The Abdus Salam International Centre
for Theoretical Physics

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Electron transport through nanostructures

Lecture 3

From Fermi liquid to Luttinger liquid

Regional School on Physics at the Nanoscale, Hanoi, 14-25 December 2009

Outline of the course:

- Quantum Dots
- Kondo effect in nano-devices
- From Fermi liquid to Luttinger liquid

For reading:

FL: A.Abrikosov, L. P.Gorkov and I.E. Dzyaloshinski. Methods of Quantum Field Theory in Statistical Physics (Pergamon, 1965)

LL: T.Giamarchi. Quantum Physics in One Dimension (Clarendon, Oxford 2003)

* Some transparencies are courtesy of Vadim Cheianov (Lancaster University)

Outline of this lecture

- Fermions and Bosons: symmetry and statistics
- Ideal Fermi Gas: quasiparticles
- Fermi liquid: effects of interaction
- Basic concepts of FL theory
- 1D structures: experiment on Luttinger Liquids
- Tomonaga-Luttinger model
- Excitations in TL model
- Basic concepts of LL

Fermions and Bosons

$$\Psi(\xi_1, \xi_2) = e^{i\alpha}\Psi(\xi_2, \xi_1)$$

$\alpha = 0$

Bosons

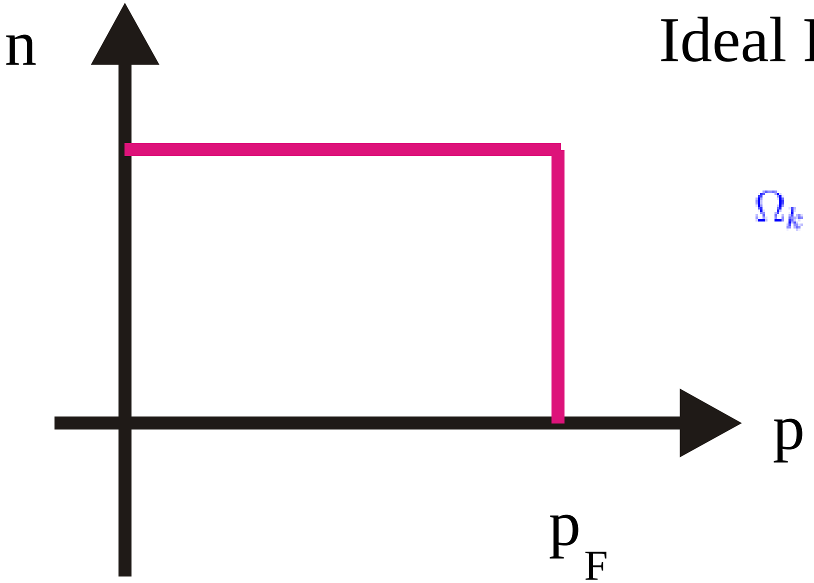
$$\Psi = \frac{1}{\sqrt{N! n_{p_1}! \ldots}} \sum P \psi_{p_1}(\xi_1)\psi_{p_2}(\xi_2)\ldots\psi_{p_N}(\xi_N)$$

$\alpha = \pi$

Fermions

$$\Psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{p_1}(\xi_1) & \psi_{p_1}(\xi_2) & \ldots & \psi_{p_1}(\xi_N) \\ \psi_{p_2}(\xi_1) & \psi_{p_2}(\xi_2) & \ldots & \psi_{p_2}(\xi_N) \\ \ldots & \ldots & \ldots & \ldots \\ \psi_{p_N}(\xi_1) & \psi_{p_N}(\xi_2) & \ldots & \psi_{p_N}(\xi_N) \end{vmatrix}$$

Ideal Fermi - gas



$$\Omega_k = -T \ln\left(\sum_k \left(e^{\frac{\mu-\epsilon_k}{T}}\right)^{n_k}\right) = -T \ln\left(1 + e^{\frac{\mu-\epsilon_k}{T}}\right)$$

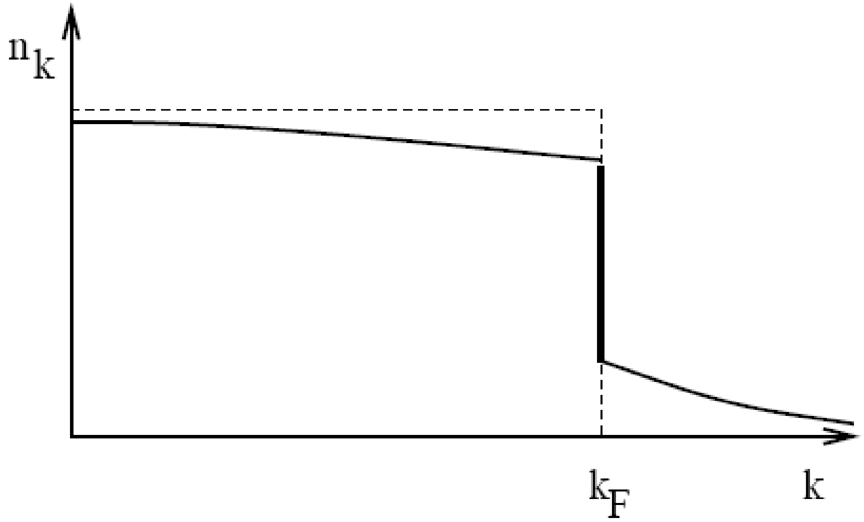
$$n_k = 0, 1$$

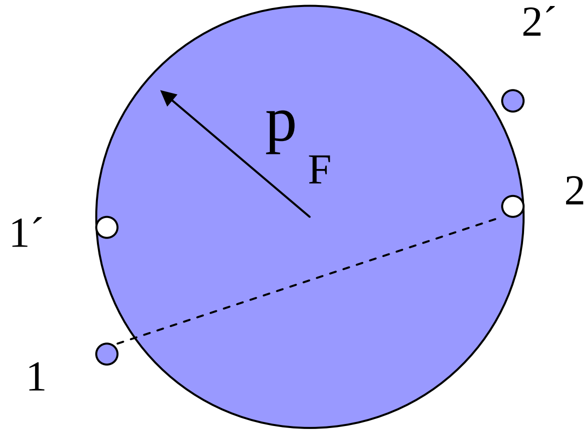
$$n(\epsilon_k) = \frac{1}{e^{\frac{\epsilon_k - \mu}{T}} + 1}$$

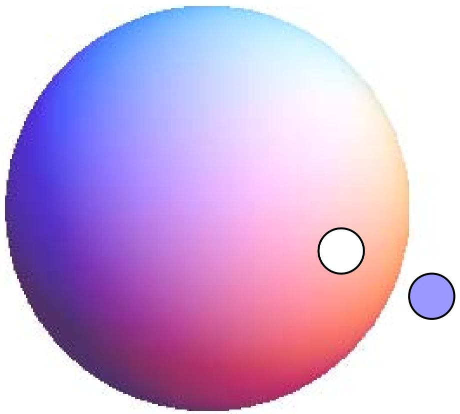
Fermi sphere

$$N = \int 2 \frac{4\pi p^2 dp V}{(2\pi\hbar)^3} = \frac{V p_F^3}{(3\pi^2\hbar^3)}$$

Fermi - liquid theory







Luttinger theorem

$$\frac{N_e}{V} = 2q\frac{N}{V} + 2\frac{V_F}{(2\pi\hbar)^3}$$

Particles and holes

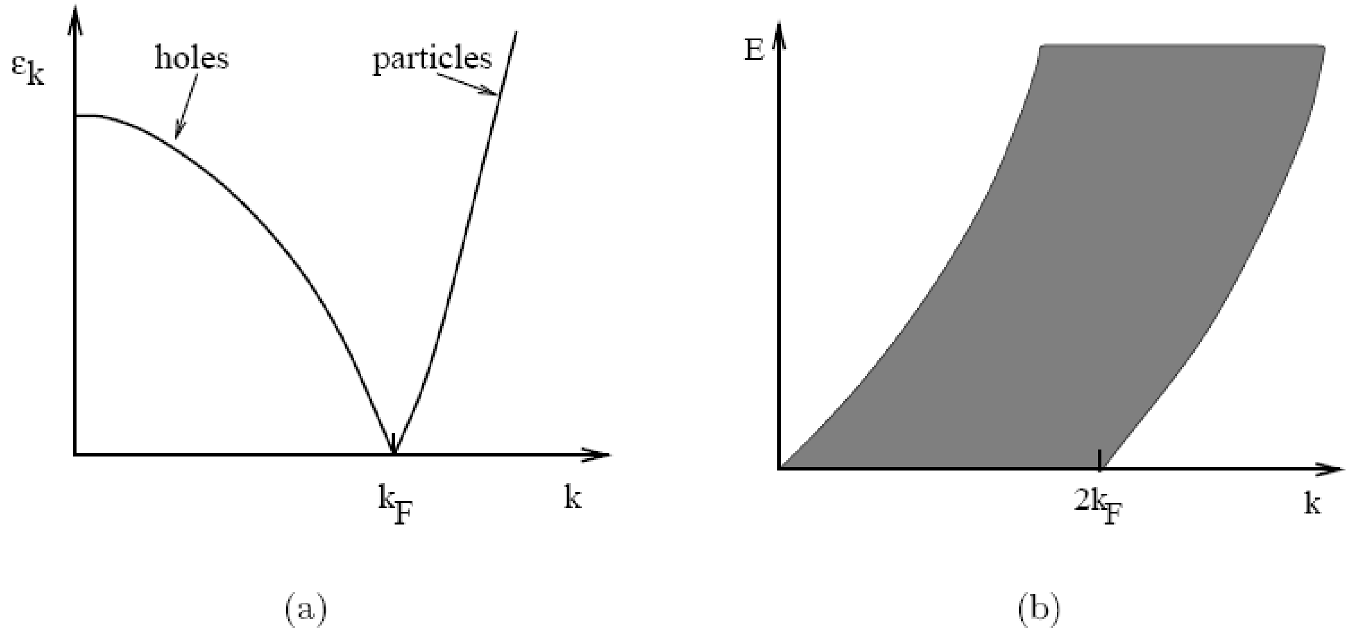
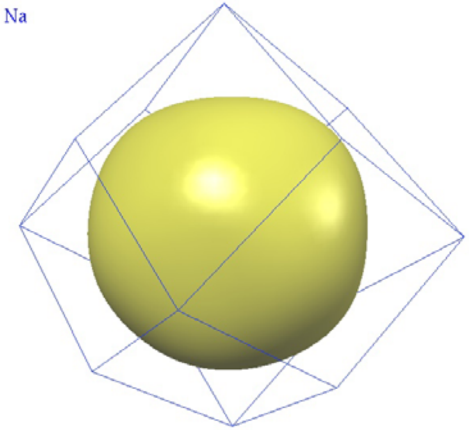
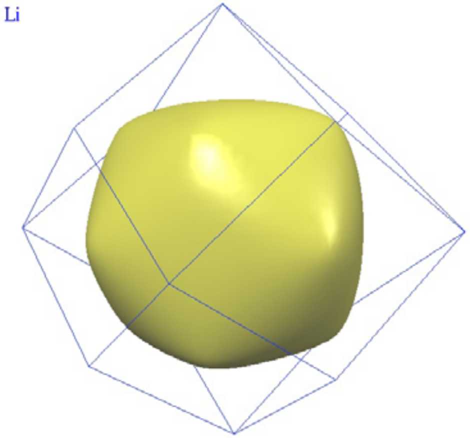


Figure 1: (a) the energy–momentum relation for the elementary particle ($k > k_{\rm F}$) and hole ($k < k_{\rm F}$) excitations; and (b) the particle–hole continuum.

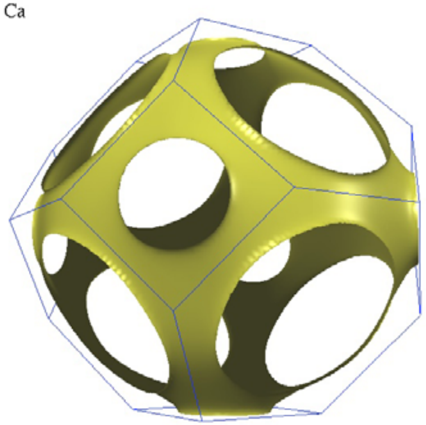
Na



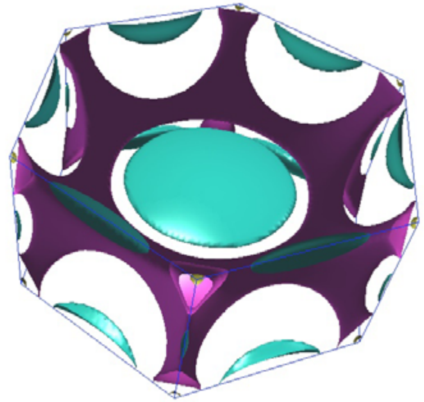
Li

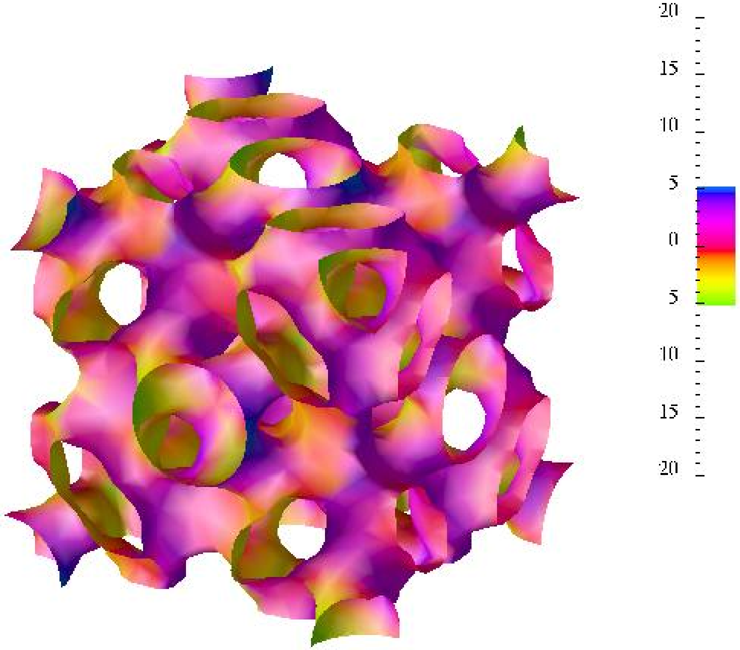


Ca

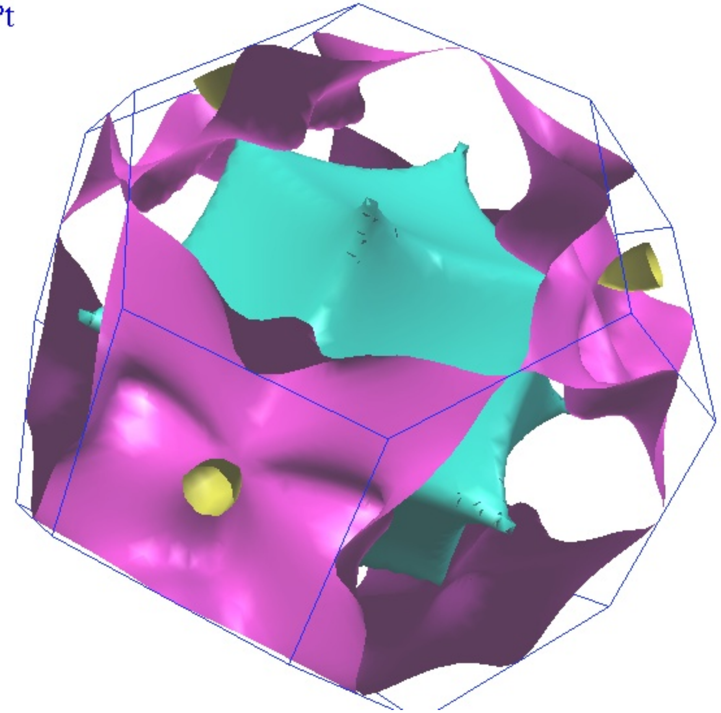


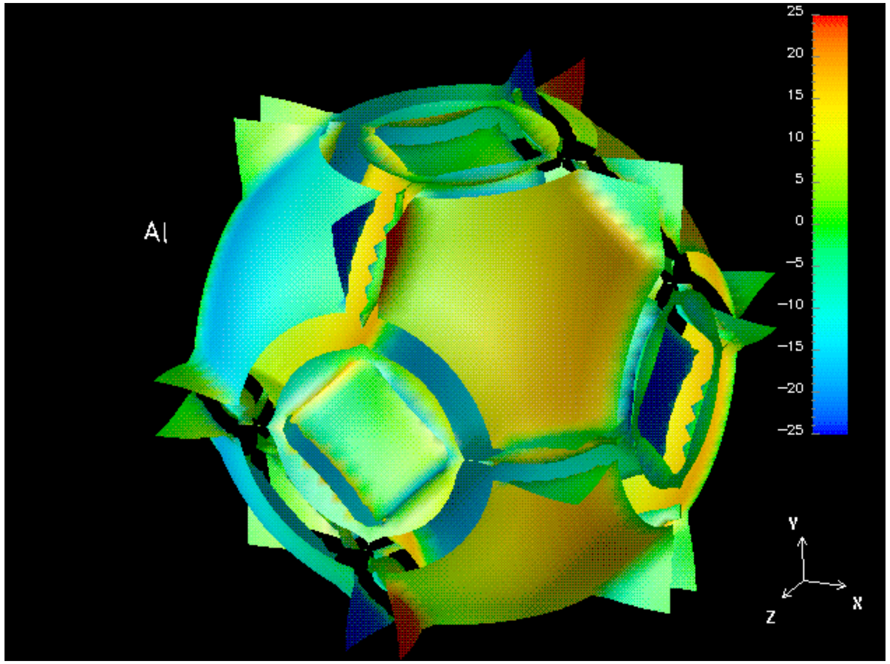
Mg

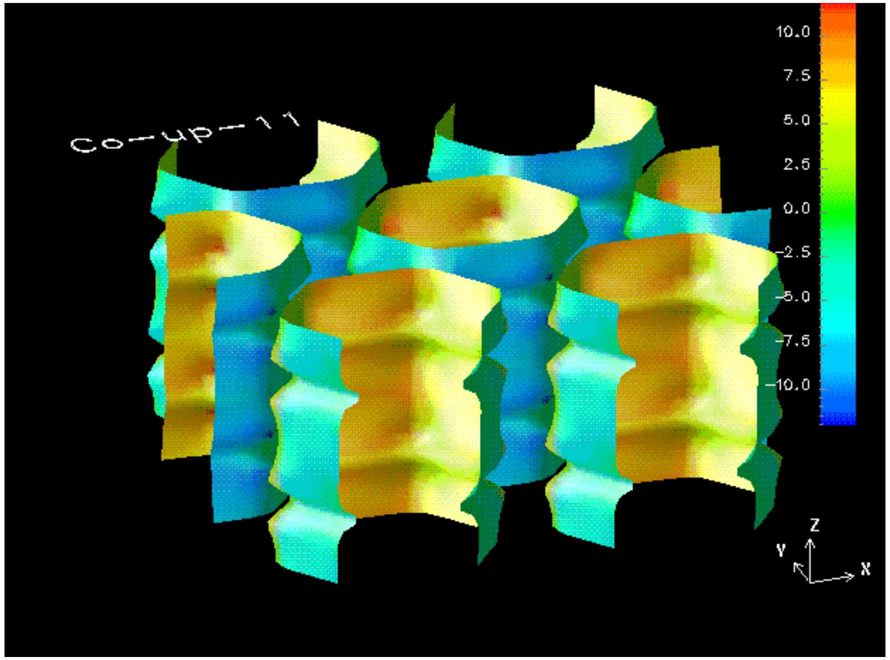


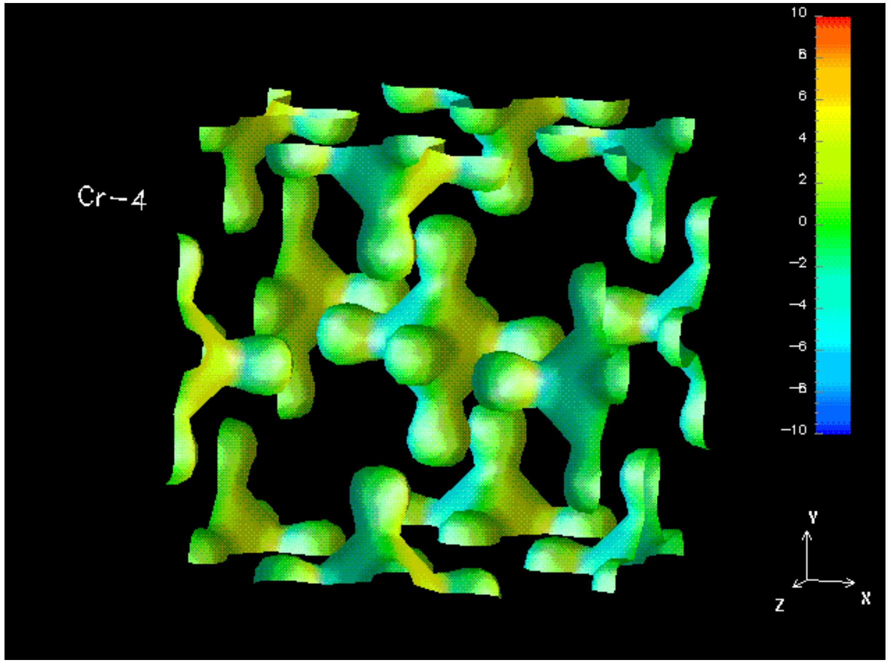


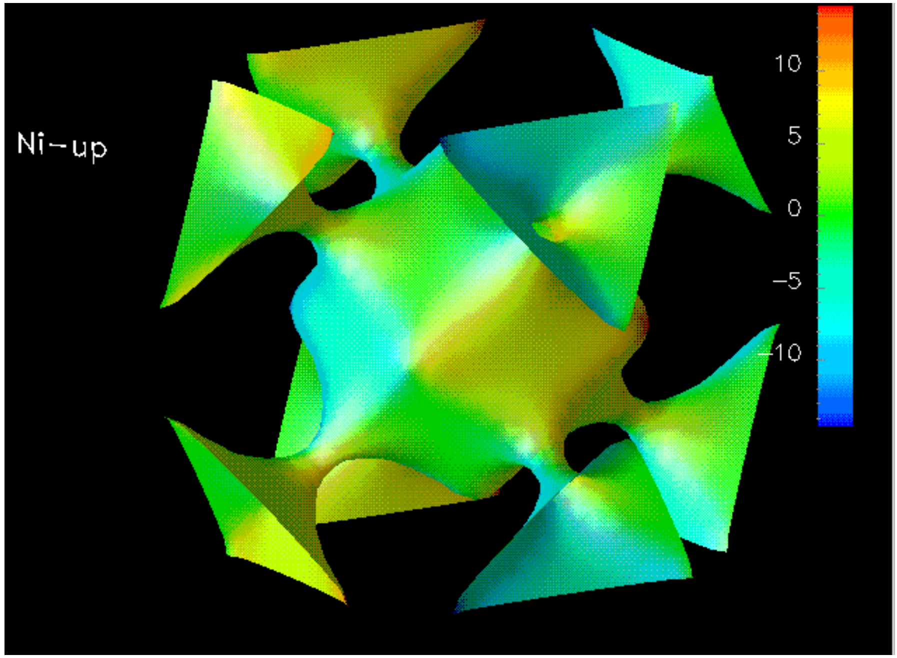
Pt



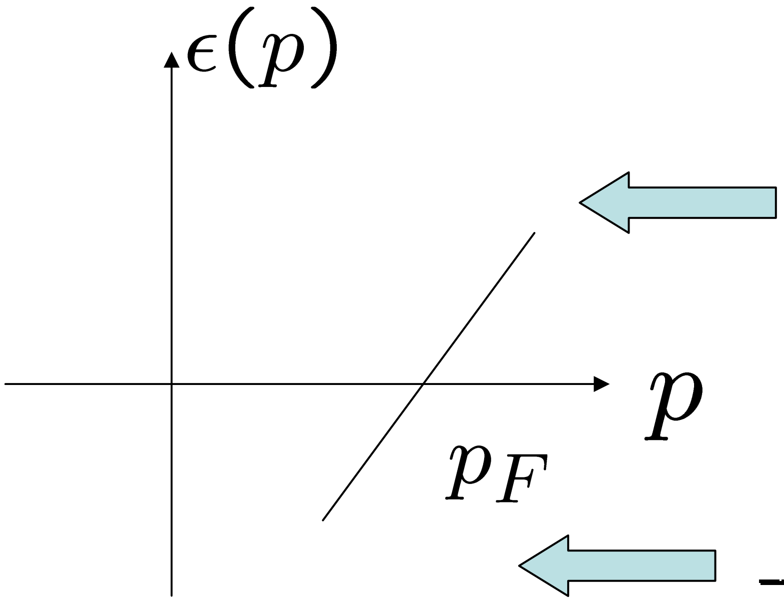

Al
25
20
15
10
5
0
−5
−10
−15
−20
−25
Y
Z
X


Co−up−11
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Z
Y
X


Cr−4
10
8
6
4
2
0
−2
−4
−6
−8
−10
Y
Z
X


Ni−up
10
5
0
−5
−10

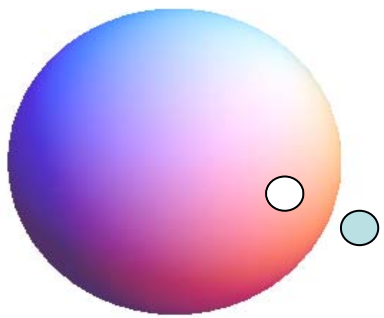
Crash course of the Fermi-liquid theory



$$\xi = \frac{p^2}{2m} - \frac{p_0^2}{2m} \approx v(p - p_0)$$

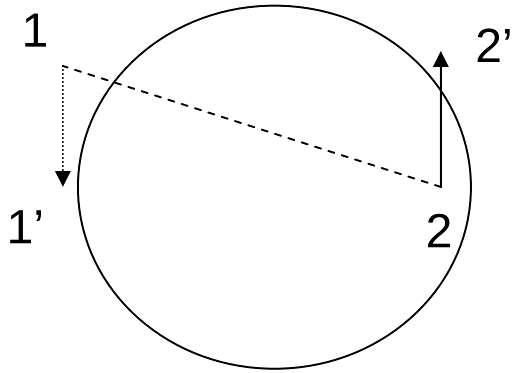
Quasiparticles (electrons)

$$-\xi = \frac{p_0^2}{2m} - \frac{p^2}{2m} \approx v(p_0 - p)\Big)$$



Quasiparticles (holes)

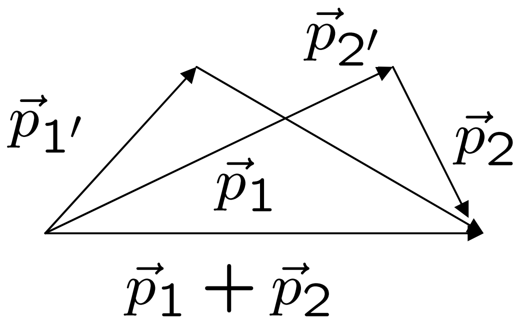
Life time of quasiparticles I



Wave Function $\sim \exp\left(-i\xi_p t - \gamma_p t\right)$

damping → γ_p

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_{1'} + \vec{p}_{2'}$$



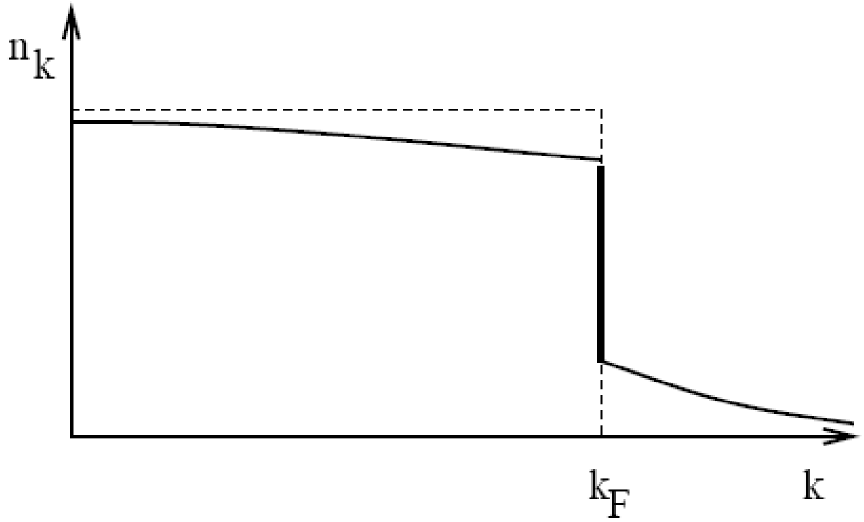
$p_1 > p_F$ $\qquad$ $p_2 < p_F$

$p_{1'} < p_F$ $\qquad$ $p_{2'} < p_F$

$$\gamma_p \sim W(p) \sim \int d\vec{p}_{1'} d\vec{p}_2 d\vec{p}_{2'} \delta(\vec{p}+\vec{p}_2-\vec{p}_{1'}-\vec{p}_{2'})\delta(\epsilon+\epsilon_2-\epsilon_{1'}-\epsilon_{2'})$$

$$\sim \int d\vec{p}_2 d\vec{p}_{2'} n(\epsilon_2)\left(1-n(\epsilon_{2'})\right)\left(1-n(\epsilon_{1'})\right)\delta(\epsilon+\epsilon_2-\epsilon_{1'}-\epsilon_{2'})$$

Life time of quasiparticles II



$$p \sim p_{1'} \sim p_2 \sim p_{2'} \sim p_F$$

$$p_2 < p_F$$

$$p_F < p_{1'} < p + p_2 - p_F$$

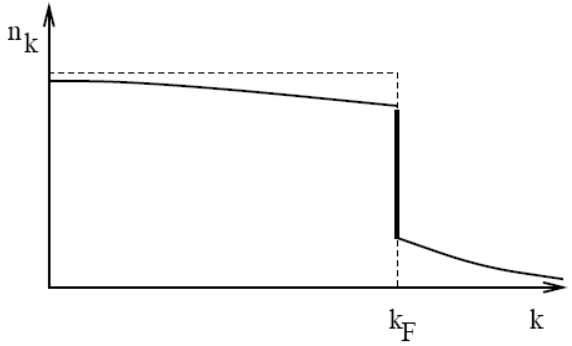
$$p_F - p < p_2 - p_F < 0 \qquad 0 < p_{1'} - p_F < p + p_2 - 2p_F$$

$$\gamma_p \sim \int_{p_F-p}^{0} d(\underbrace{p_2 - p_F}_{\tilde{p}_2}) \int_0^{p-p_F+\overbrace{(p_2-p_F)}^{\tilde{p}_2}} d(\underbrace{p_{1'} - p_F}_{\tilde{p}_{1'}}) = \frac{(p-p_F)^2}{2}$$

$$\gamma_p \sim \frac{\xi_p^2}{\epsilon_F} \ll \xi_p$$

Quasiparticles (electrons and holes) are well defined !

Life time of quasiparticles III



$$\gamma_p = \frac{1}{\tau} \sim \frac{\xi_p^2}{\epsilon_F} \ll \xi_p$$

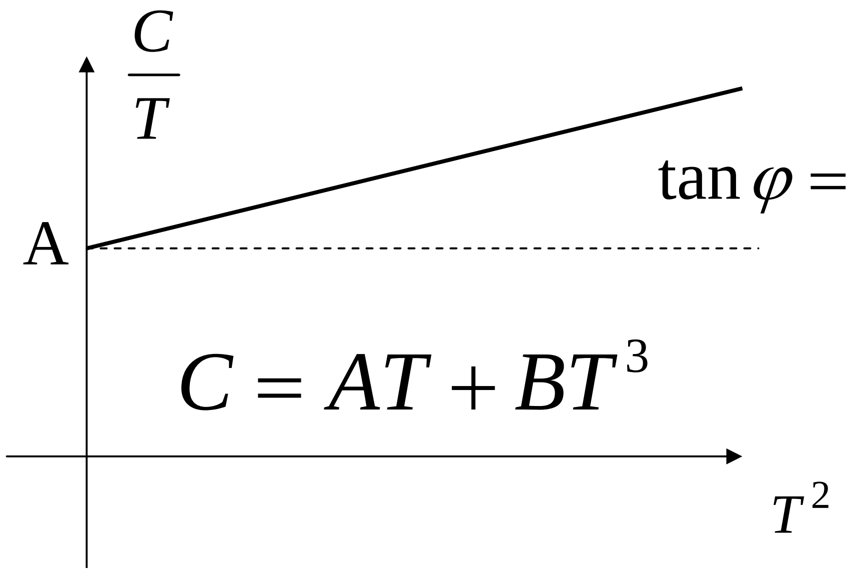
Finite temperatures $$\frac{1}{\tau} \sim \frac{T^2}{\epsilon_F} \sim T \cdot \frac{T}{\epsilon_F}$$ ⇐ Quasiparticle's number

⇑ Quasiparticle's energy

Finite frequencies $$\frac{1}{\tau} \sim \frac{\omega^2}{\epsilon_F}$$

FL Resistivity at low temperatures $$\frac{\delta\rho}{\rho} \sim \left(\frac{T}{\epsilon_F}\right)^2$$

Fermi liquid theory: specific heat



$$E = 2V \int \epsilon(p) n(\epsilon_p) \frac{d^3p}{(2\pi\hbar)^3}$$

$$C = V^{-1} \left(\frac{\partial E}{\partial T}\right)_V = 2 \int \epsilon(p) \left(\frac{\partial n(\epsilon_p)}{\partial T}\right) \frac{d^3p}{(2\pi\hbar)^3}$$

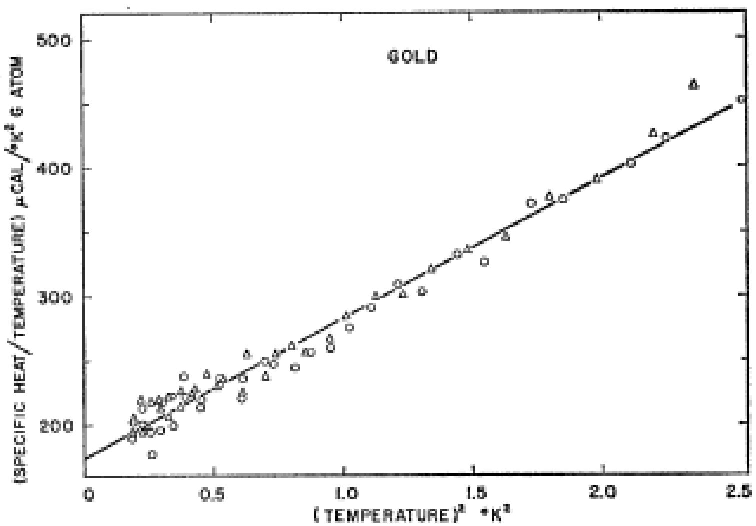


FIG. 1. Plot of (specific heat/temperature) versus (temperature²) for gold.

$$C = \frac{\pi^2}{3} \rho(0) T = \gamma T \qquad \gamma = \frac{p_F m^*}{3\hbar^3} \qquad \rho(0) = \frac{p_F m^*}{\pi^2 \hbar^3}$$

	N/V $[cm^{-3}]$	ϵ_F [eV]	T_F [K]	m^*/m
Li	4.6×10^{22}	4.7	5.5×10^4	2.3
Na	2.5	3.1	3.7	1.3
K	1.34	2.1	2.4	1.2
Cu	8.5	7.0	8.2	1.3
Ag	5.76	5.5	6.4	1.1
Au	5.9	5.5	6.4	1.1

Basic concepts of the FL theory

Similarities with Ideal Fermi Gas

- Weekly excited states of FL ⟷ weekly excited states of IFG

- Fermi momentum is connected with total electrons density exactly the same way as in IFG

- Theory is applicable at $T \ll \epsilon_F$

Differences between Ideal Fermi Gas and Fermi Liquid

- In FL quasiparticles "talk" to each other (e.g. forming SC state)

- Effective mass of excitations in FL (electrons, holes) is, in general, not equal to bare mass of quasiparticles in IFG

- There is a jump with a magnitude Z<1 in the Fermi distribution function at ϵ_F

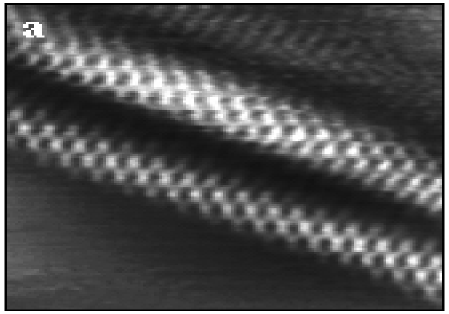
Beyond the scope of this lecture: zero sound, superconductivity, effects of disorder etc

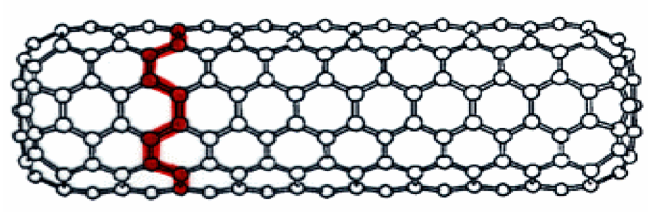
FL theory works great for description of
3D and 2D metals.

What about 1D ?

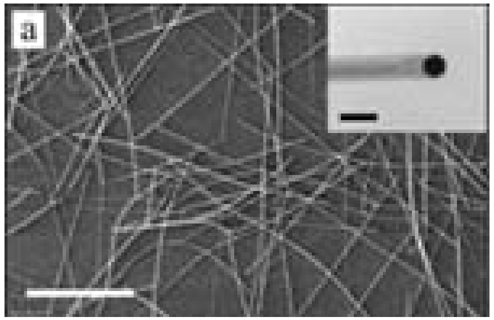
1D-Systems

1D systems – Carbon Nanotubes





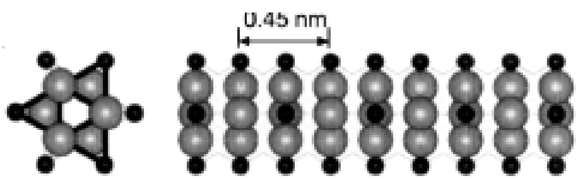
Semiconducting Nanowires



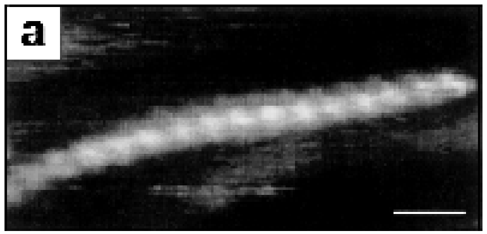
GaAs GaP InP InAs ZnS ZnSe CdS CdSe

$GaAs_{0.6}P_{0.4}$ $InAs_{0.5}P_{0.5}$ GaN(Fe)

Single molecules

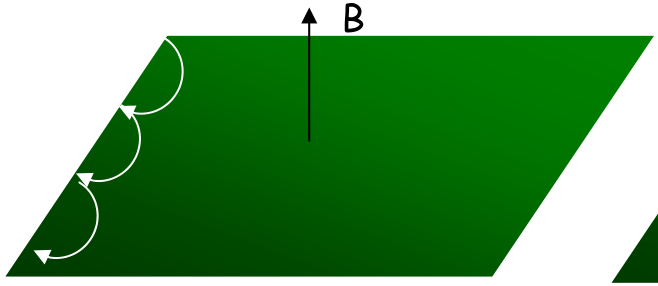


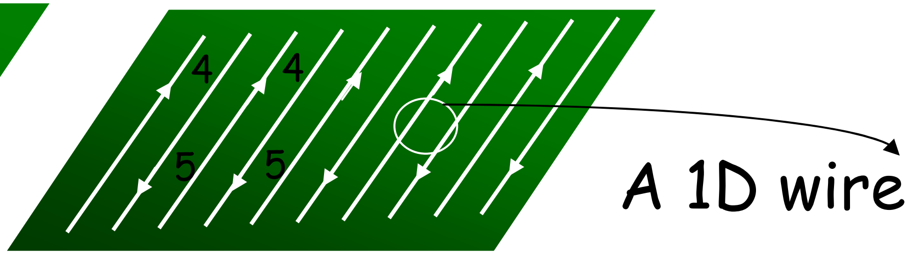
Mo_6Se_6(MoSe)



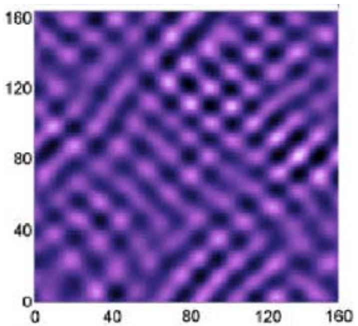
Other Systems Showing 1D Physics

2D systems – Chiral edges of the QH and FQHE
Striped phase at high Landau levels



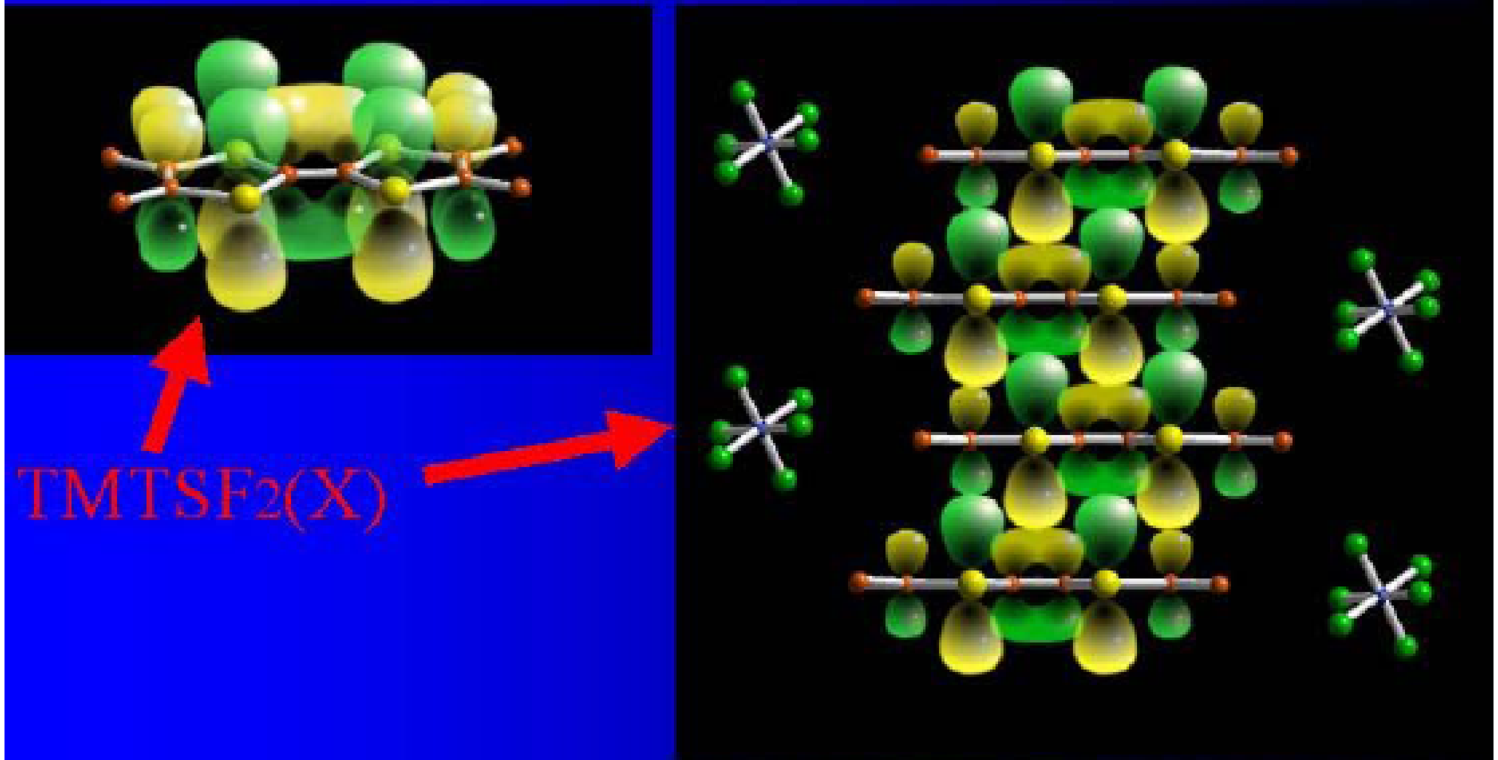


3D systems – Crystals of 1D molecules - Polyacetelene
Stripes in High Tc superconductors



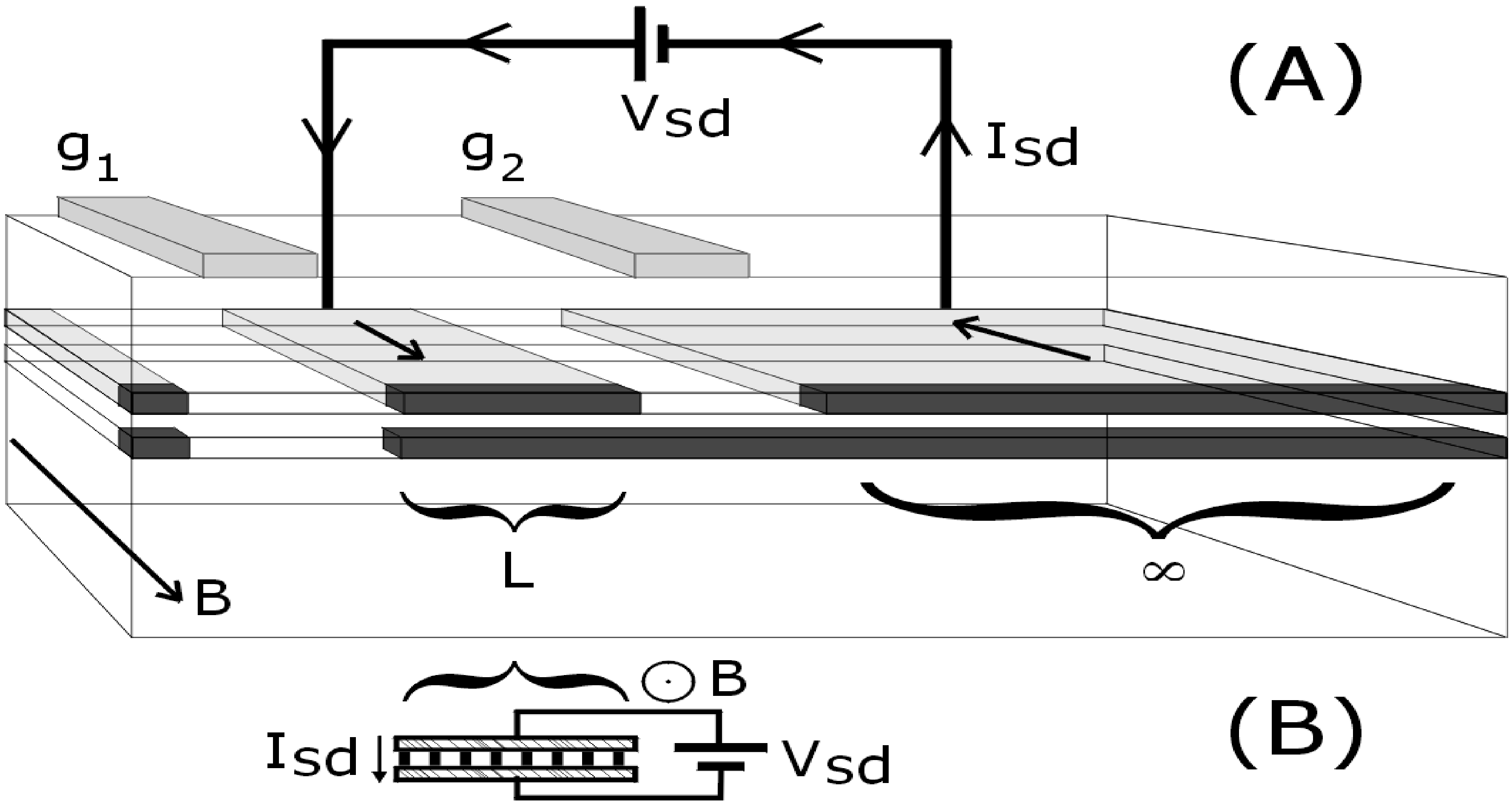
Coupled wires systems

Organic (super-) conductors



- Propagation of electrons along the chains

Circuit



Top wire in equilibrium with the source
and bottom wire in equilibrium with the drain

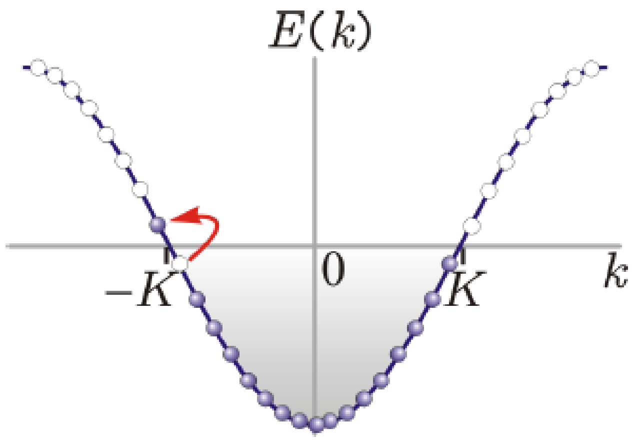
Crash course of the Luttinger-liquid theory

$$\widehat{\mathcal{H}} = \frac{1}{L}\sum_p \xi(p) a_p^+ a_p + \frac{1}{2L^2}\sum_{p_1+p_3=p_2+p_4} V_{p_1-p_2} a_{p_1}^+ a_{p_2} a_{p_3}^+ a_{p_4} \, ,$$

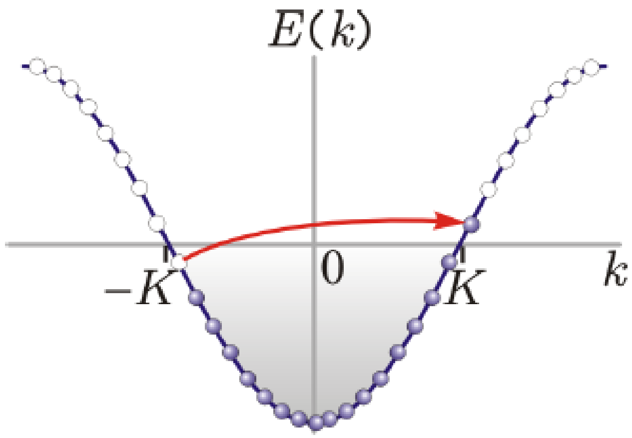
$$(1)\ p_1 \approx p_0,\ p_2 \approx p_0,\ p_3 \approx p_0,\ p_4 \approx p_0;$$
$$(2)\ p_1 \approx p_0,\ p_2 \approx p_0,\ p_3 \approx -p_0,\ p_4 \approx -p_0$$

$$g_1(k) \equiv V_k \, , \quad g_2(k) \equiv V_{2p_0+k}$$

Forward scattering



Backward scattering



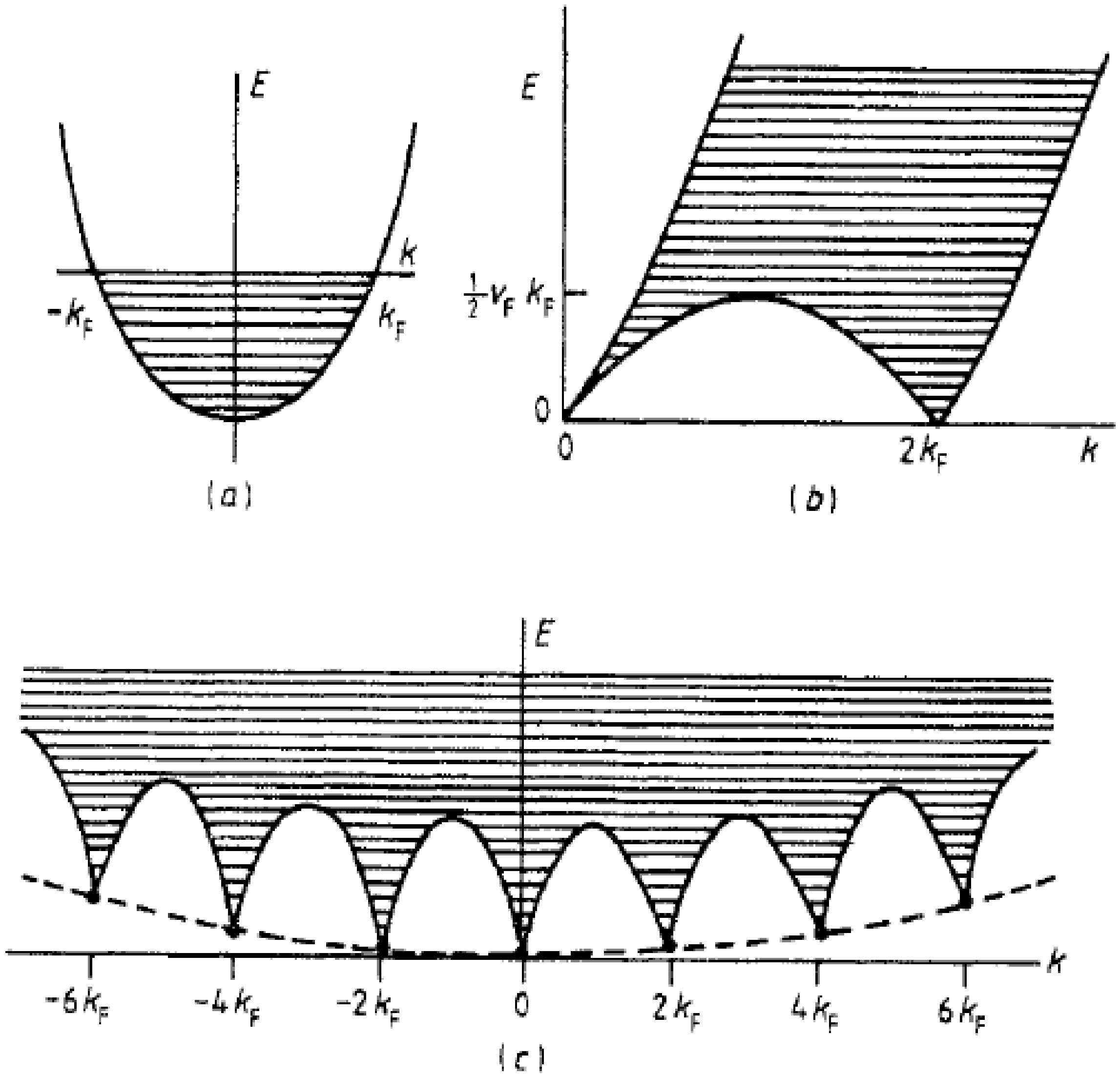
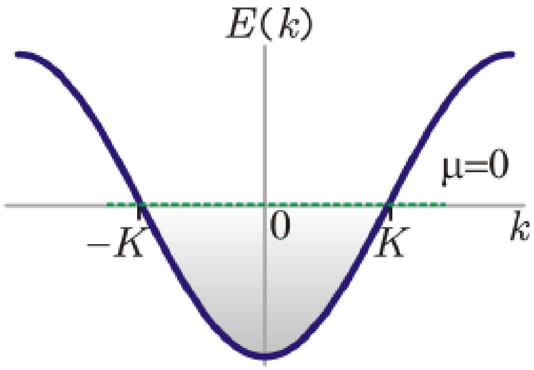
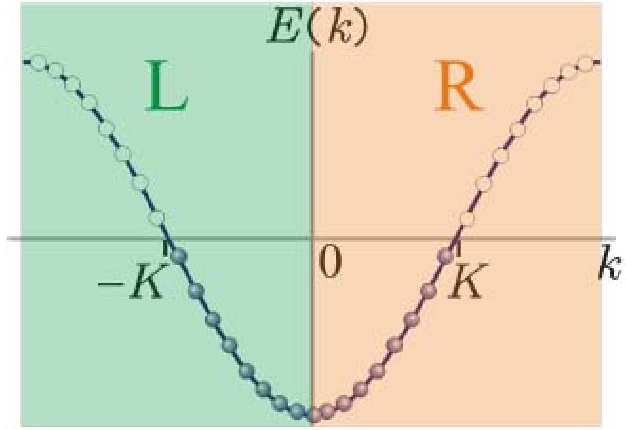


Figure 1. (*a*) Single-particle spectrum of the free Fermi gas in 1D; (*b*) Particle–hole pair spectrum; (*c*) full zero-charge (multiple particle–hole) excitation spectrum (energy differences $E(n) = 2\pi v_F n^2/L$ of extremal states at $k = 2nk_F$ greatly exaggerated).

Tomonaga-Luttinger model I



$$\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_0 + \widehat{\mathcal{H}}_1 + \widehat{\mathcal{H}}_2 \,,$$



$$\widehat{\mathcal{H}}_0 = \frac{1}{L} \sum_{k \ll p_0} \xi(p_0 + k) \left(a^+_{p_0+k} \, a_{p_0+k} + a^+_{-p_0-k} \, a_{-p_0-k} \right) \;,$$

$$\widehat{\mathcal{H}}_1 = \frac{1}{2L^2} \sum_{k_1, k_2, q} g_1(q) \left[a^+_{p_0+k_1+q/2} \, a_{p_0+k_1-q/2} \, a^+_{p_0+k_2-q/2} \, a_{p_0+k_2+q/2} \; + \right.$$

$$\left. + \; (p_0 \to -p_0) \right] \;,$$

$$\widehat{\mathcal{H}}_2 = \frac{1}{L^2} \sum_{k_1, k_2, q} g_2(q) \, a^+_{p_0+k_1+q/2} \, a_{p_0+k_1-q/2} \, a^+_{-p_0+k_2-q/2} \, a_{-p_0+k_2+q/2} \,.$$

Tomonaga-Luttinger model II

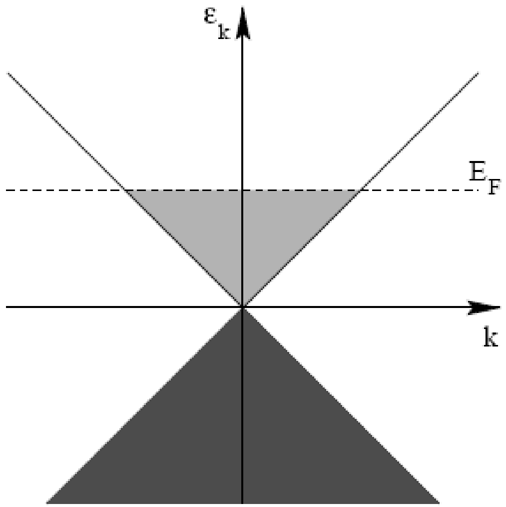


Figure 7: Single–particle energy spectrum of the Luttinger model. Occupied states are shown in grey, the dark grey area represents the states added to make the model solvable.

From Fermions to Bosons

$$\widehat{\mathcal{H}}_1 = \frac{1}{2}\sum_q g_1(q)\left(\widehat{\rho}_1(q)\,\widehat{\rho}_1(-q) + \widehat{\rho}_2(q)\,\widehat{\rho}_2(-q)\right),$$

$$\widehat{\mathcal{H}}_2 = \sum_q g_2(q)\,\widehat{\rho}_1(q)\,\widehat{\rho}_2(-q)\,.$$

What about $\mathcal{H}_0$?

$$\widehat{\mathcal{H}}_0 = \sum_k \alpha_k \left(\widehat{\rho}_1(k)\,\widehat{\rho}_1(-k) + \widehat{\rho}_2(k)\,\widehat{\rho}_2(-k)\right)$$

Problem # 1

Calculate $\left[\widehat{\rho}_1(k),\, \widehat{\mathcal{H}}_0\right] =$?

Find α_k

Spectrum of excitations

$$\widehat{\mathcal{H}} = \frac{1}{2\pi L} \sum_{k>0} \left[\Big(2\pi k v + k g_1(k)\Big) \Big(b_k^+ b_k + b^+_{-k} b_{-k}\Big) + k g_2(k) \Big(b_k^+ b^+_{-k} + b_k b_{-k}\Big) \right] .$$

Bogolubov transformation

$$\begin{aligned} \tilde{b}_k &= \operatorname{ch}\theta_k \; b_k + \operatorname{sh}\theta_k \; b^+_{-k} , \\ \tilde{b}^+_{-k} &= \operatorname{ch}\theta_k \; b^+_{-k} + \operatorname{sh}\theta_k \; b_k , \end{aligned}$$

$$\operatorname{th} 2\theta_k = g_2(k) / (g_1(k) + 2\pi v) .$$

$$\widehat{\mathcal{H}} = \frac{1}{L} \sum_k \omega(k) \, \tilde{b}_k^+ \tilde{b}_k$$

$$\omega(k) = \frac{|k|}{2\pi} \Big((2\pi v + g_1(k))^2 - g_2^2(k) \Big)^{1/2} .$$

Luttinger Hamiltonian. Standard Notations.

$$H_{\mathrm{LUT}} = \frac{v_c}{2\pi}\int dx \left[\frac{1}{K}(\partial_x\phi)^2 + K(\partial_x\theta)^2\right]$$

where

$$[\partial_x\theta(x), \phi(x')] = -i\pi\delta(x - x')$$

$$v_c = \sqrt{\left(v_{\mathrm{F}} + \frac{g_1}{2\pi}\right)^2 - \left(\frac{g_2}{2\pi}\right)^2}$$ is the sound velocity

$$K = \sqrt{\frac{2\pi v_{\mathrm{F}} + g_1 - g_2}{2\pi v_{\mathrm{F}} + g_1 + g_2}}$$ is the Luttinger parameter

Problem # 2

Using the Heisenberg equation $i\partial_t A = [A, H]$ show that field ϕ satisfies the wave equation

$$\partial_t^2\phi - v_c^2\partial_x^2\phi = 0$$

Correlation Functions: Bosons I

We start with the imaginary time correlator

$$\mathcal{G}(x,\tau) = \langle T\phi(x,\tau)\phi(x')\rangle$$

It is a Fourier transform of

$$\mathcal{G}(x,\tau) = \beta^{-1}\sum_{\omega_n}\int \frac{dk}{2\pi} e^{ikx - i\omega_n\tau}\mathcal{G}(i\omega_n, k)$$

where $G(i\omega_n, k)$ in a free Boson theory it is given by

$$\mathcal{G}(i\omega_n, k) = \frac{\pi v K}{\omega_n^2 + v_c^2 k^2}, \qquad \omega_n = \frac{2\pi n}{\beta}$$

Correlation Function of Fermions. $T = 0$

The right-moving Fermion is given by

$$\psi_R(x) = e^{i\phi_R(x)} \equiv e^{i\theta(x)+i\phi(x)}$$

Applying Gaussian Integration Formula to this expression we find

Gaussian Integration Formula

$$\langle T\psi_R(x,\tau)\psi_R^\dagger(x')\rangle = \frac{c}{(x+iv_c\tau)^{\Delta}(x-iv_c\tau)^{\bar{\Delta}}}$$

where

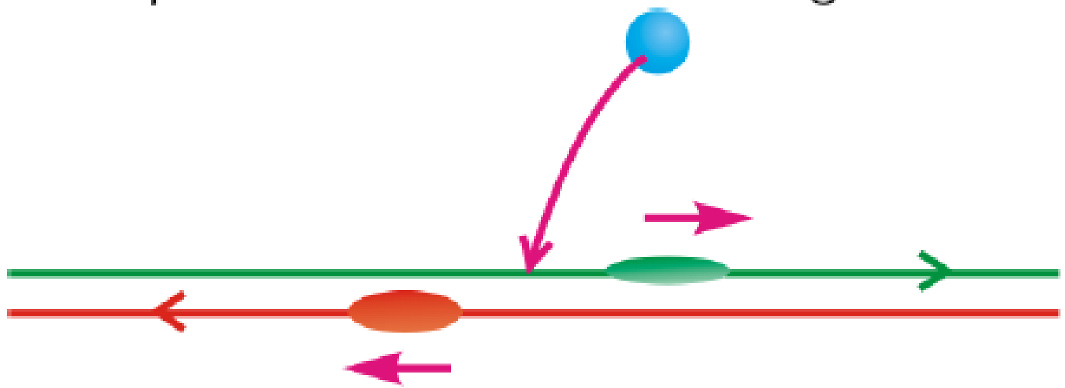
$$\Delta = \frac{(1+K)^2}{4K} \qquad \text{and} \qquad \bar{\Delta} = \frac{(1-K)^2}{4K}$$

Correlation Function of Fermions. $T = 0$

The structure of correlation function

$$\langle T\psi_R(x,\tau)\psi_R^\dagger(x')\rangle = \frac{c}{(x+iv_c\tau)^{\Delta}(x-iv_c\tau)^{\bar{\Delta}}}$$

suggests that in interacting system the "right" electron is no more a pure right-mover. It rather splits into two counterpropagating wave-packets. This is called charge fractionalization.



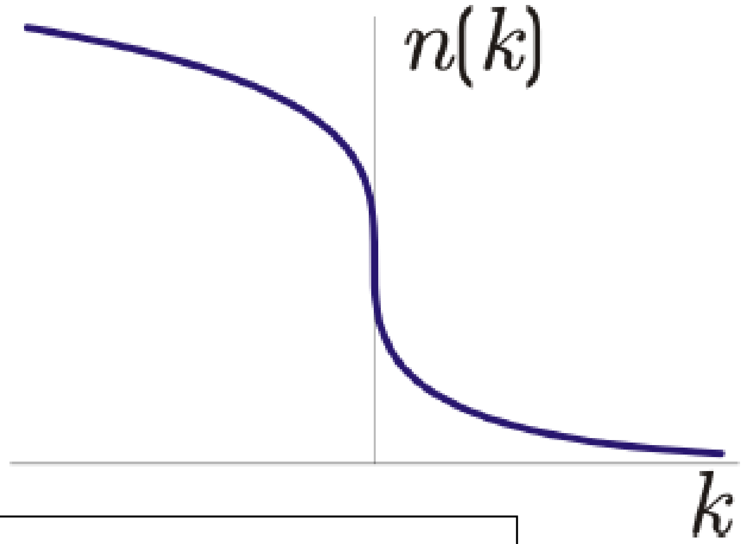
Particle occupation numbers.

The particle occupation numbers are found as

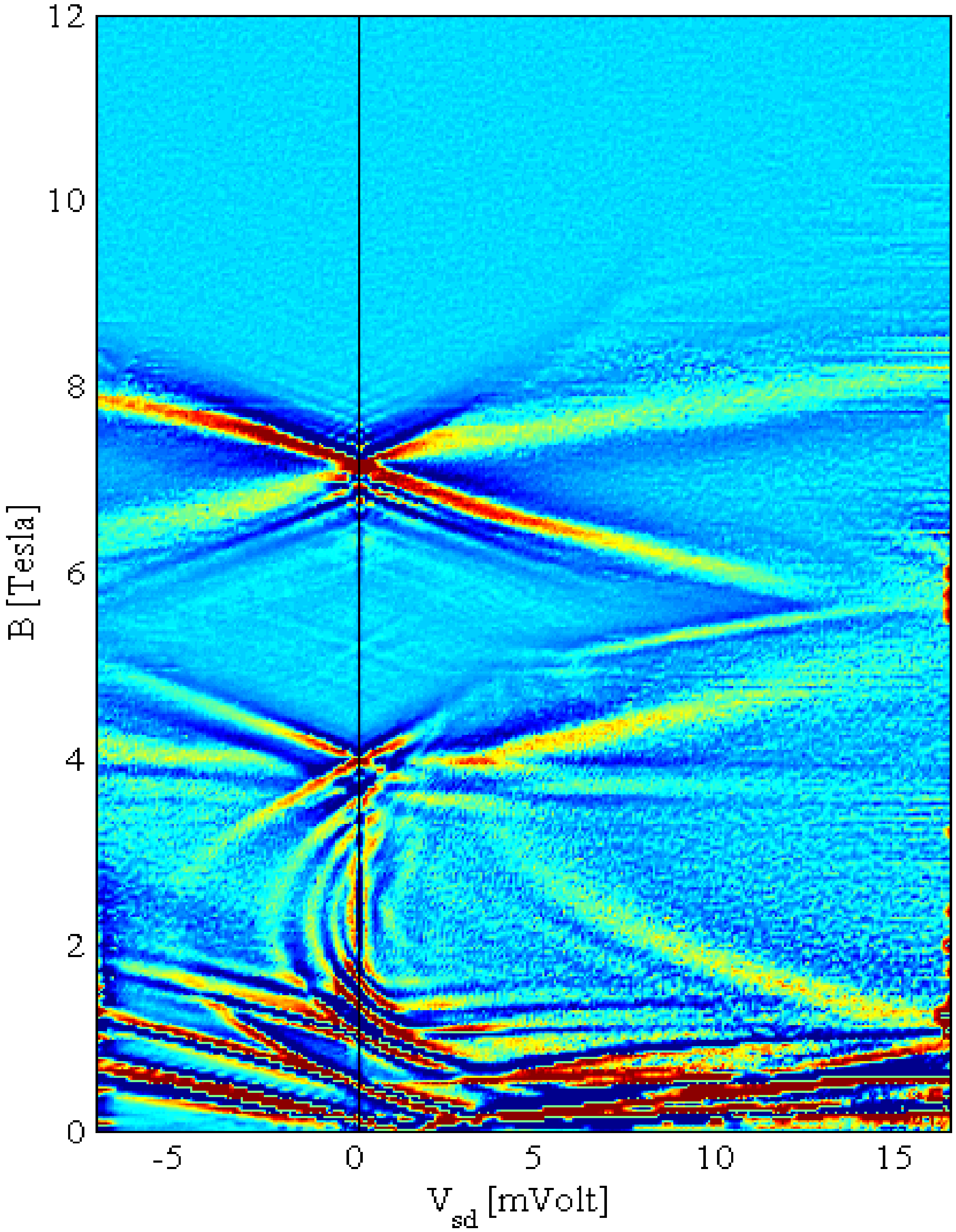
$$n_R(k) = \int dx e^{-ikx} \langle T \psi_R^\dagger(x) \psi_R(x') \rangle = n_0 + c\,\mathrm{sgn}(k)|k|^{\Delta+\bar{\Delta}-1}$$

$$\Delta + \bar{\Delta} - 1 = \frac{(K-1)^2}{2K} > 0$$

Instead of the sharp Fermi step there is a continuous distribution with a power-law singularity at $k = 0$.

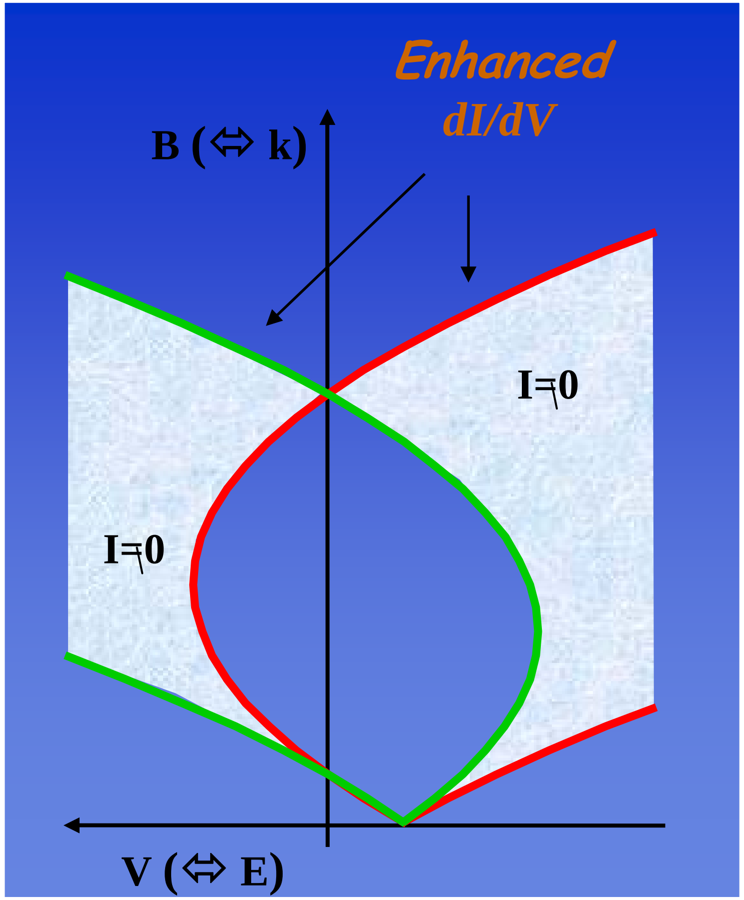


$$n_k \approx n_{k_\mathrm{F}} - \mathrm{const.} \times \mathrm{sign}(k - k_\mathrm{F})|k - k_\mathrm{F}|^\delta$$



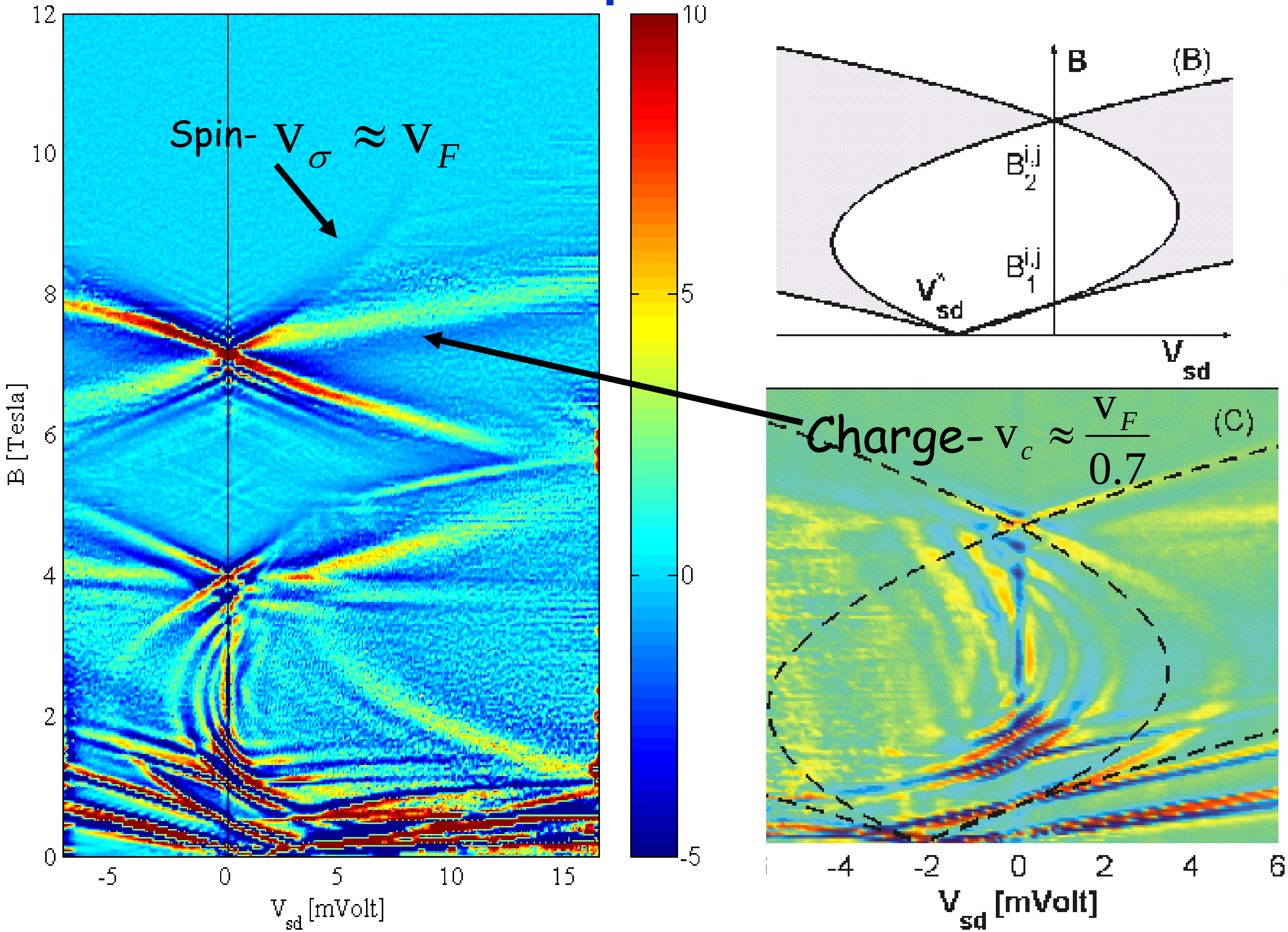
T=0.3K

O. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, *Science* **295**, 825 (2002).



Comparison With the Non-Interacting Dispersion

Observe 30% deviations



Messages to take home

• Bosons are "true" quasiparticles in Tomonaga-Luttinger model representing collective excitations

• Electron-electron interaction near p_F is strong.
The fermion's lifetime is too small

• FL theory is not applicable in 1D

• Spin and charge degrees of freedom are completely separated,
Corresponding excitations propagate with different velocities

• Interaction effects are encoded in Luttinger parameter K

• For $\mathcal{K} \neq 0$ charge fractionalization is observed