The Abdus Salam International Centre
for Theoretical Physics

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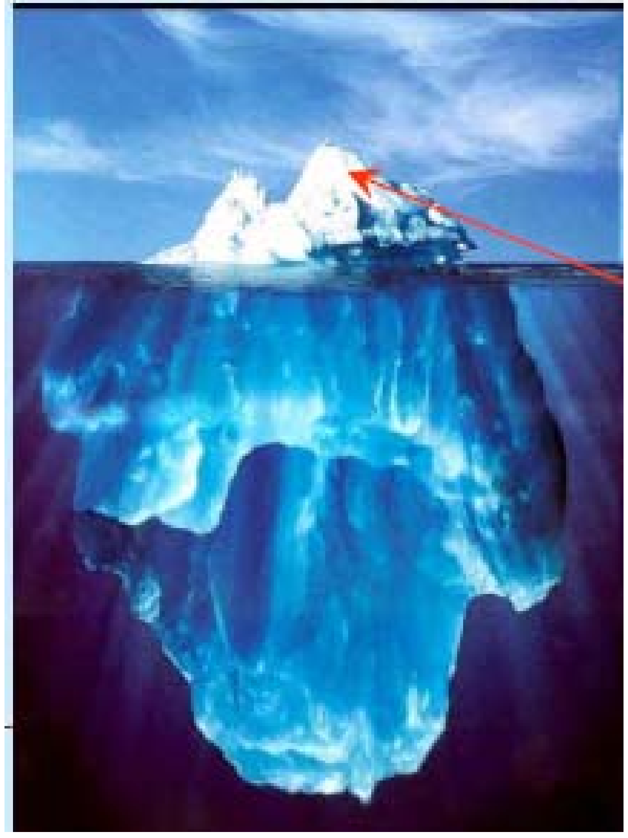
Electron transport through nanostructures

Lecture 2

Kondo effect in nano-devices

Regional School on Physics at the Nanoscale, Hanoi, 14-25 December 2009



Outline of the course:

- Quantum Dots
- Kondo effect in nano-devices
- From Fermi liquid to Luttinger liquid

For reading:

Kondo effect in QD: M. Pustilnik and L. Glazman cond-mat/0401517

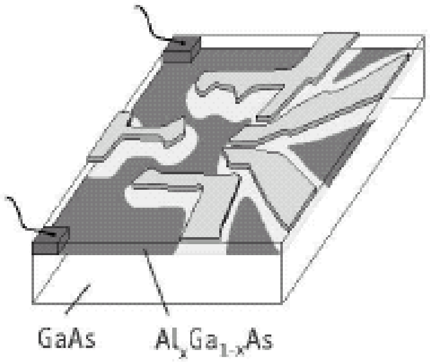
Dynamical symmetries: K.Kikoin, M.N. Kiselev and Y.Avishai, Nanotechnology Research Journal (2007) also cond-mat/0407163

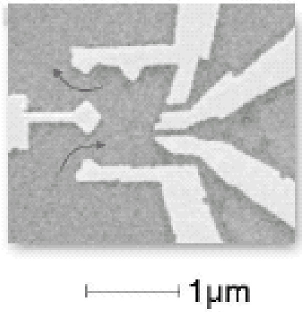
Popular reading: Leo Kouwenhoven and Leonid Glazman, Physics World 2001

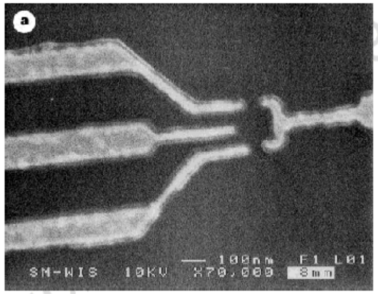
Outline of this lecture

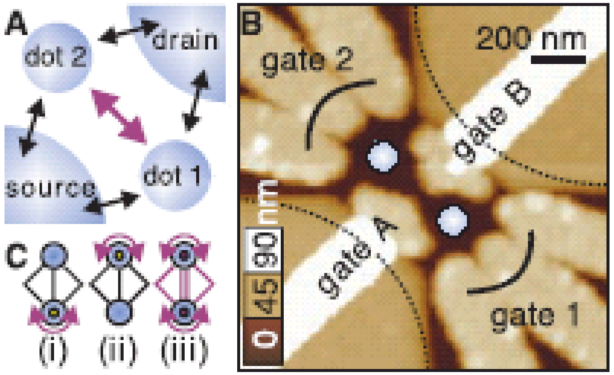
- Single Electron Transistor
- Kondo effect in SET
- Kondo effect in double dots
- Kondo effect out of equilibrium
- Kondo effect in molecular electronics
- Kondo effect in nanoelectromechanics
- Perspectives

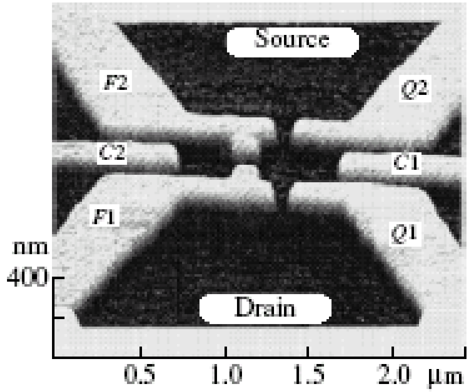
Quantum dots: from simple to complex

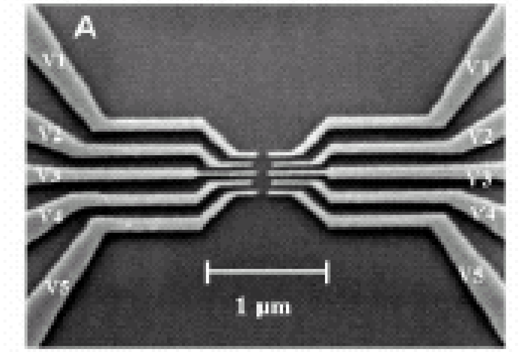












D.Goldhaber-Gordon et al (1998)

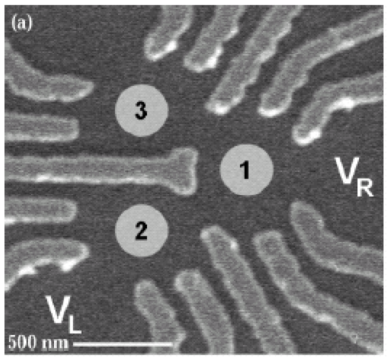
J.P.Kotthaus (1995)

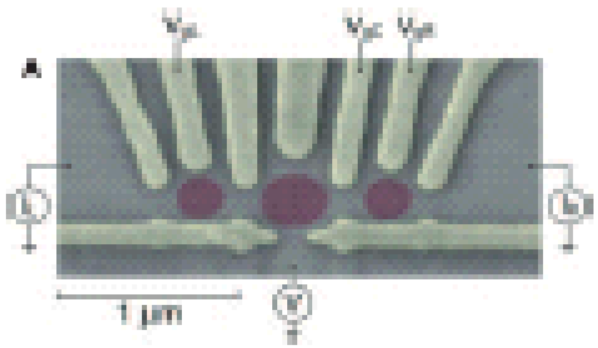
A.Holleitner et al (2002)

L.W.Molenkamp et al (1995)

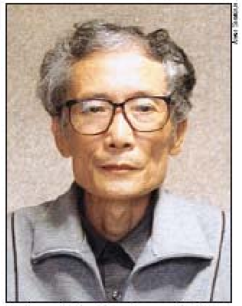
H.Jeong et al (2001)

C.Marcus et al (2003)

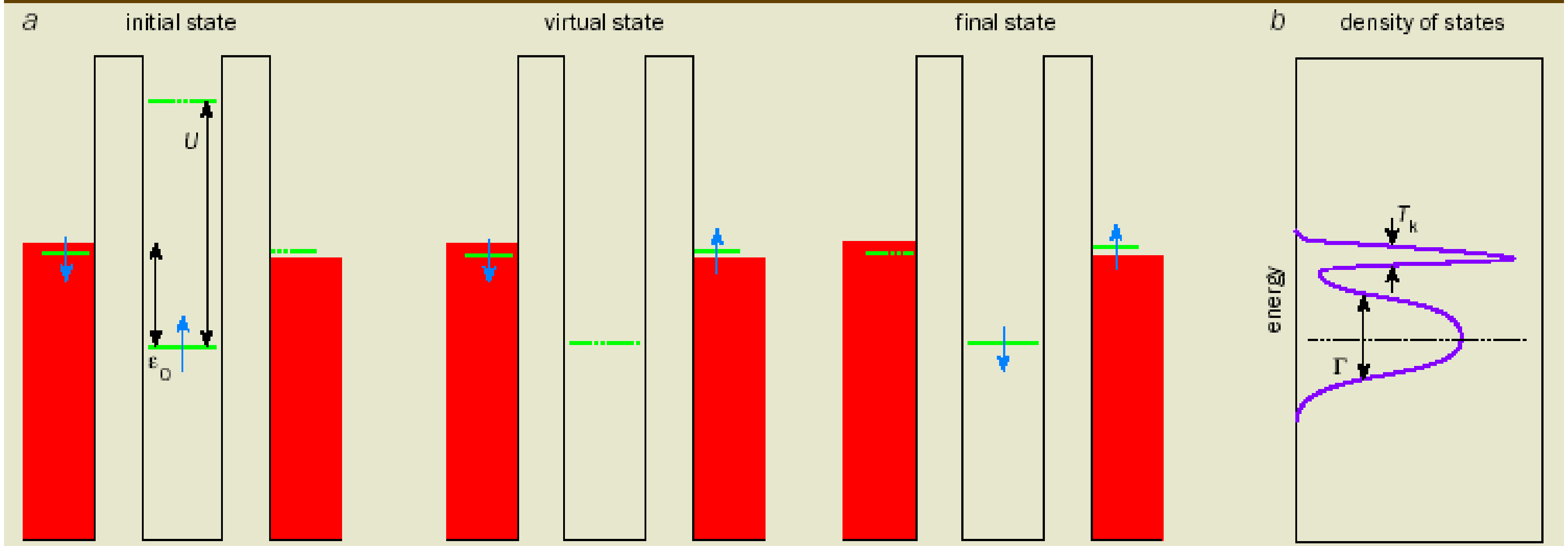




Kondo Effect in Quantum Dots

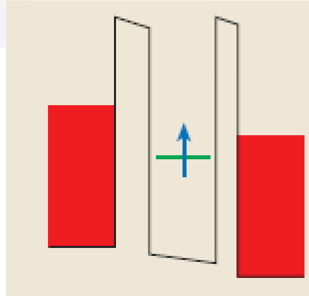


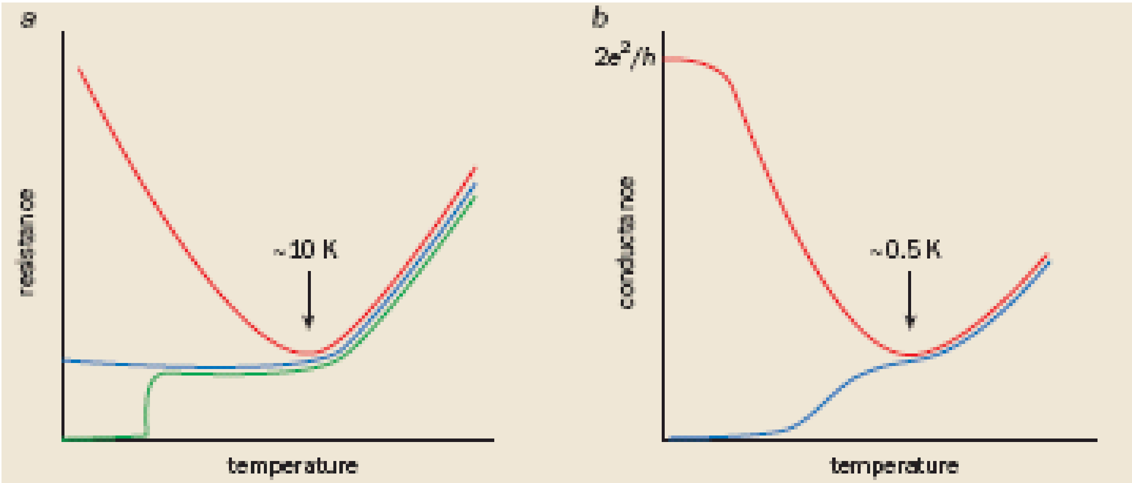
2 Spin flips



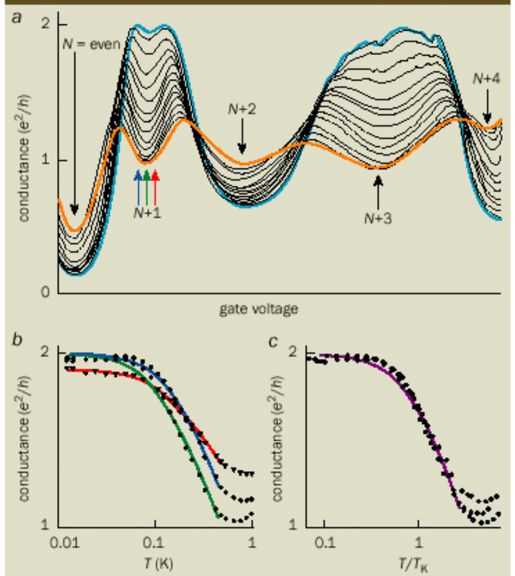
(a) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ε_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, U, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden "virtual state" outside the impurity, and then be replaced by an electron from the metal. This can effectively "flip" the spin of the impurity. (b) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

Universal Scaling





5 Universal scaling

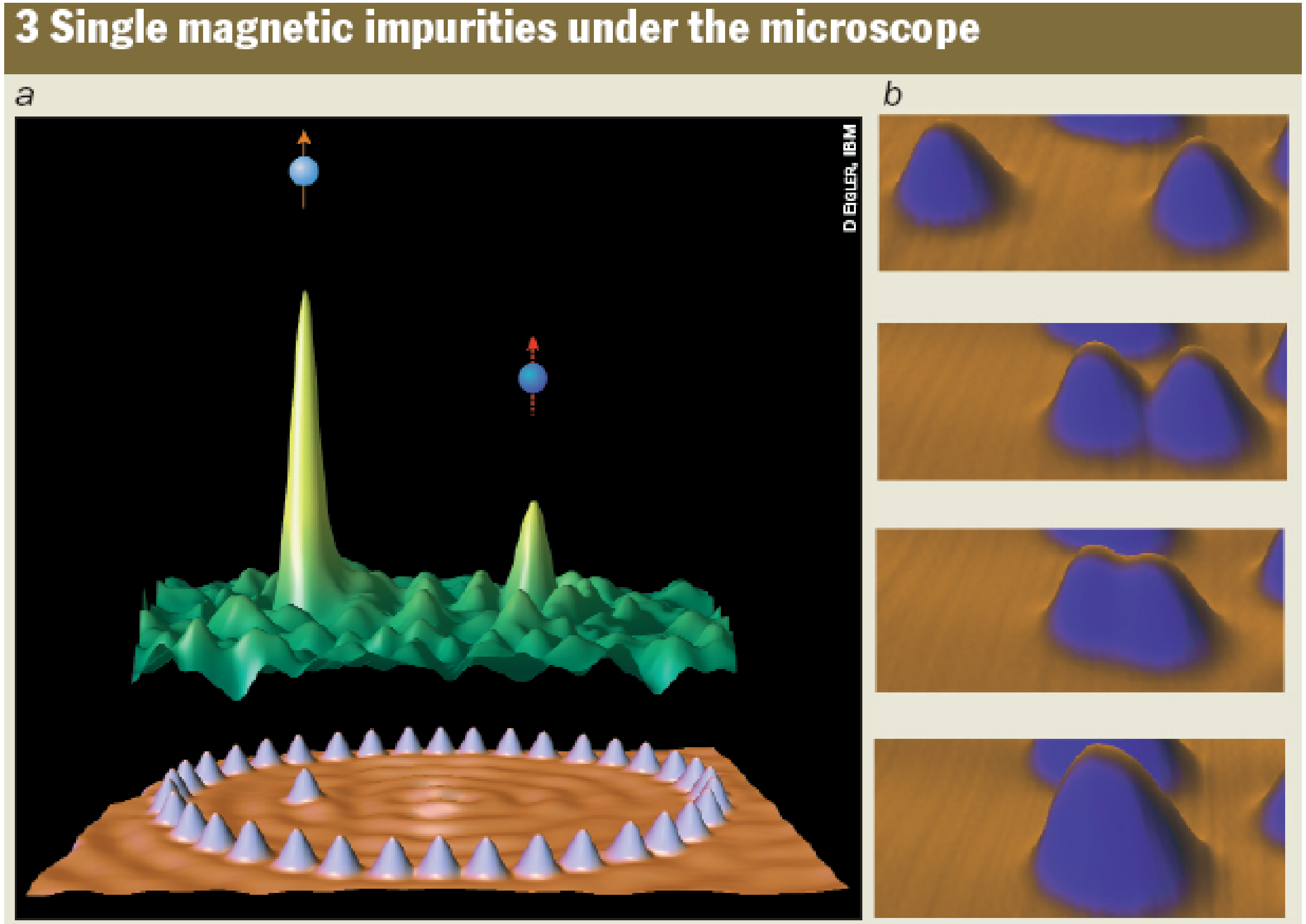


(*a*) The conductance (*y*-axis) as a function of the gate voltage, which changes the number of electrons, *N*, confined in a quantum dot. When an even number of electrons is trapped, the conductance decreases as the temperature is lowered from 1 K (orange) to 25 mK (light blue). This behaviour illustrates that there is no Kondo effect when *N* is even. The opposite temperature dependence is observed for an odd number of electrons, i.e. when there is a Kondo effect. (*b*) The conductance for $N+1$ electrons at three different fixed gate voltages indicated by the coloured arrows in (*a*). The Kondo temperature, T_K, for the different gate voltages can be calculated by fitting the theory to the data. (*c*) When the same data are replotted as a function of temperature divided by the respective Kondo temperature, the different curves lie on top of each other, illustrating that electronic transport in the Kondo regime is described by a universal function that depends only on T/T_K.

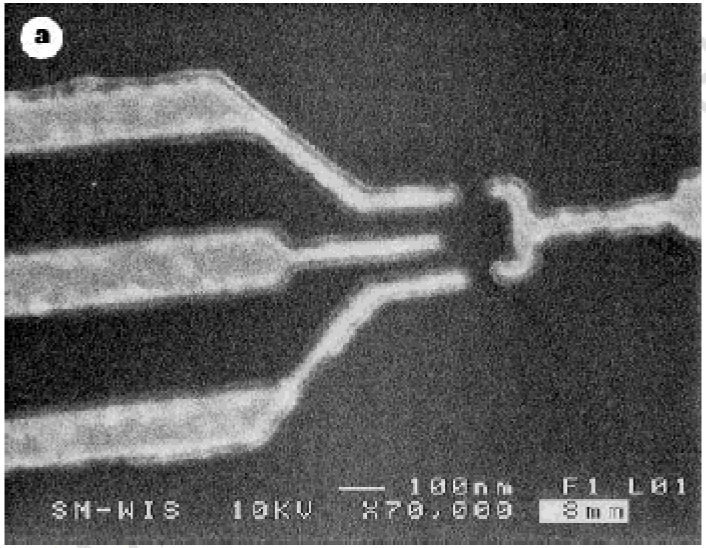
$$G/G_0 \propto \ln^{-2}\left(\max[T/T_K]\right)$$

$$T_K = \frac{1}{2}(\Gamma U)^{1/2} \exp\left(\pi\varepsilon_0 \frac{\varepsilon_0 + U}{\Gamma U}\right)$$

Quantum corals



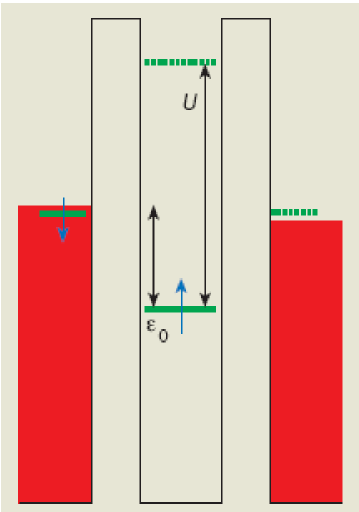
Kondo effect in single electron transistor



Single orbital level coupled to two leads

$$H = H_{leads} + H_{tun} + H_{dot}$$

$$H_{leads} = \sum_{k,\sigma\alpha=L,R} [\epsilon_k - \mu_\alpha] c^\dagger_{k,\sigma\alpha} c_{k,\sigma\alpha}$$

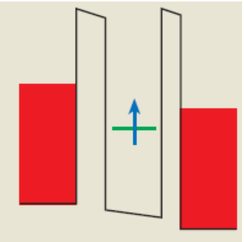


$$H_{tun} = \sum_{k,\sigma\alpha} [V_\alpha c^\dagger_{k,\sigma\alpha} d_\sigma + H.c.]$$

$$H_{dot} = \sum_\sigma \varepsilon_0 d^\dagger_\sigma d_\sigma + U(n-N)^2$$

Tunneling width $\Gamma_\alpha = \pi\rho|V_\alpha|^2$

Single orbital level coupled to two leads



Glazman-Raikh rotation

$$\begin{pmatrix} c_{k\sigma L} \\ c_{k\sigma R} \end{pmatrix} = U \begin{pmatrix} c_{k\sigma +} \\ c_{k\sigma -} \end{pmatrix} \qquad U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\tan\theta = \left|\frac{V_R}{V_L}\right| \qquad |V|^2 = |V_L|^2 + |V_R|^2$$

Only symmetric combination of the leads is coupled to the dot

Single level Anderson model is reduced to Kondo model

From Anderson model to Kondo model

$$H' = H_{dot} + H_{leads} + H_{tun}$$

$$H_K = W H' W^\dagger \qquad W = \exp(V)$$

$$V = \sum_{k\sigma\alpha} [\left(w^{(1)}_{k\alpha} (1 - n_{-\sigma}) + w^{(2)}_{k\alpha} n_{-\sigma} \right) d^\dagger_\sigma c_{k\sigma\alpha} + h.c.]$$

$$0 = H_{tun} + [V, H_{dot} + H_{leads}]$$

$$H_K = \sum_{k\alpha\sigma, k'\alpha'\sigma'} J_{\alpha\alpha'} [\vec{\sigma}_{\sigma\sigma'} \vec{S} + \frac{1}{4} \delta_{\sigma\sigma'}] c^\dagger_{k\sigma,\alpha} c_{k'\sigma',\alpha'}$$

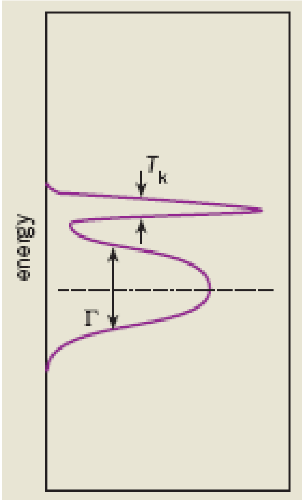
$$J_{\alpha,\alpha'} = \sqrt{\Gamma_\alpha \Gamma_{\alpha'}} / (\pi \rho_0 E_d)$$

Effective model:

$$H_K = \sum_{k\alpha\sigma,k'\alpha'\sigma'} J_{\alpha\alpha'}[\vec{\sigma}_{\sigma\sigma'}\vec{S} + \frac{1}{4}\delta_{\sigma\sigma'}]c^{\dagger}_{k\sigma,\alpha}c_{k'\sigma',\alpha'}$$

$$J \to \mathcal{J} = \frac{J}{1-\nu J \ln(D/T)}$$

$$T_K = D \exp\left(-\frac{1}{\nu J}\right)$$

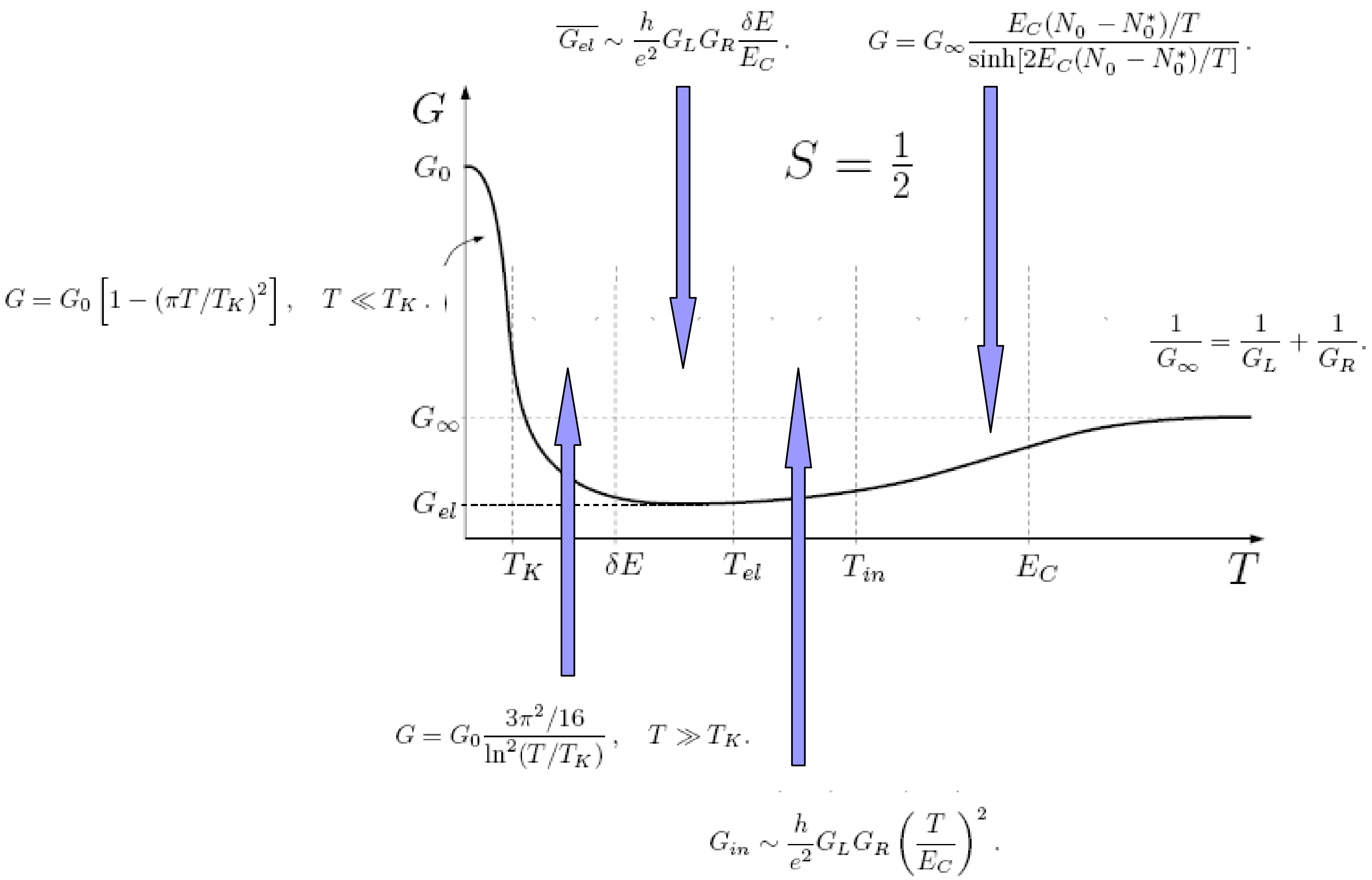


$T \gg T_K$ Weak coupling regime, accessible by perturbation theory

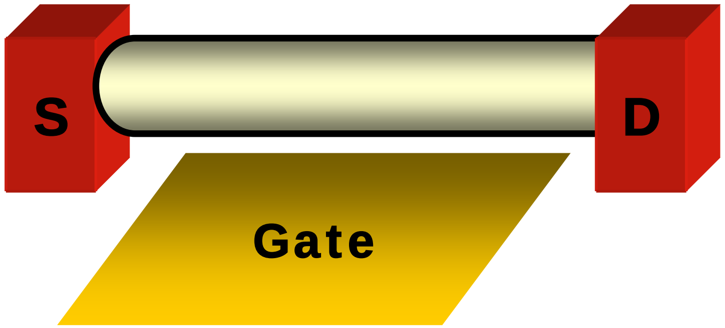
$T \ll T_K$ Strong coupling regime, non-perturbative

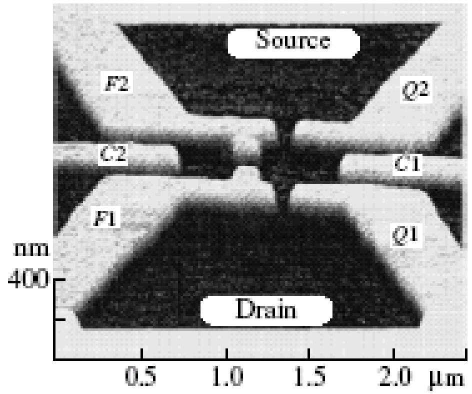
Fermi-liquid behaviour of thermodynamics and transport (see Lecture3)

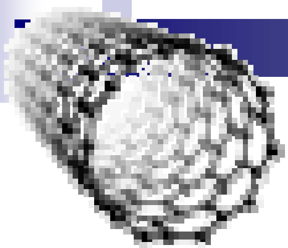
Electric transport through Kondo QD



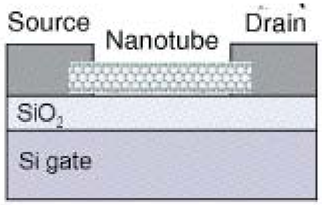
Non-equilibrium Singlet/Triplet Kondo effect

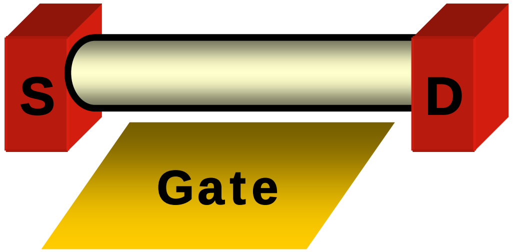


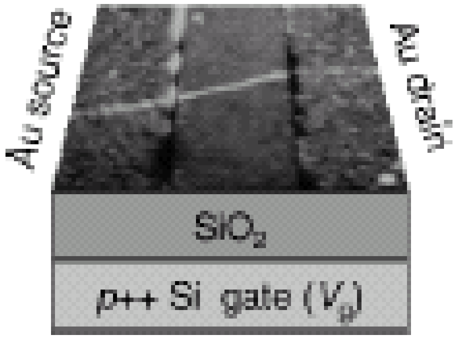


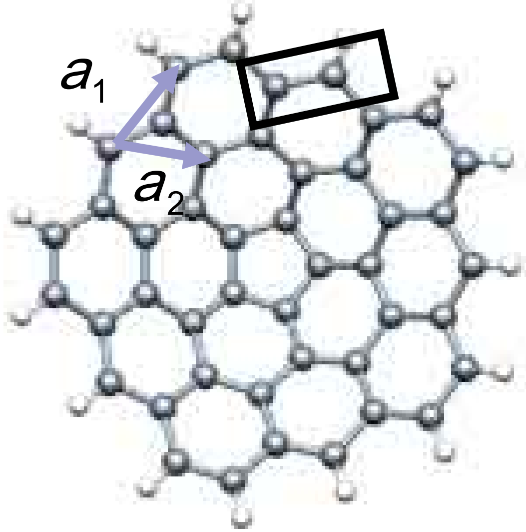


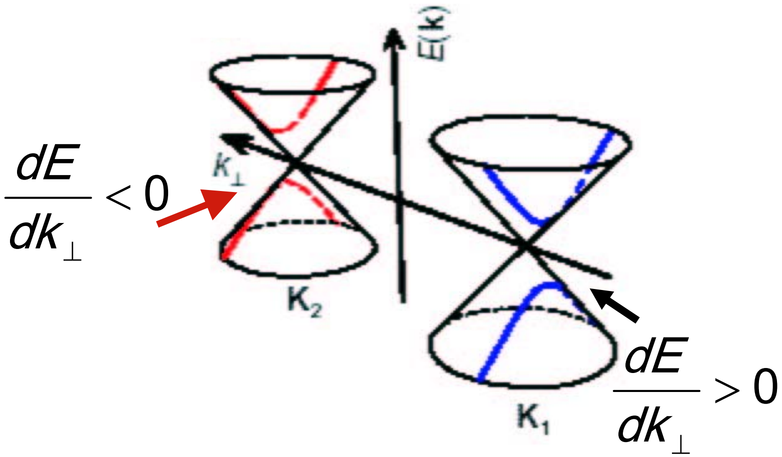
Carbon Nanotubes

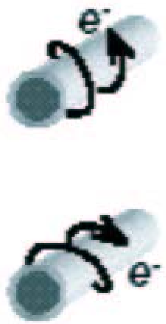


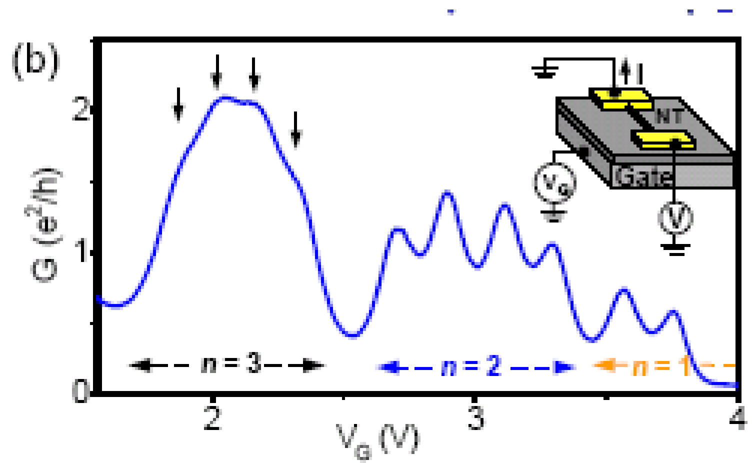




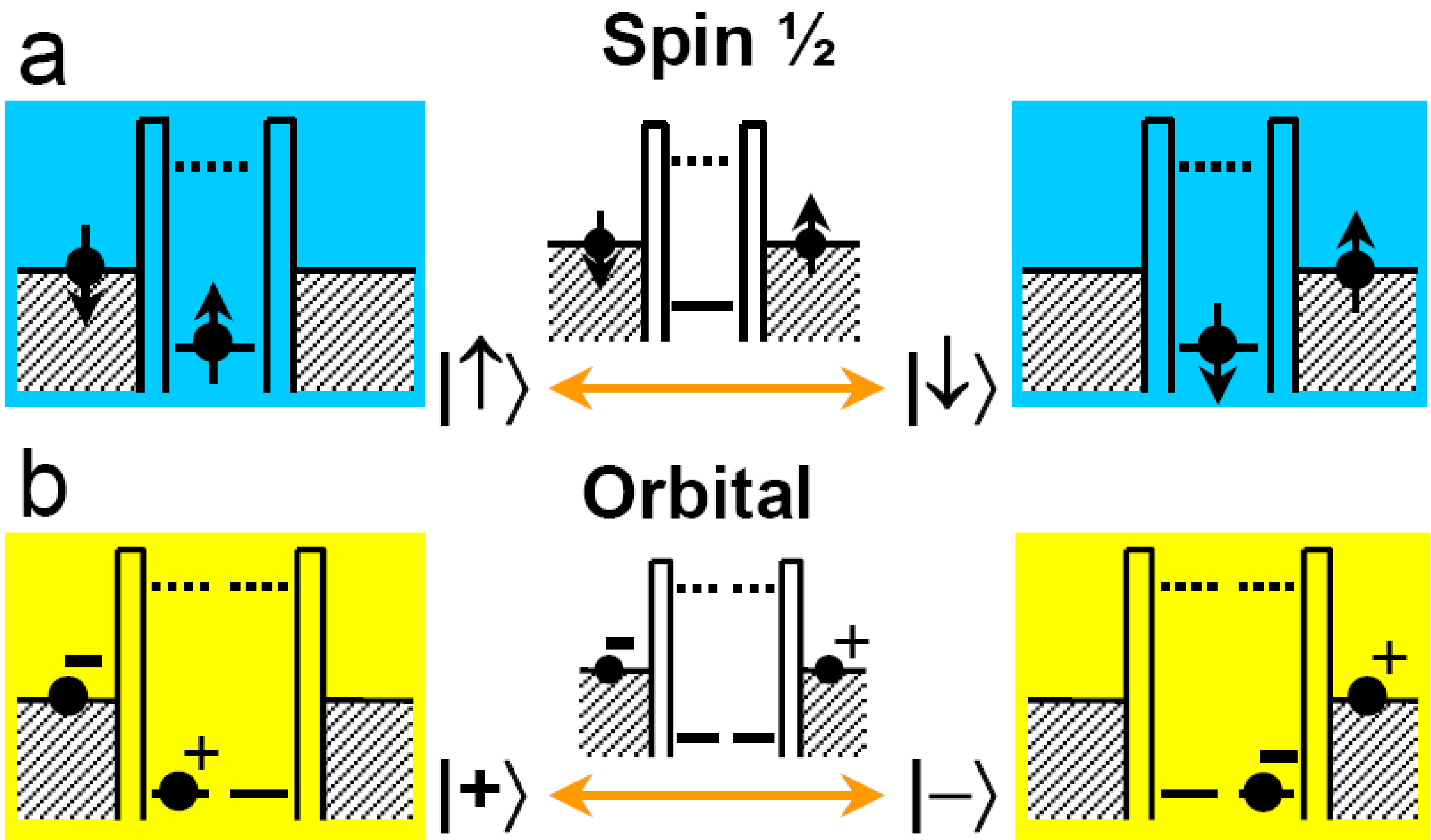


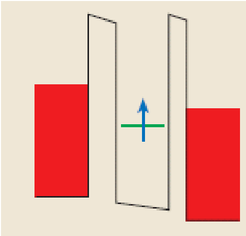




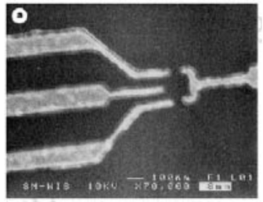


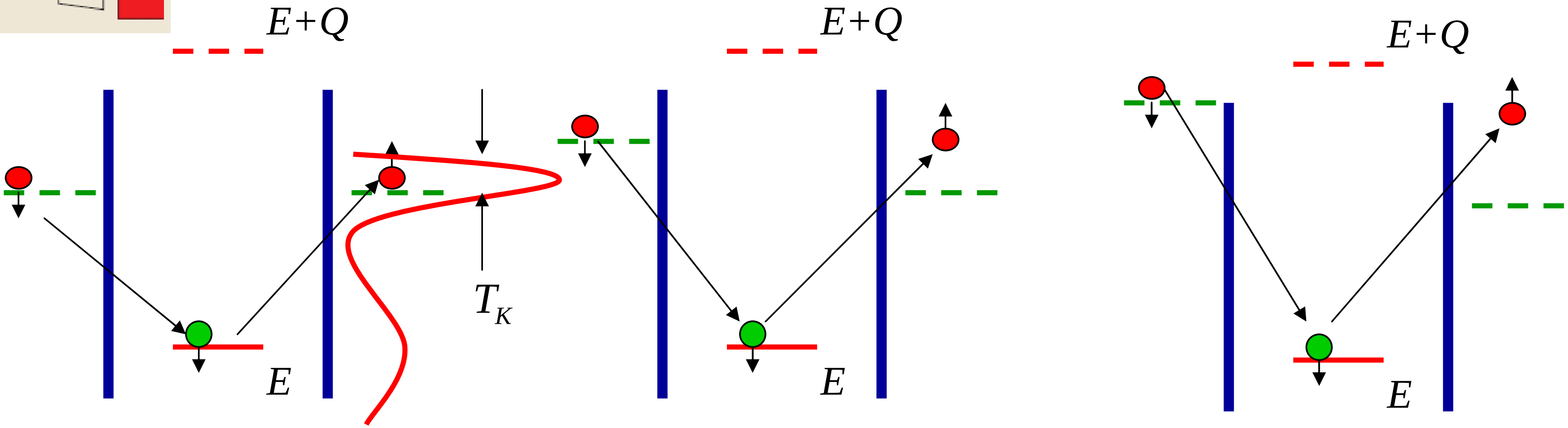
Spin and Orbital Kondo Effect





Kondo effect: coherence and decoherence





Zero-bias (equilibrium)

$$T_K$$

Small bias
(quasi-equilibrium)

$$eV \ll T_K$$

Large bias
(out of equilibrium)

$$eV \gg T_K$$

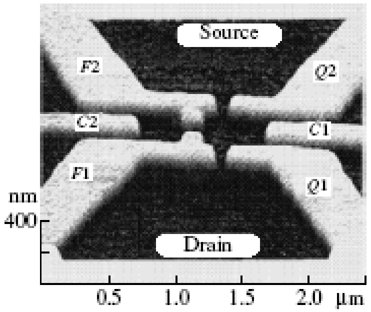
Effects of decoherence

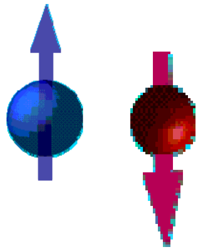
$$\Gamma_{rel} \sim eV$$

$$\Gamma_{rel} \sim eV / \ln^2 \left(eV / T_K \right)$$

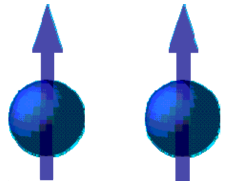
There is no strong coupling (Kondo) regime at low T in out of equilibrium

From Single Quantum Dot to Double Quantum Dot

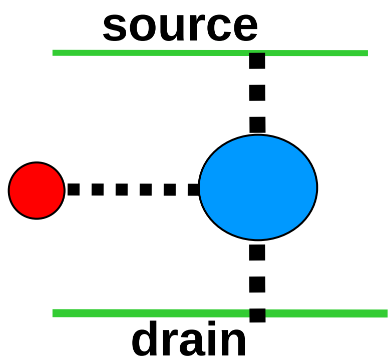




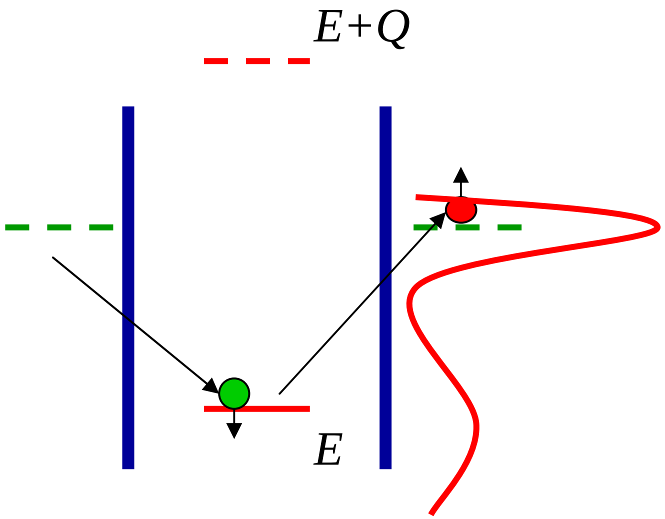
Singlet

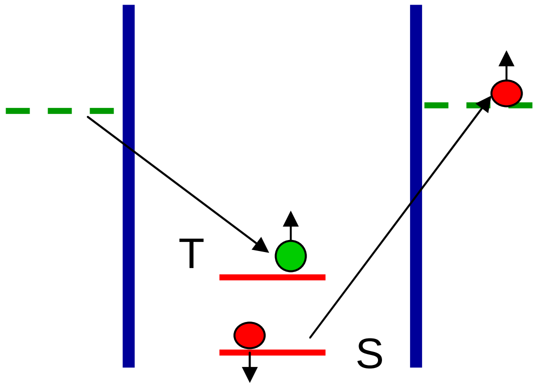


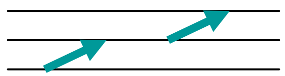
Triplet



- Kondo co-tunneling through QD: N=1
- Kondo co-tunneling through DQD: N=2

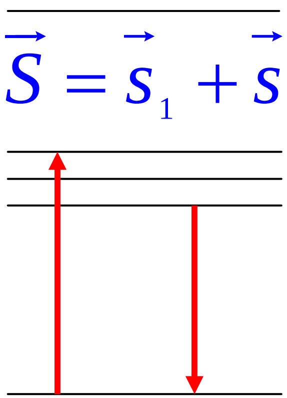






S^+

$\vec{S} = \vec{s}_1 + \vec{s}_2$



R^+

$\vec{R} = \vec{s}_1 - \vec{s}_2$

Kondo Hamiltonian

$H = J\,(\mathbf{S}\,\mathbf{s})$

$S=1/2$

Generalized Kondo Hamiltonian

$H = J_1\,(\mathbf{S}\,\mathbf{s}) + J_2\,(\mathbf{R}\,\mathbf{s})$

$S=1$ (triplet) plus $S=0$ (singlet)

Non-universal Kondo temperature

$$\Delta_{TS} \sim T_K(\Delta_{TS})$$

“Simple” knowledge about Kondo Effect

- Kondo effect exists if the total number of electrons in a dot is odd

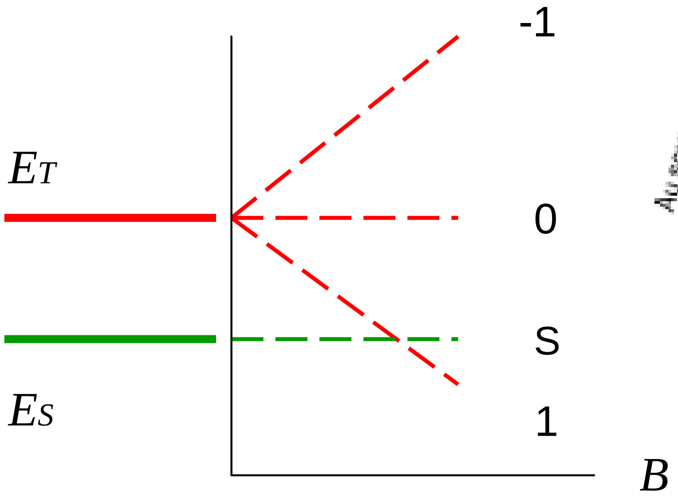
- Kondo effect is destroyed by external magnetic field

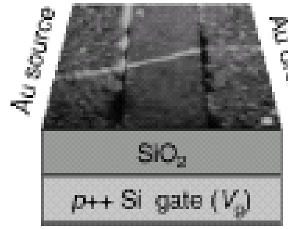
- Relaxation effects associated with the non-equilibrium conditions eliminate the Kondo peak

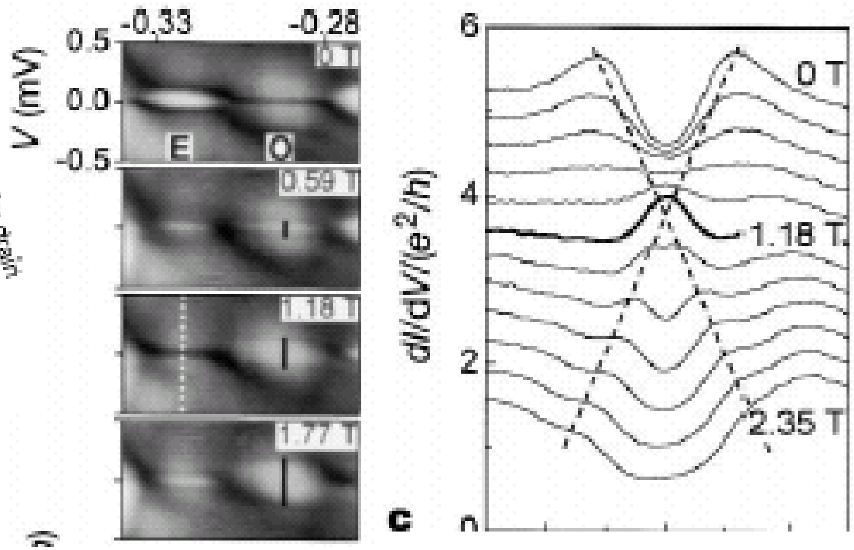
Is it always true?

S/T transition: *Magnetic field induced Kondo effect*

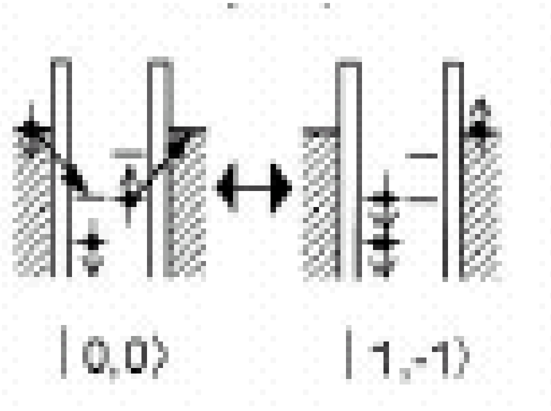
Symmetry reduction from SO(4) to SU(2)







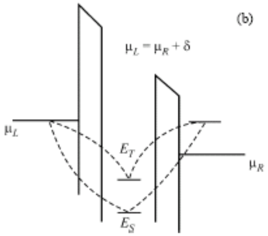
$$H_{Kondo} = J\left(\vec{R} \bullet \vec{s}\right)$$



Kondo effect due to the dynamical symmetry of DQD

M. Pustilnik, Y. Avishai & K.Kikoin (2000)

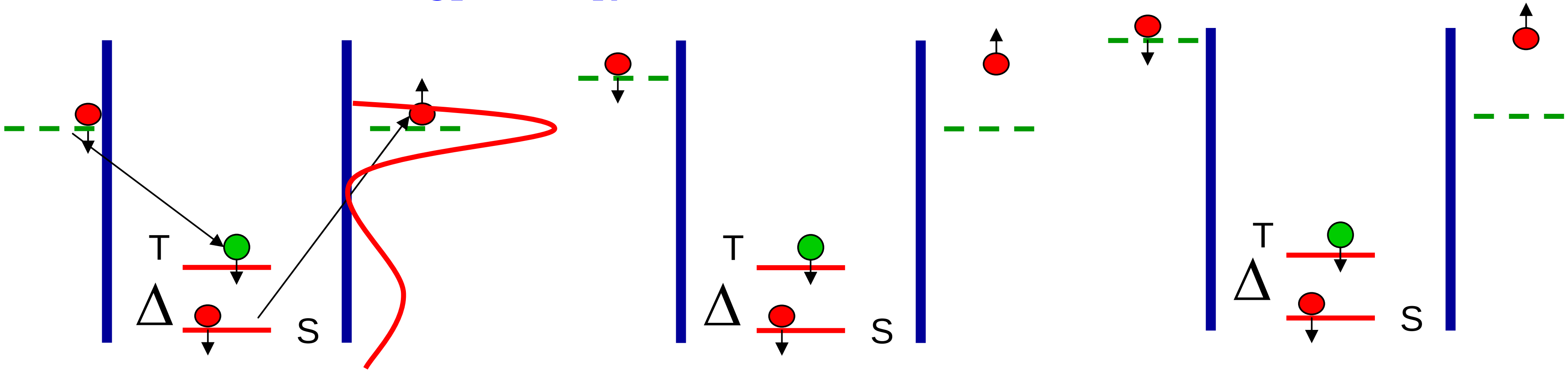
D. Kobden et al (2000)



Non-equilibrium Kondo effect in DQD

$\Delta_{ST} \ll T_K^{EQ}$ **Underscreened S=1 NEK**

$\Delta_{ST} \gg T_K^{EQ}$ **Is Kondo effect possible?**

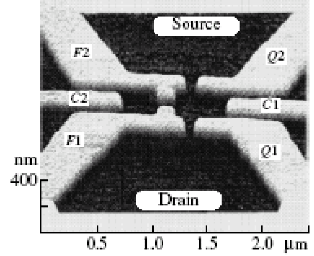


Zero-bias (equilibrium)	Small bias (quasi-equilibrium)	Large bias (out of equilibrium)
T_K^{EQ}	$eV \ll T_K^{EQ}$	$eV \gg T_K^{EQ}$
Effects of decoherence	$\Gamma_{rel} \sim eV$	**?**

What happens if $eV \sim \Delta_{ST}$ **?**

Non-equilibrium Kondo effect in DQD

$$\Delta_{ST} \gg T_K^{EQ}$$

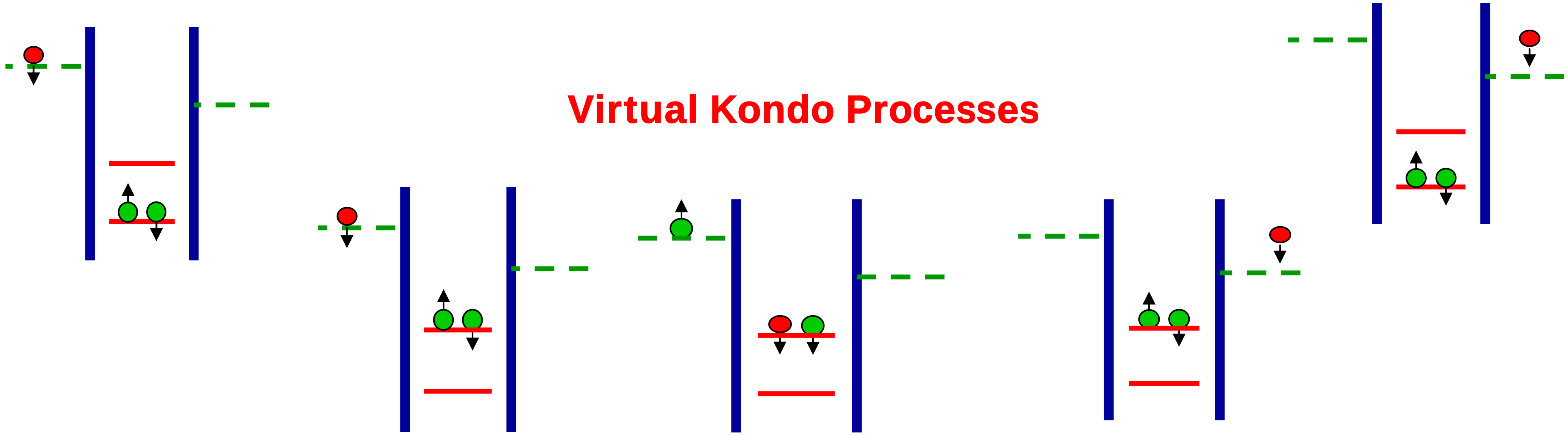


$$G/G_0 \propto \ln^{-2}\left(\max\left[(eV-\Delta),T\right]/T_K\right)$$

initial

final

Virtual Kondo Processes

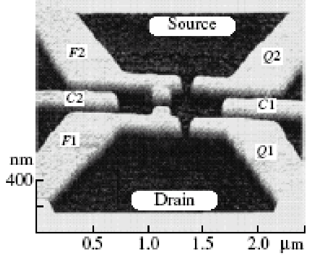


$$T_K^{NEQ} = D\exp\left(-\frac{1}{\nu J_0^T}\right) = \left(T_K^{EQ}\right)^2/D$$

MK, K.Kikoin and L.W.Molenkamp JETP Lett 2003
MK, K.Kikoin and L.W.Molenkamp, PRB 2003

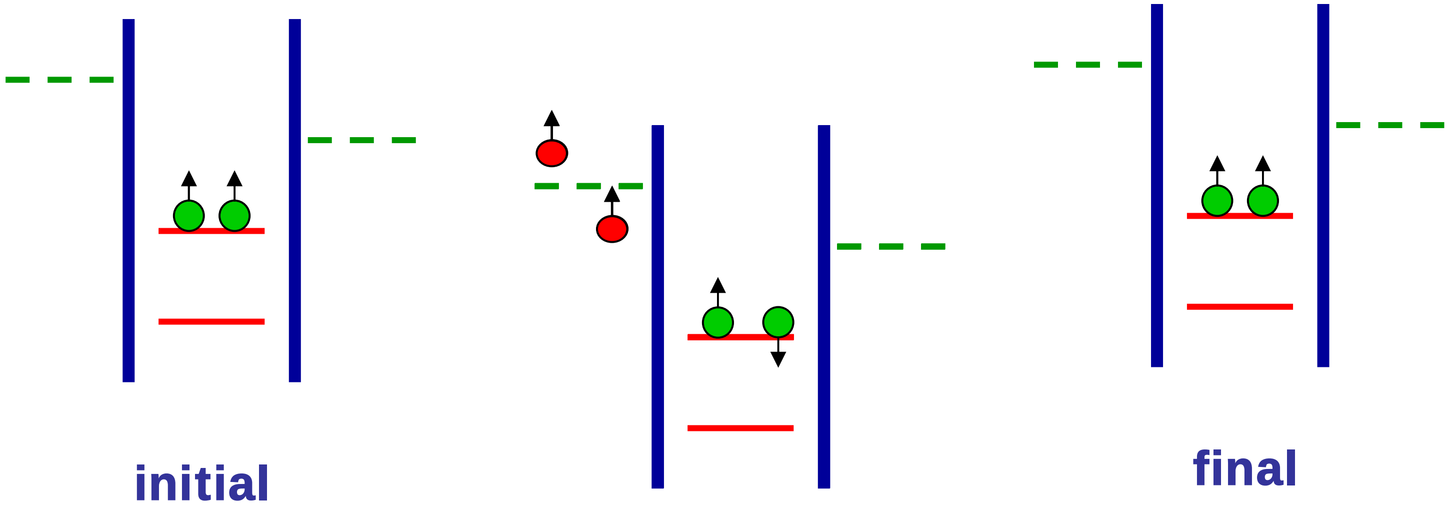
$$\Delta \gg T_K^{EQ}$$

Non-equilibrium Kondo effect in DQD



Effects of decoherence and repopulation

Triplet / Triplet Relaxation

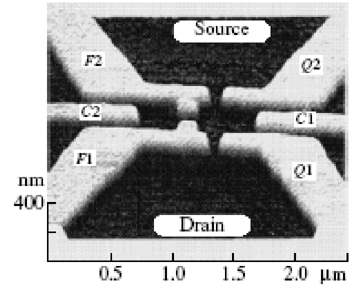


$$\hbar / \tau_d \sim eV \left(J^{ST} / D\right)^2 \left[1 + O\left(J / D \ln\{D / eV\}\right)\right]$$

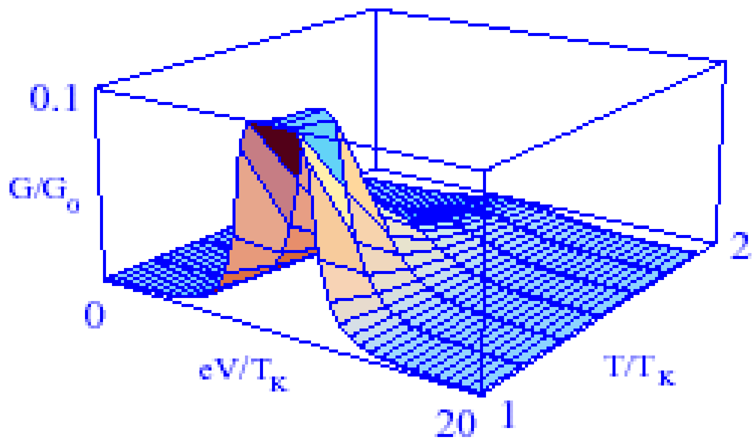
$$P_t(eV) \propto \exp\left(-\Delta^*(eV)/T\right)$$

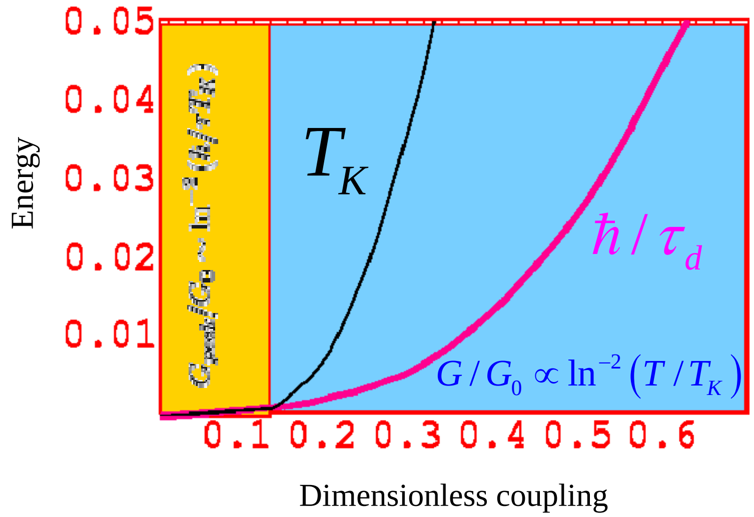
$$\left|\Delta^*(eV) - \Delta\right| \ll \Delta$$

Non-equilibrium Kondo effect in Double Quantum Dot



$$H_{\rm int} = \sum [(J^{TT}_{\alpha\alpha'}\vec{S} + J^{ST}_{\alpha\alpha'}\vec{R})\cdot \vec{s}_{\alpha\alpha'}]$$

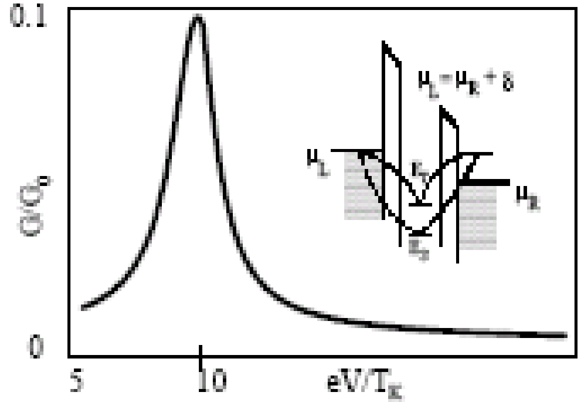




$$G_0 = 2e^2/h$$

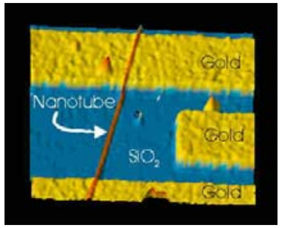
$$T_K^{NEQ} \sim \left(T_K^{EQ}\right)^2 / D$$

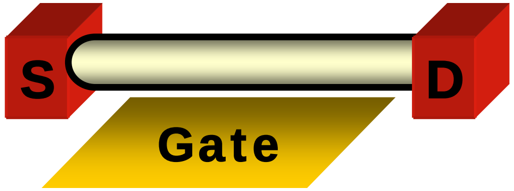
$$G/G_0 \propto \ln^{-2}\left(\max\left[(eV-\Delta),T\right]/T_K\right)$$

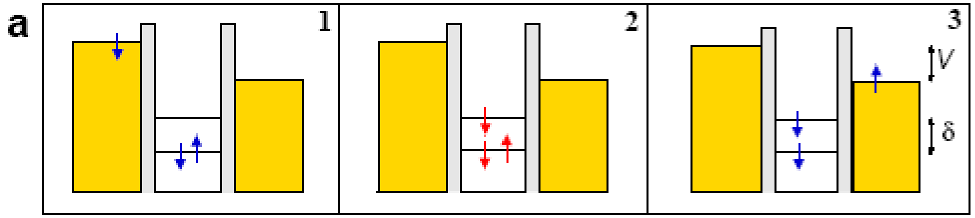


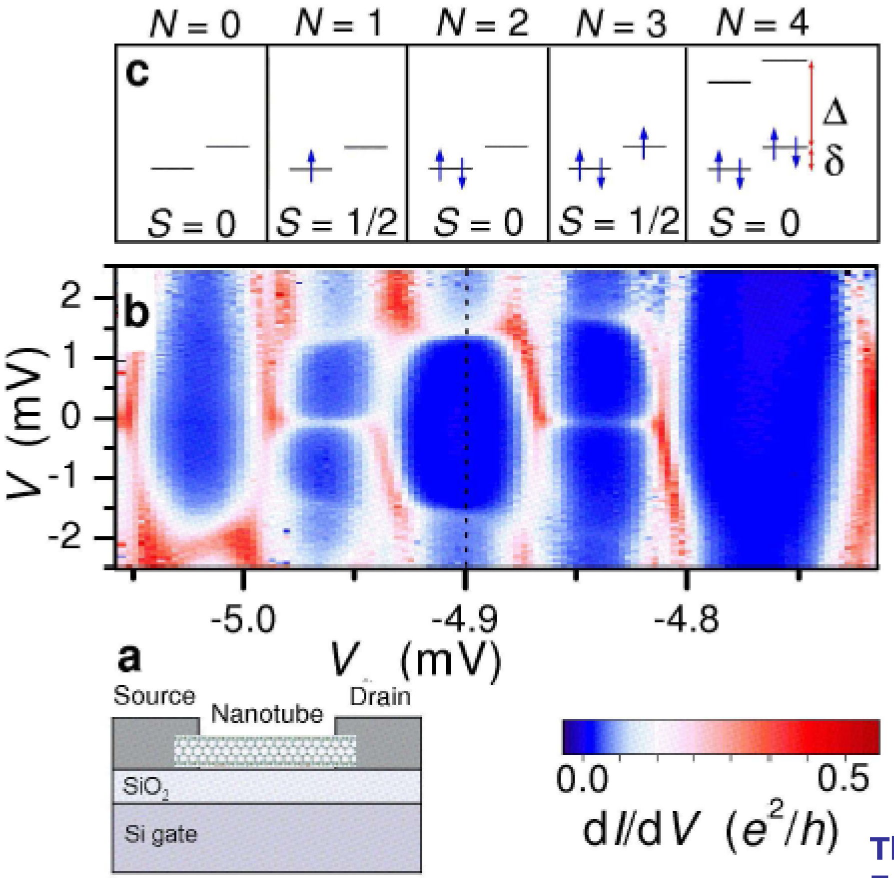
MK, K.Kikoin and L.W.Molenkamp, (2003)

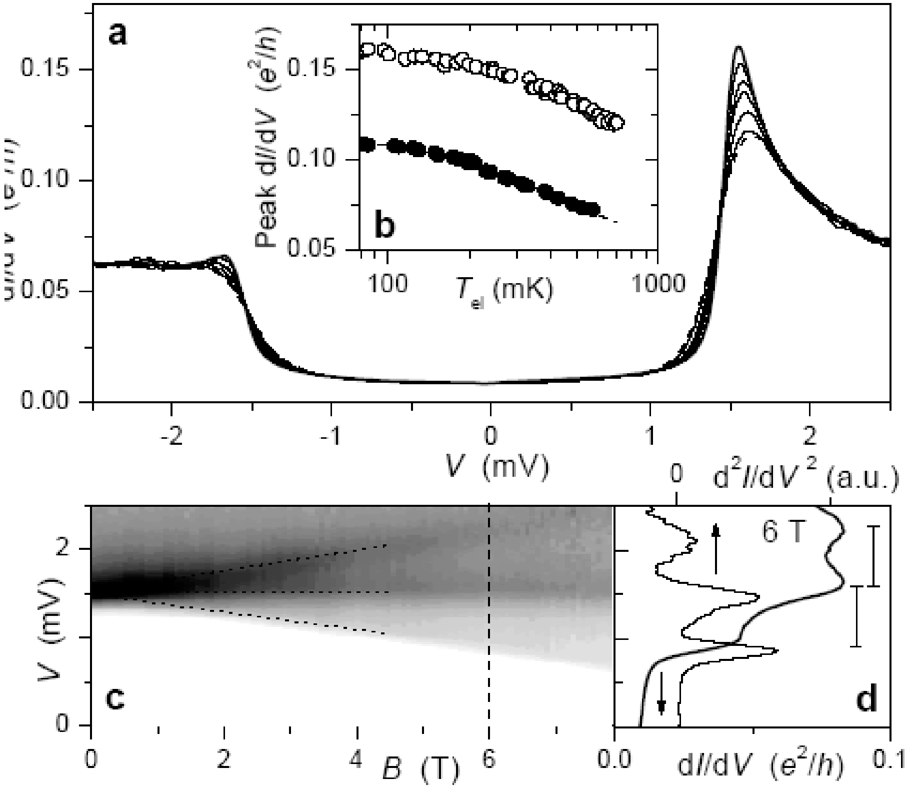
Singlet/Triplet finite bias Kondo effect in Carbon Nanotubes





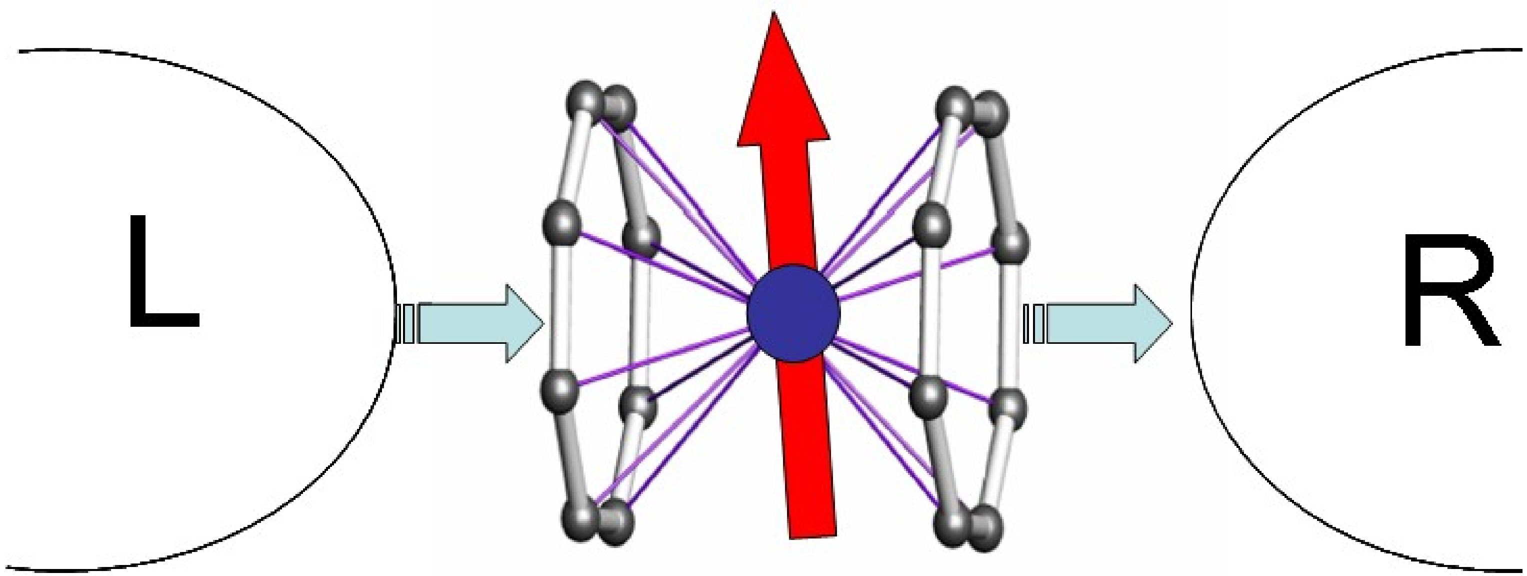






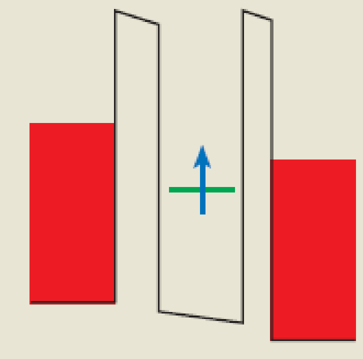
Theory DQD: MK, K.Kikoin and L.W.Molenkamp, PRB 2003
Experiment+Theory CNT: J.Paaske et al, Nature Physics 2006

Phonon induced Kondo effect in a Molecular Transistor



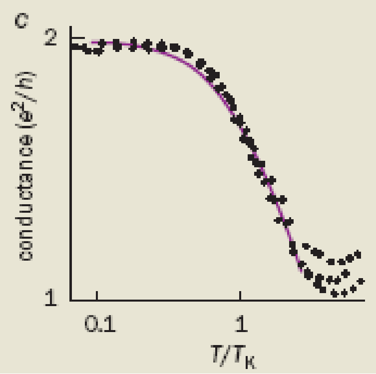
Why do we look for the Kondo effect in molecular devices ?

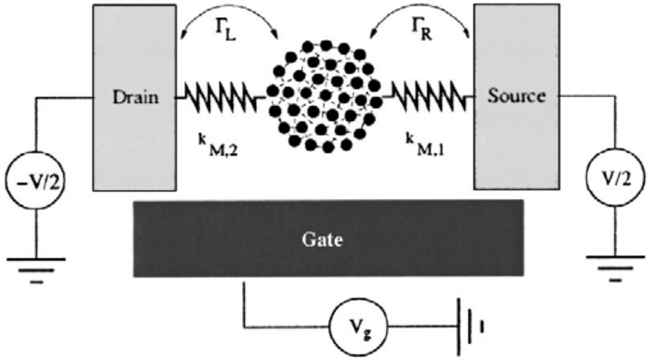
• The Kondo effect makes it easier for states belonging to the two opposite electrodes to mix



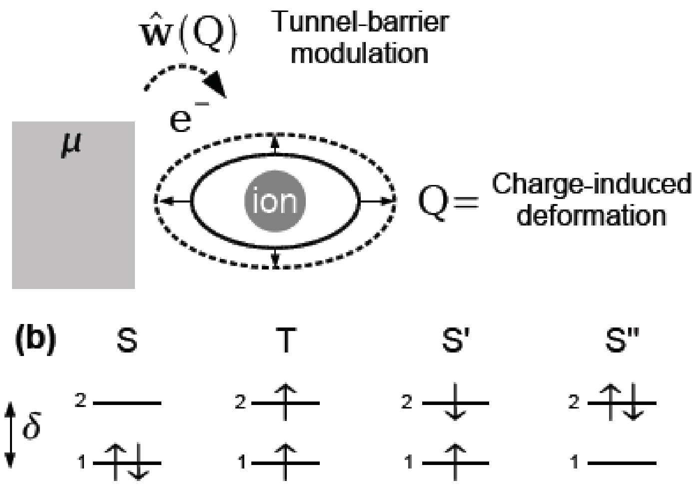
• Reasonably high Kondo temperatures > 10 K (compared to 100 mK- 1 K for QDs)

• SETs are highly controllable (by bias, magnetic field etc) devices





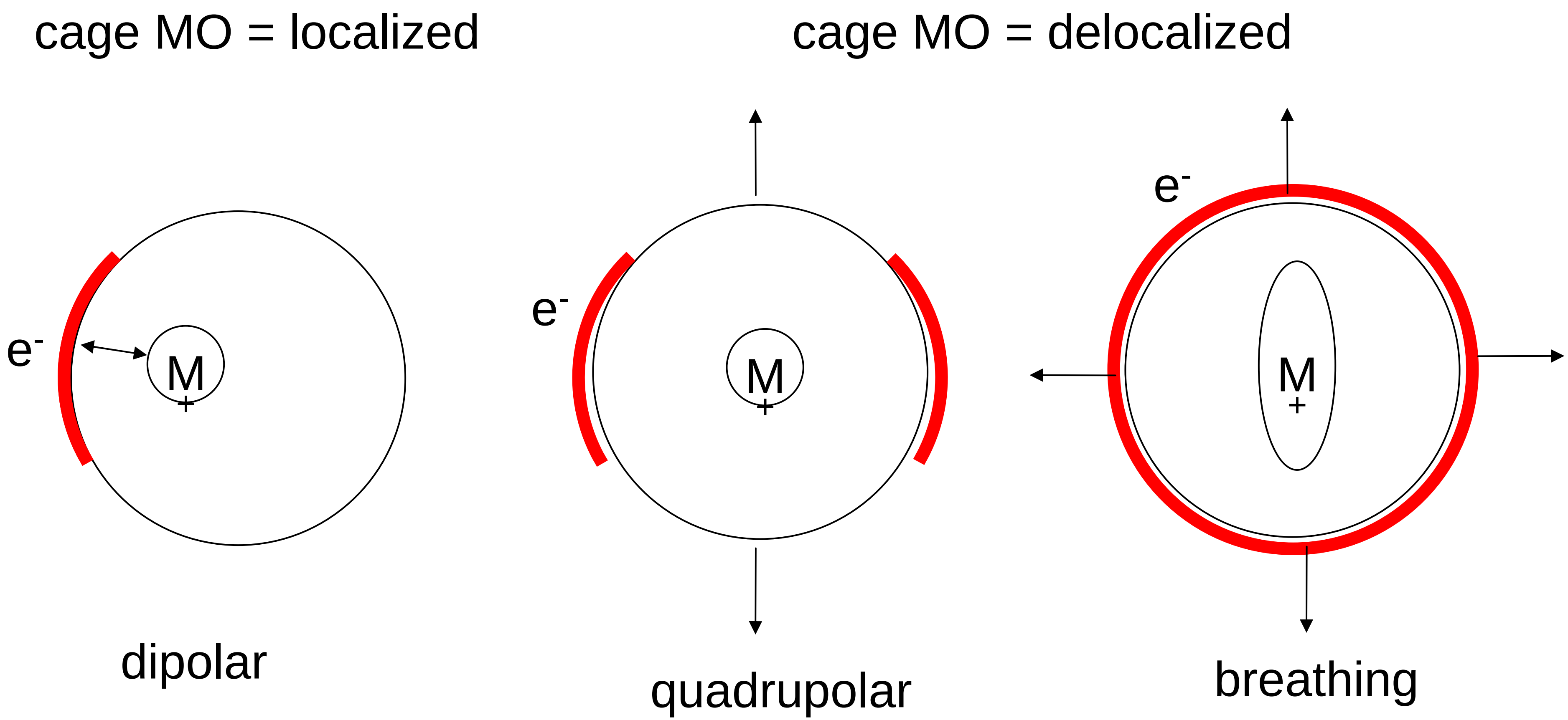
Kondo + phonons: Effective model

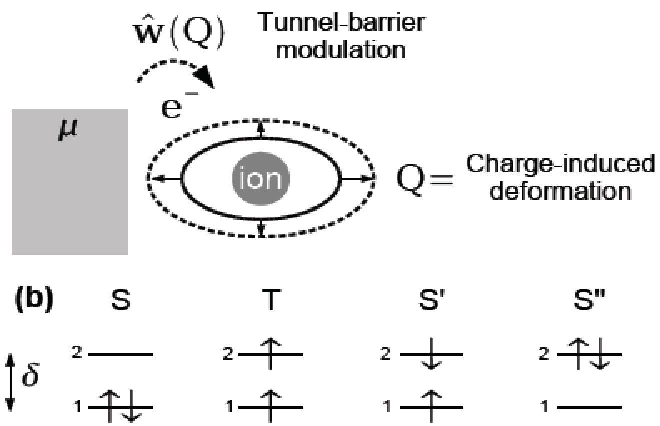


$$H = H_{mol} + H_{res} + H_{tun}$$

$$H_{mol} = H_Q^{(N)} + H_Q^{(N+1)} + H_Q^{(N-1)} + T_n$$

$$H_{res} + H_{tun} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \widehat{w}_Q \sum_{k\mu\sigma} \left(\tilde{d}^\dagger_{\mu\sigma} c_{k\sigma} + H.c. \right)$$

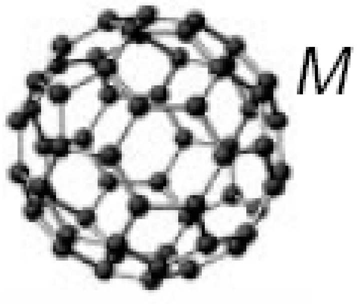




$$\Delta \equiv E_T - E_S = \delta - I > T_K$$

Triplet Singlet Exchange

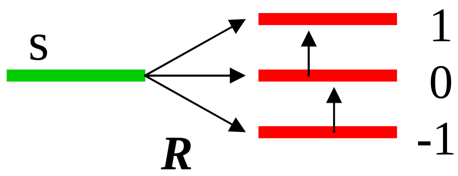
TMOC = Transition Metal + Organic Complex (cage)



$$H^{(N)}_{mol} = \sum_{\Lambda=S,T0,T\pm} E_\Lambda(Q)|\Lambda\rangle\langle\Lambda|$$

Singlet Triplet

Assumption: even electron occupation number



Singlet is a ground state

SO(4) symmetry

$$H_{tun} = \hat{w}(Q)\sum_k \sum_{\Lambda\gamma\sigma}{}' [|\Lambda\rangle\langle\gamma|c_{k\sigma} + H.c.]$$

$$H_{eff} = H_{res} + \frac{1}{2}\Delta\mathbf{S}^2 + \hat{J}_S\mathbf{S}\cdot\mathbf{s} + \hat{J}_R\mathbf{R}\cdot\mathbf{s} + \frac{\Omega}{2}P^2$$

Local phonon can be emitted or absorbed in a co-tunneling processes

The main source of phonon emission/absorption is the tunneling rate

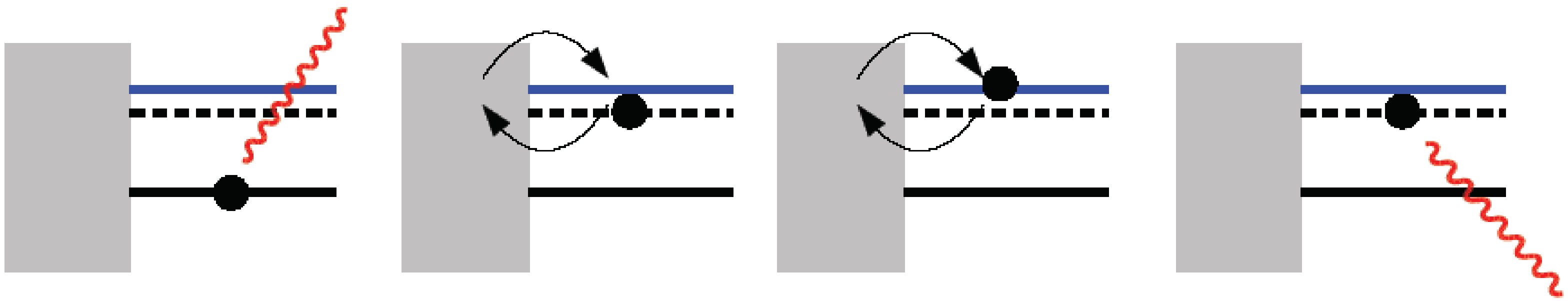
Vibration assisted tunneling

$$H_{eff} = H_{res} + \frac{1}{2}\Delta \mathbf{S}^2 + \hat{J}_S \mathbf{S}\cdot\mathbf{s} + \hat{J}_R \mathbf{R}\cdot\mathbf{s} + \frac{\Omega}{2}P^2$$

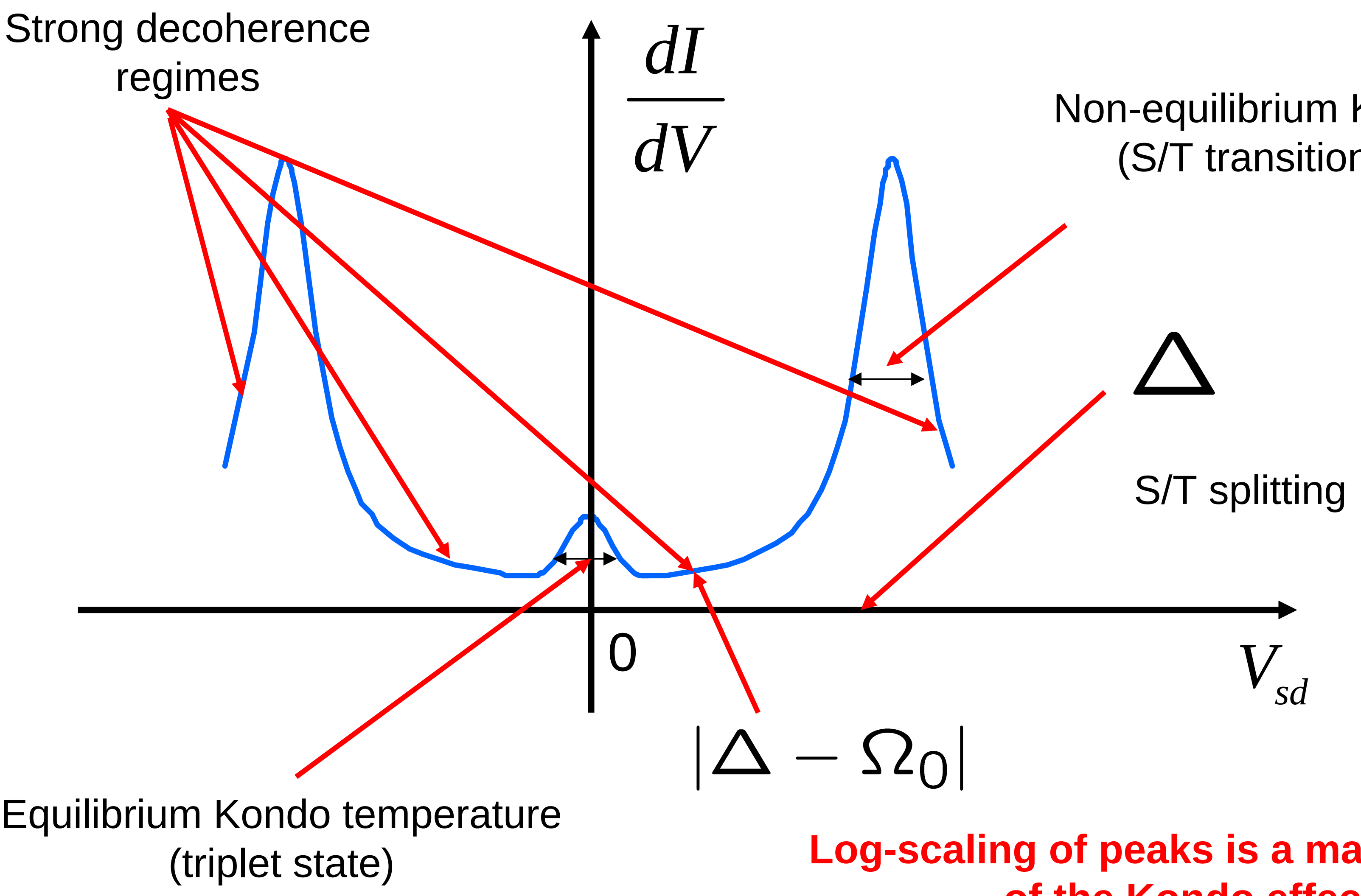
$$\hat{J}_S(Q) = J_S + j_S Q^2, \qquad \hat{J}_R(Q) = J_R + j_R Q$$

Quantized displacement operator

$$Q = \frac{(b^\dagger + b)}{\sqrt{2}}$$



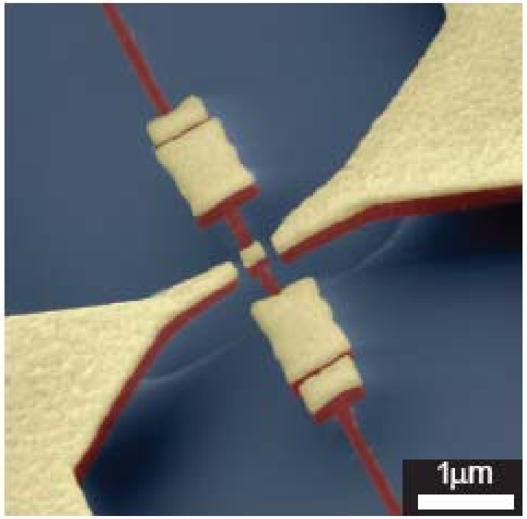
Differential Conductance

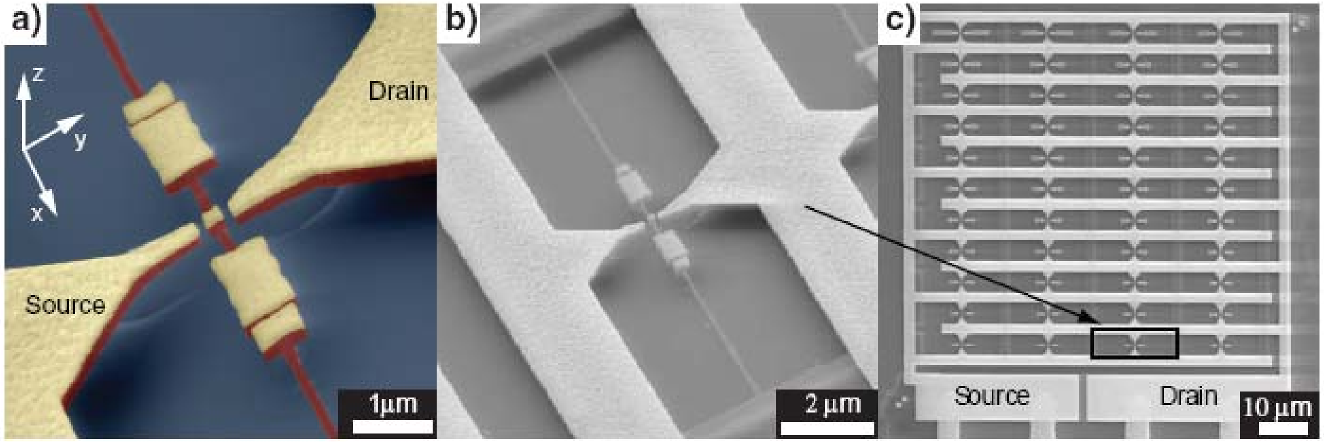


Log-scaling of peaks is a manifestation of the Kondo effect

K.Kikoin, MK, M.Wegewijs, PRL 2006

Nanoelectromechanical shuttling: QD devices





J. Kotthaus et al, Nature Nanotechnology 2008

Kondo Shuttling:

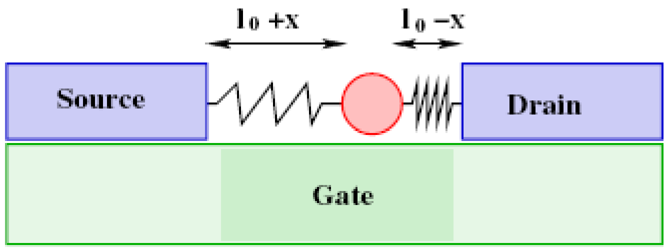
How to make the Kondo effect work in the Nanoelectromechanical devices?

How is the KE influenced by the NEM?

- **the nano-devise is nano-machined by external periodic force**
- **the nano-device changes its shape in the process of the tunneling**

K.Kikoin, MK and M.R.Wegewijs, PRL 2006
MK, K.Kikoin, R.Shekhter and V.Vinokur, PRB 2006

The model



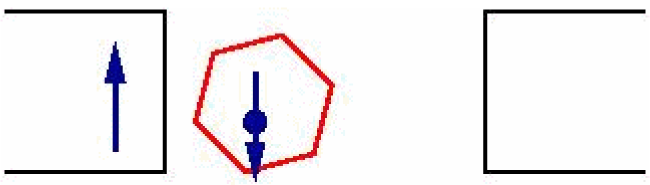
$$H_0 = \sum_{k,\alpha} \varepsilon_{k\sigma,\alpha} c^\dagger_{k\sigma,\alpha} c_{k\sigma,\alpha} + \sum_{i\sigma} [\epsilon_i - e\mathcal{E}x] d^\dagger_{i\sigma} d_{i\sigma} + Un^2$$

$$H_{tun} = \sum_{ik\sigma,\alpha} T^{(i)}_\alpha(x)[c^\dagger_{k\sigma,\alpha} d_{i\sigma} + H.c],$$

SW transformation

$$H = H_0 + \sum_{k\alpha\sigma,k'\alpha'\sigma'} \mathcal{J}_{\alpha\alpha'}(t)[\vec{\sigma}_{\sigma\sigma'}\vec{S} + \frac{1}{4}\delta_{\sigma\sigma'}] c^\dagger_{k\sigma,\alpha} c_{k'\sigma',\alpha'}$$

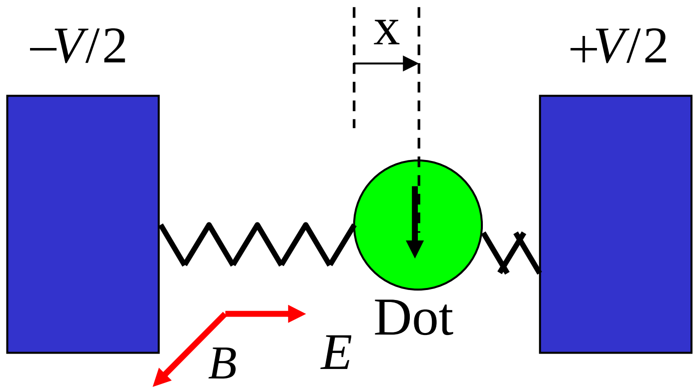
$$\mathcal{J}_{\alpha,\alpha'}(t) = \sqrt{\Gamma_\alpha(t)\Gamma_{\alpha'}(t)}/(\pi\rho_0 E_d(t)) \qquad \Gamma_\alpha(t) = 2\pi\rho_0 |T_\alpha(x(t))|^2$$



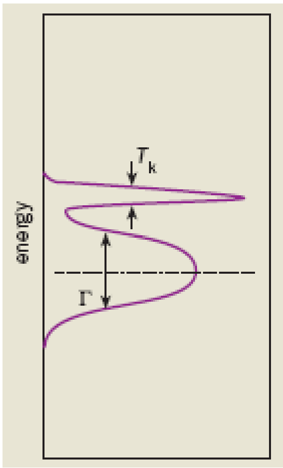
Classical shuttling trajectories

$$\langle x^2 \rangle \gg \hbar/(m\Omega)$$

Odd-spin Kondo shuttle



Competition between



Breit-Wigner Resonance

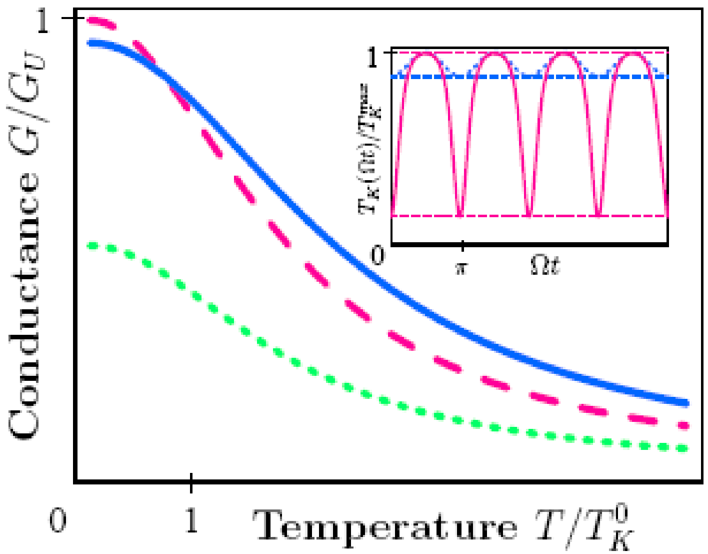
$$G = \frac{2e^2}{h}\left\langle \frac{4\Gamma_L(t)\Gamma_R(t)}{(\Gamma_L(t)+\Gamma_R(t))^2} \right\rangle$$

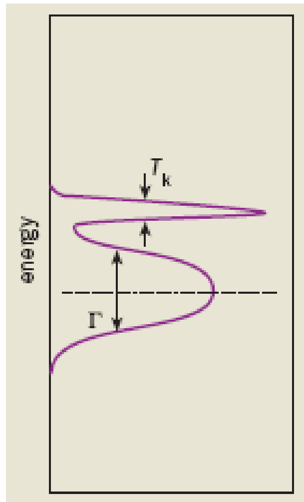
Abrikosov-Suhl Resonance

$$G(T) = \frac{3\pi^2}{16} G_U \left\langle \frac{4\Gamma_L(t)\Gamma_R(t)}{(\Gamma_L(t)+\Gamma_R(t))^2} \frac{1}{[\ln(T/T_K(t))]^2} \right\rangle$$

Adiabaticity $\quad \hbar\Omega \ll T_K \ll \Gamma$

Time-dependent Kondo temperature



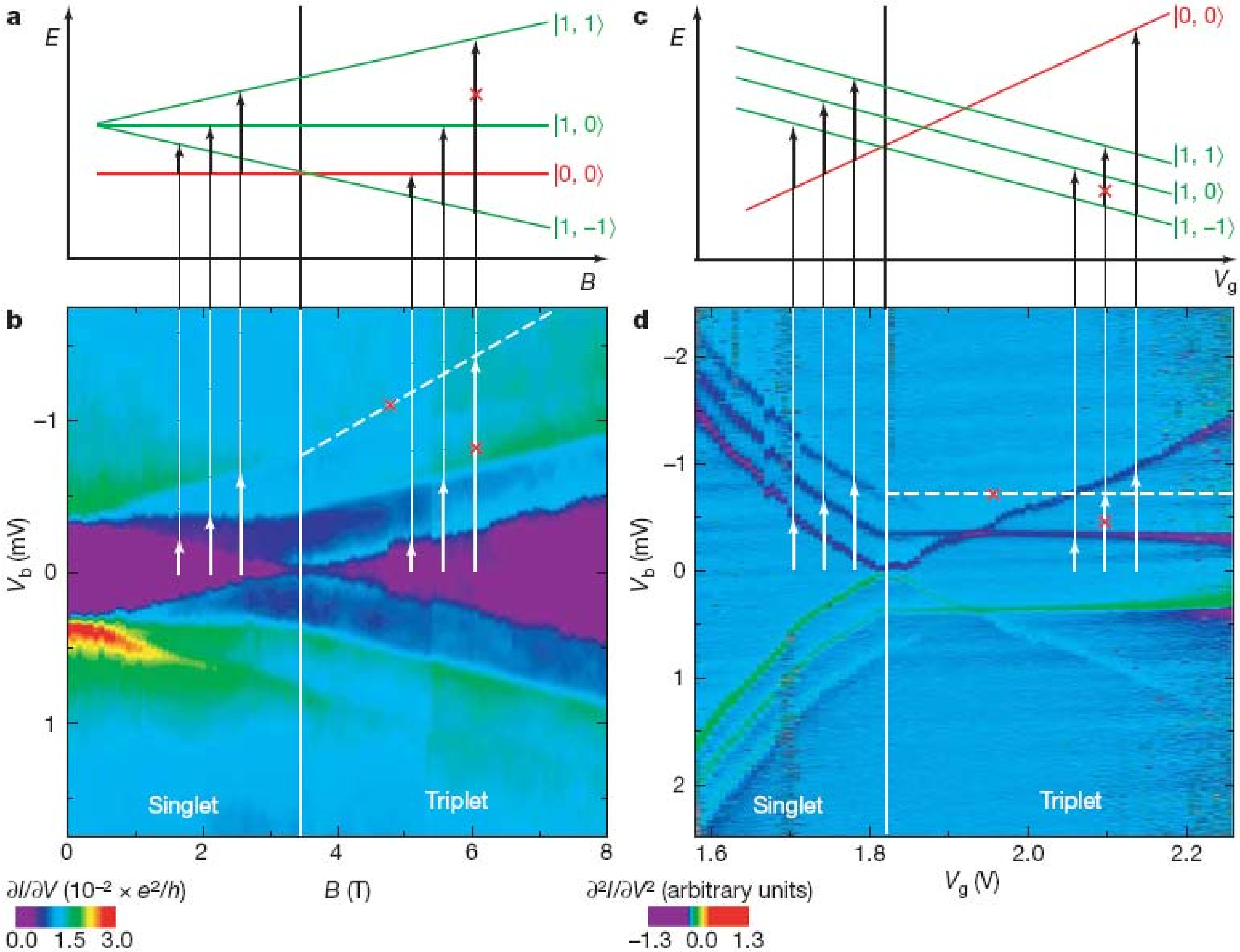


$$T_K(t) = D(t)\exp\left[-\frac{\pi U}{8\Gamma_0\cosh(2x(t)/\lambda_0)}\right]$$

$$\langle T_K\rangle = T_K^0\left\langle \exp\left[\frac{\pi U}{4\Gamma_0}\frac{\sinh^2(x(t)/\lambda_0)}{1+2\sinh^2(x(t)/\lambda_0)}\right]\right\rangle$$

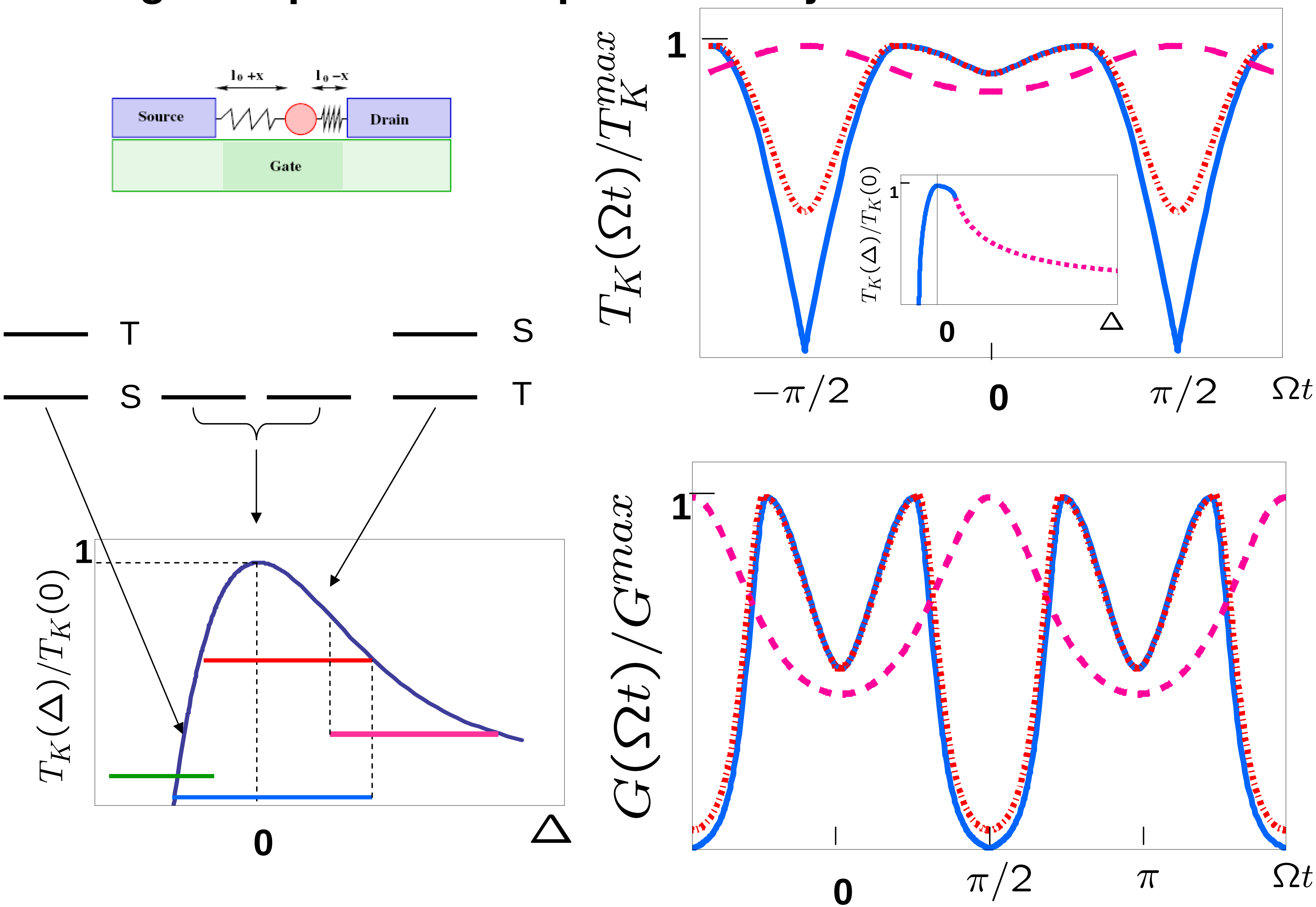
$$G(T) = G_K^0\left\langle\left(\frac{1}{1-2\alpha^2(T)\sinh^2[x(t)/\lambda_0]}\right)^2\right\rangle, \qquad \frac{\delta G_K}{G_K^0} = \frac{G(T)-G_K^0}{G_K^0} = 2\frac{\delta T_K}{T_K^0}\frac{1}{\ln(T/T_K^0)}.$$

Singlet/Triplet Shuttle

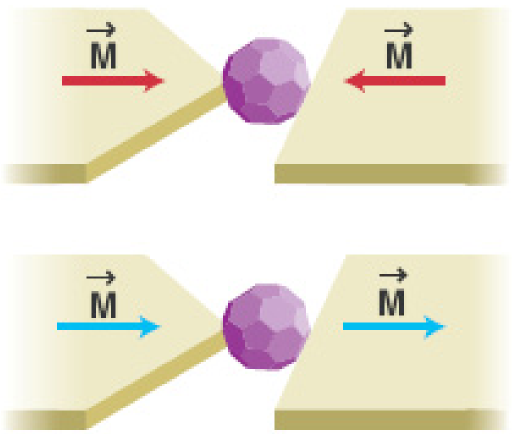


N.Roch et al Nature 2008

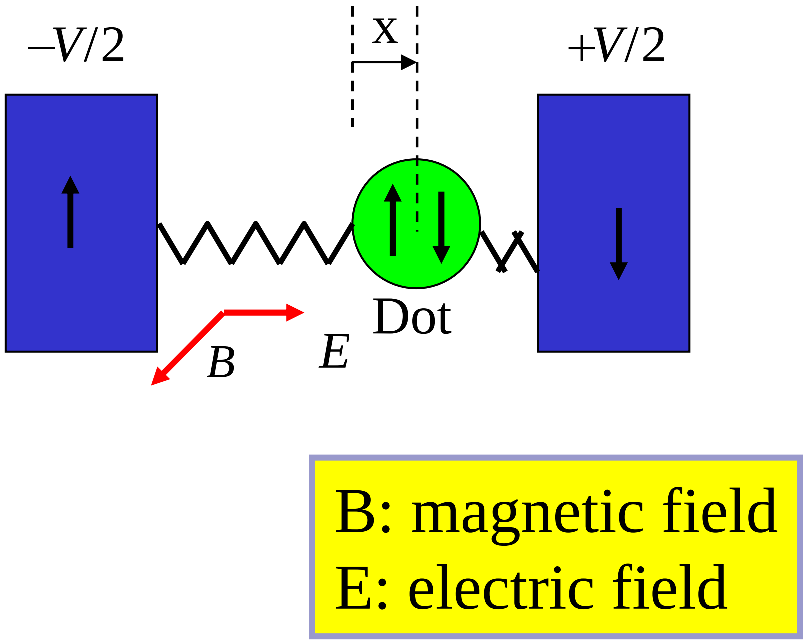
Singlet/Triplet Shuttle: possible trajectories



Perspectives



A. Pasupathy et al., Science **306**,86 (2004)



- NEM-SET between spin-polarized leads
- NEM spin manipulation
- Non-adiabatic shuttle
- Shot Noise
- Coupled NEM-SET devices (DQD, TQD)