Introduction to Supersymmetry and MSSM

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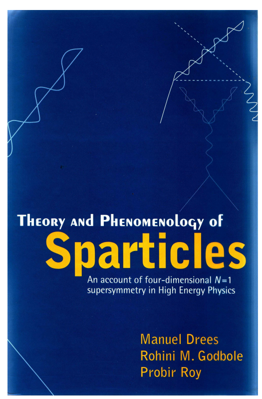
Based on:

M. Drees, R.M. Godbole and P. Roy,

Theory and phenomenology of sparticles, World Scientific, 2005



- Current state of play in HEP. SUSY: why it is the Standard Beyond the Standard Model Physics idea!
- Hierarchy problem : Raison de tré for SUSY?
- SUSY algebra, Superfields, Construction of Supersymmetric Lagrangians
- MSSM, Higgs sector, Softly broken SUSY.
- Connection of high scale physics to $\mathcal{L}_{SOFT}$
- Connection of parameters of $\mathcal{L}_{SOFT}$ to phenomenology
- Current colliders and MSSM?

There is a lot that is soo...right with the SM and there is a lot that is not so right!

Supersymmetry is an attractive cure for some of the ills!

The idea of SUSY is theoretically so appealing that even if SUSY had not provided this cure, theorists would have still explored it. But because it does do so, it is likely that it is realised at the TeV scale and hence relevant for the LHC :-)

SUSY:

1) J. Wess and J.A. Bagger, Supersymmetry and Supergravity, Princeton University Press, 2nd edition (Princeton, 1993).

2) P. West Introduction to Supersymmetry and Supergravity, World Scientific (Singapore, 1990).

SUSY + MSSM + Colliders + DM expt + Cosmology

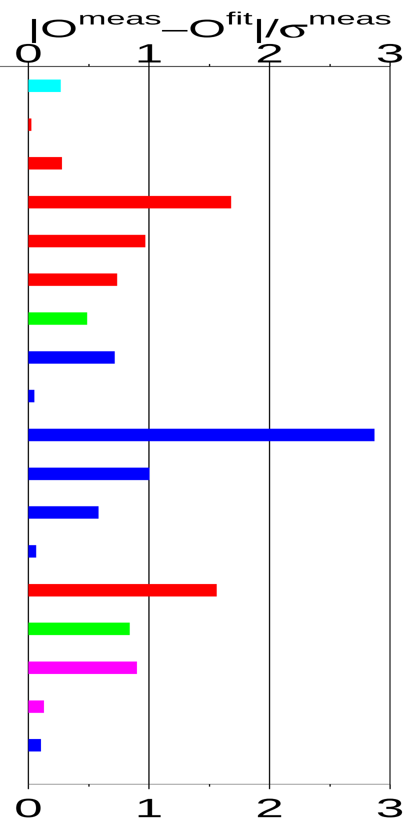
3)S.P. Martin, Supersymmetry primer: 9709356v5. Also in Perspectives of Supersymmetry, edited by G. Kane et al.

4) M. Drees, R.M. Godbole and P. Roy, Theory and phenomenology of sparticles, World Scientific, 2005.

5) H. Baer and X. Tata, "Weak scale Supersymmetry: From superfields to scattering events," *Cambridge, UK: Univ. Pr. (2006)* .

see http://lepewwg.web.cern.ch

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4958
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01644
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21582
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.025	80.376
Γ_W [GeV]	2.098 ± 0.048	2.092
m_t [GeV]	172.4 ± 1.2	172.5

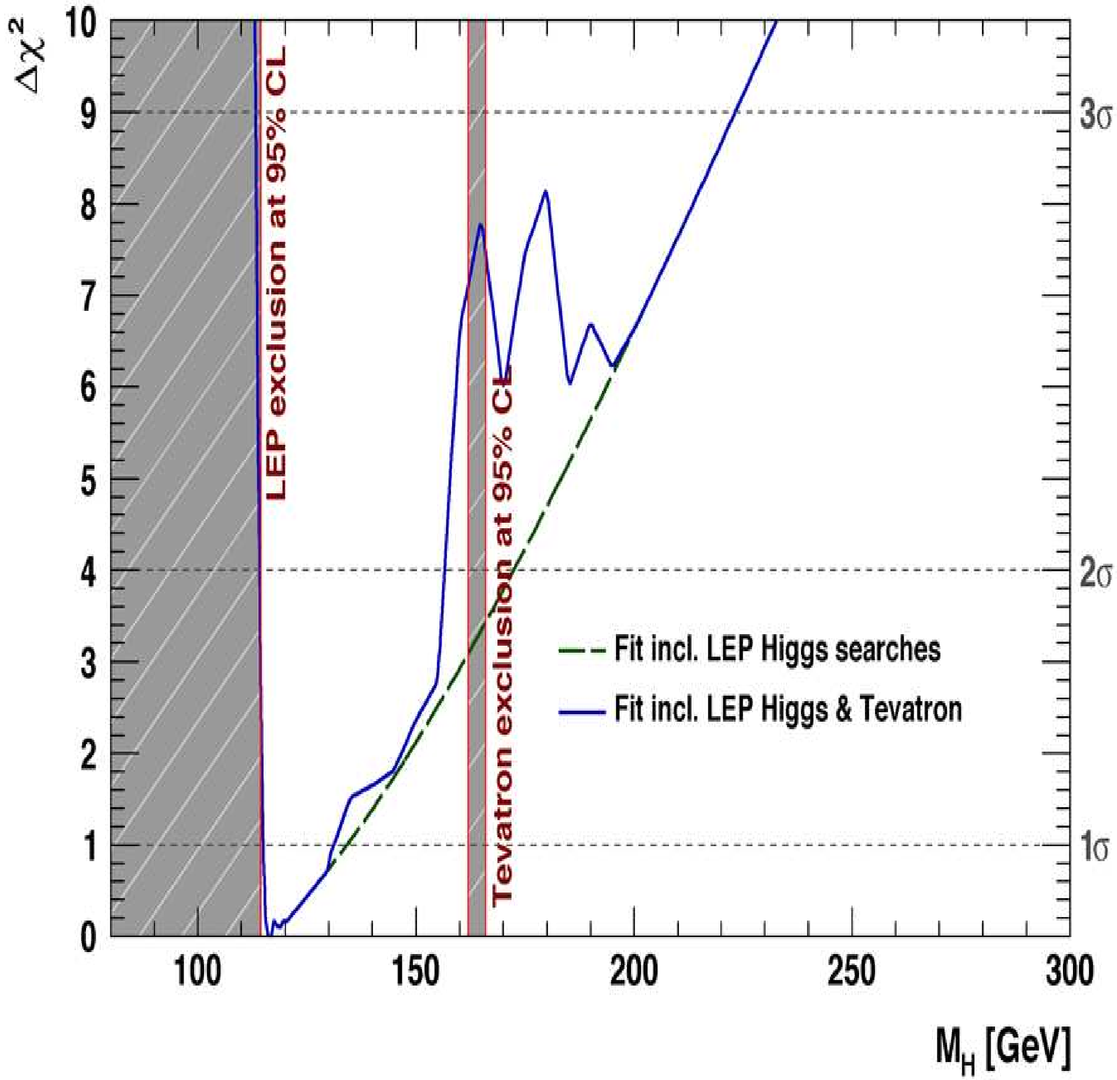


July 2008

What do the EW precision measurements mean for the Higgs?

The loop corrections depend on the Higgs mass. Since that is the only unknown these measurements indirectly constrain the Higgs mass.

If all the current information is put together the Higgs mass should be less than 150 GeV. (**indirect experimental limit!**). Higgs is **LIGHT**.



Can the SM keep the Higgs 'light' naturally?

Theoretical and esthetic issue.

One of the strong motivations for BSM.

SUSY provides arguably the most elegant solution to this deep problem!

- Dark Matter makes up 23% of the Universe.!
- Direct evidence for the nonzero ν masses (RP violation)
- Baryon Asymmetry in the Universe! (CP violation)

- **Instability of the EW scale under radiative corrections.**
- Need to get a basic understanding of the flavour problem
- Unification of couplings

SUSY offers attractive solutions to the item 4 and lends almost naturally to 1 and 6. item 2 also can be accomodated in SUSY models. For item 5, SUSY perhaps does not have any special advantages. (talks by U. Niereste)

- Why **TeV scale SUSY?**

- A possible cold dark matter candidate

- A light Higgs boson consistent with indirect constraints from precision EW measurements and some more ease in satisfying precision constraints.

- Unification of gauge couplings

- Esthetic beauty

Most important:

Hierarchy problem and stablisation of Higgs mass at EW scale.

Supersymmetry (SUSY) is a case of experiments and theorists pursuing a very pretty theoretical idea, which has eluded discovery so far!

Particle states classified according to the representation of the Poincare group they belong to.

A supersymmetry transformation connects ferrmionic and bosonic states.

Supersymmetry is the only possible extension of the known space time symmetries of the particle interactions.

Non-renormalisation theorem: a simple statement is that the vacuum energy does not receive perturbative quantum corrections in a supersymmetric theory.

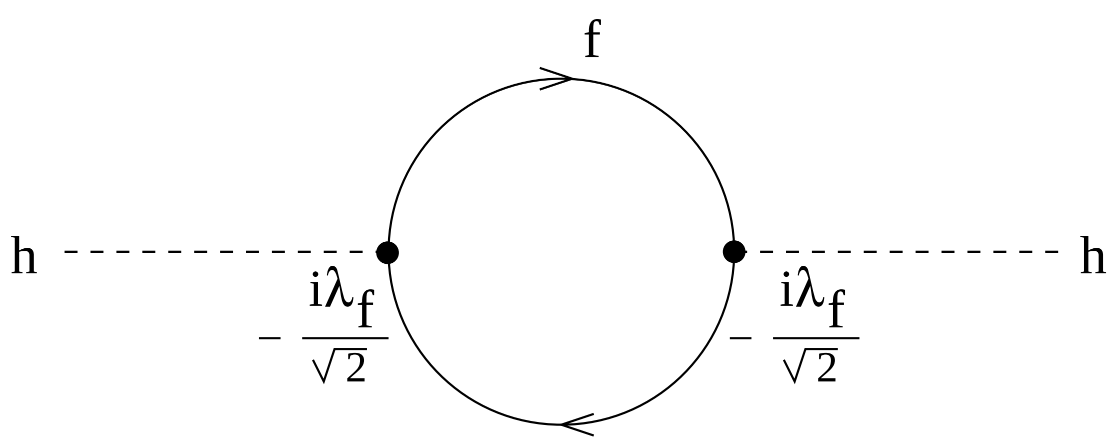
Supersymmetric quantum field theories are 'natural' in spite of the scalars in the theory.

The last two points make it a particularly attractive solution to solve 'one' of the ills of the SM and 'stabilise' the EW symmetry breaking scale against radiative corrections.

A toy model to explain 'stablisation' :

$$\mathcal{L} = -\lambda_f h \bar{f} f - \lambda_f v \bar{f} f$$

Consider the corrections to the $(mass)^2$ due to fermion loops.



$$\Pi^f_{hh}(0) = -2\lambda_f^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2}\right].$$

First term is quadratically divergent and independent of m_S. Renormalising quadratic divergence gives corrections proportional to loop factor $\times m_f^2$ (for GUTS $10^{32} GeV^2$).

m_S required to be bounded above by unitarity argument and hence less than 1 TeV. Small m_S not 'natural' and thus destabilised by loop corrections away from the EW scale.

Thus fine tuning to one part in 10^{26} if we have GUTS.

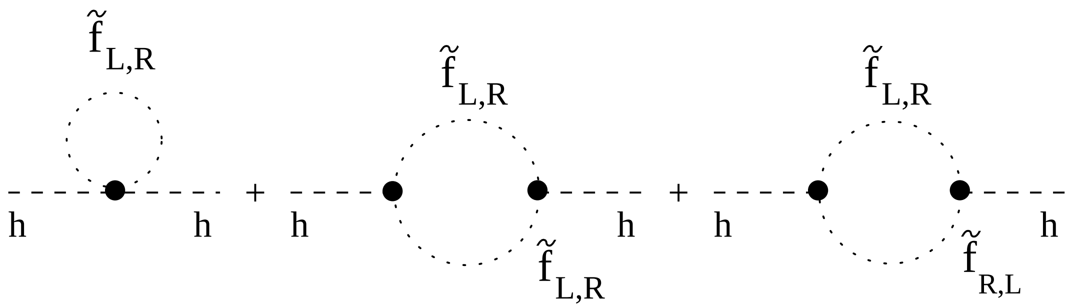
Cure provided by SUSY:

W. Fischler, H.P. Nilles, J. Polchinski, S. Raby and L. Susskind, Phys. Rev. Lett. **47** (1981) 575;

R. Kaul and P. Majumdar, Print 81-0373 unpublished and Nucl. Phys. **B199**

Add two complex scalar fields and hence additional interaction terms between these and h.

$$\begin{aligned}
\mathcal{L}_{\tilde{f}\tilde{f}\phi} &= \tilde{\lambda}_f |\phi|^2 (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) + (\lambda_f A_f \phi \tilde{f}_L \tilde{f}_R^\star + \text{h.c.}) \\
&= \frac{1}{2}\tilde{\lambda}_f h^2 (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) + v\tilde{\lambda}_f h (|\tilde{f}_L|^2 + |\tilde{f}_R|^2) \\
&\quad + \frac{h}{\sqrt{2}} (\lambda_f A_f \tilde{f}_L \tilde{f}_R^\star + \text{h.c.}) + \cdots .
\end{aligned}$$



The contributions due to fermion loops are cancelled by those from the scalar loops, if certain conditions among the couplings are obeyed, along with equal masses of the scalars and the fermion f.

SUSY gurantees the equalities and thus Higgs mass will not be destabilised by loop corrections.

Quadratic divergence cancelled if

$$\tilde{\lambda}_f = -\lambda_f^2$$

Independent of $m_{\tilde{f}_L}, m_{\tilde{f}_R}, A_f$

In addition logarithmic divergence **also** cancelled if $m_{\tilde{f}_R} = m_{\tilde{f}_L} = m_f$ and $A_f = 0$.

As long as $\tilde{\lambda}_f = -\lambda_f^2$

• If $m_f \neq m_{\tilde{f}_L} \neq m_{\tilde{f}_R}$, $A \neq 0$, the loop corrections have a logarithmic divergence. Renormalise.

• The loop corrections to mass then should be of the order $(TeV)^2$ if the scalar has to be 'naturally' light.

• Some symmetry should gurantee the equality of couplings!

• Criticism: introducing more particles to cure the problem!

But this is not the first time we did it. Recall the positron which was postulated due to the space time invariance.... Also cured a divergence.

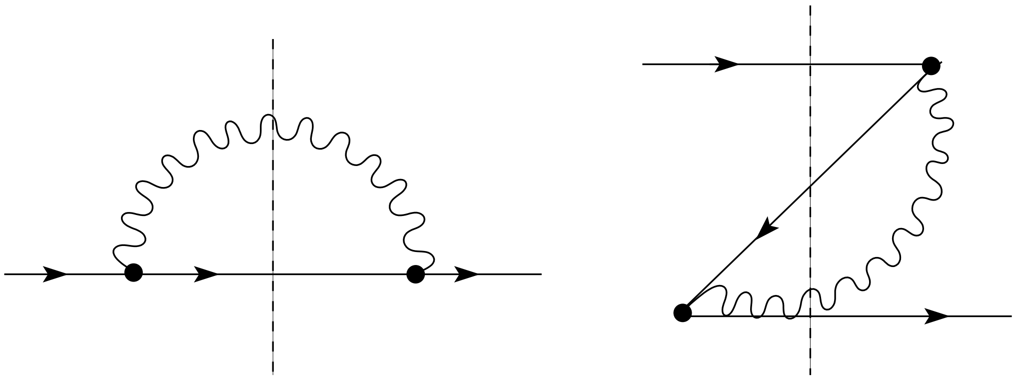
Analogy : highlighted by Murayama : hep-ph/9410285, hep-ph/0002242.

Correction to electron mass due to self energy:

$$E^{cl.}_{\mathrm{self}} = \frac{3}{5}(e^2/4\pi r_e) = 1(MeV/c^2)\left(\frac{0.86 \times 10^{-15} meters}{r_e}\right)$$

Smaller the value of r_e more the fine tuning needed to have the physical mass of 0.511 MeV.

If $r_e \simeq 10^{-17}$ meters, fine tuning of the elecron mass to one part in 100.



1.1 (a) 1.1 (b)

$$E_{\text{self}}^{quant.} = \frac{3e^2 m_e}{16\pi^2} \ln(m_e r_e)$$

Linear divergence cured by positron contribution to logarithmic divergence. Lorentz invariance reduced the degreee of divergence.

Proportional to m_e which is 'naturally' small due to chiral symmetry!

Even with replacing r_e by $\lambda_{Pl} = M_{Pl}^{-1}$, the Quantum correction will be only 10% of m_e.

Supersymmetry transforms Bosons to Fermions and vice versa.

$Q|\text{boson}, \text{fermion}\rangle = |\text{fermion}, \text{boson}\rangle$

Q is an anticommuting spinor.

Easy to check:

$[Q, M_{\mu\nu}] \neq 0$ **where** $M_{\mu\nu}$ **are the generators of the homogeneous Lorentz group**

Form of theory restricted by Haag Łopuszánski, Sohnius extension of the Coleman-Mandula theorem.

Poincare algebra gets extended:

$$[Q_a, Q_b]_+ = -2(\gamma^\mu C)_{ab} P_\mu, \quad [Q_a, P_\mu] = 0$$

$$[M_{\mu\nu}, Q_a] = -(\Sigma_{\mu\nu})_{ab} Q_b, \quad [Q_a, R] = (\gamma_5)_{ab} Q_b$$

a is the spinorial index, R is the additional $U(1)$ invariance that the algebra has.

Schematically,

$[T_i, Q] = [T_i, Q^\dagger] = 0$

T_i are generators of the gauge group.

Single particle states of this theory are described by irreducible representation of this algebra, called supermulitplet.

P_μ and gauge generators commute with Q.

All members of a supermultiplet will have the same $(\text{mass})^2$ and gauge quantum numbers for unbroken supersymmetry.

SUSY breaking will split the masses of different members of a given multiplet.

The number of fermionic and bosonic degress of freedom should be equal in a given supermultiplet.

Will mention only chiral(matter) and vector (gauge) supermulitplets here.

Physical interpretation of scalar field: spin $= 0$ and

$$\begin{aligned} \phi(x): &\quad \text{particle} \\ \phi^\dagger(x): &\quad \text{antiparticle} \end{aligned}$$

Physical interpretation of spinor fields: spin $= \frac{1}{2}$ and

$$\begin{aligned} \xi_A(x): &\quad \text{left-handed fermion} \\ \xi^\dagger{}_{\dot{A}}(x) \equiv (\xi_A(x))^\dagger: &\quad \text{right-handed antifermion} \\ \overline{\xi}_A(x): &\quad \text{left-handed antifermion} \\ \overline{\xi}^\dagger{}_{\dot{A}}(x) \equiv (\overline{\xi}_A(x))^\dagger: &\quad \text{right-handed fermion} \end{aligned}$$

Physical interpretation of vector fields:spin $= 1$ and

$$\begin{aligned} A_{A\dot{B}}(x): &\quad \text{particle} \\ (A_{A\dot{B}})^\dagger(x): &\quad \text{antiparticle} \end{aligned}$$

A $\dagger$ denotes complex conjugation but a $\overline{}$ denotes an independent field.

SUSY : bosons and fermions belong to the same multiplet.

Consider the free massive action for a complex scalar and a Majorana fermion of the same mass:

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \phi^\dagger - m^2 \phi \, \phi^\dagger + \frac{i}{2} \xi \sigma^\mu \overleftrightarrow{\partial}_\mu \xi^\dagger - \frac{m}{2} (\xi\xi + \xi^\dagger \xi^\dagger)$$

This action is invariant upto a total derivative under the infinitesimal variations:

$$\begin{aligned} \delta\phi &= \epsilon \, \xi \\ \delta\xi &= -i\sigma^\mu \partial_\mu \phi \, \epsilon^\dagger - m\phi^\dagger \, \epsilon \end{aligned}$$

where ϵ is a (constant) anticommuting spinor.

This is a (free) supersymmetric theory with the physical fields ϕ, ξ of a chiral multiplet.

Note that the propagating degrees of freedom are 2 bosonic + 2 fermionic, equality being a precondition for supersymmetry.

Use of auxiliary fields necessary so that algebra closes off shell as well.

To discuss Supersymmetric theories with interactions necessary to talk in the Superfield language where SUSY is manifest.

For this, adjoin to the spacetime coordinates x^μ the anticommuting coordinates $\theta_A, \theta^{\dagger\dot{A}}$.

A general superfield is a field $f(x, \theta, \theta^\dagger)$ depending on all the coordinates (commuting and anticommuting) of superspace. and will be a Taylor expansion in $\theta, \theta^\dagger$ with a finite number of terms. A given superfield will involve lots of ordinary fields with different spins.

Combined invariance under SUSY and Lorentz transformations means the superfields need to satisfy some constraints.

Two types of Superfields: Chiral Superfield and Vector Superfield.

The Taylor expansion of a chiral superfield is:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\,\theta\xi(y) + \theta\theta F(y)$$

where $y^\mu = x^\mu - i\theta\sigma^\mu\theta^\dagger$.

If ξ represents a conventional fermion (quark/lepton) then ϕ will be a sfermion (squark/slepton).

Similarly the Taylor expansion of a vector superfield in the Wess-Zumino gauge is:

$$\begin{aligned} V(x,\theta,\theta^\dagger) &= \theta\sigma^\mu\theta^\dagger A_\mu(x) + \theta\theta\,\theta^\dagger\lambda^\dagger(x) + \theta^\dagger\theta^\dagger\theta\lambda(x) \\ &\quad + \tfrac{1}{2}\,\theta\theta\,\theta^\dagger\theta^\dagger D(x) \end{aligned}$$

Since A_μ is a gauge field λ is a gaugino.

Vector superfield written in Wess Zumino gauge, using the Gauge transformation freedom.

Thus a chiral supermultiplet contains fermions and their scalar partners and vector supermultiplet contains the gauge bosons and the gauginos.

Supersymmetric Lagrangians: For this we need the rules for integrating over anticommuting coordinates:

$$\int d\theta_1\,\theta_1 = \int d\theta_2\,\theta_2 = 1$$
$$\int d\theta_1 = \int d\theta_2 = 0$$

and similarly for the dotted coordinates.

Define $d^2\theta = \frac{1}{2}d\theta_1 d\theta_2$. Then:

$$\int d^2\theta\ \theta\theta = \int \tfrac{1}{2}d\theta_1 d\theta_2\,.\,2\theta_2\theta_1 = 1$$

Under the supersymmetry transformations we described, the F-term of a chiral superfield and the D-term of a vector superfield transform as total derivatives. Therefore their variations vanish under $\int d^4x$.

Prescription for generating Supersymmetric actions is simple: $\int d^2\theta$ on a chiral superfield, and $\int d^2\theta d^2\theta^\dagger$ on a vector superfield, pick out the "last" terms (F-term and D-term respectively).

Combining the above facts, general actions of the form:

$$\int d^4x\, d^2\theta \;\; \text{(chiral superfield)}, \;\; \int d^4x\, d^2\theta d^2\theta^\dagger \;\; \text{(vector superfield)}$$

are automatically supersymmetric.

- As the canonical dimension of θ is $-\frac{1}{2}$, and that of a chiral superfield Φ is $+1$, one possible free (quadratic) action is:

$$S_D = \int d^4x\, d^2\theta\, d^2\theta^\dagger \quad \Phi^\dagger(y^\dagger, \theta^\dagger)\, \Phi(y, \theta)$$

  where recall that $y^\mu = x^\mu - i\theta\sigma^\mu\theta^\dagger$.

- First shift $y^\mu \to x^\mu$ in the integrand under $\int d^4x$ and thereby remove the $\theta^\dagger$ dependence. After this, i$\int d^2\theta$ picks out the term proportional to $\theta\theta$ as desired. With the equation of motion of an auxiliary field, we arrive at the familiar action:

$$S = \int d^4x \left\{ \partial_\mu\phi\, \partial^\mu\phi^\dagger - |m|^2\phi\,\phi^\dagger + \frac{i}{2}\,\xi\sigma^\mu\overleftrightarrow{\partial}_\mu\xi^\dagger - \frac{m}{2}(\xi\xi + \xi^\dagger\,\xi^\dagger\,) \right\}$$

- The payoff is that we can now generalise this to an interacting supersymmetric action.

- For a set of chiral superfields $\Phi^i, i = 1, 2, \cdots, n$, the most general action is:

$$S = \int d^4x\, d^4\theta\, K(\Phi^i, \Phi^{\dagger i}) + \int d^4x (d^2\theta\, \mathcal{W}(\Phi^i) + \text{c.c.})$$

- Here K is an arbitrary real function of $\Phi^i, \Phi^{\dagger i}$, called the Kähler potential, and $\mathcal{W}$ is an arbitrary analytic function of Φ^i alone, called the superpotential.

Not presented details of writing the free gauge lagrangian as well as the interactions.

Needless to say gauge invariance gave us interaction of matter fields with gauge fields, here also gauge invariance and SUSY together, predict precisely interactions of all the (s)particles with gauge fields and among themselves.

Important: there are two sources of scalar self couplings and two sources of the Yukawa couplings.

In the SM, corrections to the Higgs mass are quadratically divergent. This is the root of the naturalness problem in grand unified theories.

With supersymmetry, the non-renormalisation theorem says that perturbatively, superpotential terms receive no quantum corrections.

As an example, $\int d^2\theta \, m \, \Phi^2$ is uncorrected.

This does not mean there is no mass renormalisation in these theories! Rather, it means that the mass renormalisation is equal and opposite to the wave-function renormalisation, which comes from D-terms and is logarithmic.

We can now introduce the Minimal Supersymmetric Standard Model (MSSM).

The motivation, as said before, amelioration of the naturalness problem.

In the MSSM, all left-handed fermions and left-handed antifermions of the SM are promoted to chiral superfields:

$$L_1 = \begin{pmatrix} L_{\nu_e} \\ L_e \end{pmatrix}, \overline{E}_1, \overline{N}_1; \ L_2 = \begin{pmatrix} L_{\nu_\mu} \\ L_\mu \end{pmatrix}, \overline{E}_2, \overline{N}_2; \ L_3 = \begin{pmatrix} L_{\nu_\tau} \\ L_\tau \end{pmatrix}, \overline{E}_3, \overline{N}_3$$

$$Q_1 = \begin{pmatrix} Q_u \\ Q_d \end{pmatrix}, \overline{U}_1, \overline{D}_1; \ Q_2 = \begin{pmatrix} Q_c \\ Q_s \end{pmatrix}, \overline{U}_2, \overline{D}_2; \ Q_3 = \begin{pmatrix} Q_t \\ Q_b \end{pmatrix}, \overline{U}_3, \overline{D}_3$$

We will ignore the existence of left-handed anti-neutrinos $\overline{N}_i$ since technically they are not part of the MSSM.

The component expansions of these superfields are, for example:

$$
\begin{aligned}
L_e(y,\theta) &= \tilde{e}(y) + \sqrt{2}\theta\, e(y) + \cdots \\
\overline{E}_1(y,\theta) &= \tilde{\bar{e}}(y) + \sqrt{2}\theta\, \bar{e}(y) + \cdots
\end{aligned}
$$

The scalar components of these superfields are the left selectron $\tilde{e}$ and the left spositron $\tilde{\bar{e}}$.

Being scalars they do not have a handedness. The term left refers not to their chirality but to that of their superpartners.

Also, clearly $\tilde{\bar{e}}$ is **not** the antiparticle of $\tilde{e}$.

The antiparticles are found in the anti-chiral superfields:

$$
\begin{aligned}
L_e^\dagger(y^\dagger,\theta^\dagger) &= \tilde{e}^\dagger(y^\dagger) + \sqrt{2}\theta^\dagger\, e^\dagger(y^\dagger) + \cdots \\
\overline{E}_1^\dagger(y^\dagger,\theta^\dagger) &= \tilde{\bar{e}}^\dagger(y^\dagger) + \sqrt{2}\theta^\dagger\, \bar{e}^\dagger(y^\dagger) + \cdots
\end{aligned}
$$

with $\tilde{e}^\dagger$ being the right spositron and $\tilde{\bar{e}}^\dagger$ the right selectron.

Similarly we can write the superfields corresponding to quarks.

• Next we extend the gauge fields of the SM to vector superfields:

$$\begin{array}{rcl} B_\mu(x) & \rightarrow & V_Y(x,\theta,\theta^\dagger) \\ W^a_\mu(x) & \rightarrow & V^a_W(x,\theta,\theta^\dagger), \quad a=1,2,3 \\ g^{a'}_\mu(x) & \rightarrow & V^{a'}_g(x,\theta,\theta^\dagger), \quad a'=1,2,\cdots,8 \end{array}$$

• The $SU(2)$ and $SU(3)$ vector superfields are converted to matrices $\mathbf{V}_W, \mathbf{V}_g$ as described earlier.

• The component expansions of these superfields contain fermion partners called gauginos:

$$\begin{aligned} V_Y(x,\theta,\theta^\dagger) &= \theta\sigma^\mu\theta^\dagger\, B_\mu(x) + \theta\theta\,\theta^\dagger\lambda^\dagger_B(x) + \theta^\dagger\theta^\dagger\,\theta\lambda_B(x) + \cdots \\ \mathbf{V}_W(x,\theta,\theta^\dagger) &= \theta\sigma^\mu\theta^\dagger\, \mathbf{W}_\mu(x) + \theta\theta\,\theta^\dagger\boldsymbol{\lambda}^\dagger_W(x) + \theta^\dagger\theta^\dagger\,\theta\boldsymbol{\lambda}_W(x) + \cdots \\ \mathbf{V}_g(x,\theta,\theta^\dagger) &= \theta\sigma^\mu\theta^\dagger\, \mathbf{g}_\mu(x) + \theta\theta\,\theta^\dagger\boldsymbol{\lambda}^\dagger_g(x) + \theta^\dagger\theta^\dagger\,\theta\boldsymbol{\lambda}_g(x) + \cdots \end{aligned}$$

• $\lambda_B, \boldsymbol{\lambda}_W, \boldsymbol{\lambda}_g$ are known as binos, winos and gluinos respectively.

Finally, we need to introduce chiral superfields for the Higgs boson.

• A novel feature of supersymmetry: while in the SM a single Higgs doublet gives masses to all quarks and leptons, in the MSSM that is not possible.

• The reason is that chiral superfields couple through a complex analytic superpotential.

Thus out of the SM couplings:

$$-f^{(u)}(q\cdot\phi)\,\bar{u} - f^{*\,(u)}(q^\dagger\cdot\phi^\dagger)\,\bar{u}^\dagger$$
$$-f^{(d)}(q\cdot\phi^\dagger)\,\bar{d} - f^{*\,(d)}(q^\dagger\cdot\phi)\,\bar{d}^\dagger$$

where q is the first-generation quark doublet and ϕ is the Higgs doublet, only the first line can be promoted to a superfield coupling.

• The second line, where $\phi^\dagger$ couples to left-handed fermions, cannot be made into a superfield coupling.

• For this reason there have to be two Higgs doublets in the MSSM (of course there could be more than two, but that would not be the MSSM).

• One gives mass to the down quark (and down squark) while the other gives mass to the up quark (and up squark).

• Thus: $-f^{(u)}(q\cdot\phi)\,\bar{u} \;\rightarrow\; -\,f^{(u)}(Q_1\cdot H_2)\,\overline{U}_1$

where H_2 is a Higgs chiral superfield of hypercharge $+1$ (like the SM Higgs). It is also called the "up-type" Higgs.

• Next, introduce a Higgs H_1 of hypercharge -1 and couple it by replacing:

$$-f^{(d)}(q\cdot\phi^\dagger)\,\bar{d} \;\rightarrow\; -\,f^{(d)}(Q_1\cdot H_1)\,\overline{D}_1$$

H_1 is also called the "down-type" Higgs.

• The component expansion of the Higgs superfields is:

$$H_I(y,\theta) = h_I(y) + \sqrt{2}\theta\,\tilde{h}_I(y) + \cdots, \quad I = 1,2$$

and the fermions $\tilde{h}_I$ are called Higgsinos.

• The Higgsinos $\tilde{h}_1$ and $\tilde{h}_2$ are chiral fermions of hypercharges -1,+1 respectively.

• Being chiral, they contribute to the triangle anomaly for the hypercharge, but because they carry equal and opposite hypercharge, the anomalies cancel out: $\sum_{\tilde{h}} Y_{\tilde{h}}^3 = 0$

• This provides an independent reason for two Higgs doublets, and puts a constraint on extensions of the MSSM.

• Anticipating the Higgs mechanism, the charge assignments of the two Higgs doublets have to be: $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$

• Let the scalar components of the superfields H_1, H_2 be $h_1(x), h_2(x)$. The possible VEV's they can develop, consistent with charge conservation, are: $\langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

• It is possible to rotate the fields so that both VEV's are real. Then we define:

$$\tan\beta = \frac{v_2}{v_1}$$

and this is the important β -parameter of the MSSM.

• We have discussed the results of the Higgs mechanism, but not yet written down the Higgs potential that achieves this.

• Actually, except for a few terms, we haven't yet written down any interactions of the MSSM!

• Partly this is because of supersymmetry breaking. Because supersymmetry must be broken in nature, we need to consider interactions that break it spontaneously or explicitly.

• Supersymmetry breaking will have some interplay with the rest of the MSSM and in particular the Higgs mechanism.

Keeping that in mind, we start by writing down the action of the supersymmetric part of the MSSM:

$$S_{\mathsf{susy}} = S_{\mathsf{gauge}} + S_{\mathsf{matter}} + S_{\mathsf{Higgs}}$$

Here,

$$S_{\mathsf{gauge}} = \frac{1}{8\pi}\mathrm{Im}\int d^4x\, d^2\theta(\tau_Y\, W_Y W_Y + \frac{1}{4T(\mathcal{R}_W)}\mathrm{tr}\,\tau_W\,\mathbf{W}_W\mathbf{W}_W + \frac{1}{4T(\mathcal{R}_g)}\mathrm{tr}\,\tau_g\,\mathbf{W}_g\mathbf{W}_g)$$

while

$$S_{\mathsf{matter}} = \int d^4x\, d^2\theta\, d^2\theta^\dagger(L_i^\dagger e^{g_Y V_Y Y + \mathbf{V}_W} L_i + \overline{E}_i^\dagger L_i^\dagger e^{g_Y V_Y Y + \mathbf{V}_W} L_i + \overline{E}_i^\dagger e^{g_Y V_Y Y}\overline{E}_i + Q_i^\dagger e^{g_Y V_Y Y + \mathbf{V}_W + \mathbf{V}_g} Q_i + \overline{U}_i^\dagger e^{g_Y V_Y Y + \mathbf{V}_g}\overline{U}_i + \overline{D}_i^\dagger e^{g_Y V_Y Y + \mathbf{V}_g}\overline{D}_i)$$

• To conclude the SUSY part of the action we write down:

$$S_{\text{Higgs}} = \int d^4x\, d^2\theta\, d^2\theta^\dagger (H_1^\dagger e^{-g_Y V_Y + \mathbf{V}_W} H_1 + H_2^\dagger e^{g_Y V_Y + \mathbf{V}_W} H_2) + \int d^4x\, d^2\theta\, \mathcal{W}_{\text{MSSM}}$$

where

$$\mathcal{W}_{\text{MSSM}} = \mu H_1 \cdot H_2 - f_{ij}^{(e)} (L_i \cdot H_1)\, \overline{E}_j - f_{ij}^{(d)} (Q_i \cdot H_1)\, \overline{D}_j - f_{ij}^{(u)} (Q_i \cdot H_2)\, \overline{U}_j$$

The last three terms in $\mathcal{W}_{\text{MSSM}}$ are evident from our previous discussions.

• The bilinear interaction between the Higgs fields is new. This is the famous μ-term.

• In principle there could be additional terms of the form:

$$L_i \cdot H_2, \quad (L_i \cdot L_j)\,\overline{E}_k, \quad (L_i \cdot Q_j)\,\overline{D}_k, \quad \overline{U}_i \overline{D}_j \overline{D}_k$$

• However they are forbidden by R-parity, a discrete symmetry under which $\theta \rightarrow -\theta$.

• We can assign an R-parity to a superfield. This is equal to the R-parity of the bosonic components, and minus the R-parity of the fermionic components. In the MSSM it is natural to choose the following R-parity assignments:

$$\begin{array}{ll} L_i,\ \overline{E}_i,\ Q_i,\ \overline{U}_i,\ \overline{E}_i : & \text{odd} \\ H_1,\ H_2,\ V_Y,\ \mathbf{V}_W,\ \mathbf{V}_g : & \text{even} \end{array}$$

• This has the pleasant effect that all known particles have $R = +1$ while all superpartners have $R = -1$.

• It is easy to check that all the terms on the previous slide are allowed by R-parity while all the ones at the top of this slide are forbidden.

- SUSY has to be broken.

- The supersymmetry algebra in the rest frame is:

$$[Q_A, Q^{\dagger}{}_{\dot{B}}]_{+} = 2\delta_{A\dot{B}}E$$

It follows that:

$$Q_A|0\rangle,\ Q^{\dagger}{}_{\dot{B}}|0\rangle \quad \Leftrightarrow \quad E = 0.$$

LHS vanishes if and only if Supersymmetry is not spontaneously broken.

Supersymmetry breaking is then indicated by a vacuum which is not SUSY invariant and nonero vacuum energy.

Non renormalisation theorem says that if Supersymmetry is unbroken at the tree level then Supersymmetry breaking term can not be generated in perturbation theory.

• It is possible to break supersymmetry spontaneously at tree level using either F-terms or D-terms.

• Let us give an example of each one. The classic model of F-term breaking is the O'Raifeartaigh model:

$$\mathcal{W}(\Phi_0, \Phi_1, \Phi_2) = \Phi_1 f_1(\Phi_0) + \Phi_2 f_2(\Phi_0)$$

Here, f_1, f_2 are two polynomials chosen such that $f_1(\Phi_0) = 0,\ f_2(\Phi_0) = 0,$ do not have a common solution for Φ_0.

The potential for the scalar components of these superfields is:

$$\sum_{i=0,1,2}\left|\frac{\partial \mathcal{W}}{\partial \phi_i}\right|^2 = |\phi_1 f_1'(\phi_0) + \phi_2 f_2'(\phi_0)|^2 + |f_1(\phi_0)|^2 + |f_2(\phi_0)|^2$$

For the vacuum energy to vanish, all three terms in this potential must vanish.

However the last two cannot vanish together since the relevant polynomials have no common zero.

It follows that the vacuum energy is > 0 and supersymmetry is spontaneously broken. There is a massless fermion ("goldstino") in the spectrum.

Note that the auxiliary fields, by their equations of motion, are set equal to:

$$F_i = -\frac{\partial \mathcal{W}^\dagger}{\partial \phi_i^\dagger}, \qquad i = 0, 1, 2$$

and therefore term by term, the potential above can be written:

$$|F_0|^2 + |F_1|^2 + |F_2|^2$$

Now we have just shown that at least one of F_1, F_2 develops a vev, thereby illustrating the general principle that spontaneous supersymmetry breaking is signalled by a vev of the auxiliary field.

Next we consider a model of D-term breaking.

Consider a theory with an Abelian gauge field and a single chiral superfield Φ of charge $t = 1$. Because of this choice Now the scalar potential comes from the D-term, and is:

$\frac{1}{2}(\eta + g\phi^\dagger\phi)^2$

If $\eta > 0$, the vacuum configuration is $\phi = 0$ and the vacuum energy is positive, so supersymmetry is spontaneously broken.

On the other hand if $\eta < 0$, the vacuum configuration is any ϕ satisfying

$$g\phi^\dagger\phi = |\eta|.$$

In this case the vacuum energy is zero, and supersymmetry is unbroken. However, since ϕ gets a vev, the gauge symmetry is spontaneously broken.

This illustrates an amusing, and generally complex, interplay between supersymmetry and gauge symmetry.

• It is possible to break supersymmetry spontaneously at tree level using either F-terms or D-terms.

• However it can be shown that spontaneous supersymmetry breaking within the MSSM at tree level leads to sum rules on particle masses that are in conflict with experiment.

• Let M_e^2, M_u^2 etc. be the square mass matrices for the charge -1, for the charge $+\frac{2}{3}$ etc.

• We can show, $S\mathrm{Tr}M_e^2 + S\mathrm{Tr}M_\nu^2 = S\mathrm{Tr}M_u^2 + S\mathrm{Tr}M_d^2 = 0$

Remember in Supertrace terms with different spins come with opposite signs. For the above equation to be satisfied, we would need in each family some slepton/squark field lighter than lepton/quark. This is in clear conflict with experiments.

• Therefore the current attitude is to assume supersymmetry breaking takes place by some other mechanism, usually involving extensions beyond the MSSM.

• Without knowing the details of this mechanism, we can still discuss the impact of the breaking on the MSSM, in the form of soft supersymmetry breaking terms.

• Even if SUSY is broken we still want to use the symmetry to give a solution to the hierarchy problem. Recall we needed equality of couplings indicated by the symmetry, not necessarily the masses.

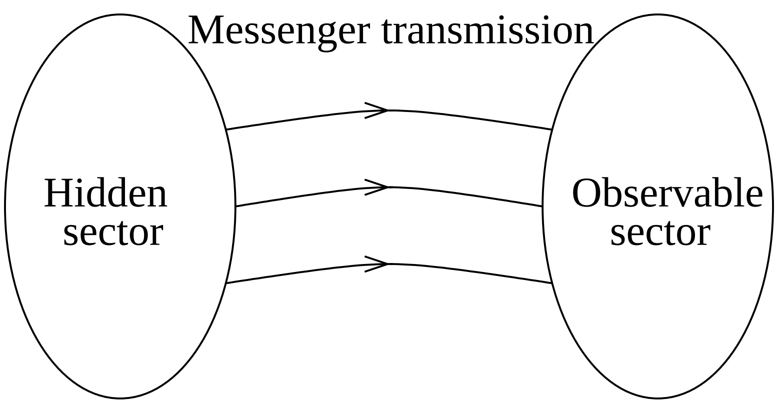
• **'SOFT'** SUSY breaking.

• Spontaneous breakdown of Superrsymmetry, though theoretically favoured, has to involve fields outside the MSSM.

• Phenomenologically such fields will have to be much heavier than the EW scale and hence also the sparticles.

• Spontaneous Supersymmetry Breakdown (SSB) needs to be effected in a sector of fields which are singlets with respect to the SM gauge group and known as the hidden or secluded sector. SSB can take place there at a distinct scale Λ_s.

• Supersymmetry breaking is then transmitted to the Gauge nonsinglet observable or visible sector by a messenger sector (associated with a typical mass scale M_M that could, but need not, be as high as the Planck mass M_{Pl}); this may or may not require additional gauge nonsinglet messenger superfields.



Transmission of SUSY breaking from hidden sector to the observable sector.

Two broad classes of the messenger sector

1] Higher dimensional operators suppressed by inverse powers of the Planck mass (gravity mediated). $M_s \sim \Lambda_s^2/M_{Pl}$.

or

2) fields with gauge interactions at lower energy scales (Gauge mediated..). $M_s \sim$ (gauge coupling $)^2\ \Lambda_s^2/M_M$.

The basic idea is to allow non-supersymmetric interactions of scaling dimension ≤ 3, moreover restricted so that they do not induce dimension 4 operators via loop diagrams.

- This ensures that the non-renormalisation theorems of supersymmetry continue to apply in an approximate, but still useful, form.

- These are called "soft terms". The following types of terms qualify as soft:

$$
\begin{aligned}
S_{\text{soft}} \; = \; & \int d^4x (- \phi^{\dagger I} (m^2)_{IJ} \phi^J \\
& + (C_I \phi^I - \tfrac{1}{2} B_{IJ} \phi^I \phi^J + \tfrac{1}{6} A_{IJK} \phi^I \phi^J \phi^K + \text{c.c.}) \\
& - \tfrac{1}{2} (M_\alpha \lambda^\alpha \lambda^\alpha + \text{c.c.}))
\end{aligned}
$$

where we have used the generic notation ϕ^I for all scalars, and λ^α for all gauginos, in the theory.

• Let us make the soft terms more explicit in terms of MSSM fields. The first term becomes:

$$
\begin{aligned}
-\phi^{\dagger I}(m^2_{IJ})\phi^J \;\rightarrow\; & -(\tilde{\ell}^\dagger_i\,(M^2_{\tilde{\ell}})_{ij}\,\tilde{\ell}_j + \tilde{\bar{e}}^\dagger_i\,(M^2_{\tilde{\bar{e}}})_{ij}\,\tilde{\bar{e}}_j \\
& + \tilde{q}^\dagger_i\,(M^2_{\tilde{q}})_{ij}\,\tilde{q}_j + \tilde{\bar{u}}^\dagger_i\,(M^2_{\tilde{\bar{u}}})_{ij}\,\tilde{\bar{u}}_j + \tilde{\bar{d}}^\dagger_i\,(M^2_{\tilde{\bar{d}}})_{ij}\,\tilde{\bar{d}}_j \\
& + m_1^2\,h_1^\dagger h_1 + m_2^2\,h_2^\dagger h_2)
\end{aligned}
$$

and so on...

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SOFT}}$$

Most of the discussion will center around terms in $\mathcal{L}_{\mathsf{SOFT}}$.

$$
\begin{aligned}
-\mathcal{L}_{\mathsf{SOFT}} \;=\;& \tilde{q}^{\star}_{iL}(\mathcal{M}^2_{\tilde{q}})_{ij}\tilde{q}_{jL} + \tilde{u}^{\star}_{iR}(\mathcal{M}^2_{\tilde{u}})_{ij}\tilde{u}_{jR} + \tilde{d}^{\star}_{iR}(\mathcal{M}^2_{\tilde{d}})_{ij}\tilde{d}_{jR} + \tilde{\ell}^{\star}_{iL}(\mathcal{M}^2_{\tilde{\ell}})_{ij}\ell_{jL} \\
&+\tilde{e}^{\star}_{iR}(\mathcal{M}^2_{\tilde{e}})_{ij}\tilde{e}_{jR} + [h_1\cdot\tilde{\ell}_{iL}(f^e A^e)_{ij}\tilde{e}^{\star}_{jR} + h_1\cdot\tilde{q}_{iL}(f^d A^d)_{ij}\tilde{d}^{\star}_{jR} \\
&+\tilde{q}_{iL}\cdot h_2(f^u A^u)_{ij}\tilde{u}^{\star}_{jR} + \mathsf{h.c.}] + m_1^2|h_1|^2 + m_2^2|h_2|^2 + (B\mu h_1\cdot h_2 + \mathsf{h.} \\
&+\frac{1}{2}(M_1\bar{\tilde{\lambda}}_0 P_L\tilde{\lambda}_0 + M_1^{\star}\bar{\tilde{\lambda}}_0 P_R\tilde{\lambda}_0) + \frac{1}{2}(M_2\bar{\tilde{\vec{\lambda}}}P_L\tilde{\vec{\lambda}} + M_2^{\star}\bar{\tilde{\vec{\lambda}}}P_R\tilde{\vec{\lambda}}) \\
&+\frac{1}{2}(M_3\bar{\tilde{g}}^a P_L\tilde{g}^a + M_3^{\star}\bar{\tilde{g}}^a P_R\tilde{g}^a) \\
\equiv\;& V_{\mathsf{SOFT}} + \text{gaugino mass terms.}
\end{aligned}
$$

Recall:

$\overline{\tilde{e}_L}$ for the right spositron. The antisfermionic fields denoted by conjugation of the sfermionic fields.

Assumes $R_p = (-1)^{(3(B-L)+2S)} = (-1)^{(3B-L+2S)}$ is conserved.

The μ term from the $\mathcal{L}_{SUSY}$ plays an important role as well.

There are two aspects:

1) Mass spectra for sparticles, mass bounds for the Higgs:

The mass bounds for the Higgs not crucially dependent on details of SUSY breaking, but do depend on the breaking scale, that too through loop corrections. The mass spectra for others depend on the details of the SUSY breaking scenario.

2)Couplings of sparticles with the standard model particles.

SUSY equates couplings of the interaction eigenstates. They need not be mass eigenstates as a result of SUSY breaking. Alternatively mass eigenstates couplings depend on SUSY breaking parameters.

The raison de trê of SUSY is to keep the Higgs mass stable under radiative corrections. How does it happen exactly in MSSM? In spite of the breaking of SUSY there exist **upper** bound for the mass of the lightest Higgs.

This is the only sector of Sparticles where theory has predictive power for sparticle *mass* which is robust with respect to the details of Soft SUSY breaking mechanism.

The scalar potential for the Higgs sector is generated from terms in the Superpotential and the quartic coupling is determined in terms of the gauge couplings.

$V_H = \frac{1}{8}(g_Y^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \frac{g_2^2}{2}|h_1^\dagger h_2|^2 + m_{1h}^2|h_1|^2 + m_{2h}^2|h_2|^2 + (m_{12}^2 h_1 \cdot h_2 + h.c.)$,

$m_{1,2h}^2 = m_{1,2}^2 + |\mu|^2$ and $m_{12}^2 = B\mu$.

V_H comes from $V_{SOFT} = -\mathcal{L}_{SOFT}$ - gaugino mass terms from $\mathcal{W}_{MSSM}$ and D terms.

Only three parameters compared to the six parameters plus one phase for the general two Higgs doublet potential.

The potential develops the 'wrong' sign of the mass term required for EW symmetry breaking through RG evolution (we will see below).

Two complex Higgs Doublets.

After EW symmetry breaking five physical Higgs boson states are left:

Three neutrals: Scalars h,H and a pseudoscalar A which couples to fermions with a γ_5 and two charged Higgs bosons $H^{\pm}$.

The potential involving only the neutrals

$V_H^0 = \frac{1}{8}(g_Y^2 + g_2^2)^2(|h_1^0|^2 - |h_2^0|^2)^2 + m_{1h}^2|h_1^0|^2 + m_{2h}^2|h_2^0|^2 - m_{12}^2(h_1^0 h_2^0 +$ h.c.) , Two conditions:

1)coming from demanding that V_H^0 must be bounded from below

$m_{1h}^2 + m_{2h}^2 = m_1^2 + m_2^2 + 2|\mu|^2 > 2|m_{12}^2|$

Valid at all scales.

2)from requiring that EW symmetry breaking should happen:

$m_{12}^4 > m_{1h}^2 m_{2h}^2 = (m_1^2 + |\mu|^2)(m_2^2 + |\mu|^2)$ valid at and below the scale where EW breaking is operative.

In the supersymmetric limit $m_{1h}^2 = m_{2h}^2 = \mu^2$, the two conditions are incompatible with each other. No EW breaking with unbroken SUSY. Intimate connection between SUSY breaking and the EW symmetry breaking in MSSM.

What does the above result mean?

This means that the EW symmetry is unbroken when SUSY is broken.

With EW symmetry unbroken the SM particles are massless. In the world with broken EW they should have masses $\sim v = 174 GeV$.

The Sparticles (SUSY partners of all the SM particles) can be heavier, receiving masses from SUSY breaking and not required to be light.

But if we want SUSY still to provide solution to the Hierarchy problem some spartners must be around TeV, ie. LHC has a good chance to find SUSY.

For EW symmetry breaking to occur V_H should have a minimum for

$$\langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

Consistency conditions that this be a minimum gives two relations and thus constraints mentioned before:

$$-2B\mu = -2m_{12}^2 = (m_1^2 - m_2^2) \tan 2\beta + M_Z^2 \sin 2\beta$$

$$|\mu|^2 = (\cos 2\beta)^{-1} (m_2^2 \sin^2 \beta - m_1^2 \cos^2 \beta) - \frac{1}{2} M_Z^2 \ .$$

Example of the constraints from Higgs sector which reduces number of parameters.

A relation between Supersymmetric paramter μ, SUSY breaking mass parameters and **accurately measured mass M_Z.**

This is μ problem so to say... if SUSY breaking parameters are large then one needs to have a small tuning that this relation is satisfied.

Recall the logarithmic corrections to M_H^2, when SUSY is broken.

Range of $\tan\beta$ values restricted to $1, \tan\beta < 60$ for radiatively induced symmetry breaking to work.

The light higgs mass is bounded from above.

Discussion of Higgs mass spectra Non minimal SSM?

Partly in lecture by Djouadi.

Implications of the failure of LEP to find Higgs for SUSY May be one transparency if time is left.

What we discussed so far were interaction eigenstates. In reality we need to deal with mass eigenstates.

Just EW breaking, **even without SUSY breaking**, will cause mixing between the charged EW gauginos $(\tilde{\lambda}_1, \tilde{\lambda}_2)$ and the charged higgsinos ($\tilde{h}_1^-, \tilde{h}_2^+$), to produce the mass eigenstates $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$. One needs the two Higgs doublets also to make sure that there are enough degrees of freedom to have two massive Dirac spinors.

EW and SUSY breaking $\Rightarrow \tilde{\lambda}_0, \tilde{\lambda}_3, \tilde{h}_1^0, \tilde{h}_2^0$ mix $\Rightarrow \tilde{\chi}_i^0, i = 1, 4$.

The mixing in this sector depends on $\mu, M_i, i = 1, 2, \tan\beta$**.**

For R_p conservation $\tilde{\chi}_1^0$ absolutely stable and a DM candidate.

• The two doublet superfields H_1, H_2 are left chiral ones and they contain the left chiral higgsinos

$$\tilde{h}_{1L} \equiv \begin{pmatrix} \tilde{h}_1^1 \\ \tilde{h}_1^2 \end{pmatrix} = \begin{pmatrix} \tilde{h}_1^0 \\ \tilde{h}_1^- \end{pmatrix}_L \; ; \; \tilde{h}_{2L} \equiv \begin{pmatrix} \tilde{h}_2^1 \\ \tilde{h}_2^2 \end{pmatrix} = \begin{pmatrix} \tilde{h}_2^+ \\ \tilde{h}_2^0 \end{pmatrix}_L .$$

• Conjugate superfields $H_1^\dagger, H_2^\dagger$ contain the corresponding right chiral ones.

• The left chiral charginos comprise four orthogonal states: the positively charged $\tilde{\chi}^+_{1L}, \tilde{\chi}^+_{2L}$ and the negatively charged $\tilde{\chi}^-_{1L}, \tilde{\chi}^-_{2L}$.

• Define charged gaugino (wino) fields

$$\tilde{\lambda}^\pm = \frac{1}{\sqrt{2}}\left(\tilde{\lambda}_1 \mp i\tilde{\lambda}_2\right) = \tilde{\lambda}^\pm_L + \tilde{\lambda}^\pm_R \; ,$$

• Superscripts $1, 2$ are Cartesian $SU(2)_L$ indices.

- The massive $\tilde{\chi}^+_{1L}$ and $\tilde{\chi}^+_{2L}$ are orthogonal linear combinations of $\tilde{\lambda}^+_L$ and $\tilde{h}^+_{2L}$ while $\tilde{\chi}^-_{1L}$ and $\tilde{\chi}^-_{2L}$ are formed with $\tilde{\lambda}^-_L$ and $\tilde{h}^-_{1L}$.

- There is *no* $\tilde{h}^-_{2L}$ or $\tilde{h}^+_{1L}$!

- the right chiral charginos $\tilde{\chi}^-_{1R}$, $\tilde{\chi}^-_{2R}$ and $\tilde{\chi}^+_{1R}$, $\tilde{\chi}^+_{2R}$ are orthogonal linear combinations of the charge conjugates of the above pairs of gauginos and higgsinos, viz. $\tilde{\lambda}^-_R$, $\tilde{h}^-_{2R}$ and $\tilde{\lambda}^+_R$, $\tilde{h}^+_{1R}$ respectively.

- Require both $\tilde{h}^+_{2L}$ and $\tilde{h}^-_{1L}$, as appear in the two Higgs doublets, otherwise some chargino field, lacking a partner to make a Dirac mass term in the Lagrangian density, would remain massless.

The mass term in the Lagrangian looks like:

$$\mathcal{L}^c_{MASS} = -\frac{g_2}{\sqrt{2}}(v_1\lambda^+\tilde{h}^2_1 + v_2\lambda^-\tilde{h}^1_2 + \text{h.c.}) - (M_2\lambda^+\lambda^- + \mu\tilde{h}^2_1\tilde{h}^1_2 + \text{h.c.}) \; .$$

- $\psi^+ \equiv \begin{pmatrix} \lambda^+ \\ \tilde{h}^1_2 \end{pmatrix}, \; (\psi^+)^T \equiv (\lambda^+ \; \tilde{h}^1_2) \; , \psi^- \equiv \begin{pmatrix} \lambda^- \\ \tilde{h}^2_1 \end{pmatrix}, \; (\psi^-)^T \equiv (\lambda^- \; \tilde{h}^2_1)$
- $-\mathcal{L}^c_{MASS} = (\psi^-)^T \mathbf{X} \psi^+ + \text{h.c.} \; ,$

with

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} .$$

$$\mathcal{M}^n = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix},$$

The character and mass of the four neutralinos,

$\chi_l^0 = Z_{ln} \psi_n^0$

decided by $M_i, i = 1, 3, \mu$.

The SUSY breaking terms induce mixing in the sfermion sector too!

$\tilde{f}_R - \tilde{f}_L$ mixing decided by $A, \mu, \tan\beta$ **and** m_f.

Sfermion generation mixing: mainly controlled by soft parameters.

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{q}_3} + (1/2 - 2/3\sin^2\theta_W)M_Z^2\cos 2\beta + m_t^2 & -m_t(A^{t\star} + \mu\cot\beta) \\ -m_t(A^t + \mu^\star\cot\beta) & m^2_{\tilde{t}} + 2/3 M_Z^2 \cos 2\beta \sin^2\theta_W + m_t^2 \end{pmatrix}.$$

Sfermions		Gauginos and higgsinos	
Name	Symbol	Name	Symbol
(left, right) selectron	$\tilde{e}_{L,R}$	gluinos	$\tilde{g}^a$
(left, right) smuon	$\tilde{\mu}_{L,R}$		
(left, right) stau	$\tilde{\tau}_{L,R}$	lighter charginos	$\tilde{\chi}_1^{\pm}$
e-sneutrino	$\tilde{\nu}_e$		
μ-sneutrino	$\tilde{\nu}_\mu$	heavier charginos	$\tilde{\chi}_2^{\pm}$
τ-sneutrino	$\tilde{\nu}_\tau$		
(left, right) u-squark	$\tilde{u}_{L,R}$	lightest neutralino	$\tilde{\chi}_1^0$
(left, right) d-squark	$\tilde{d}_{L,R}$		
(left, right) c-squark	$\tilde{c}_{L,R}$	next-to-lightest neutralino	$\tilde{\chi}_2^0$
(left, right) s-squark	$\tilde{s}_{L,R}$		
(left, right) stop	$\tilde{t}_{L,R}$	next-to-heaviest neutralino	$\tilde{\chi}_3^0$
(left, right) sbottom	$\tilde{b}_{L,R}$	heaviest neutralino	$\tilde{\chi}_4^0$

LSP: various candidates possible. But neutralino is the most common one.

These mixings decide the interactions of the mass eigenstates and hence their decay patterns.

They also affect the indirect effects caused in the loops.

Since they are controlled by soft SUSY breaking parameters their determination will also afford a handle on these parameters.

The parameters $\mu, \tan\beta, M_1, M_2$ and A for the third generations affect the search strategies and determination of the mixing can be used to get information on the Lagrangian parameters as well.

• 124 total parameters and 105 are new :-)! Complex Gaugino masses: 6, sfermion mass matrices: 45, trilinear couplings : 54, bilinears: 4, Higgs sector: 2

111 - 6 (constraints from Higgs sector) = 105!

• Luckily most processes, at tree level, depend only on a few of these 105 parameters.

• But severe phenomenological problems for 'generic' values of these parameters! FCNC, unacceptable CP violation (large electric dipole moments for neutron (for example)). In radiatively driven EW symmetry breaking one gets colour/charge breaking vacua! Impossible to get right values of M_Z..!

• Discussions of the SUSY phenomenology tractable only in **Constrained** MSSM (CMSSM) where assumptions are made to reduce the parameters drastically!

• The assumptions guided by SUSY breaking scenarios.

• Finally experiments will tell us. The day of reckoning is now at hand!

• 44 of these are phases and they can NOT be rotated away by field redefinition. In most discussions these are put to zero by hand, as the constraints from low energy phenomenology are quite severe. But in few cases large CP phases MAY be allowed. $\not{C}\not{P}$ MSSM discussions very interesting.

• R_p conservation assumed in most discussions.

Phenomenologically motivated choices:

- All new $\not{CP}$ phases to be zero.
- Masses and trilinear coupling diagonal in flavour basis. **NO FCNC.**
- Universal first and second generation squarks. I.e no problems with K_0–$\bar{K}_0$ mixing!

Now number of parameters:

Three gaugino masses: M_1, M_2 and M_3 : 3,

Higgs mass parameters: m_1^2, m_2^2 : 2,

First two generation sfermion masses: $m_{\tilde{q}}, m_{\tilde{u}_R}$: 5 parameters;

three trilinear couplings A_u, A_d, A_e: 3,

Third generation masses and Trilinear parameters: 8

Usually collider phenomenology insensitive to A_u, A_d, A_e (which play an important role in neutron edm, $(g-2)_\mu$ etc.)

Very often possible to justify using common value for the soft-SUSY breaking terms. Such a CMSSM much more predictive.

Even more constrained: mSUGRA!!

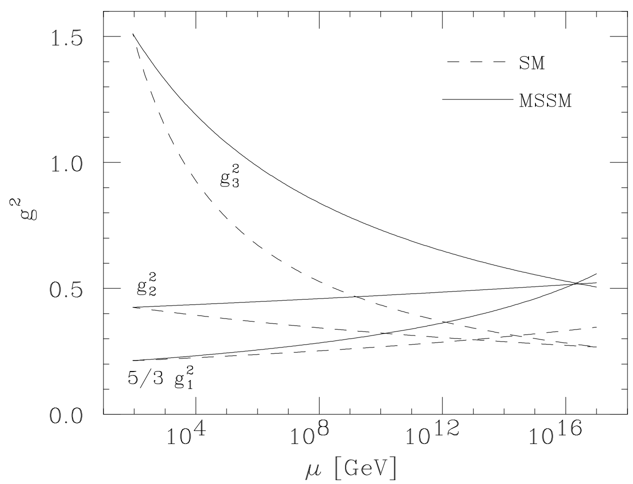
Various SUSY breaking mechanisms give values of soft parameters at a high scale of SUSY breaking and transmission to the visible sector. These are **boundary conditions** on these parameters at some high scale.

They determine the sparticle masses which are $\mathcal{O}$ TeV.

We need to know value of these parameters at the **low** scale to test predictions of the **high scale** theory in experiments and also to translate information on low energy constraints/values of the SUSY breaking parameters to the highs scales to find restrictions/values at high scale

RGE for gauge couplings can be computed only from vacuum polarisation (super)graphs.

Superpotential Yukawa couplings RGE too can be computed from just two point functions.



Gaugino masses evolve the same way as the gauge couplings at one loop.

Even if one starts from $m_1^2 = m_2^2$, the RGE for m_2 will receive large contributions from $t\bar{t}$ loops (recall H_2 was the 'up' Higgs) and the corresponding term in the scalar potential can be driven negative triggering EW symmetry breaking.

So in supersymmetric theories the EW symmetry breaking is 'natural', with a heavy top quark. One does not have to choose the scalar potential with the 'wrong' sign.

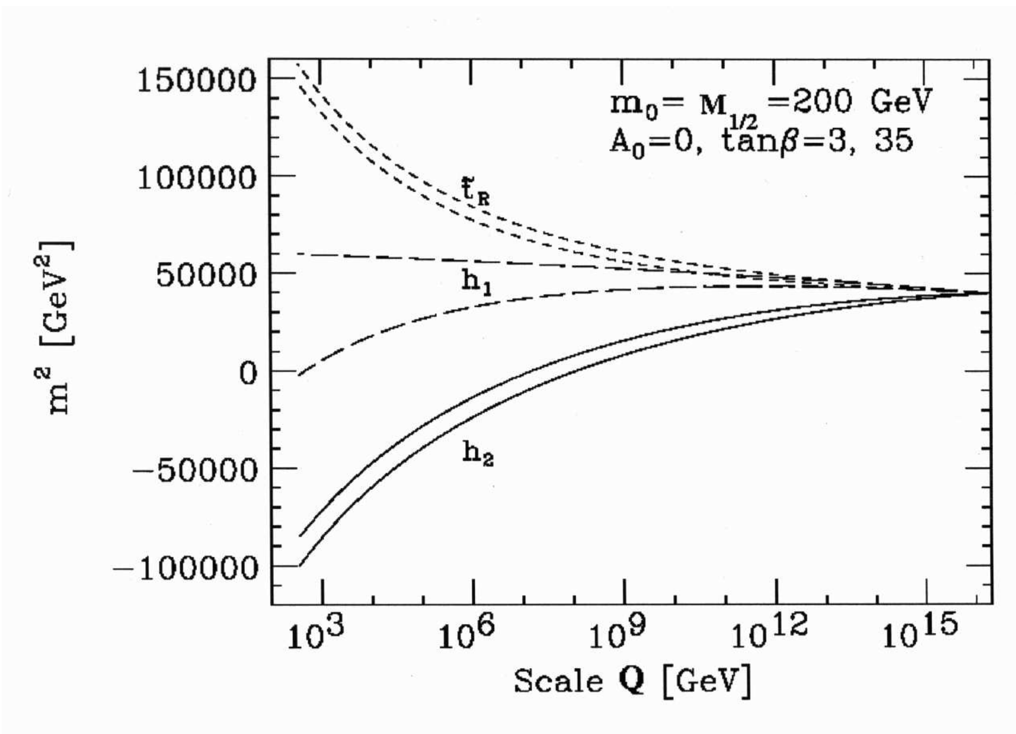
As advertised in mSUGRA, all the scalar SUSY breaking parameters are **Universal**.

$$m_{ij}^2 = m_0^2 \, \delta_{ij}, A_{ijk} = A_0, \quad \forall i,j,k \ , B_{ij} = B_0 \ \forall \ i,j.$$

Of course the RGE's will be different for different SUSY breaking masses and A parameters. These involve gaugino masses!

The Gaugino masses need a boundary condition: one imposes GUT scale unification on gaugino masses, following approximate unification of the gauge couplings.

Parameters of mSUGRA: $m_0, M_{1/2}, A_0, B_0$. One trades off B_0 for $\tan\beta$, EW symmetry breaking condition determines μ^2 in terms of all the others. Then the parameters are $m_0, M_{1/2}, A_0, \tan\beta$ and $sgn(\mu)$.



Main ideas discussed here: Gravity meditation (mSUGRA), modulii mediation, GMSB and AMSB. 10 TeV $< \Lambda_s \leq M_M \leq M_{Pl}$ and $M_M \simeq \Lambda_s^2/M_s$

Mediation mechanism	Model	Gravitino mass $m_{3/2}$	Gaugino mass M_α	Sfermion mass squared m_i^2
Gravity mediated	mSUGRA	$\leq \mathcal{O}$ (TeV)	$(g_\alpha/g_2)^2 M_2$	$m_0^2 + G_i M_{1/2}^2 +$ D-terms
	$\tilde{\text{C}}$MSSM	$\leq \mathcal{O}$ (TeV)	$(g_\alpha/g_2)^2 M_2$	$\tilde{q}_\ell$: $m_{\tilde{q}_L}^2 + G_{\tilde{q}_L} M_{1/2}^2 +$ D-terms $\tilde{l}_L$: $m_{\tilde{\ell}}^2 + G_{\tilde{\ell}} M_{1/2}^2 +$ D-terms $\tilde{e}_R$: $m_{\tilde{e}_R}^2 + G_{\tilde{e}_R} M_{1/2}^2 +$ D-terms $\tilde{u}_R$: $m_{\tilde{u}_R}^2 + G_{\tilde{u}_R} M_{1/2}^2 +$ D-terms $\tilde{d}_R$: $m_{\tilde{d}_R}^2 + G_{\tilde{d}_R} M_{1/2}^2 +$ D-terms
	AMSB	20–100 TeV	$(g_\alpha b_\alpha/g_2 b_2)^2 M_2$	$m_0^2 + C_i (16\pi^2)^{-2} m_{3/2}^2$
Gauge mediated	mGMSB	10^{-5} eV – 1 keV	$(g_\alpha/g_2)^2 M_2$	$\tilde{q}_L$: $M_3^2\ G'_{\tilde{q}_L} +$ D-terms $\tilde{l}_L$: $M_2^2\ G'^2_{\tilde{\ell}_L} +$ D-terms $\tilde{e}_R$: $M_2^2\ G'_{\tilde{e}_R} +$ D-terms $\tilde{u}_R$: $M_3^2\ G'_{\tilde{u}_R} +$ D-terms $\tilde{d}_R$: $M_3^2\ \tilde{G}'_{\tilde{d}_R} +$ D-terms

mSUGRA $M_1 : M_2 : M_3 \simeq 1 : 2.8 : 7$, $m_0, M_{1/2}, A, \tan\beta, sgn(\mu)$

mGMSB : similar, subject to some corrections depending on couplings of the messenger fields. $M_M, \Lambda_s, sgn(\mu), \tan\beta, n_q, n_l$ and $m_{3/2}$.

AMSB: $M_1 : M_2 : M_3 \simeq 2.8 : 1 : 8.3$, $m_0, M_{3/2}, \tan\beta, sgn(\mu)$.

mSUGRA: LSP is $\tilde{\chi}_1^0$.

mGMSB Gravitino is LSP, NLSP: $\tilde{\chi}_1^0, \tau_1, \tilde{e}_R$. NLSP can be long lived and quasi stable! Cosmological constraints on $m_{3/2}$ and hence on scale of SUSY breaking.

mSUGRA,mGMSB: Once LEP constraints are imposed, $\tilde{\chi}_1^0$ is an almost pure $U(1)$ gaugino and $\tilde{\chi}_2^0 \sim$ pure $SU(2)$ gaugino. ($|M_1| < |\mu|$).

Note : These things have important implications for viability of the LSP as the DM. Higgsinos annihilate too efficiently and can be a good DM candidate only if heavier than $\sim$ TeV.

AMSB: Both $\tilde{\chi}_1^{\pm}, \tilde{\chi}_1^0$ are pure $SU(2)$ gauginos and degenerate. Loop effects need to be included to lift the degeneracy.

Most of the constraints come in the form of inequalities.

The first two generation squarks can not be much lighter than the gluinos.

$\left(m_{\tilde{q}}/m_{\tilde{l}}\right)|_{GMSB} > \left(m_{\tilde{q}}/m_{\tilde{l}}\right)|_{mSUGRA}$;

$m_{\tilde{e}} < m_{\tilde{q}}$ for mSUGRA;

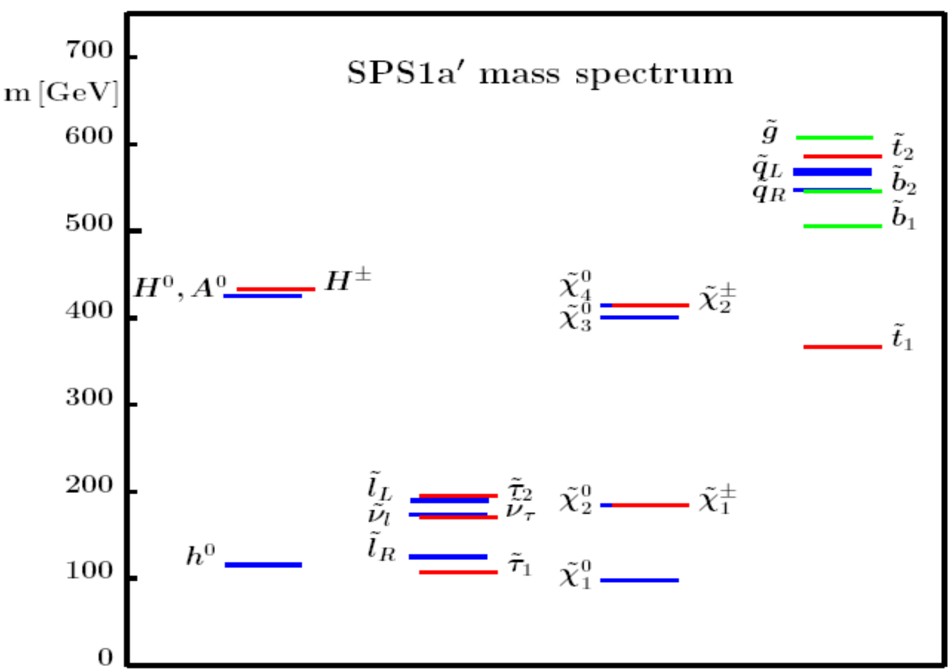
$m_{\tilde{e}_L} \simeq m_{\tilde{e}_R}$ for GMSB;

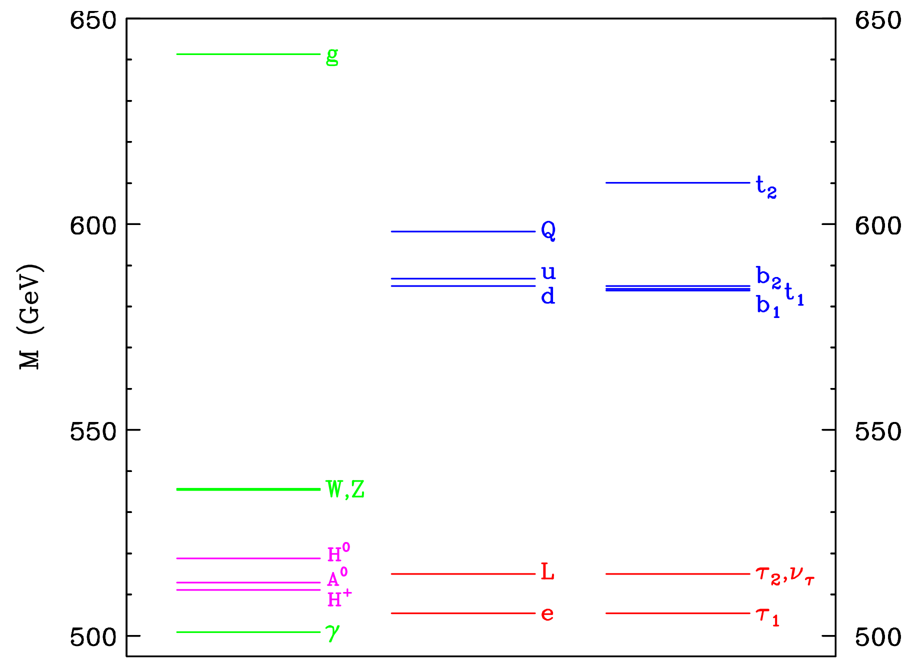
$m_{\tilde{e}_R} < m_{\tilde{e}_L}$ for mSUGRA.

Third generation sfermions are lighter than the other two due to the larger Yukawa coupling contribution to the running.

It is already indicative that a lepton collider will be, in fact, a much better probe to discriminate between different SUSY breaking ideas.

A representative spectrum. One of the benchmark points for LHC analyses. In fact the UED models predict similar spectra. So one needs to determine the spins then!





- **Discovery of sparticles** and determination of their quantum numbers.

- Quantitative verification of coupling equalities implied by supersymmetry.

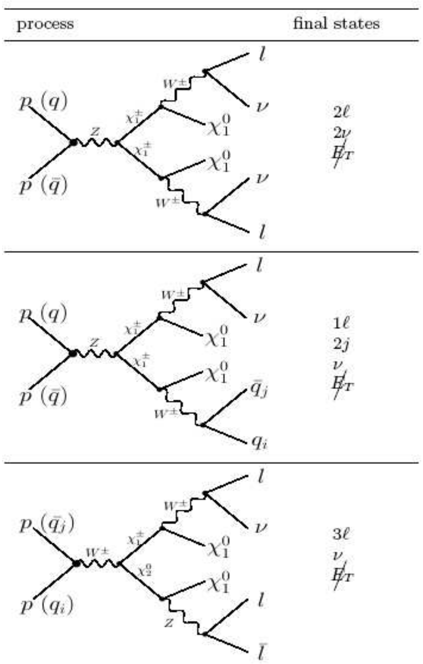
- **Measurement of the masses of scalars (including Higgs) as well as gauginos.**

- Determination of the gaugino-higgsino mixing parameters.

- Study of the properties of third generation sfermions including L-R mixing.

One would then like to use these to reconstruct the lagrangian parameters of SUSY.

LHC can address highlighted points to some extent and also point 2 indirectly. To achieve the rest we need ILC and LHC + ILC.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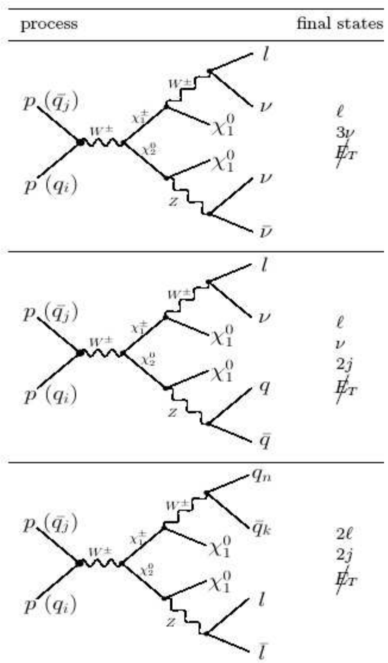
final states
2ℓ 2ν $\not{E}_T$
1ℓ $2j$ ν $\not{E}_T$
3ℓ ν $\not{E}_T$


process
final states
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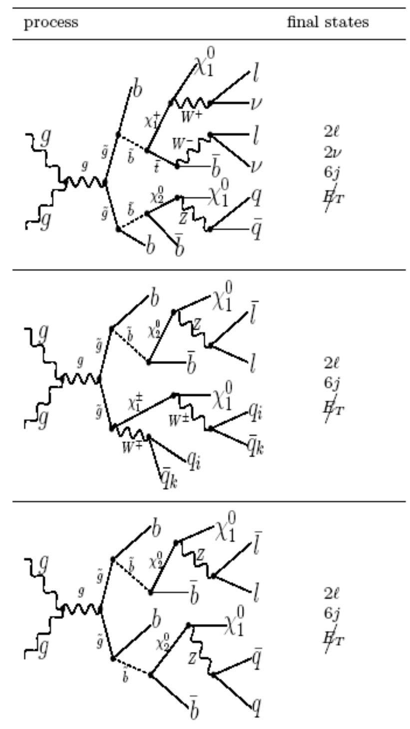
ℓ ν $2j$ $\not{E}_T$
2ℓ $2j$ $\not{E}_T$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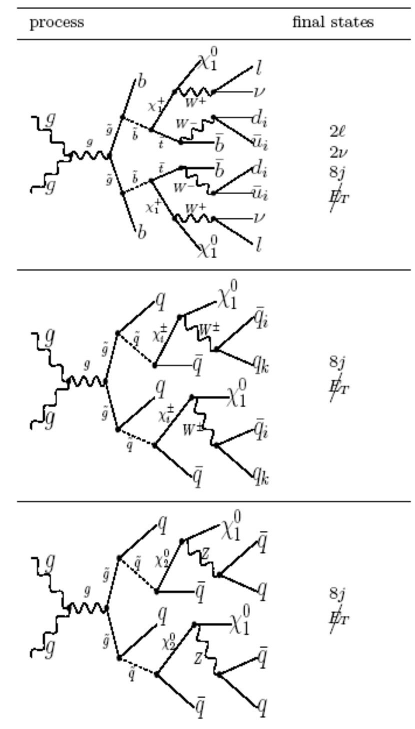
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$8j$ $\not{E}_T$
$8j$ $\not{E}_T$

Missing transverse energy signature: $\not{E}_T$

Because of R_p conservation sparticles are produced in pairs and will contain LSP which is **neutral and stable**. At e^+e^- colliders it can be just missing energy: $\not{E}$

Decay patterns of $\tilde{\chi}_i^0/\tilde{\chi}_j^{\pm}$ very important.

Generically m jets, n leptons and $\not{E}_T$.

In case of GMSB: hard photons which come from decays of the NLSP $\tilde{\chi}_1^0 \rightarrow \gamma\tilde{G}$. Large life times of the NLSP can give rise to pointing photons. So all the above + photons! If $\tilde{\tau}_1$ is the NLSP, heavy longlived charged particle tracks is the signature.

AMSB : difficult. $165 MeV < \Delta M(m_{\tilde{\chi}_1^{\pm}} - m_{\tilde{\chi}_1^0}) < 1$ GeV and $\tilde{\chi}_1^{\pm} \rightarrow \tilde{\chi}_1^0 + \pi$. Stopping track in the vertex detector or the soft pion is to be detected.

$\not{R}_p$: Even then due to very energetic neutrinos missing ET signal is not gone + large number of jets and leptons.

Where do the limits come from?

Direct searches:

NEWS!! **No sparticle has been observed yet!**

Analysis done in the context of mSUGRA, Pheno. MSSM, mGMSB, AMSB, $\not{R}_p$.

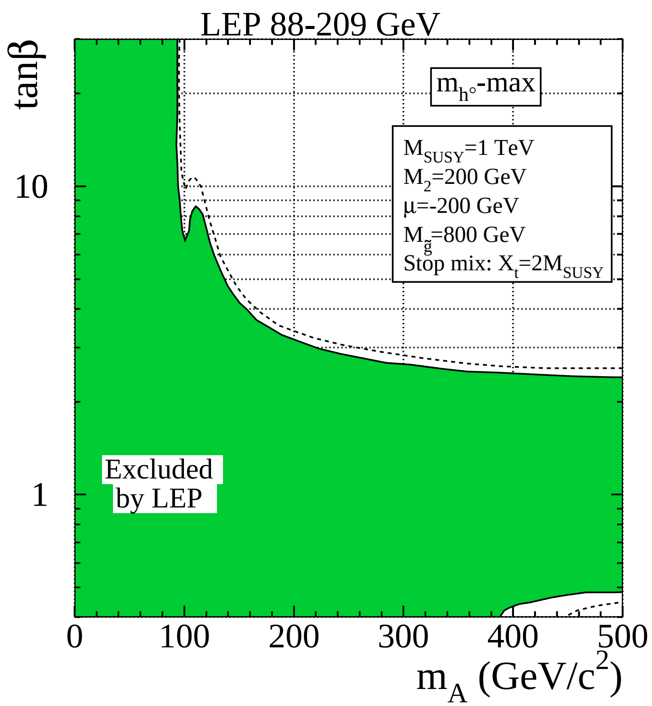
Tevatron: gluinos, squark searches. $m_{\tilde{g}} > 300$ GeV, $m_{\tilde{q}} > 260$ GeV for all flavours but top, $m_{\tilde{t}_1} > 90 - 100$ GeV. Stop different as it can not decay into a t and $\tilde{\chi}_1$

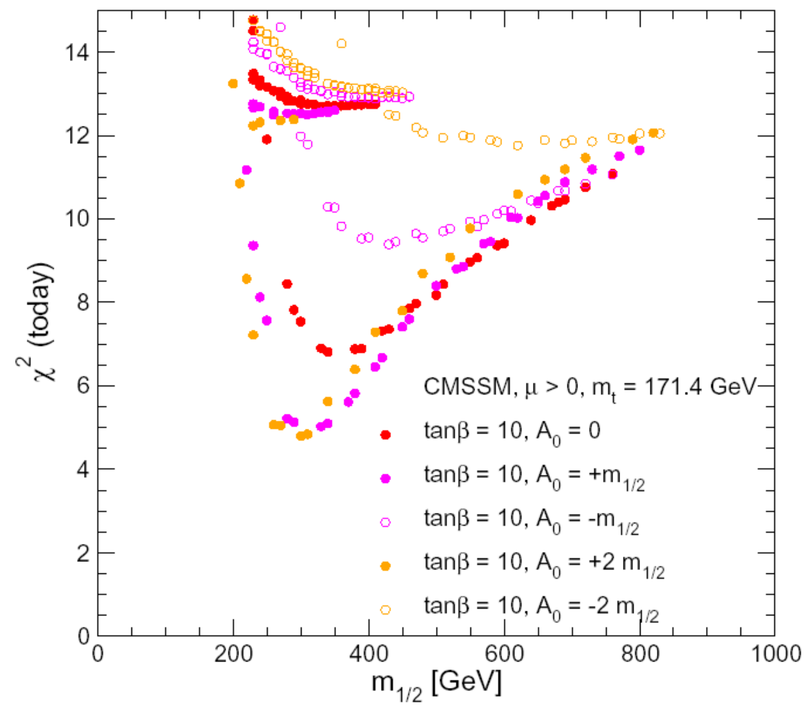
LEP: direct searches for $\tilde{\chi}, \tilde{l}$, Supersymmetric Higgs. $m_{\tilde{\chi}^{\pm}} > 103$ GeV, $m_{\tilde{t}_1}$, $m_{\tilde{l}} \gtrsim 90$ GeV.

Large M_A : $m_h > 114$GeV $M_A \simeq M_Z$: $m_h, m_A > 92$ GeV. This means a bound on $M_{H^{\pm}}$. Can be relaxed in NMSSM.

Constraints from $B \to s\gamma$. Expt. agrees with the SM. Constraints on the mSUGRA, may be alleviated with small amount of flavour violation.

Higgs limits can also be translated on limits on SUSY parameter space which determines the Higgs sector, sensitively depend on m_t used. Theoretical activity in calculating Higgs mass accurately to do draw these conclusions. Limits from Higgs mass limits and precision measurements.





In general LHC can

1)give accurate information about sparticle Mass differences and some not so accurate information on actual masses

2)discussions about how to establish spin have now started. May be possible to get information on spin of sparticle involved in decay chains.

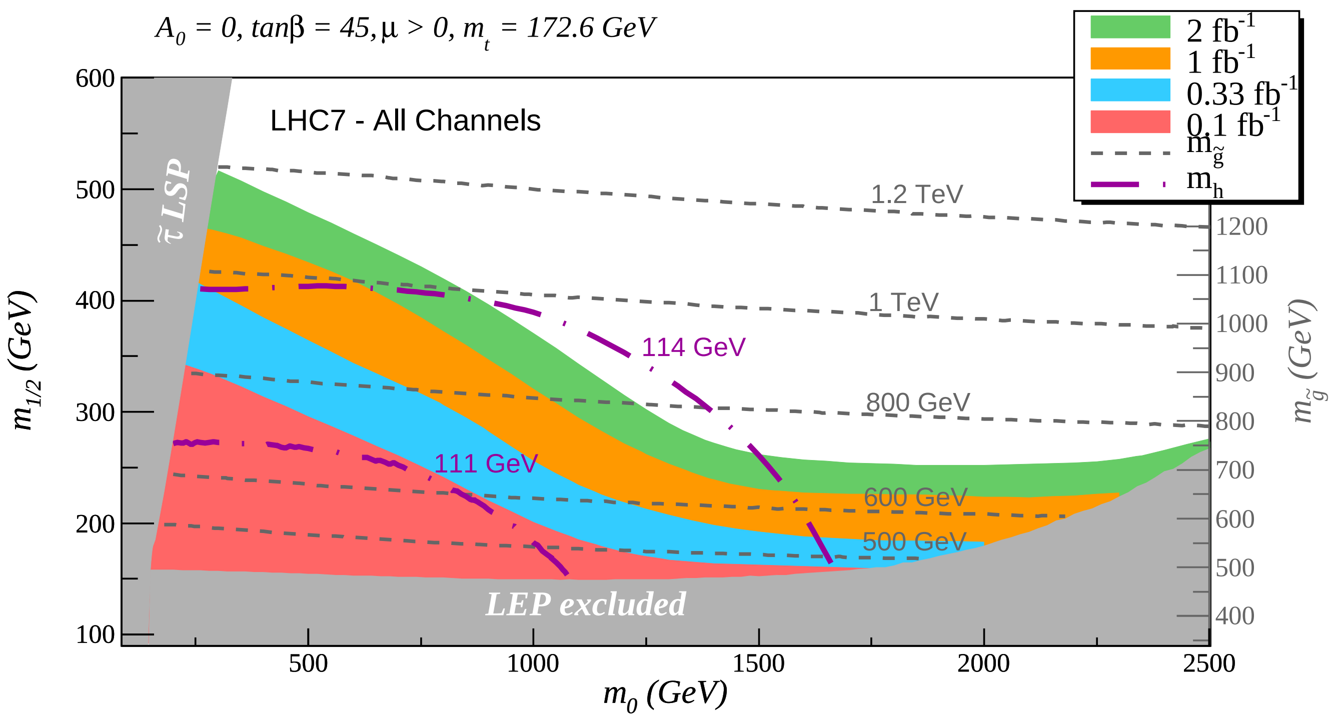
3)But for anything more we have to turn to ILC.

4)Most of the analyses were in the framework of a model. Model indep. analyses have begun now.

What LHC can do with 7 TeV and $1 fb^{-1}$.

Baer, Barger, Lessa and Tata, 1004.3594 (JHEP XX)

LHC will tell if SUSY exists in this mass range! If it does so indeed it proves relevance of SUSY for EW physics!



LEP has ruled out existence of a Higgs lighter than M_Z.

Does it mean MSSM is ruled out? no! large radiative Corrections due to large top mass, proportional to m_t^4

For last 15 years radiative corrections have been computed. Two loop results are available.

For $\tan\beta > 1$, the mass eigenvalue of h increases monotonically with m_A, saturating to its maximum Upper bound $m_h < (M_Z^2 \cos^2 2\beta + \epsilon_h)^{1/2}$ with $\epsilon_h = \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2}$.

The upper bound is further relaxed when $\tilde{t}_L - \tilde{t}_R$ mixing is included and maximal when $A^t = \sqrt{6 m_{\tilde{t}_1} m_{\tilde{t}_2}}$.

The absolute upper bound is 132 GeV in MSSM for $M_s = 1$ TeV.

Relaxed in Non minimal SSM upto 165–170 GeV.

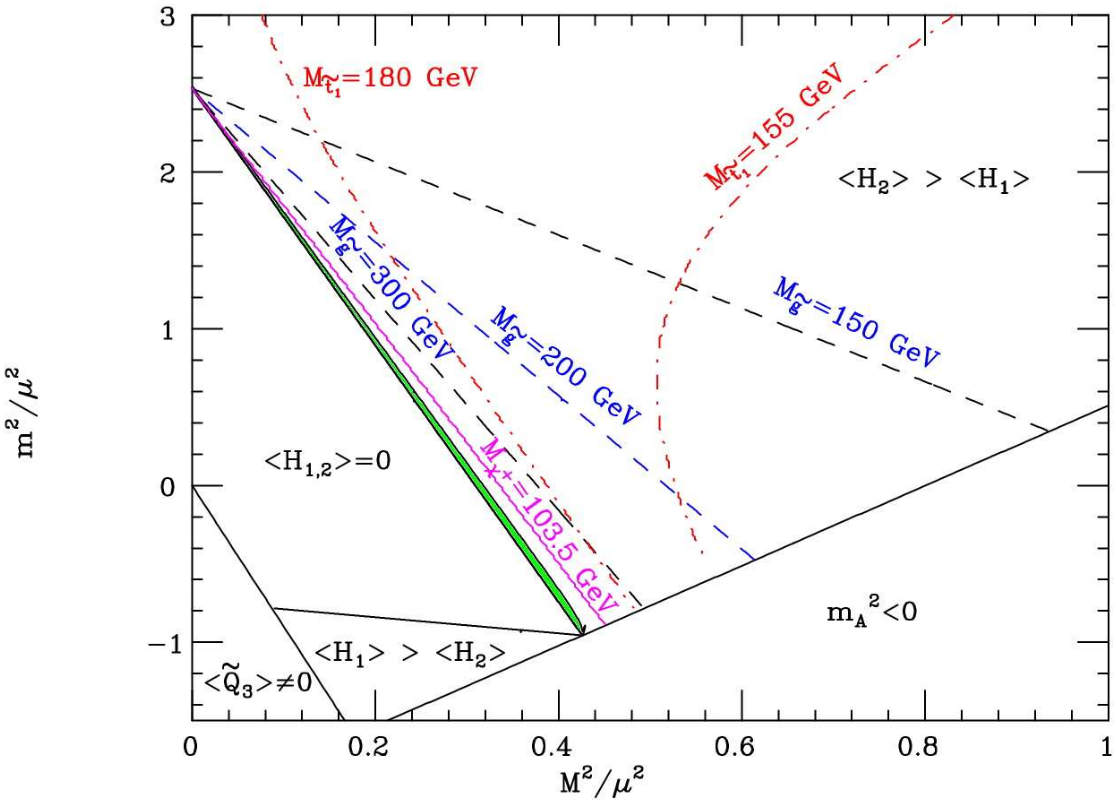
Recall:

If, $m_{\mathsf{h}}^2 = m_{\mathsf{bare}}^2 + \delta m_{\mathsf{h}}^2$ the top loop (e.g.) gives

$$\delta m_{\mathsf{h|top}}^2 \sim -\frac{3G_{\mathsf{F}}}{2\sqrt{2}\pi^2} m_t^2 \Lambda^2 \sim -(0.2\Lambda)^2.$$

The light higgs is 'natural' then only if $\Lambda \sim$ TeV.

Current experimental bounds on Higgs mass very close to the theoretical upper bound.



This can be achieved only with heavy sparticle masses.

Alternatively, the RG scale Q_C at which higgs mass turns negative causing EW symmetry breakdown and M_s need to be close to each other.

Still alternatively fine tuning required to satisfy relation between M_Z and μ and m_i^2 required by EWSB.

LHC will tell!

In the meanwhile..

To be able to understand what LHC is telling us and see things in more detail.. you can begin with: