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Mesoscopic Nano-Electro-Mechanics of Shuttle Systems

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- Lecture1: Mechanically assisted single-electronics
- Lecture2: Quantum coherent nano-electro-mechanics
- Lecture3: Mechanically assisted superconductivity

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Lecture 3
Mechanically Assisted Superconductivity

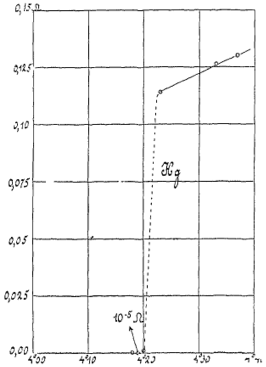
Outline

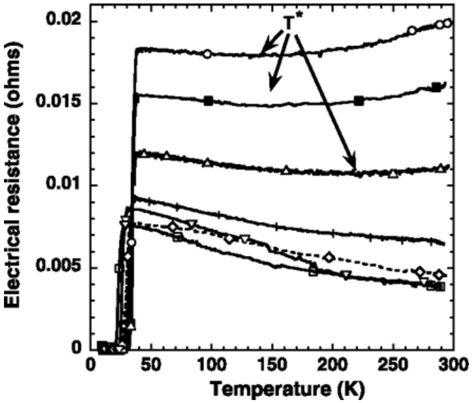
- Superconductivity – Basic facts
- Nanomechanically asisted superconductivity

Superconductivity: Basic Experimental Facts

1. Zero electrical resistance (1911)

Heike Kammerlingh-Onnes
(1853 - 1926)

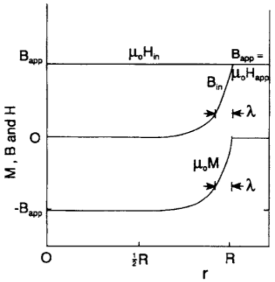




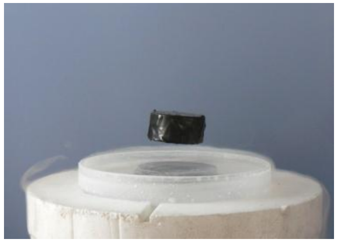
$$\delta T \sim 10^{-4}\ \text{K}$$

2. Ideal diamagnetism (Meissner effect, 1933)

Walther Meissner
(1882-1974)



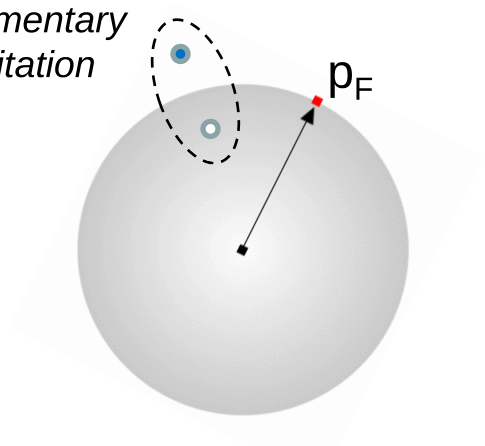
Plot of the fields B and H and of the magnetization $\mu_o M$ inside ($r < R$) and outside ($r > R$) a Type I superconducting cylinder of radius R in an axial applied magnetic field $B_{app} = \mu_o H_o$. [From Poole *et al.* (1995), p. 42.]



A magnet levitating above a high-temperature superconductor, cooled with liquid nitrogen. A persistent electric current flows on the surface of the superconductor, acting to exclude the magnetic field of the magnet (the Meissner effect).

Part 1
Mesoscopic Superconductivity
(Basic facts)

Ground State and Elementary Excitations in Normal Metals



Fermi sphere in momentum space

$$\left|N\right\rangle = \prod_{\substack{p<p_F \\ \sigma}} a^{+}_{p,\sigma} \left|0\right\rangle$$

Ground state wave function

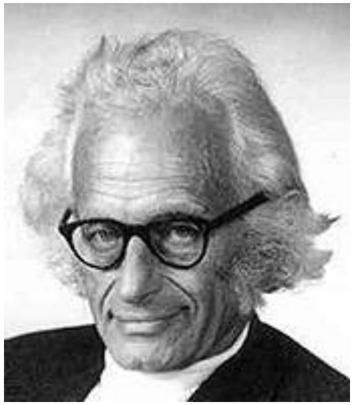
$$\xi_p = \frac{p^2}{2m} - \varepsilon_F \simeq v_F\,(p - p_F)$$

Energy of an elementary excitation

Enrico Fermi, 1901 - 1954

Cooper Instability

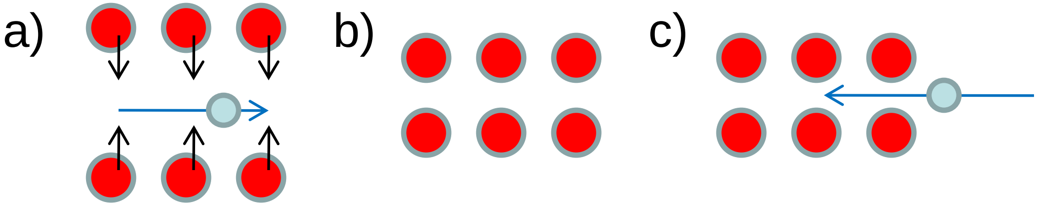
Leon Cooper, b.1930



Herbert Fröhlich, 1905 - 1991

The Fermi ground state of free electrons becomes unstable if an – even infinitesimally weak – attractive interaction between the electrons is switched on. This is the Cooper Instability (**1955**). The result is a radical rearrangment of the ground state.

Phonon mediated electron-electron attraction



a) The interaction between an electron and the lattice ions attracts the ions to the electron.

b) The resulting lattice deformation relaxes slowly and leaves a cloud of uncompensated positive charge

c) This cloud attracts, in its turn, another electron leading to an indirect attraction between electrons. In some metals this phonon-mediated attrcation can overcome the repulsive Coulomb interaction between the electrons

Quantum Fluctuations of Cooper-Pair Number

$$|BCS\rangle \equiv \prod_p (u_p + v_p e^{i\varphi} a^+_{p\uparrow} a^+_{-p\downarrow})|0\rangle = \sum_{N=1}^{\infty} \prod_{\substack{l=1 \\ \{p_l\}}}^{N} \alpha_{l,\{p_l\}} a^+_{p_l\uparrow} a^+_{-p_l\downarrow} |0\rangle$$

$$|BCS\rangle_\varphi = \sum_{N=1}^{\infty} e^{in\varphi} |n\rangle_{BCS} ; \quad |n\rangle_{BCS} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi e^{-in\varphi} |BCS\rangle_\varphi$$

$|n\rangle_{BCS} \Rightarrow$ Ground state with a given number (n) of Cooper pairs

Cooper-pair number operator ($\hat{n} = (1/2)\sum a^+_{p,\sigma} a_{p,\sigma}$ **)**

It can be proven that: $\hat{n}|BCS\rangle = \frac{1}{i}\frac{\partial}{\partial\varphi}|BCS\rangle$ It follows that $\hat{n} = -i\,\partial/\partial\varphi$ and consequently: $[\hat{n},\varphi] = -i$ from which the uncertainty relation $\delta n \delta\varphi \geq 1/2$ follows.

*Note the analogy with the momentum **p** and coordinate **x** of a quantum particle.*

$$x \Leftrightarrow \varphi; \quad \hat{n} \Leftrightarrow \hat{p}; \qquad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}; \quad [\hat{p}, x] = \frac{\hbar}{i}; \quad \delta p \delta x \geq \hbar/2$$

Quantum fluctuations of the superconducting phase ϕ occur if fluctuations of the pair number is restricted. This is the case for small samples where the Coulomb blockade phenomenon occurs.

Superconducting Current Flow

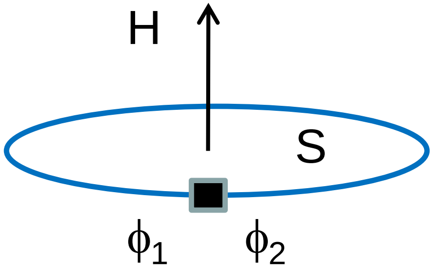
In contrast to nonsuperconducting materials where the flow of an electrical current is a nonequilibrium phenomenon, in superconductors an electrical current is a ground state property.

A supercurrent flows if the superconducting phase is spatially inhomogeneous. Its density is defined as:

$$j = nev_s; \quad v_s = \frac{\hbar}{m}\frac{\partial \varphi}{\partial x}$$

How to arrange for a spatially nonhomogeneous supeconducting phase?

One way is just to inject current into a homogeneous sample. Another way is to switch on an external magnetic field.



$$\varphi = \varphi_1 - \varphi_2 = 2\pi \frac{\Phi}{\Phi_0}; \qquad \Phi = HS; \quad \Phi_0 = \frac{h}{2e}$$

Quasiparticle Excitations in a Superconductor

We have discussed ground state properties of a superconductor. What about its excited states? These may contribute at finite temperatures or when the superconductor is exposed to external time dependent fields. Similary to a normal metal, low energy excited states of a superconductor can be represented as a gas of non-interacting quasiparticles. The energy spectrum for a homogeneous superconductor takes the form

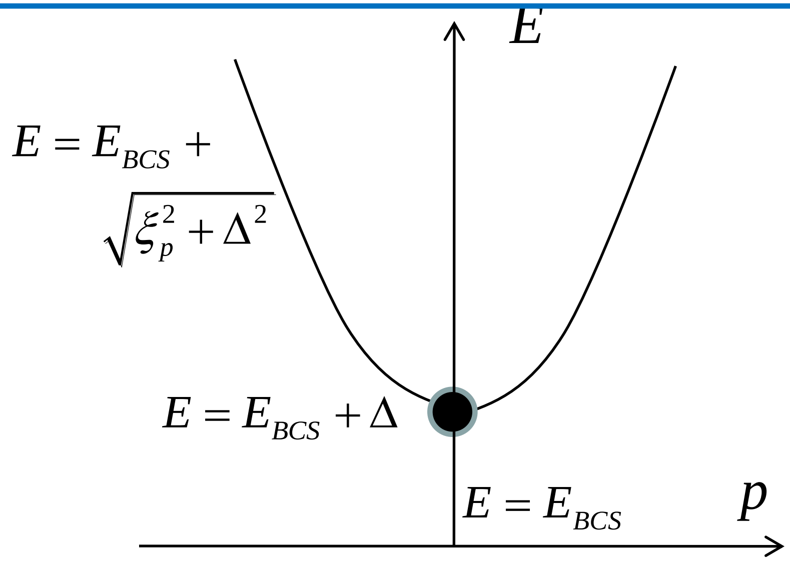
$$E = E_{BCS} + \sum n_F\left(\varepsilon_q\right)\varepsilon_q; \qquad \varepsilon_q = \sqrt{\xi_q^2 + \Delta^2}; \qquad n_F(\varepsilon) = \frac{1}{e^{\varepsilon/T}+1}$$

Important features:

- The spectrum of the elementary excitations has a gap which is given by the superconducting order parameter Δ. This is why the number of quasiparticles $\mathbf{n_F(\varepsilon_p)}$ is exponentially small at low temperatures $\mathbf{T \ll \Delta}$.
- It is important that at such low temperatures a superconductor can be considered to be a single large quantum particle or molecule which is characterized by a single (BCS) wave function.
- A huge amount of electrons is incorporated into a single quantum state. This is not possible for normal electrons due to the Pauli principle. It is the formation of Cooper pairs by the electrons that makes it possible.

Parity Effect

The **BCS** ground state is a superposition of states with different integer numbers of Cooper pairs. It does not contain contributions from states with an odd number of electrons. ***What happens if we force one more electron into a superconductor?*** The **BCS** state would not be the ground state of such a system. ***What will it be?*** The only option is to put the extra electron into a quasiparticle state. Then the ground state would correspond to the lowest-energy quasiparticle state being occupied (see figure).



$$E(N) = \begin{cases} E_{BCS} & N = 2n \\ E_{BCS} + \Delta & N = 2n+1 \end{cases}$$

Now the ground state energy depends on the parity of the electron number **N** (parity effect).

Note that the **BCS** ground state energy does not depend on the superconducting phase ϕ. Next we will see that quantum tunneling of Cooper pairs will remove this degeneracy.

Josephson Effect

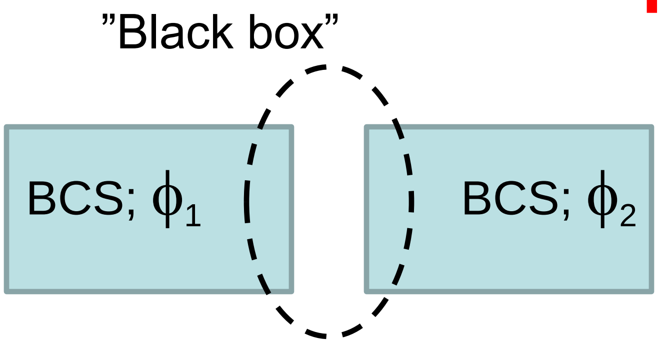
Mesoscopic effects in normal metals are due to phase coherent electron transport, i.e. the phase coherence of electrons is preserved during their propagation through the sample.

Is it possible to have similar mesoscopic effects for the propagation of Cooper pairs? To be more precise: ***What would be the effect if Cooper pairs are injected into a normal metal and are able to preserve their phase coherence?***

One possibility is to let Cooper pairs travel from one superconductor to another through a non-superconducting region. This situation was first considered by Brian Josephson, who in 1961 showed that it would lead to a supercurrent flowing through the non-superconducting region (Josephson effect, Nobel Prize in 1973).

This was the beginning of the era of macroscopic quantum coherent phenomena in solid state physics.

Josephson Coupling



$$E(\varphi) = -E_J \cos(\varphi); \quad \varphi = \varphi_1 - \varphi_2$$

$$E_J = \frac{\Delta}{4}\frac{G}{G_0}; \qquad G_0 = \frac{2e^2}{h} = R_0^{-1}$$

$$I = \frac{2e}{\hbar}\frac{\partial E(\varphi)}{\partial \varphi} = I_c \sin\varphi;$$

$$I_c = \frac{\pi\Delta}{2eR} \quad ; \qquad R = G^{-1}$$

E_J: Josephson coupling energy
I_c: Josephson critical current

- There is a small but finite probability for a phase coherent transfer of electrons between the two superconductors.
- Temperature is much smaller than Δ so quasiparticles can be neglected. Therefore only Cooper pairs can transfer charge.
- Due to the Heisenberg uncertainty principle spatial delocalization of quantum particles reduces quantum fluctuations of their momentum and hence lower their kinetic energy. Similarly, letting Cooper pairs be spread over two superconductors lower their energy.
- This lowering depends on Δ and the barrier transparency (through the conductance *G*) and can be viewed as a coupling energy caused by Cooper pair transfer.

Superconducting Ground State

Normal metal

$$|N\rangle = \prod_{p<p_F;\sigma} a^{+}_{p,\sigma}|0\rangle$$

Superconductor

$$|BCS\rangle = \prod_{p}(u_p + v_p e^{i\varphi} a^{+}_{p\uparrow} a^{+}_{-p\downarrow})|0\rangle$$

$$u_p^2 + v_p^2 = 1; \quad v_p = \frac{1}{2}\left[1 - \frac{\xi_p}{\sqrt{\xi_p^2 + \Delta^2}}\right]$$

$$\Delta e^{i\varphi} = g\langle BCS|a^{+}_{p\uparrow} a^{+}_{-p\downarrow}|BCS\rangle$$

- In the ground state of a normal metal, each single-electron state p is occupied if **$p<p_F$** .
- In the **BCS** state, on the other hand, the occupation of all states **p** have quantum fluctuations and **v_p** is the probability amplitude for state **p** to be occupied.
- In addition the occupation of state **p** is coupled to the occupation of state **–p**, so that the single-electron states fluctuate in pairs **(p;σ,-p;-σ)**
- One says that the **BCS** state forms a condensate of pairs of electrons, so called Cooper pairs.

The complex parameter $\Delta e^{i\phi}$, which controls the quantum fluctuations in the occupation of paired states, determines a new symmetry achieved by the formation of the super-conducting ground state. It is called the superconducting order parameter and has to be calculated self consistently using the condition that the ground state energy is minimized. This leads to the self consistency equation above (last equation on this slide).

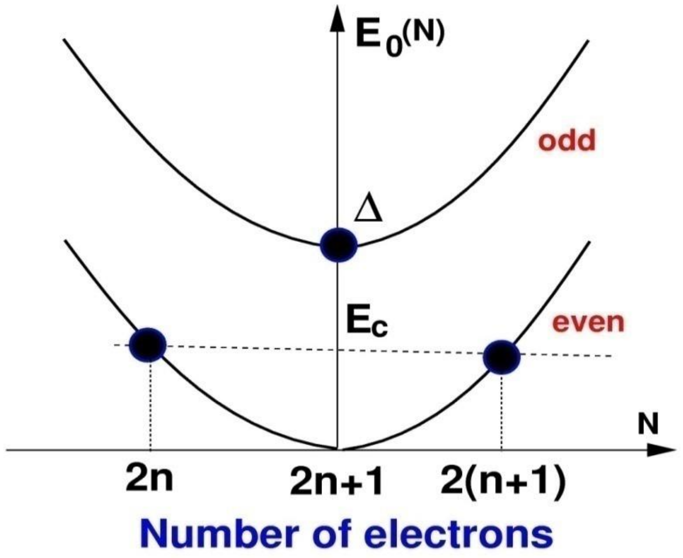
Charging Effects in Small Superconductors

Another situation where the degeneracy of the ground state energy of a superconductor with respect to the superconducting phase ϕ occurs in small superconductors, where charging effects (Coulomb blockade) are important. Still ignoring the elementary excitations in the superconductor we express the charging energy operator as

$$\hat{H}_c = \frac{e^2}{2C}(2\hat{n} - N_g)^2; \quad \hat{n} = -i\frac{\partial}{\partial\varphi}$$

This operator is nondiagonal in the space of **BCS** wave functions with different phases. This leads to quantum fluctuations of the phase ϕ whose dynamics is governed by the Hamiltonian $\hat{H}_c$.

Lifting of the Coulomb Blockade of Cooper Pair Tunneling



$$E_0(N) = E_c\,(N - \alpha V_g)^2 + \Delta_N$$

$$\Delta_N = \begin{cases} 0, & N = 2n \\ \Delta, & N = 2n+1 \end{cases}$$

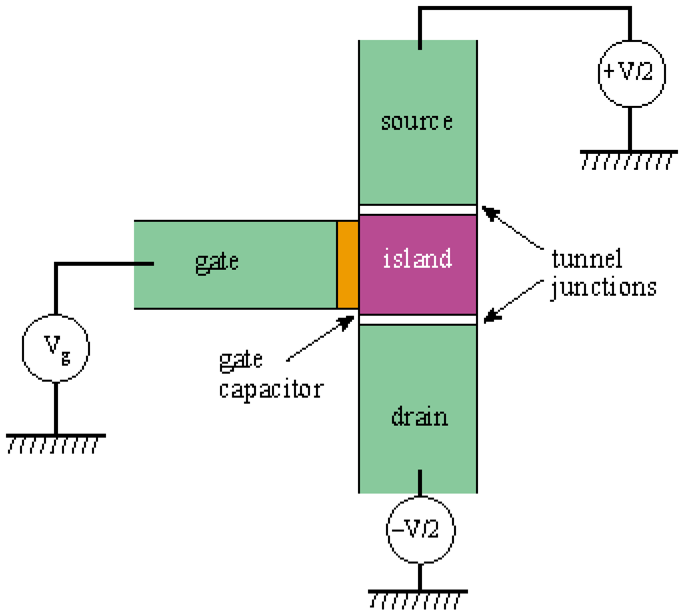
Parity Effect

At $\alpha V_g = 2n+1$ Coulomb Blockade is lifted, and the ground state is degenerate with respect to addition of one extra Cooper Pair

$$|\Psi\rangle = \gamma_1 |n\rangle + \gamma_2 |n+1\rangle$$

Single-Cooper-Pair Hybrid

Single-Cooper-Pair Transistor



The device in the picture incorporates all the elements we have considered: tunnel barriers between the central island and the leads form two Josephson junctions, while the small dot is affected by Coulomb-blockade dynamics. The Hamiltonian which includes all these elements is expressed in terms of the given superconducting phases in the leads, **ϕ_1, ϕ_2,** and the island-phase operator **ϕ**:

$$\hat{H} = \frac{e^2}{2C}(2\hat{n} - N_g)^2 - E_{J1}\cos(\varphi - \varphi_1) - E_{J2}\cos(\varphi - \varphi_2); \quad \hat{n} = -i\frac{\partial}{\partial\varphi}; \quad N_g = \alpha V_g$$

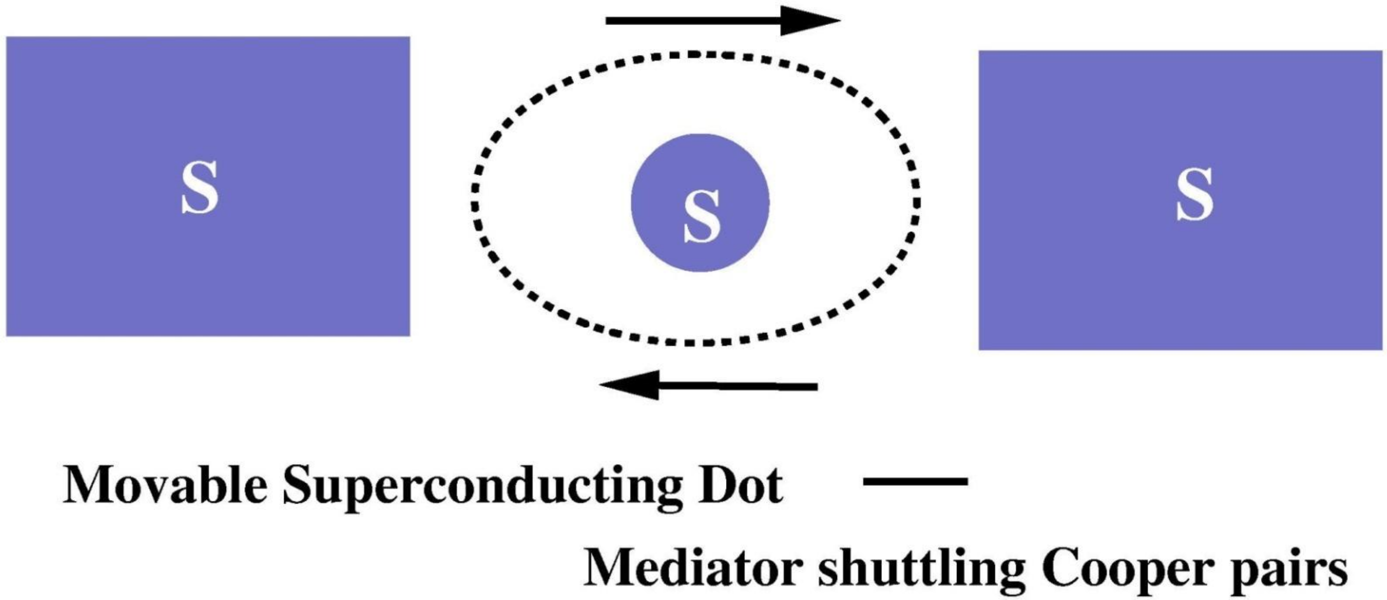
The lowest-energy eigenvalue of this Hamiltonian gives the coupling energy **$E(\phi=\phi_1-\phi_2)$** due to the flow of Cooper pairs through the Coulomb-blockade island. The Josephson current is the given as $I = (2e/h)\left(\partial E(\varphi)/\partial\varphi\right)$

Part 2

Nanomechanically assisted superconductivity

How Does Mechanics Contribute to Tunneling of Cooper Pairs?

Is it possible to maintain a mechanically-assisted supercurrent?



L.Gorelik et al. Nature 2001; A. Isacsson *et al.* PRL **89**, 277002 (2002)

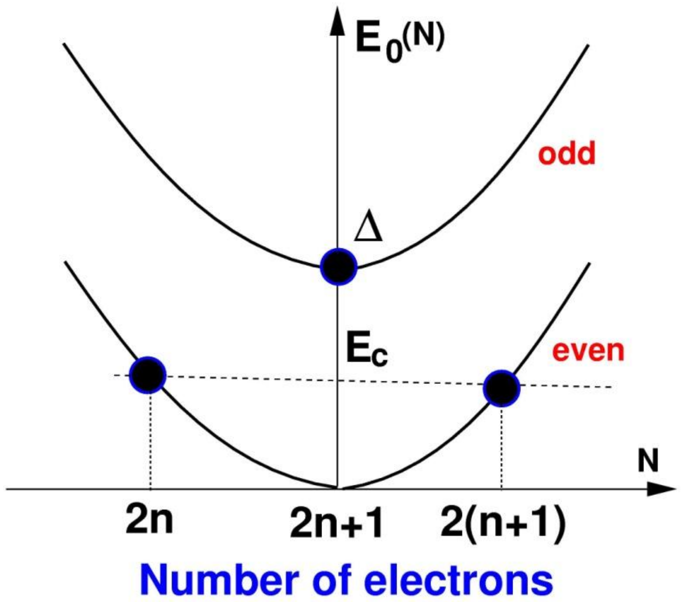
How to Avoid Decoherence?

To preserve phase coherence only few degrees of freedom must be involved.

This can be achieved or provided:

- No quasiparticles are produced
- Large fluctuations of the charge are suppressed by the Coulomb blockade:

Coulomb Blockade of Cooper Pair Tunneling



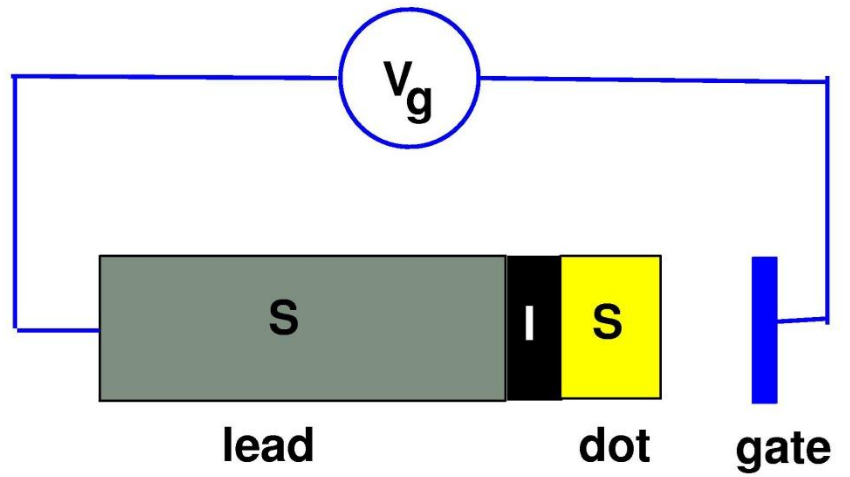
$$E_0(N) = E_c\,(N - \alpha V_g)^2 + \Delta_N$$

$$\Delta_N = \begin{cases} 0, & N = 2n \\ \Delta, & N = 2n+1 \end{cases}$$ Parity Effect

At $\alpha V_g = 2n+1$ Coulomb Blockade is lifted, and the ground state is degenerate with respect to addition of one extra Cooper Pair

$$|\Psi\rangle = \gamma_1 |n\rangle + \gamma_2 |n+1\rangle$$ Single-Cooper-Pair Hybrid

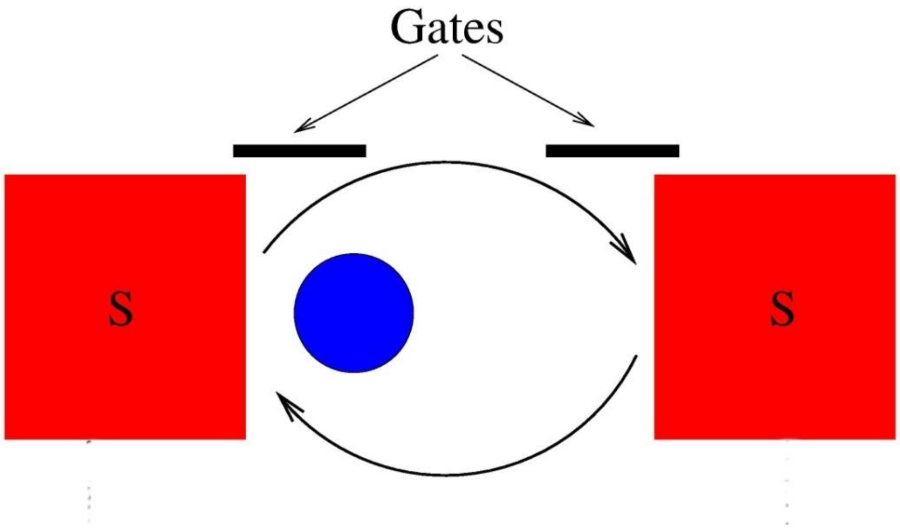
Single Cooper Pair Box



Coherent superposition of two succeeding charge states can be created by choosing a proper gate voltage which lifts the Coulomb Blockade.

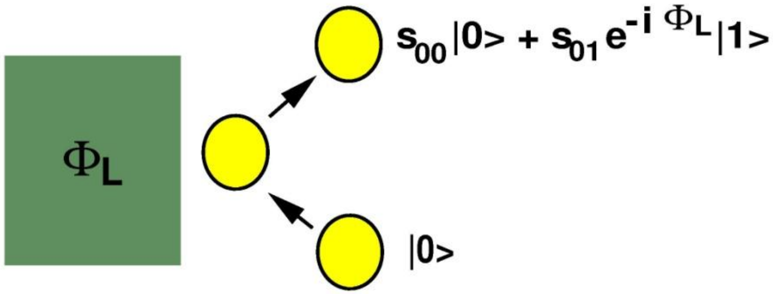
Nakamura et al., Nature 1999

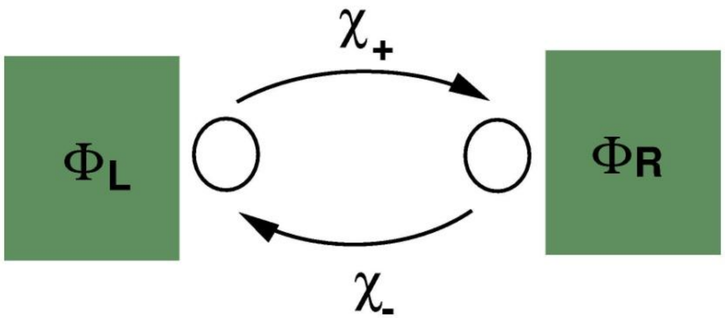
Movable Single-Cooper-Pair Box



Josephson hybridization is produced at the trajectory turning points since near these points the CB is lifted by the gates.

How Does It Work?





Between the leads Coulomb degeneracy is lifted producing an additional "electrostatic" phase shift

$$\chi_{\pm} = \int dt \left[E_0(1) - E_0(0) \right]$$

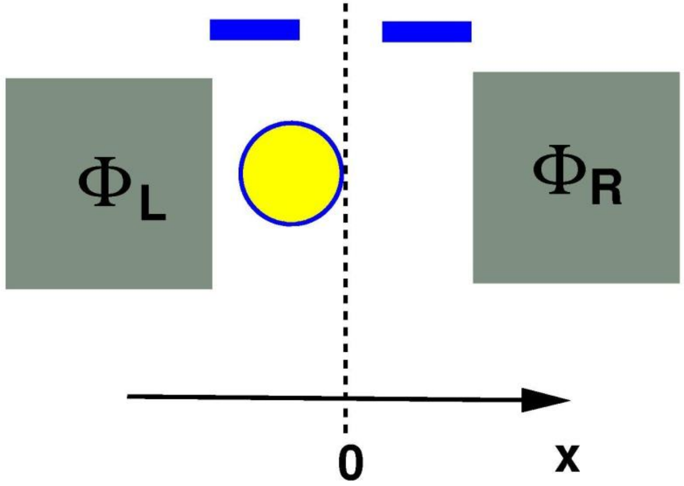
Shuttling Between Coupled Superconductors

$$H = H_C + H_J$$

$$H_C = \frac{e^2}{2C(x)}\left[2n + \frac{Q(x)}{e}\right]^2$$

$$H_J = -\sum_{s=L,R} E_J^s(x)\cos(\Phi_s - \hat{\Phi})$$

$$E_J^{L.R}(x) = E_0 \exp\left(\pm\frac{\delta x}{\lambda}\right)$$



Dynamics: Louville-von Neumann equation

$$\frac{\partial\rho}{\partial t} = -i[H,\rho] - \nu[\rho - \rho_0(H)]$$

Relaxation suppresses the memory of initial conditions.

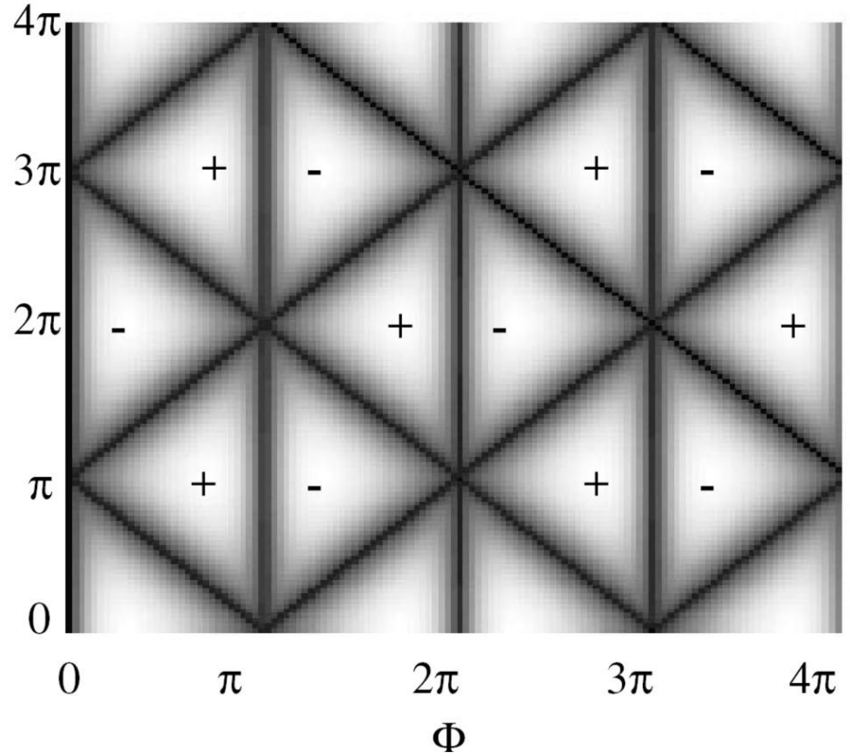
Resulting Expression for the Current

$$\frac{\bar{I}}{I_0} = \frac{\cos\vartheta\,\sin^3\vartheta\,\sin\Phi\,(\cos\chi + \cos\Phi)}{1 - (\cos^2\vartheta\,\cos\chi - \sin^2\vartheta\,\cos\Phi)^2}\,.$$

Main features:

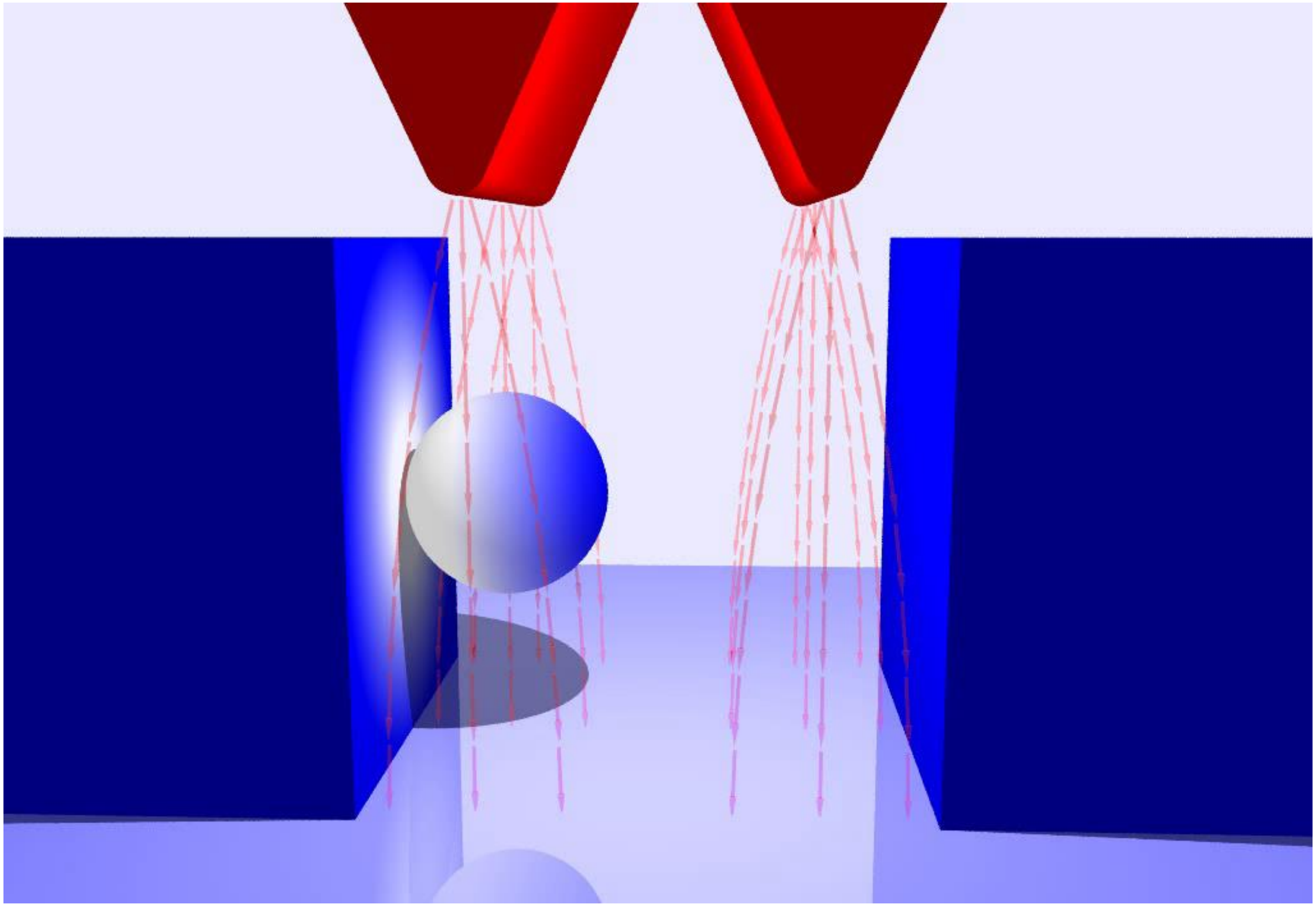
- The oscillating dependence of the dc current on the phase difference $\Phi_R - \Phi_L$ $\rightarrow$ the coherent states are controlled by the phase difference Φ;
- If there is no phase difference, $\Phi_L = \Phi_R$, but the grain's trajectory is *asymmetric*, $\chi_+ \neq \chi_-$, the current still does not vanish.
- If the grain's trajectory embeds some magnetic flux created by external magnetic field with vector-potential $\mathbf{A}(\mathbf{r})$, an extra item $(2\pi/\Phi_0)\oint \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$ enters the expression for the phase difference Φ which must be gauge-invariant.

Average Current in Units $I_0=2ef$ as a Function of Electrostatic, χ, and Supercondudting, Φ, Phases



Black regions – no current. The current direction is indicated by signs.

Shuttling of Cooper Pairs



General conclusion from the course:

Mesoscopic effects in the electronic subsystem and quantum coherent dynamics of the mechanical displacements qualitatively modify the NEM operating principles, bringing new functionality determined by quantum mechanical phases and the discrete charge of the electron.