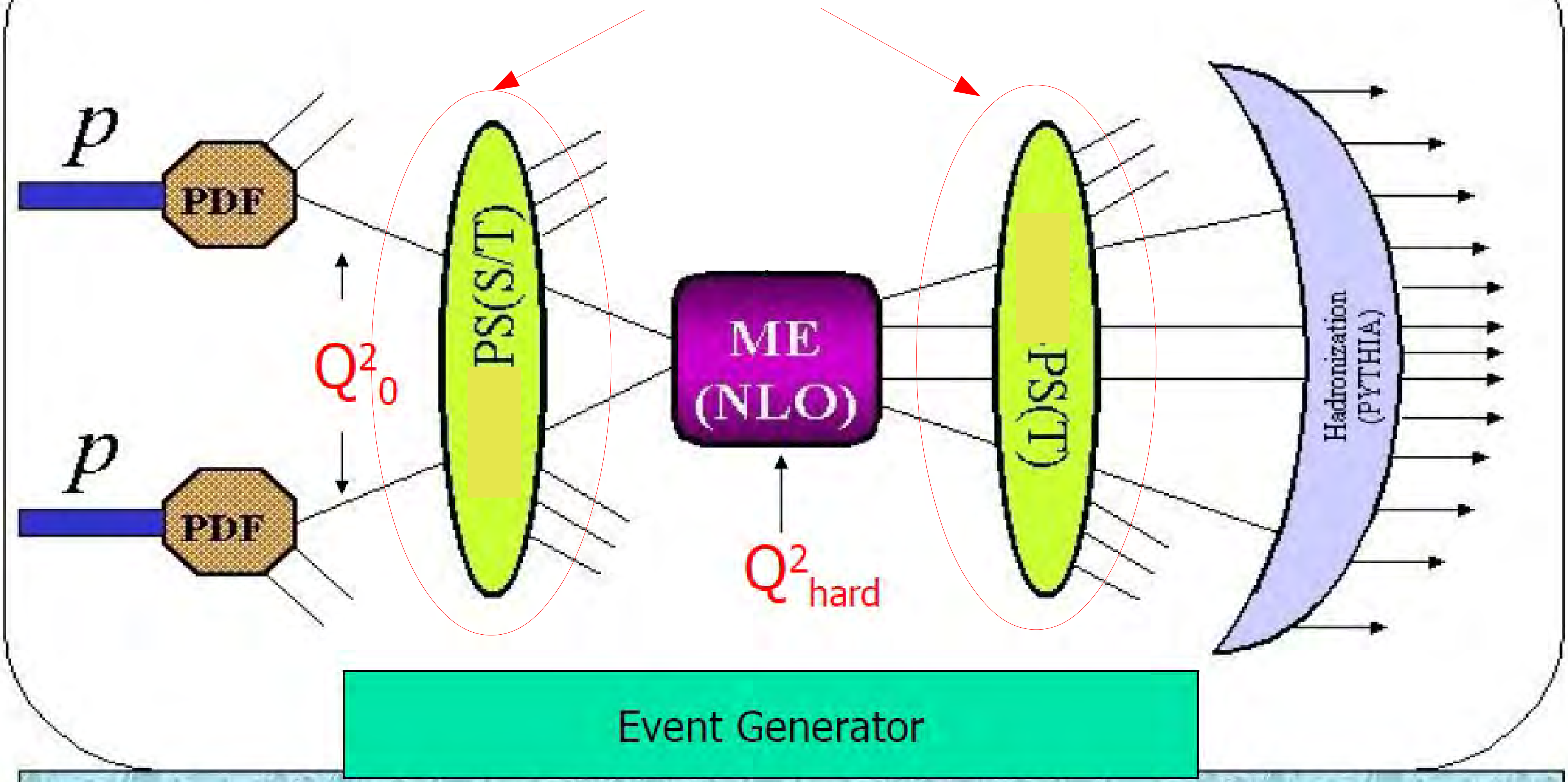
Introduction/p-QCD

Soft part = low Q2 part ← non perturbative effect



Introduction/p-QCD

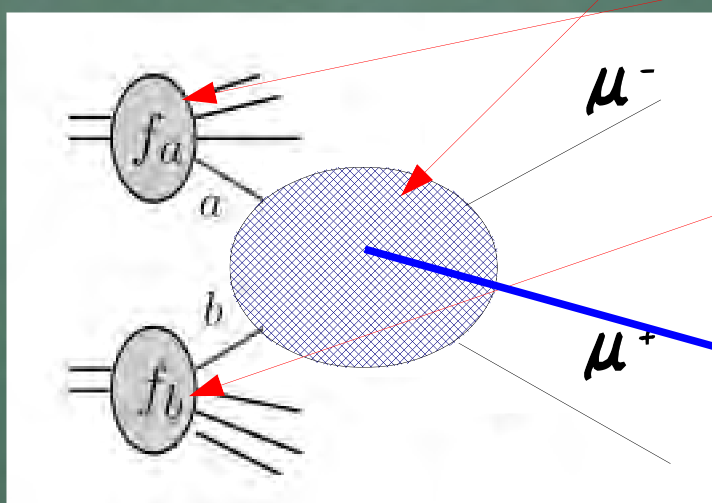
connection between low and high Q2 parts ← parton shower

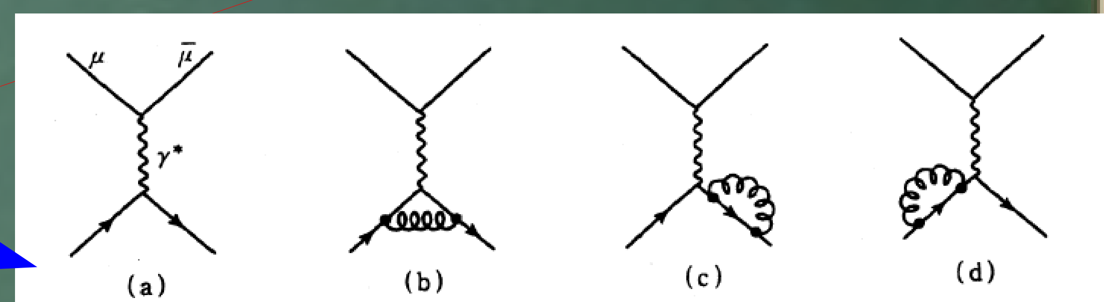


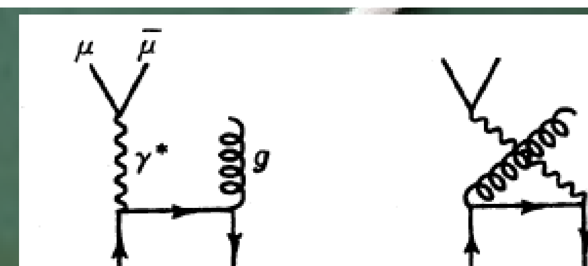
Introduction/p-QCD

Drell-Yan Process

$$d\sigma^{PP\to l^+l^-+X}(Q) \approx \sum_{a,b} d\hat{\sigma}^{ab\to l^+l^-+X}(x_1, x_2, z, Q, \mu) \otimes f_a(x_1,\mu) \otimes f_b(x_2,\mu)$$

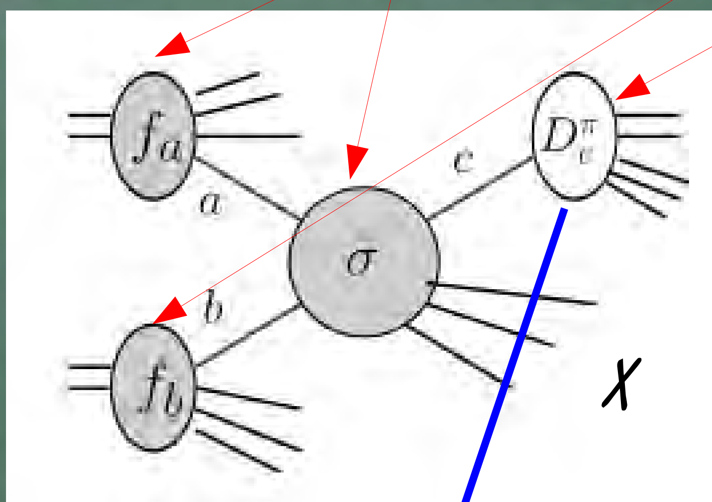


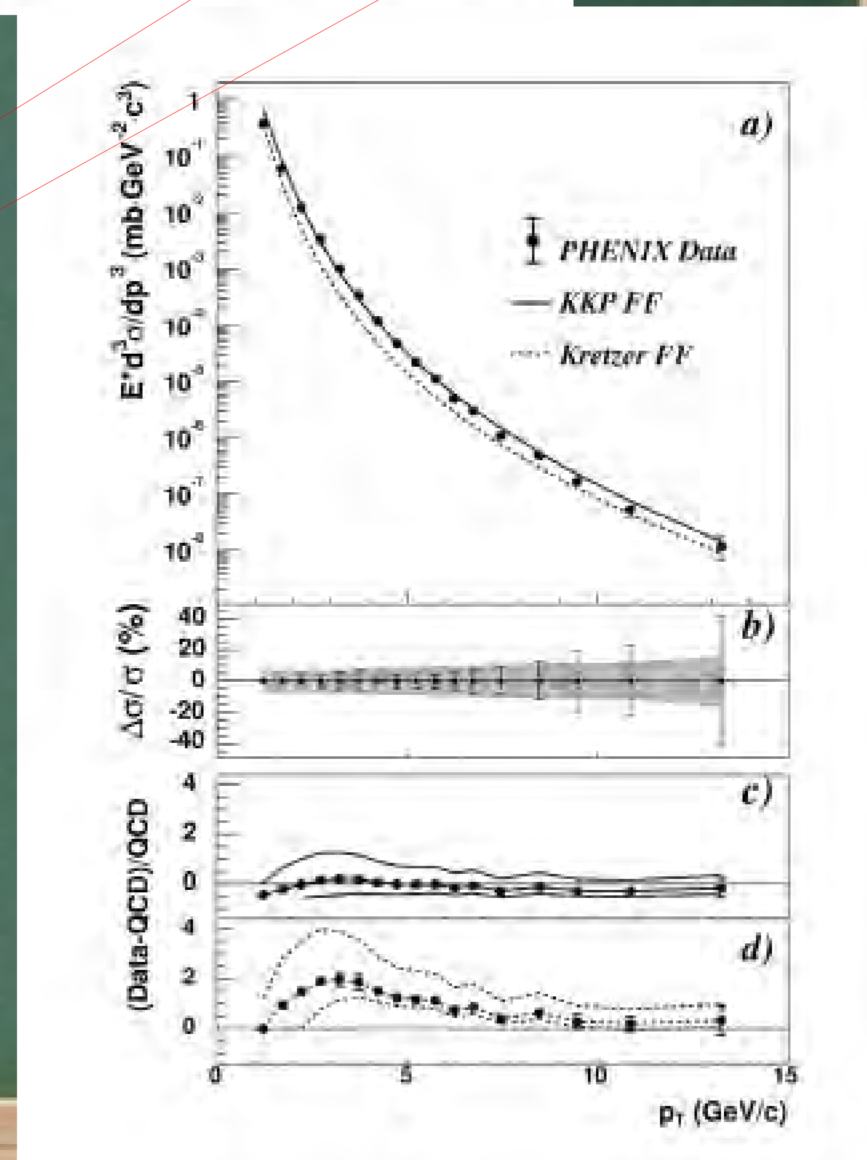




Introduction/p-QCD

$$d\sigma^{PP\to h+X}(p_T) \approx \sum_{a,b} d\hat{\sigma}^{ab\to cX}(x_1, x_2, z, p_T, \mu) \otimes f_a(x_1,\mu) \otimes f_b(x_2,\mu) \otimes D_c^{\pi}(z,\mu)$$

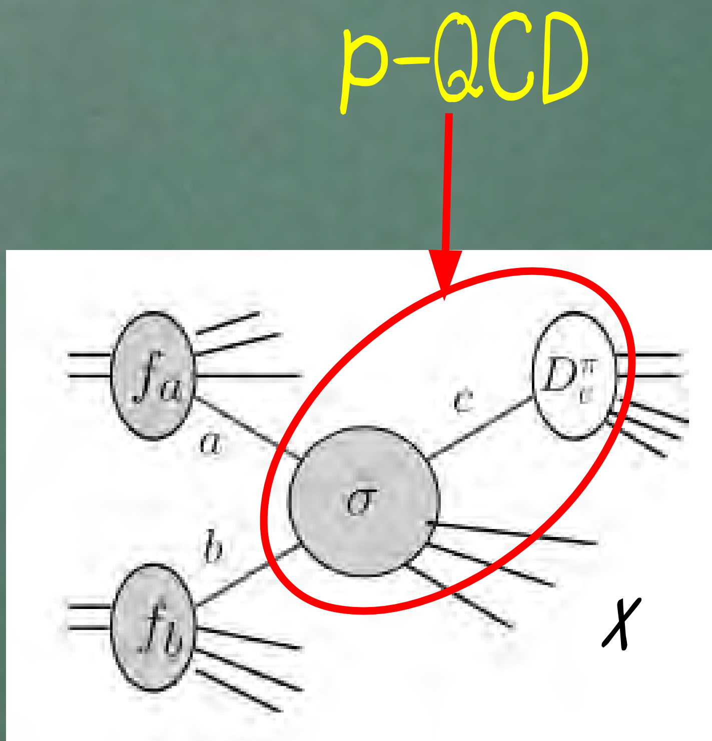


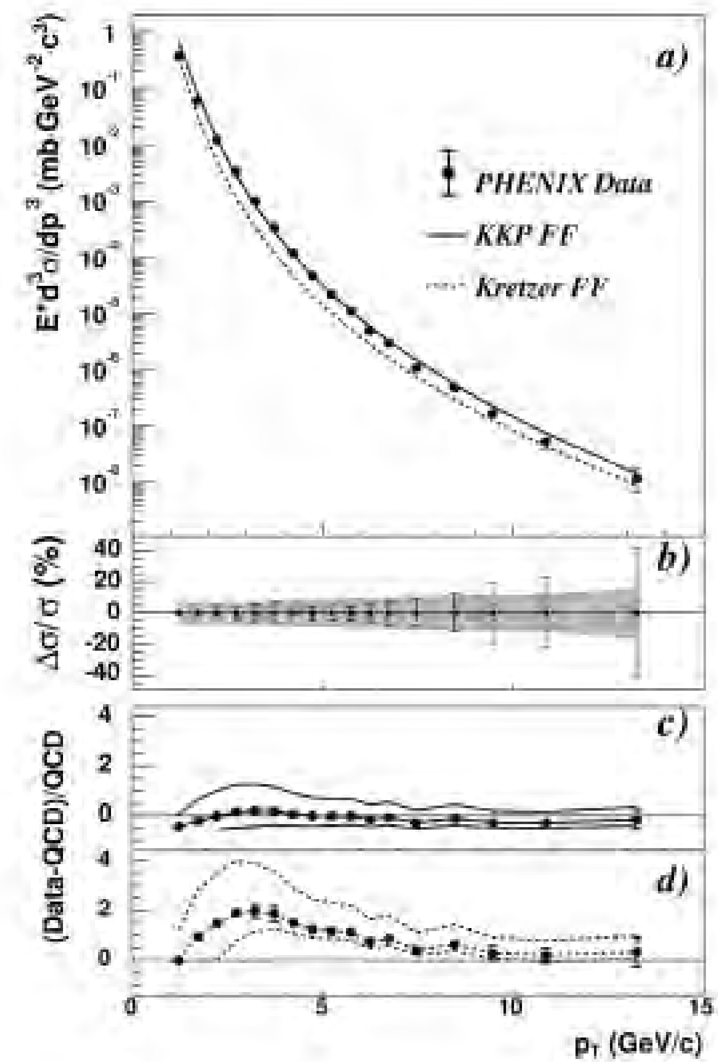


Fragmentation function

Introduction/p-QCD

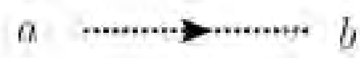
$$d\sigma^{PP\to h+X}(p_T) \approx \sum_{a,b} d\hat{\sigma}^{ab\to cX}(x_1, x_2, z, p_T, \mu) \otimes f_a(x_1,\mu) \otimes f_b(x_2,\mu) \otimes D_c^{\pi}(z,\mu)$$





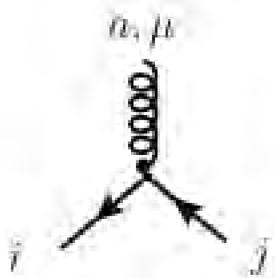
Introduction/Feynman Rule

$\frac{i}{k-m}\delta_{ij}$

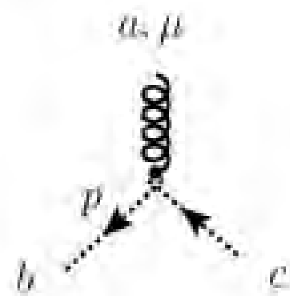


$\frac{-i}{k^2}\delta ab$

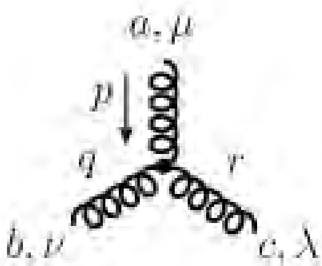
$\frac{-i}{k^2}\delta_{ab}\left(g_{\mu\nu}-(1-a_0)\frac{k_\mu k_\nu}{k^2}\right)$



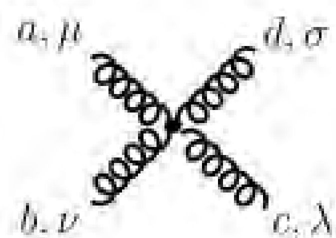
$ig_0 t^a_{ij}\gamma^\mu$



$g_0 f_{abc} p^\mu$



$-g_0 f^{abc} V^{\mu\nu\lambda}(p,q,r)$



$$-ig_0^2 f^{abe}f^{cde}(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda})$$
$$-ig_0^2 f^{ace}f^{bde}(g_{\mu\nu}g_{\lambda\sigma}-g_{\mu\sigma}g_{\nu\lambda})$$
$$-ig_0^2 f^{ade}f^{bce}(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\nu}g_{\lambda\sigma})$$

$$V_{\mu\alpha\beta}(p,-k-p,k)=(2k+p)_\mu g_{\alpha\beta}-(k-p)_\alpha g_{\beta\mu}-(k+2p)_\beta g_{\mu\alpha}$$

Outline

- Lesson 2
  - Renormalization
    - Toy example
    - quark self-energy
    - gluon vacuum-polarization
    - gluon vertex (3-point)
    - MS/$\overline{\text{MS}}$ Scheme

→ Divergent terms

GOAL: derivation of β-function
Running α_s
Asymptotic free

Renormalization/ β-function

Scale independent observables: $R(Q^2/\mu^2, \alpha_s(\mu^2))$

Renormalization Group Equation

$$\mu^2 \frac{d}{d\mu^2} R\left(Q^2/\mu^2, \alpha_s(\mu^s)\right) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right) R = 0$$

Renormalization Scale

Renormalization/ β-function

Scale independent observables: $R(Q^2/\mu^2, \alpha_s(\mu^2))$

Renormalization Group Equation

$$\mu^2 \frac{d}{d\mu^2} R\left(Q^2/\mu^2, \alpha_s(\mu^s)\right) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right) R = 0$$

β-function

Renormalization/ β-function

- What is β-function?

$$\beta(\alpha_s) = \alpha_s \frac{d\ln Z_A(\alpha_s(\mu))}{d\ln\mu^2}$$

$$\frac{d}{d\log\mu^2}\left(\frac{g_0^2}{(4\pi)^{d/2}} \equiv \mu^{2\epsilon}\frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma_E \epsilon}\right)$$

$$0 = \left(\epsilon + \frac{d\ln\alpha_s}{d\ln\mu^2} + \frac{d\ln Z_\alpha}{d\ln\mu^2}\right)\mu^{2\epsilon}\frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma_E \epsilon}$$

$$\frac{d\alpha_s}{d\ln\mu^2} = -\epsilon\alpha_s - \alpha_s\frac{d\ln Z_\alpha(\alpha_s(\mu))}{d\ln\mu^2}$$

Renormalization/ β-function

- What is β-function?

$$\beta(\alpha_s) = \alpha_s \frac{d \ln Z_A(\alpha_s(\mu))}{d \ln \mu^2}$$

$\varepsilon \to 0$

$$\frac{d\alpha_s}{d \ln \mu^2} = -\beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \beta_3 \alpha_s^5 + \cdots$$

β includes a μ^2 dependence of α_s!

Renormalization/ β-function

- 4-loop beta function

Ritbergen, Vermaseren, Larin ('97)

$$\beta(\alpha_s(Q^2)) = -\beta_0\alpha_s^2(Q^2) - \beta_1\alpha_s^3(Q^2) - \beta_2\alpha_s^4(Q^2) - \beta_3\alpha_s^5(Q^2) + \mathcal{O}(\alpha_s^6)$$

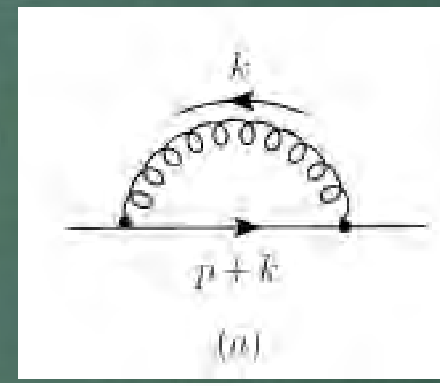
'73 '74 '80 '97

Loops & Leggs '04

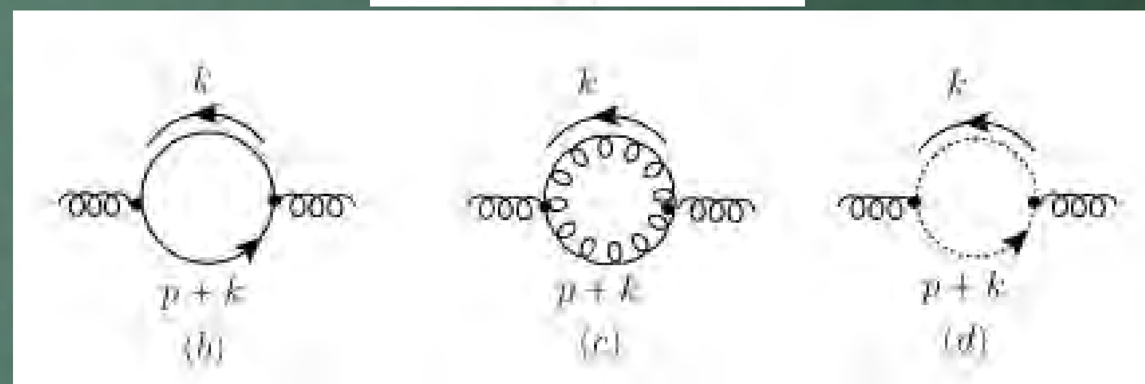
$$\begin{aligned}
\beta_0 &= \frac{33-2N_f}{12\pi}\,,\\
\beta_1 &= \frac{153-19N_f}{24\pi^2}\,,\\
\beta_2 &= \frac{77139-15099N_f+325N_f^2}{3456\pi^3}\,,\\
\beta_3 &\approx \frac{29243-6946.3N_f+405.089N_f^2+1.49931N_f^3}{256\pi^4}\,,
\end{aligned}$$

Introduction/Feynman Rule

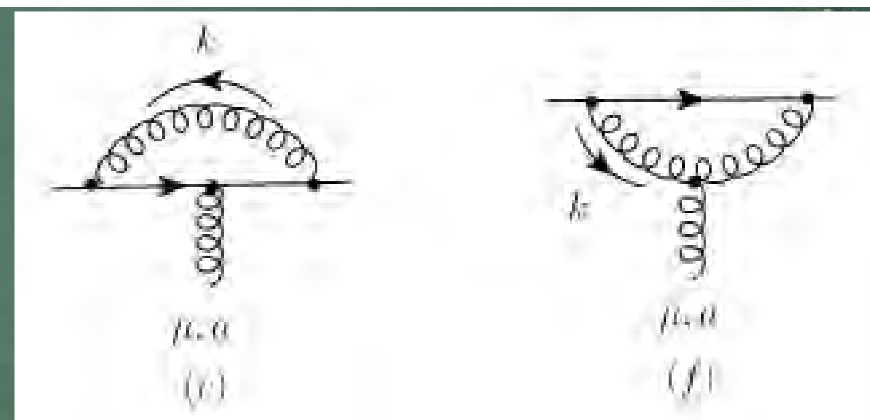
quark self-energy



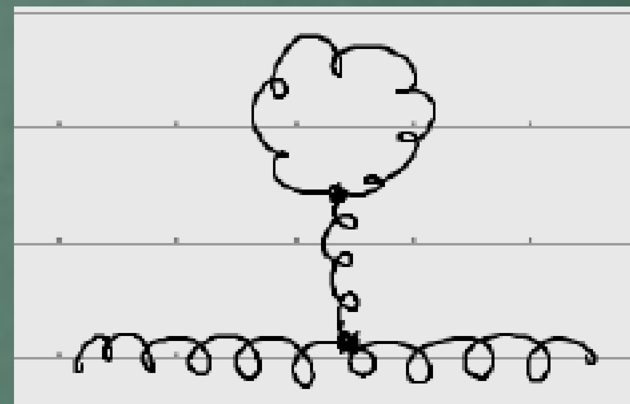
gluon
vacuum-polarization



quark gluon vertex



Introduction/Feynman Rule



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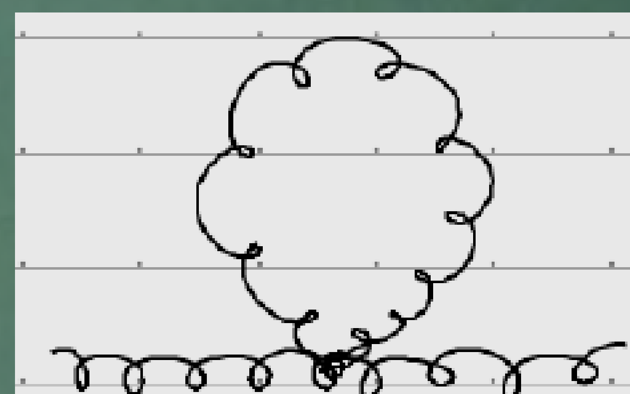
gluon
vacuum-polarization

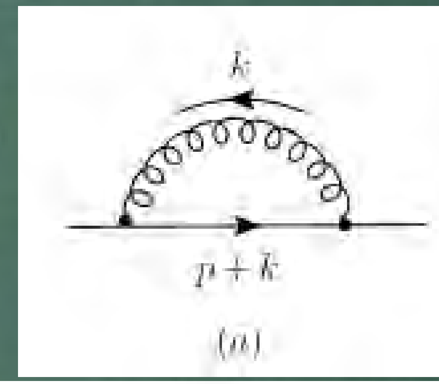


=0

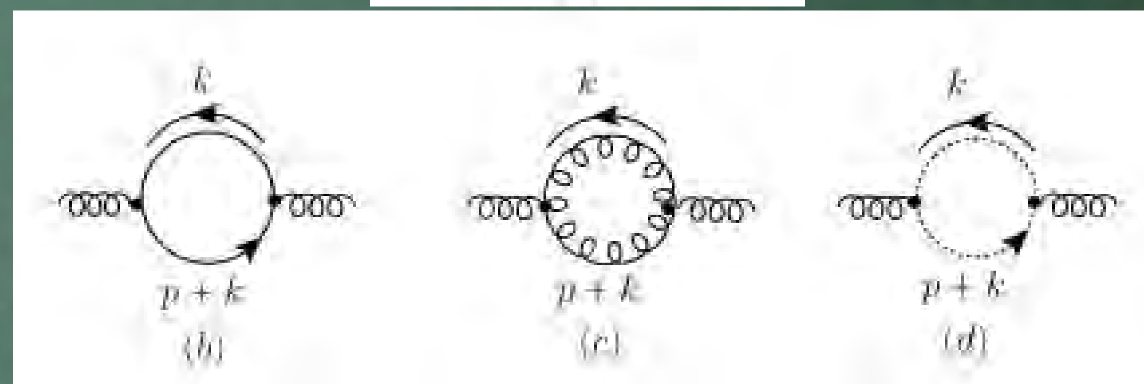
Exercise: Show this results.

Introduction/Feynman Rule

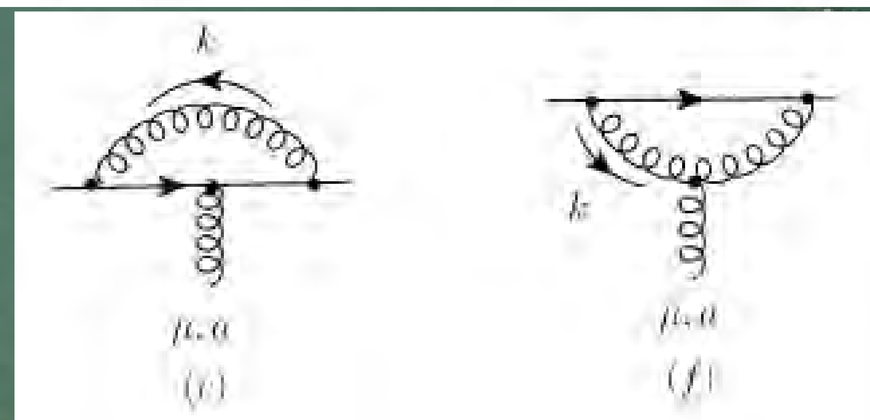
quark self-energy



gluon
vacuum-polarization



quark gluon vertex



Introduction/Feynman Rule

quark self-energy

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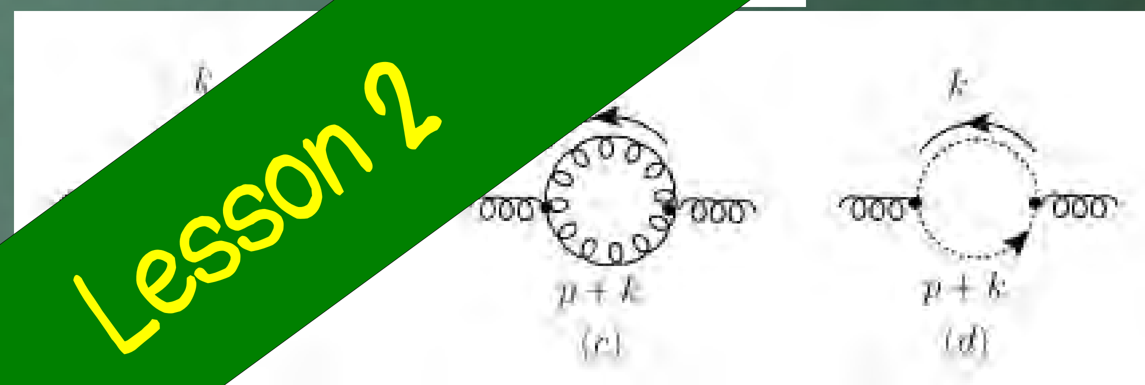
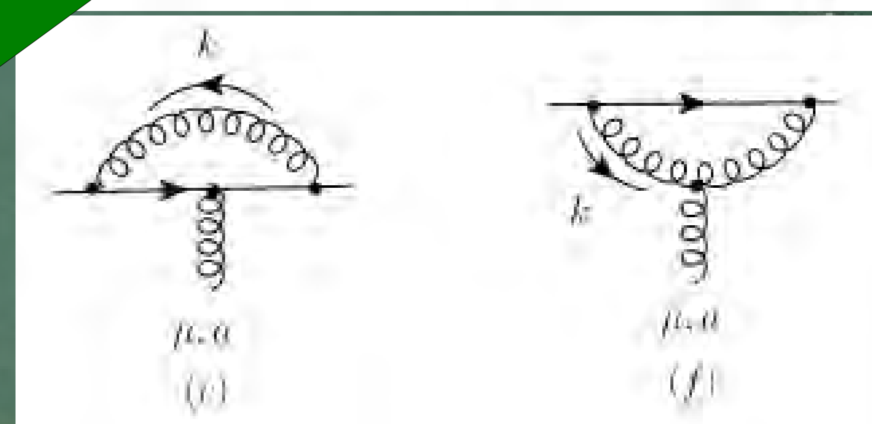


quark g x



Lesson 2