The Abdus Salam International Centre
for Theoretical Physics

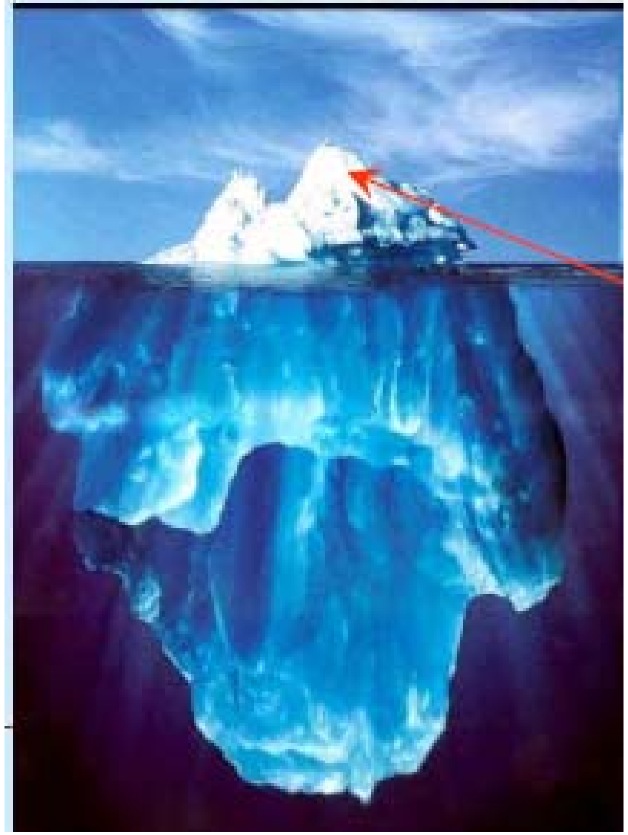
M.N.Kiselev

Electron transport through nanostructures

Lecture 1

Quantum Dots: Coulomb blockade, tunneling, cotunneling

Regional School on Physics at the Nanoscale, Hanoi, 14-25 December 2009



Outline of the course:

- Quantum Dots
- Kondo effect in nano-devices
- From Fermi liquid to Luttinger liquid

For reading:

Transport through QDs: W.G. van der Wiel et al, RMP 75 (2003)
SET and Coulomb blockade: M.A.Kastner, RMP 64 (1992)
Popular reading: Leo Kouwenhoven and Charles Marcus, Physics World 1998
See also in the web Lecture courses of Ya. Blanter, Y. Gefen, Yu. Galperin

* Some transparencies are courtesy of Yuval Gefen, Yaroslav Blanter and Yuri Galperin

Connection with other lectures:

This Lecture Course

Robert Shekhter:

Mechanically assisted single-electronics

Quantum coherent nano-electromechanics

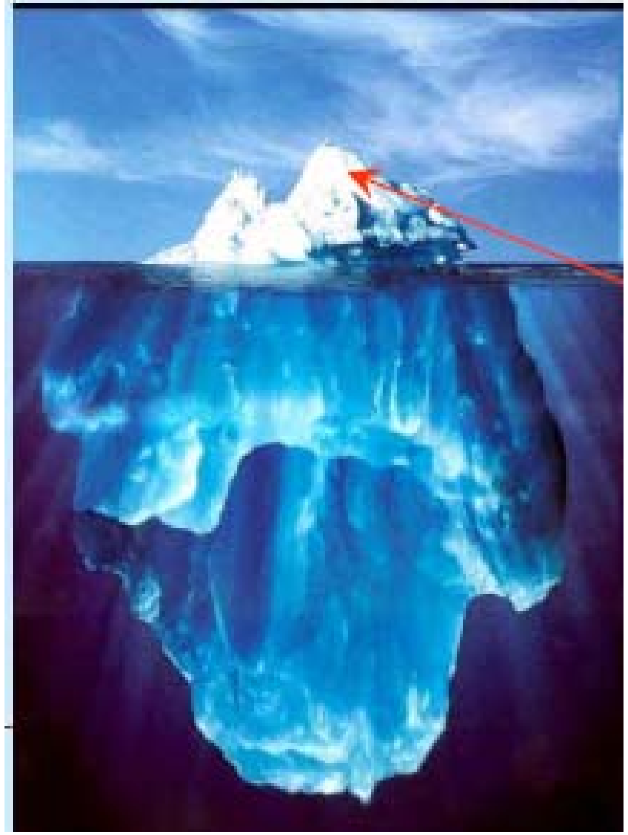
Vladimir Kravtsov

Quantum dots and random matrices

Peter Fulde:

Electron correlations on nano- and subnano scale

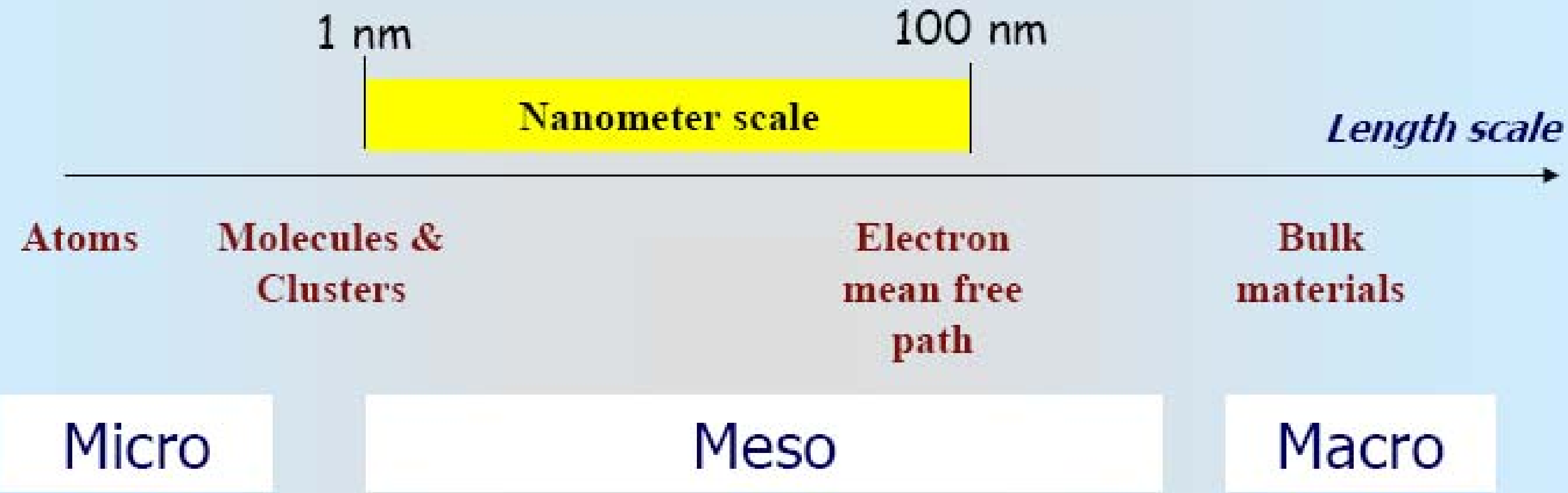
Outline of this lecture



- What is Nano?
- Examples of QD:
- Vertical vs lateral QDs
- Diffusive vs ballistic QDs
- Metallic vs semiconductor QDs
- Open vs close QDs
- Coulomb blockade
- Sequential tunneling
- Elastic vs inelastic cotunneling
- “Universal” Hamiltonian

Characteristic scales in nanoscience

$$1\ \text{nm} = 10^{-9}\ \text{m}$$



1 nm

100 nm

Nanometer scale

Length scale

Atoms

Molecules & Clusters

Electron mean free path

Bulk materials

Micro

Meso

Macro

Modern electronic devices belong to mesoscopic scale

Nano means **Big** !?

Nanoscale objects do not fully belong to the microcosm

Many atoms, electrons, etc., are involved

↓

Number of degrees of freedom is large

Size →

Micro

Nuclei

Atoms

Small molecules

Meso

Nano-objects

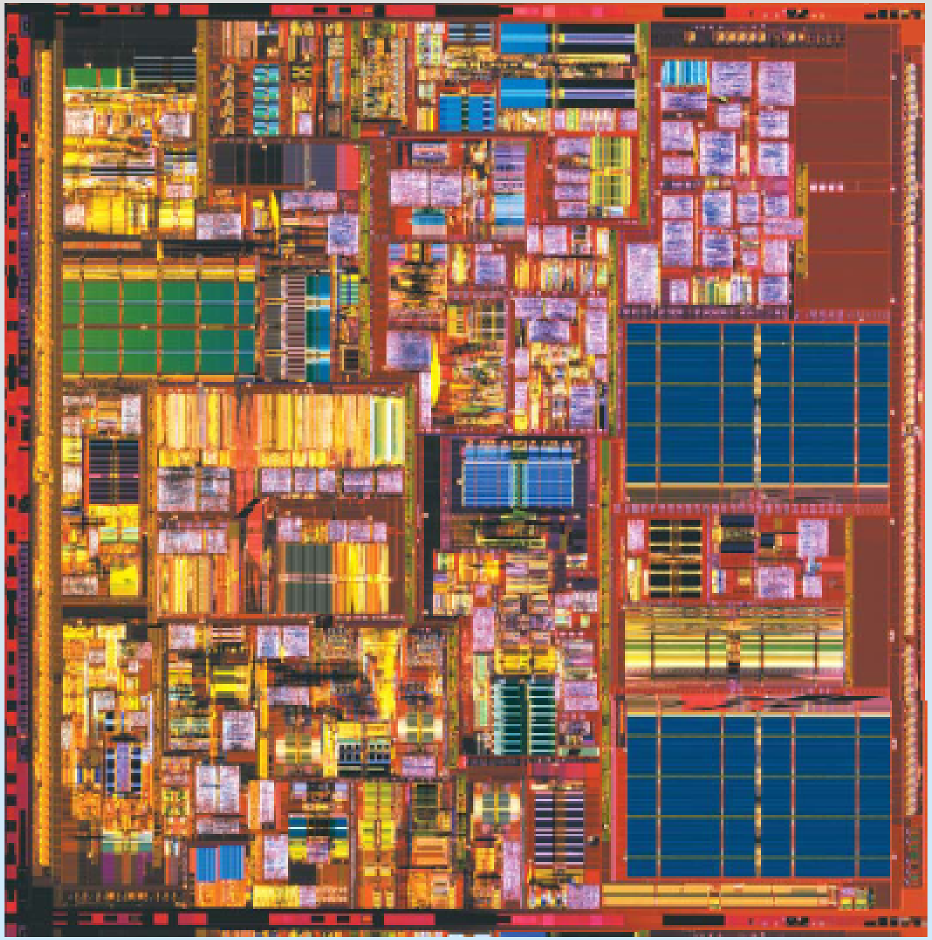
Macro

Fluids

Crystals

Glasses

CMOS TECHNOLOGY



Intel's Norwood (Pentium 4 - 130 nm) processor

Intel's Prescott processor

(released March 2004):

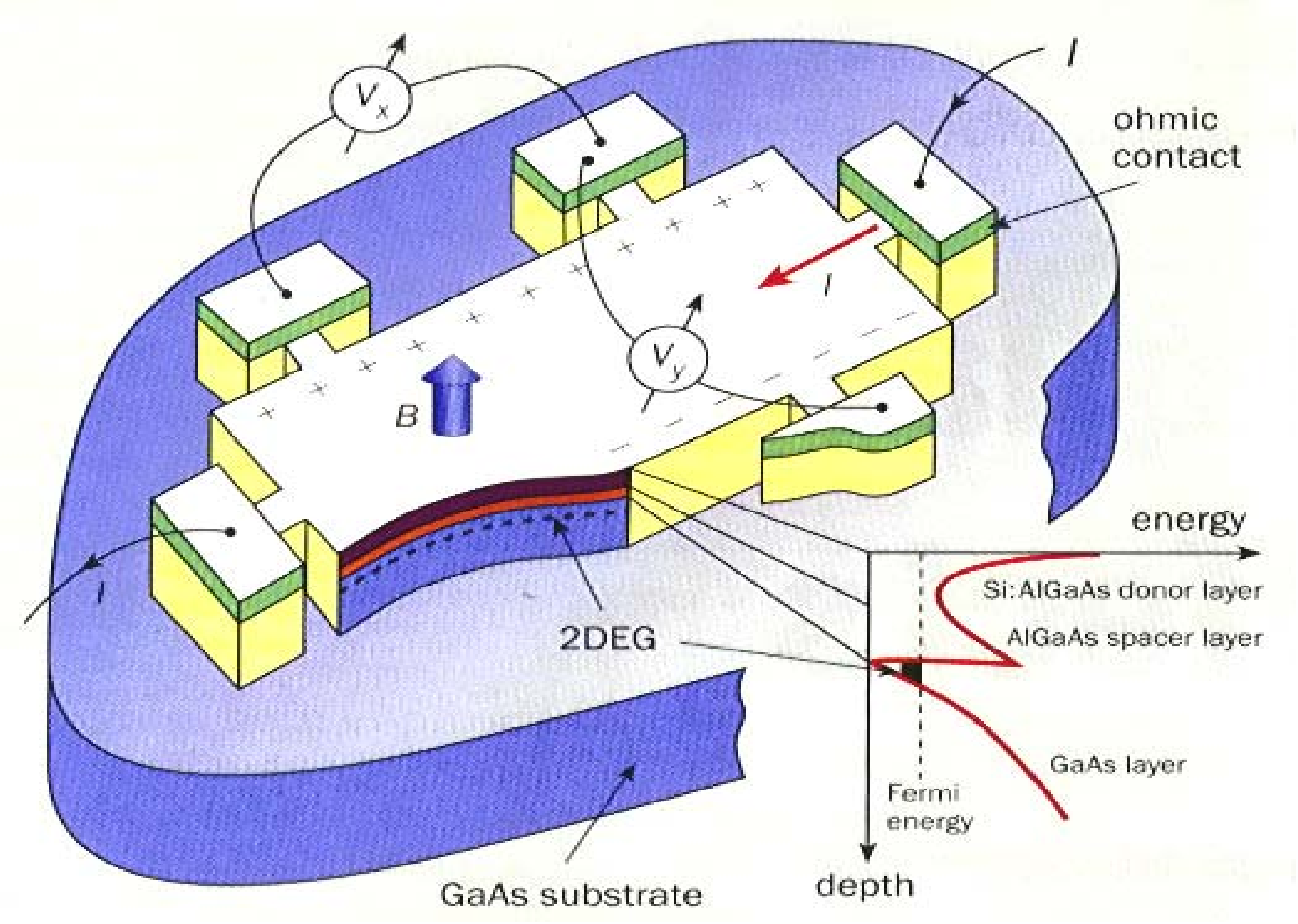
- 150 million transistors
- 90 nm design rules
- 3.4 GHz clock frequency

DRAM chips:

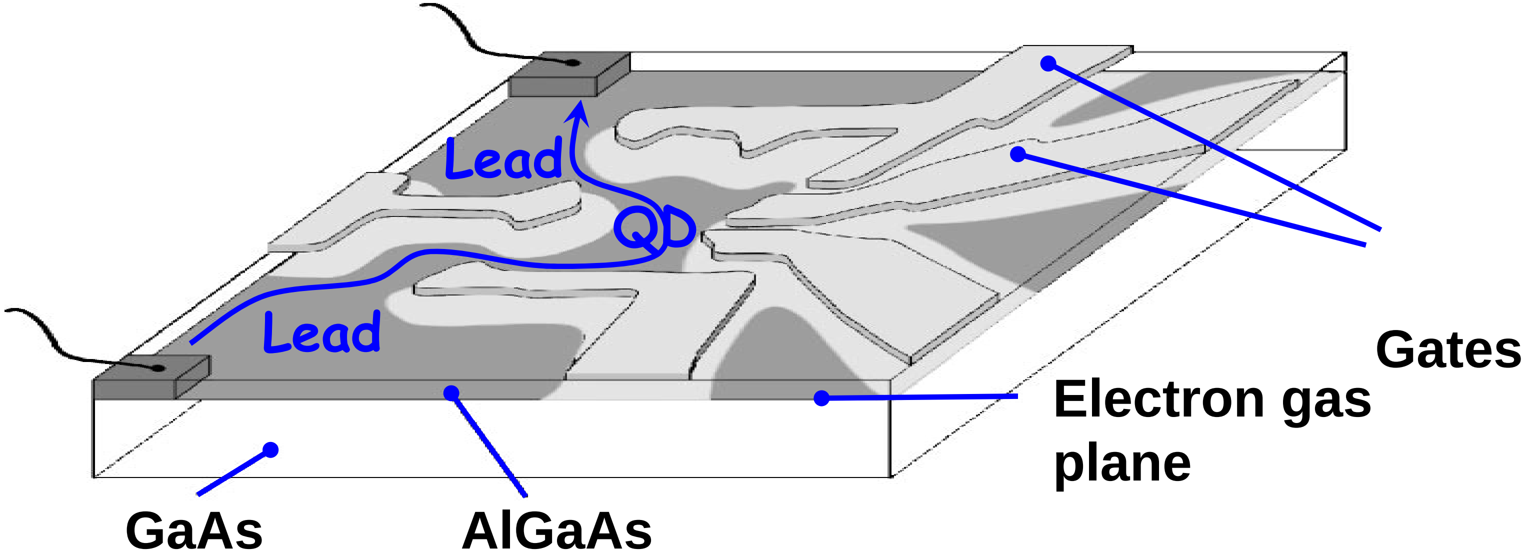
4 Gb chips demonstrated

(~ 10^9 transistors/cm^2)

Two Dimensional Electron Gas (2DEG)

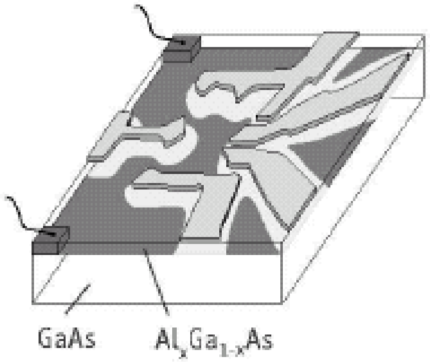


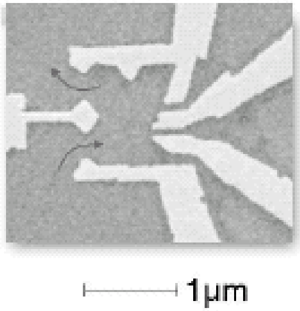
Quantum dots

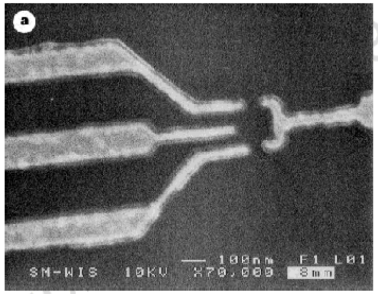


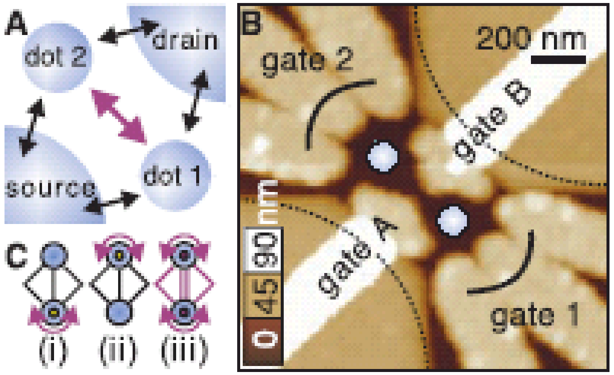
- Tune: gate potentials, temperature, field…
- Measure: I-V curves, conductance G…
- Aharonov-Bohm interferometry, dephasing, coherent state manipulation…

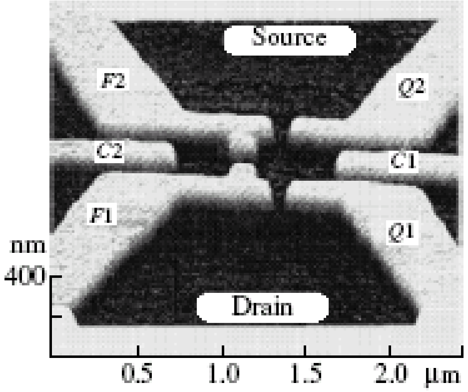
Quantum dots: from simple to complex

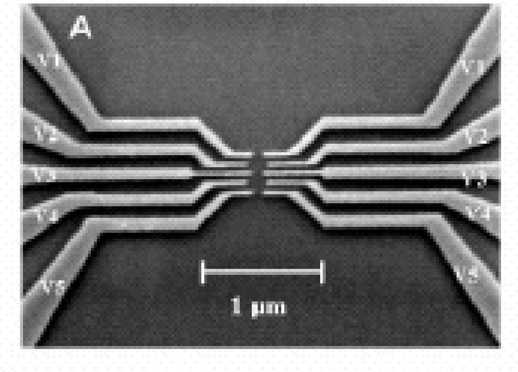












D.Goldhaber-Gordon et al (1998)

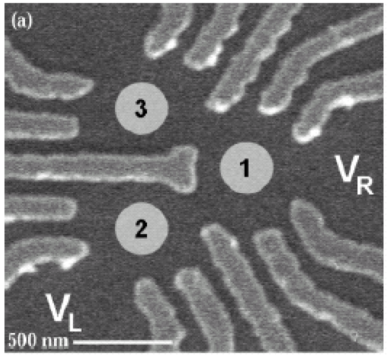
J.P.Kotthaus (1995)

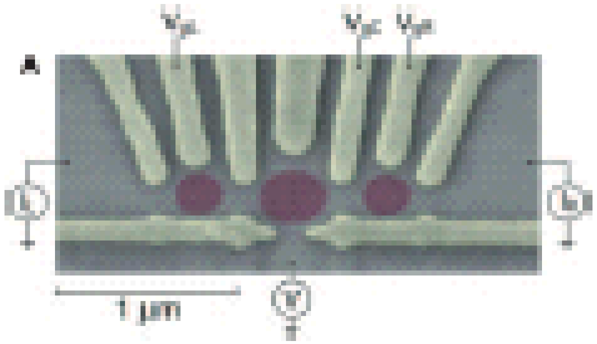
A.Holleitner et al (2002)

L.W.Molenkamp et al (1995)

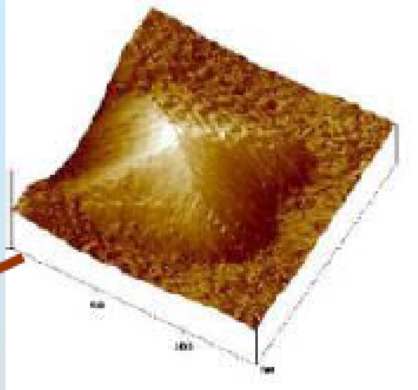
H.Jeong et al (2001)

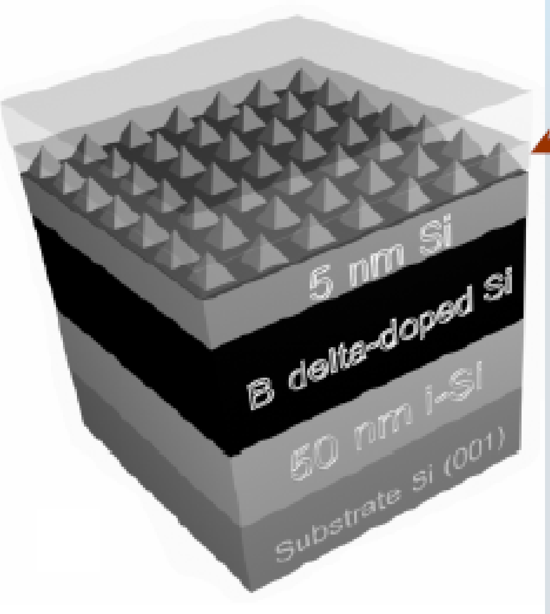
C.Marcus et al (2003)





Self-assembled QD

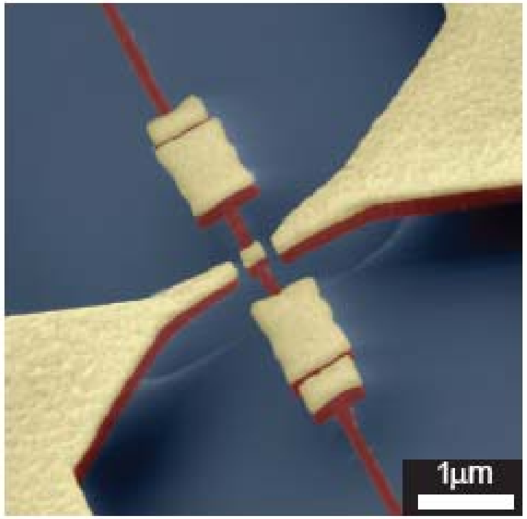


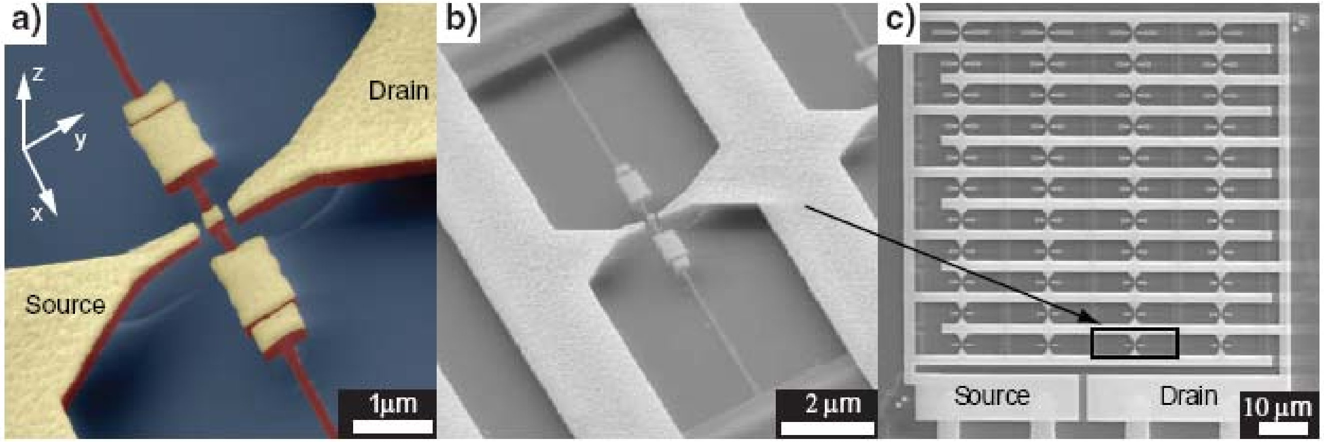


Self-assembled quantum dots are periodic arrays of “artificial atoms”.

They are considered to be promising systems for heterostructure lasers.

Nanoelectromechanical shuttling: QD devices

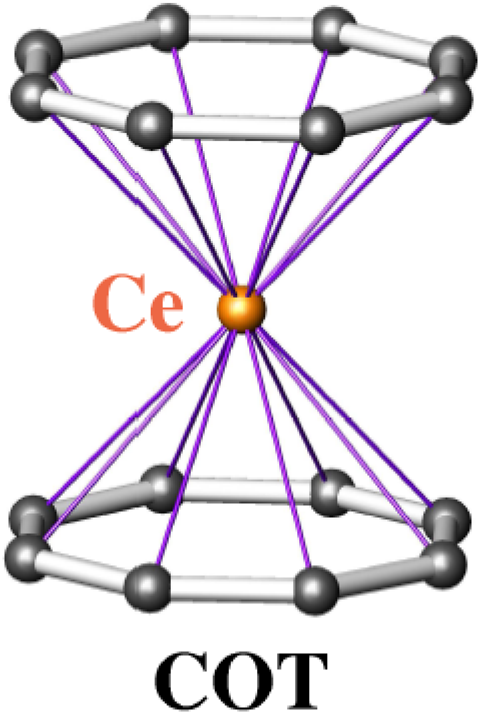




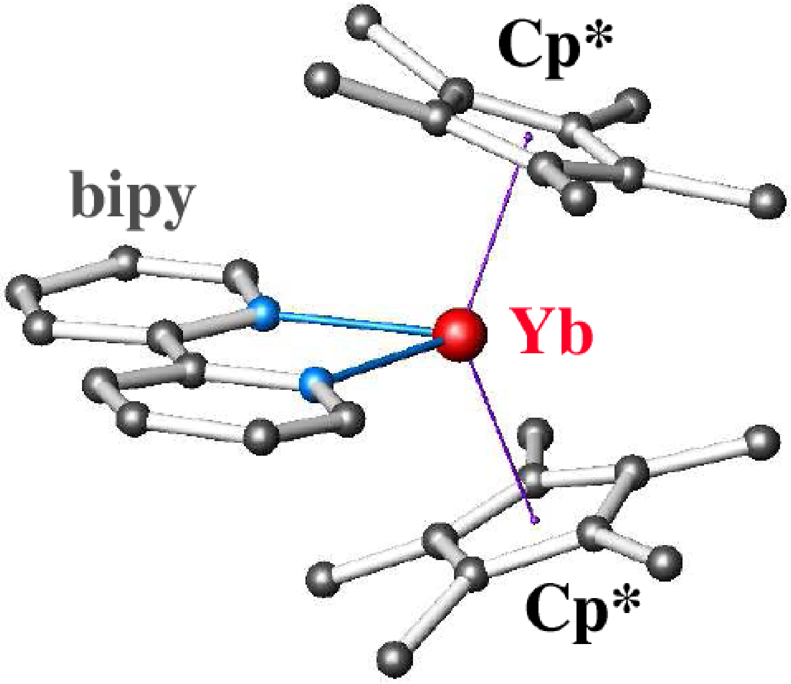
J. Kotthaus et al, Nature Nanotechnology 2008

Cerocene $Ce(C_8H_8)_2$ Ytterbocene

$Ce(COT)_2$



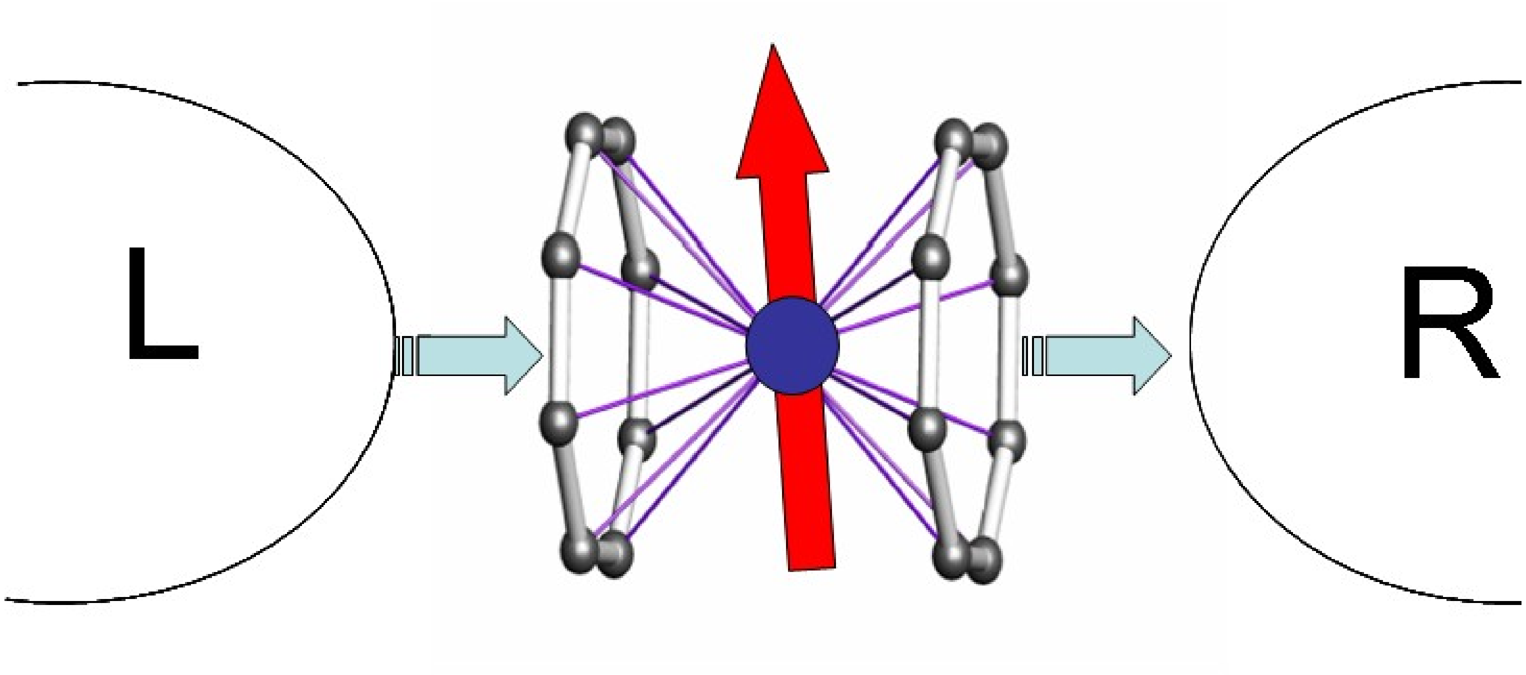
$Cp^*_2Yb(bipy)$



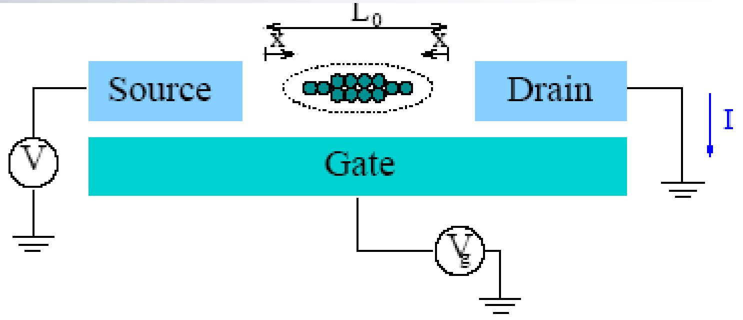
COT = C8H8

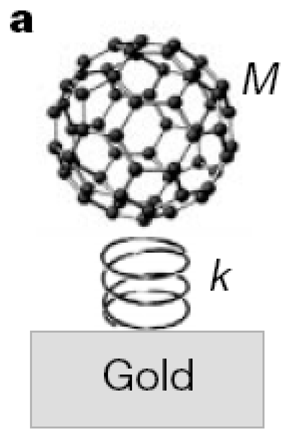
Cp* = C5Me5, bipy = (NC5H4)2]

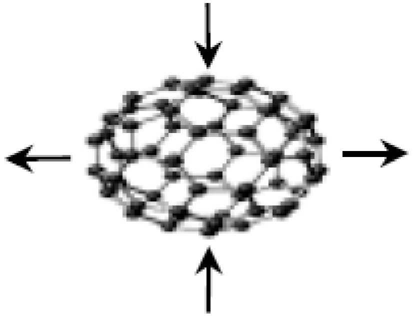
MolecularTransistor

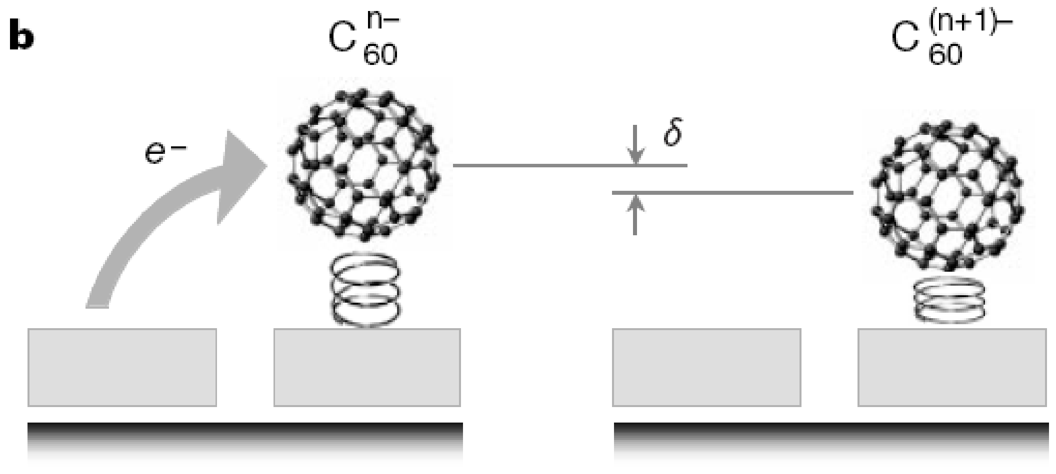


Fullerenes



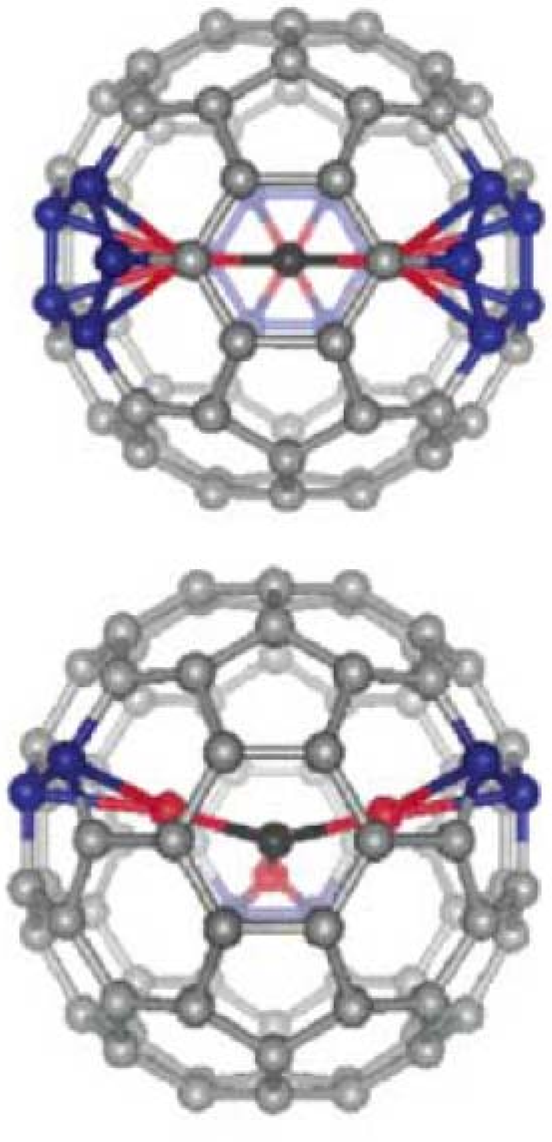


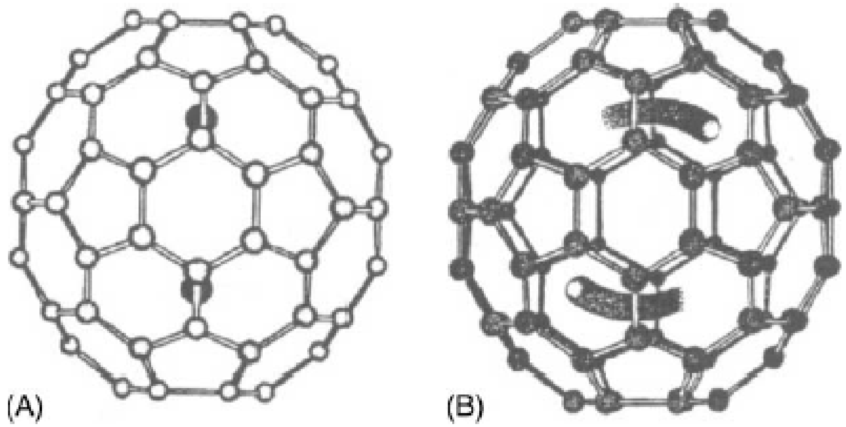


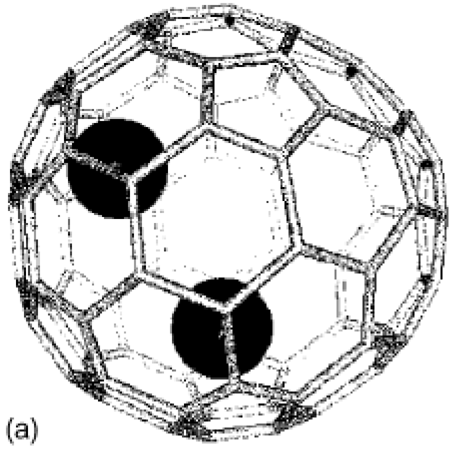


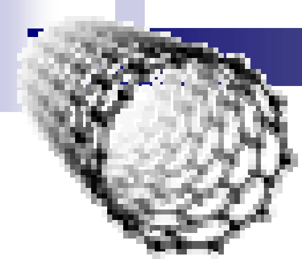
H.Park et al, Nature 2000

Transition metals inside fullerens

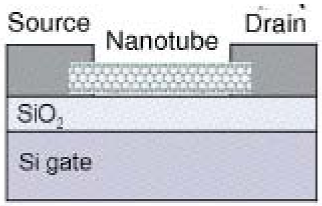


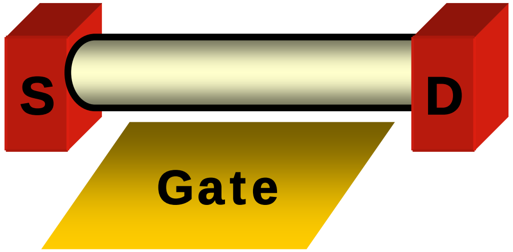


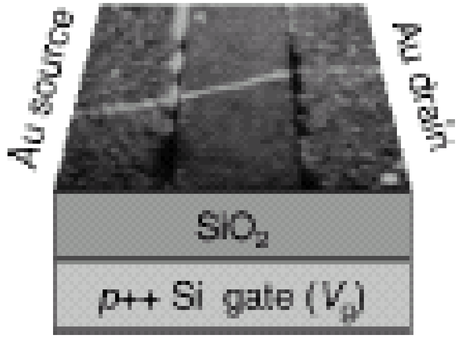


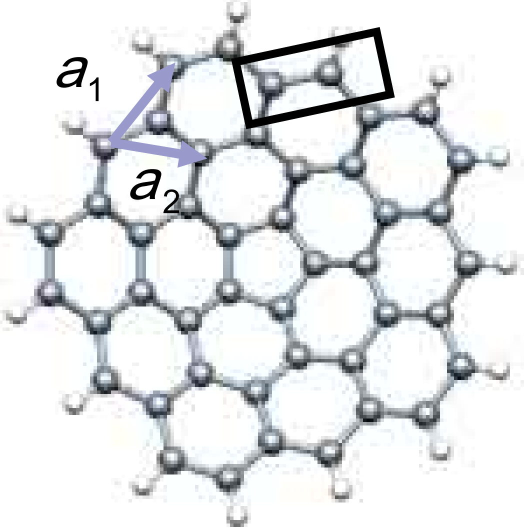


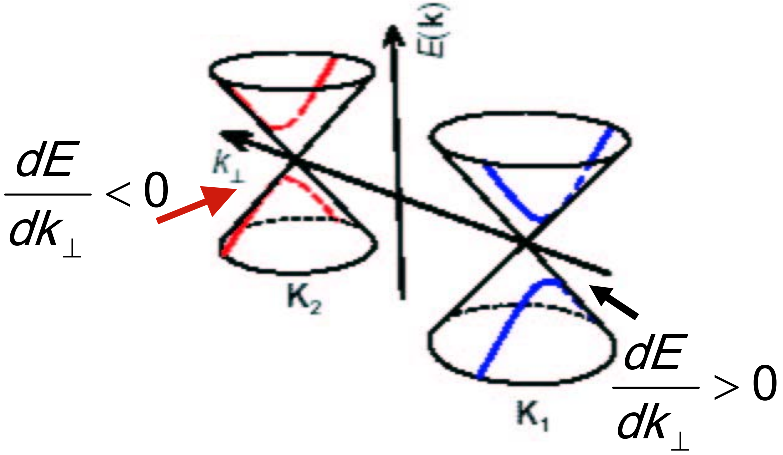
Carbon Nanotubes

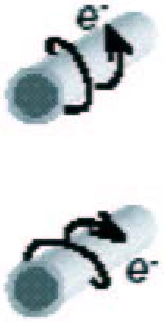


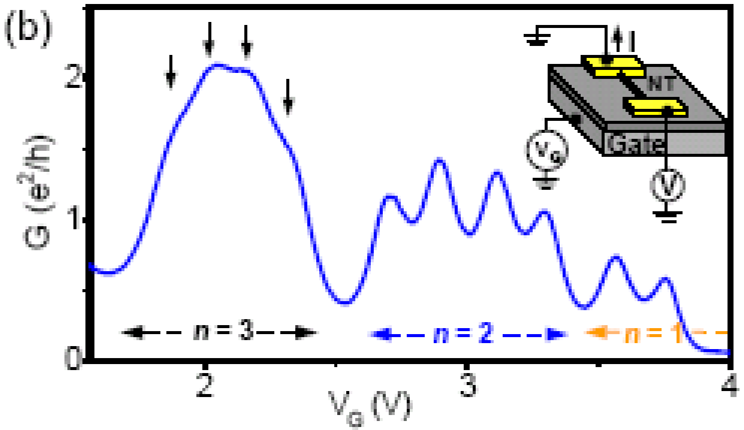




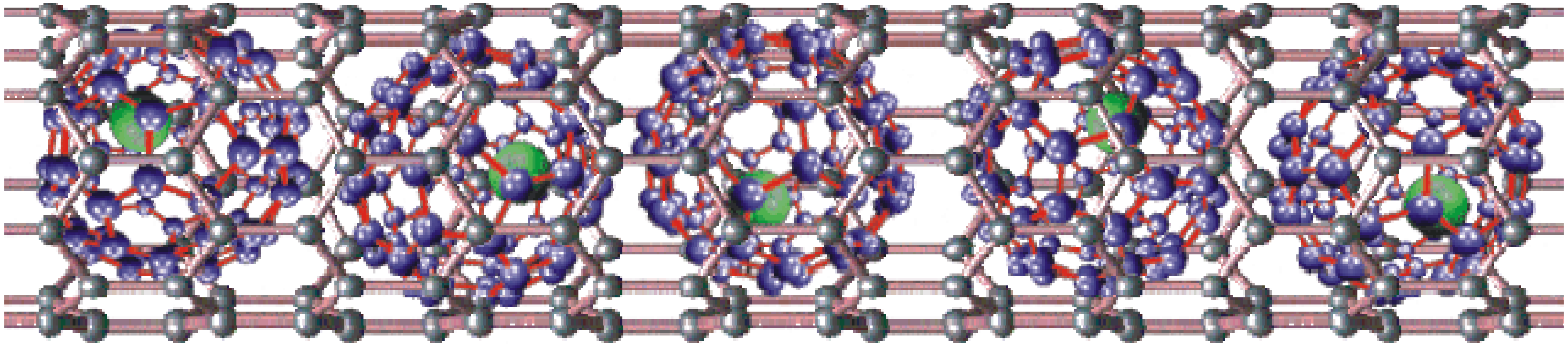


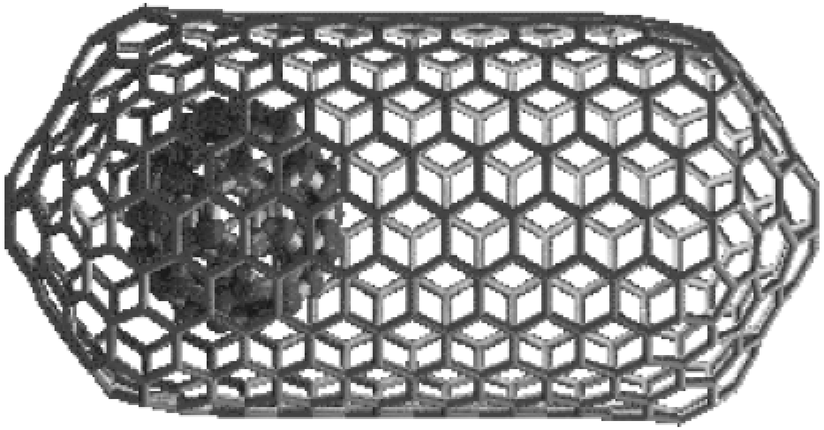


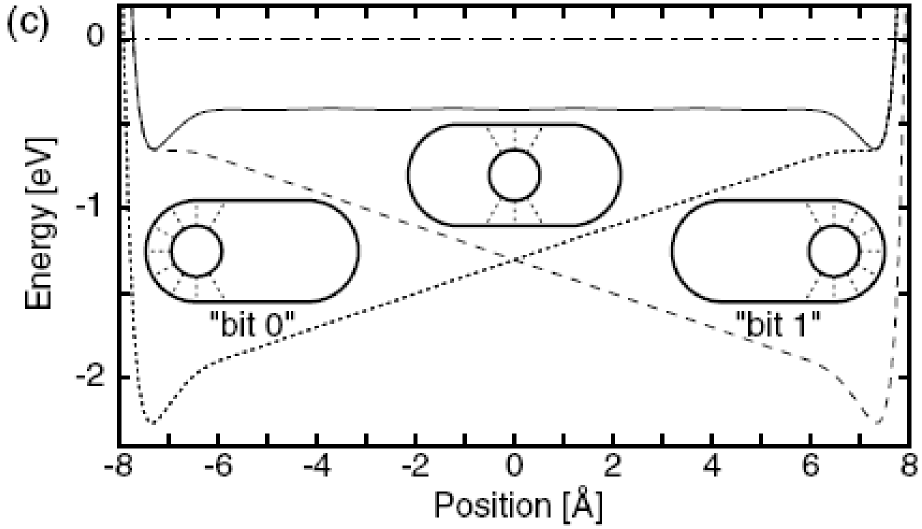




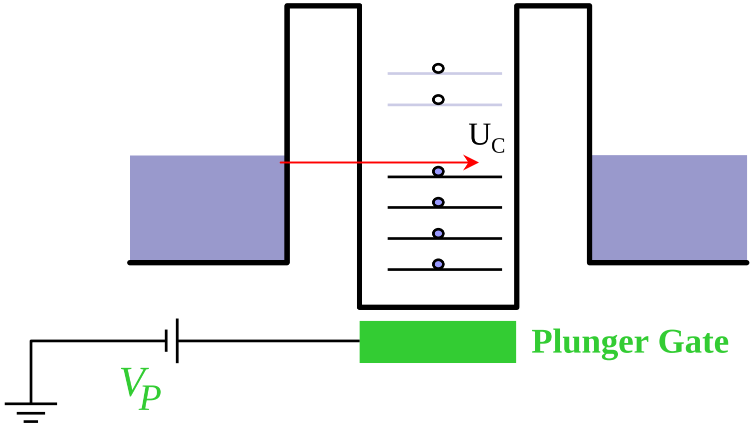
Nanotube peapods: C_{60} @ CNT

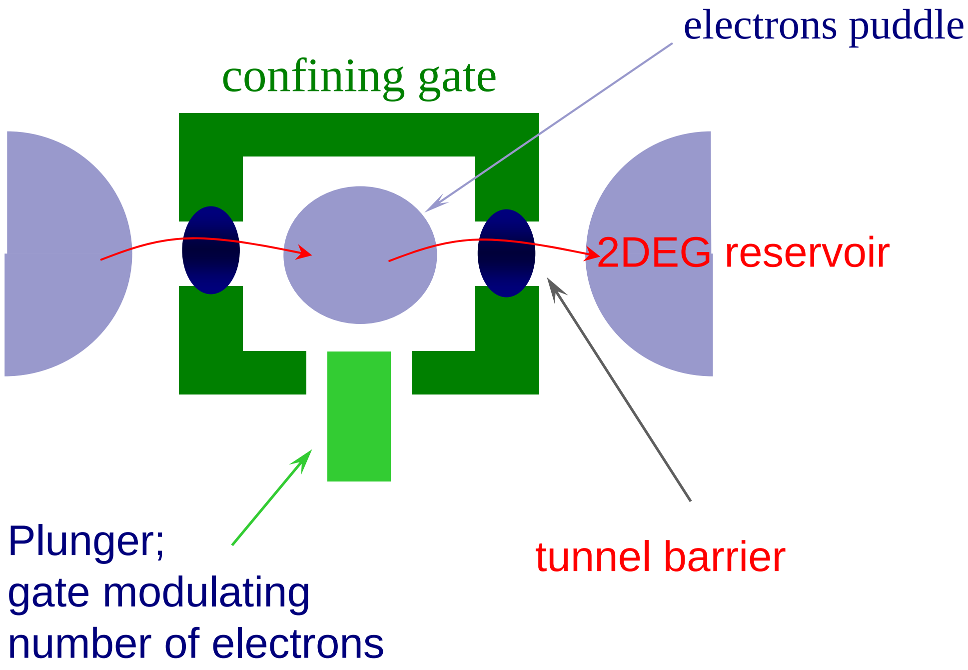






Realization of lateral QD in 2DEG



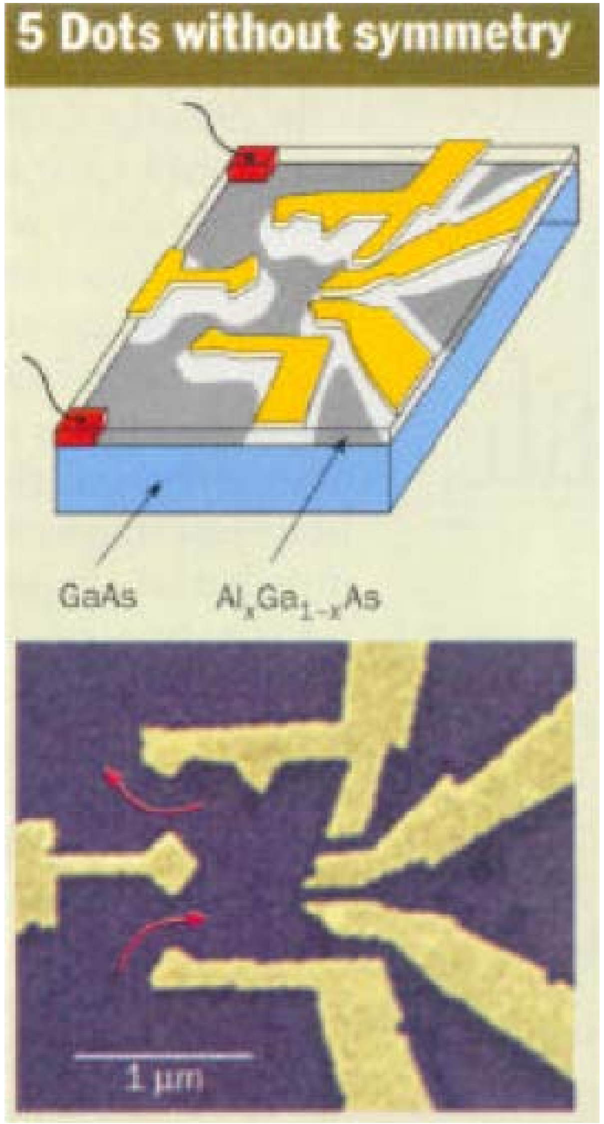

electrons puddle

confining gate

2DEG reservoir

tunnel barrier

Plunger;
gate modulating
number of electrons



Characteristic parameters:

size: $100\ \text{A}^{\circ} \rightarrow 2\mu m$

 # electrons: 0 → hundreds

mobility

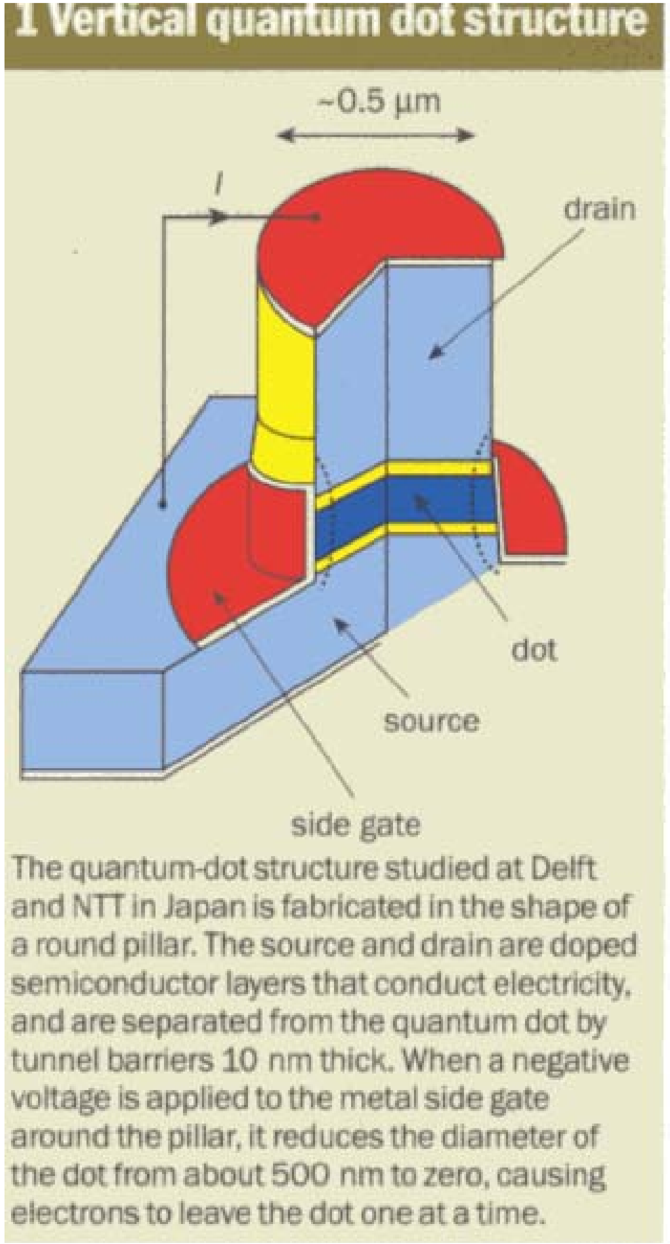
(of 2DEG in strong magnetic fields)
original Integer Quantum Hall Effect $\sim (30-50)\cdot 10^{3}\ \ \text{cm}^{2}/V\cdot\text{sec}$

current world record (Weizmann) $\sim 36\cdot 10^{6}$

CONTROL:

- ***size of QD***
- ***density of electrons → # electrons***
- ***(mobility; disorder)***
- ***shape***
- ***contact to leads***

Vertical QDs



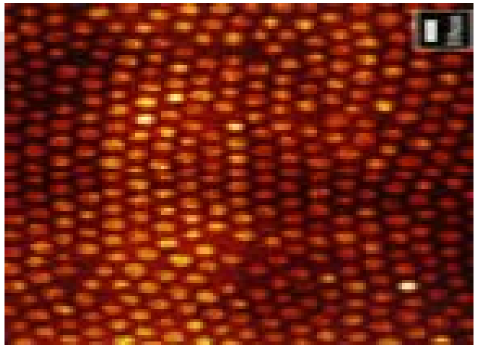
1 Vertical quantum dot structure

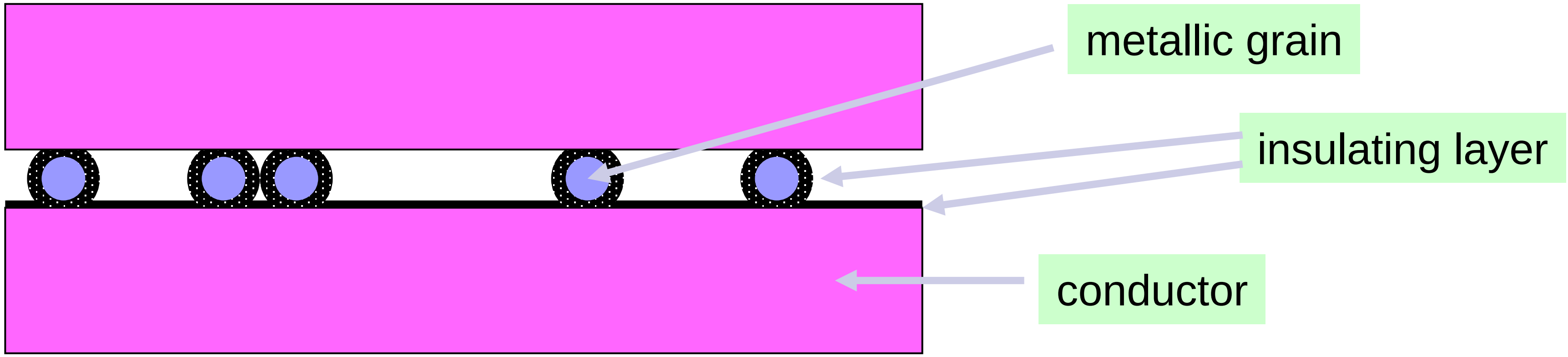
The quantum-dot structure studied at Delft and NTT in Japan is fabricated in the shape of a round pillar. The source and drain are doped semiconductor layers that conduct electricity, and are separated from the quantum dot by tunnel barriers 10 nm thick. When a negative voltage is applied to the metal side gate around the pillar, it reduces the diameter of the dot from about 500 nm to zero, causing electrons to leave the dot one at a time.

advantages: easy access to small
\# electrons,
symmetric QDs

disadvantages: hard to control
shape/size;
dot-lead coupling

Metallic QDs





metallic grain

insulating layer

conductor

size: $30A^{\circ}$ and up

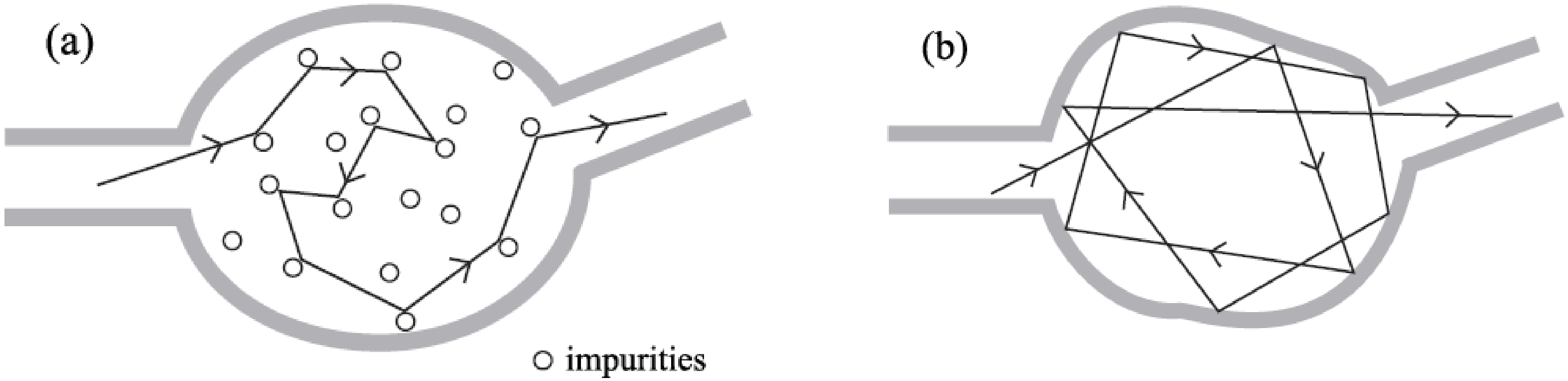
λ_F : a few A°

 # electrons: > many hundreds

originally: statistics of an ensemble
today: can attach leads to a single QD
little control: QD-lead coupling; size of QD
special appeal: QDs with special properties: SC; magnetic…

NON INTERACTING ELECTRONS

diffusive vs. ballistic

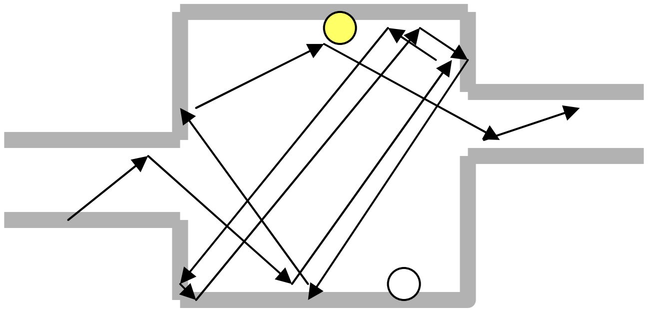


diffusive

$$E_{th} = \hbar/(\text{diffusion time}) = \hbar/(L^2/D)$$

ballistic

$$E_{th} = \hbar/(\text{time of flight}) = \hbar/(L/v_F)$$

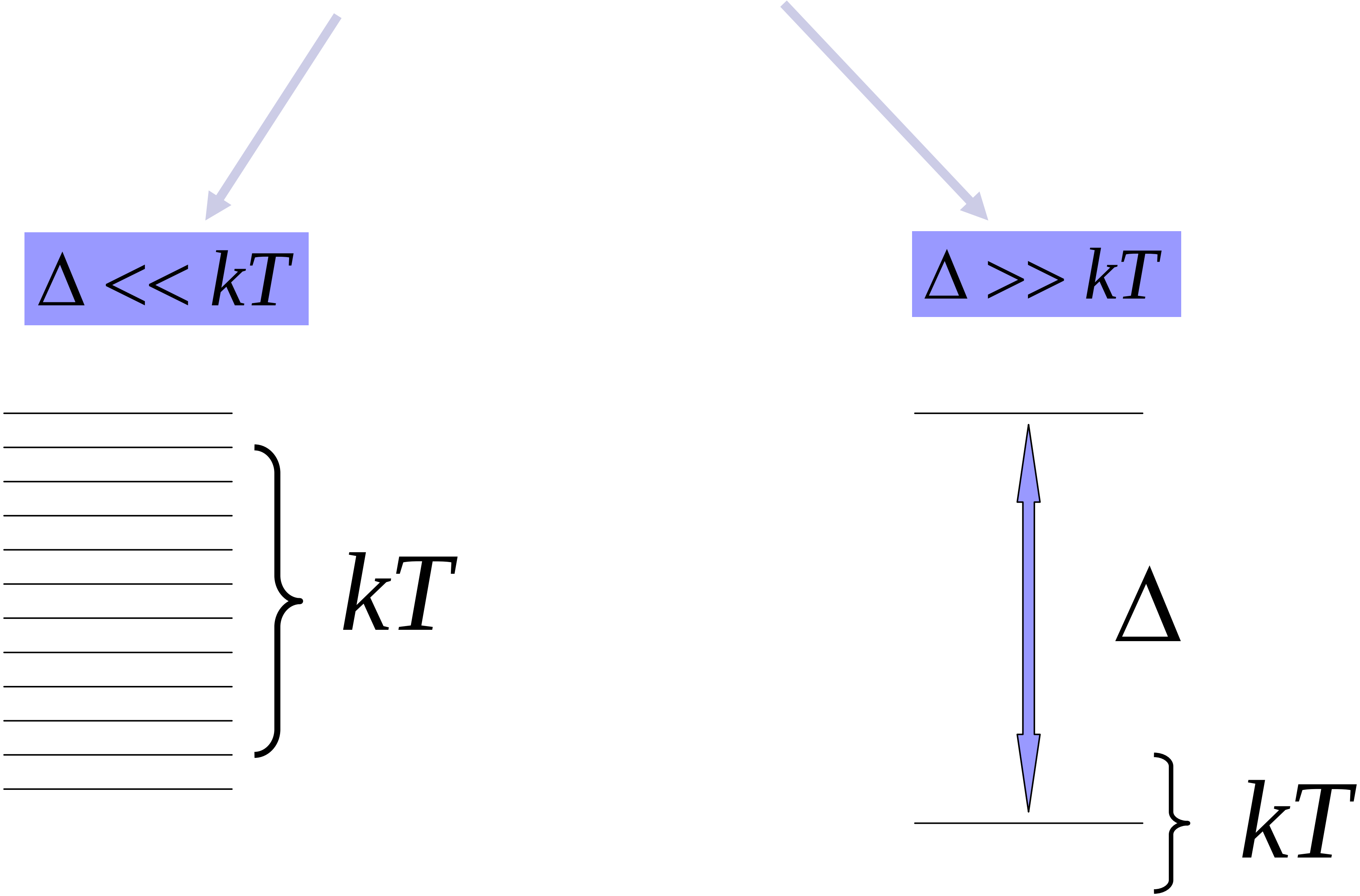


dirty ballistic

time of flight ≠ Thouless energy

(Altland , Gefen, Montambaux)

“Metallic” vs. discrete spectrum



$\Delta \ll kT$

$\Delta \gg kT$

Open vs. Closed

$$\frac{\Gamma_c}{\hbar} = \text{decay rate of a QD level into channel c}$$

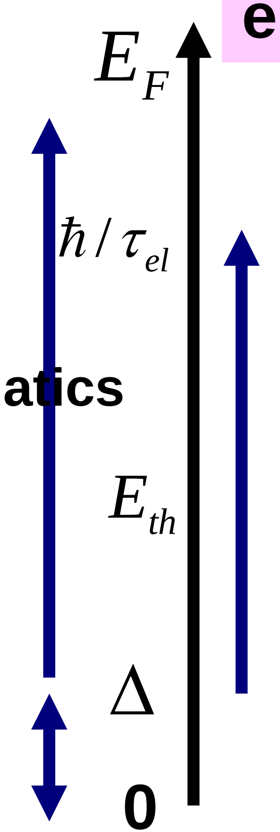
$$\text{total level width} = \Gamma = \sum_{\text{c}} \Gamma_c$$

closed QD (charge on the dot is nearly quantized)

$$\Gamma < E_c$$

NON INTERACTING ELECTRONS

energy measured from μ



E_F

$\hbar / \tau_{el}$

diagrammatics

E_{th}

Δ

non-perturbative

0

diffusive approximation (diffusons, Cooperons)

$\Big\}$

universal (RMT) regime

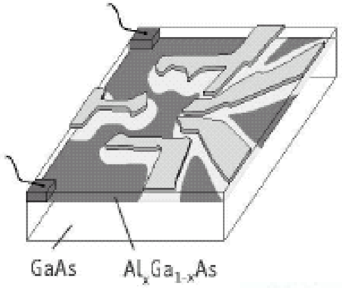
$$\Delta < \omega < E_{th}$$

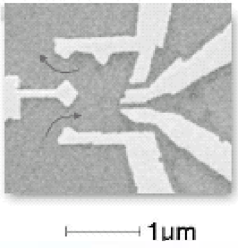
$$\omega < E_{th}$$

diagrammatics, universal

Metallic quantum dots: many-electron system

Random Matrix Theory

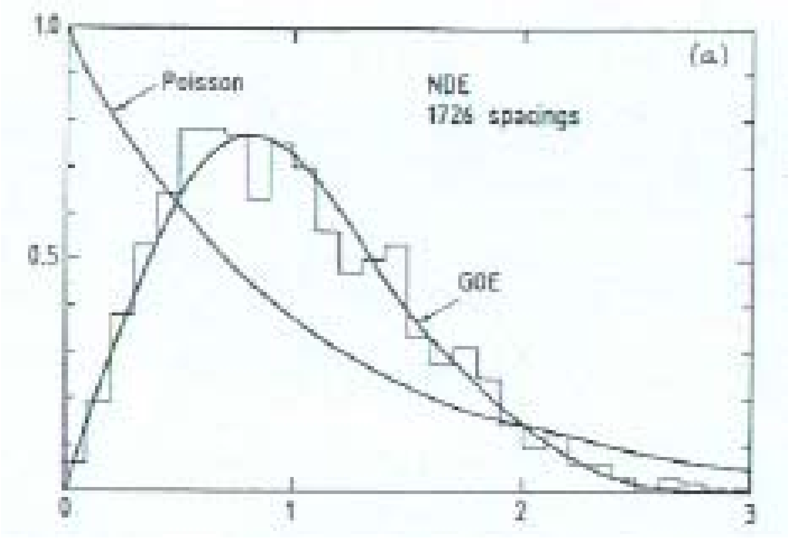




= artificial atom

GOE GUE

Wigner-Dyson statistics



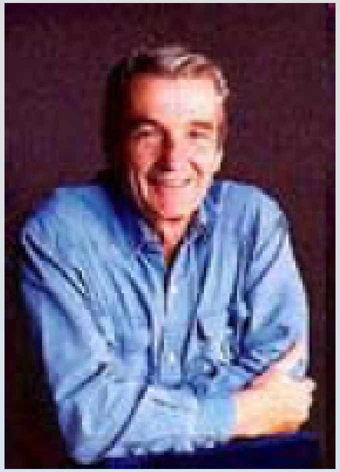
$$P\left(\left\{E_\mu\right\}\right) \propto \exp\left(\frac{\beta}{2}\sum_{\nu\neq\mu}\ln\left[\frac{|E_\mu - E_\nu|}{\delta}\right]\right)$$

$\beta = 1$ Orthogonal (GOE)

$\beta = 2$ Unitary (GUE)

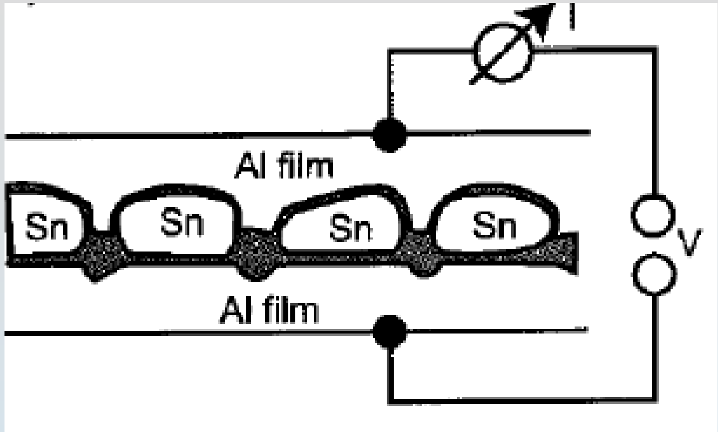
$\beta = 4$ Simplectic (GSE)

Coulomb blockage



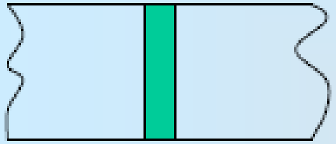
Ivar Giaever

1973



To move an electron to a confined region one has to pay for its repulsion from existing electrons

The principle of the Coulomb blockade



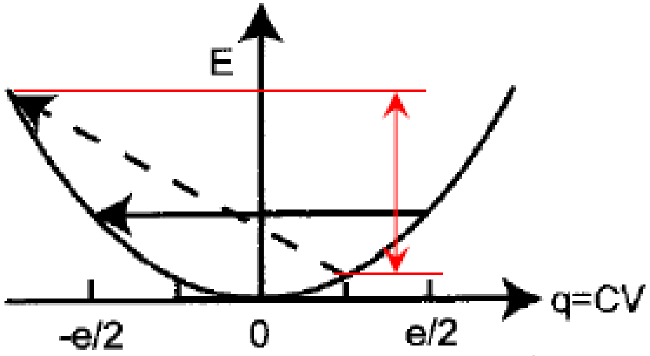
Why R matters?

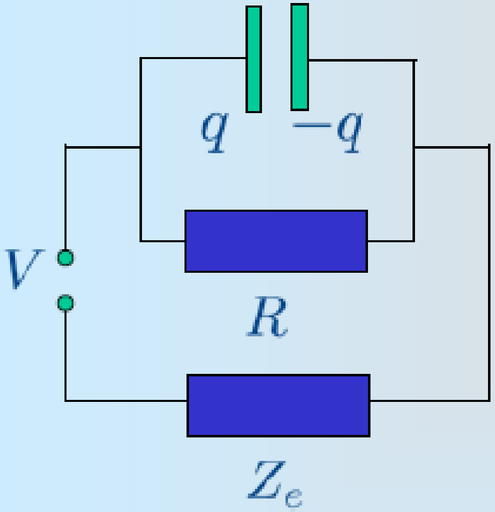
time delay $\delta t = eR/V$

duration $\tau \sim \hbar/eV$

$\delta t \gg \tau \quad \rightarrow \quad R \gg \hbar/e^2$

Energy stored is $q^2/2C$

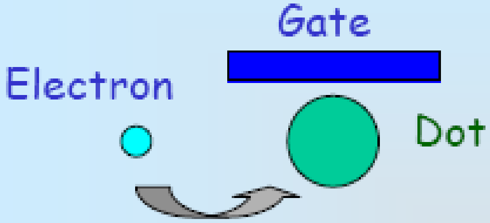




Because of environment capacitances it is difficult to observe CB in single junctions

At $|q| < e/2$ the electron tunneling will increase the energy stored in the barrier - one has to pay for the tunneling by the bias voltage

Coulomb blockade



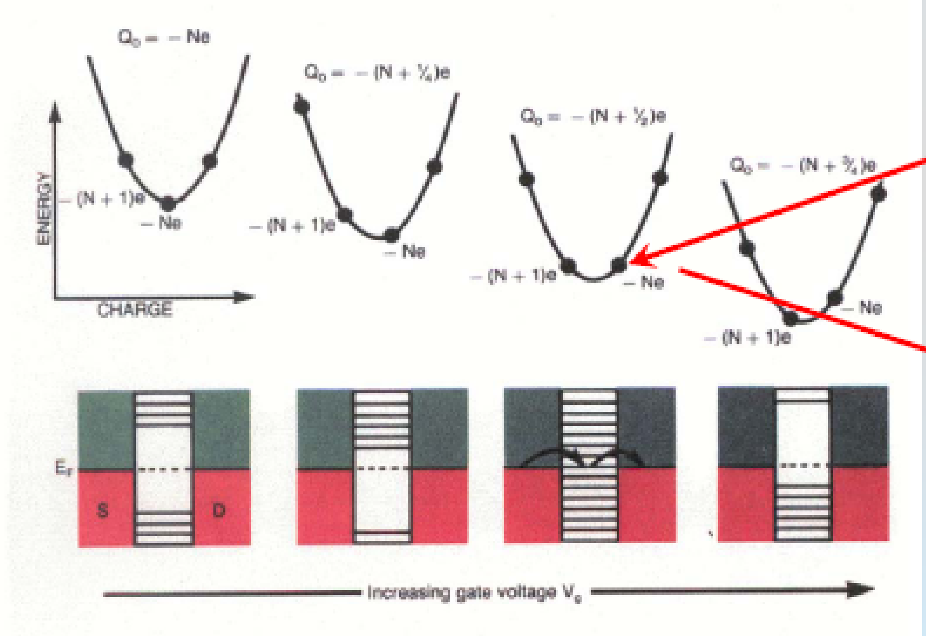
Repulsion at the dot

Cost

$$E = QV_g + \frac{Q^2}{2C}$$

Attraction to the gate

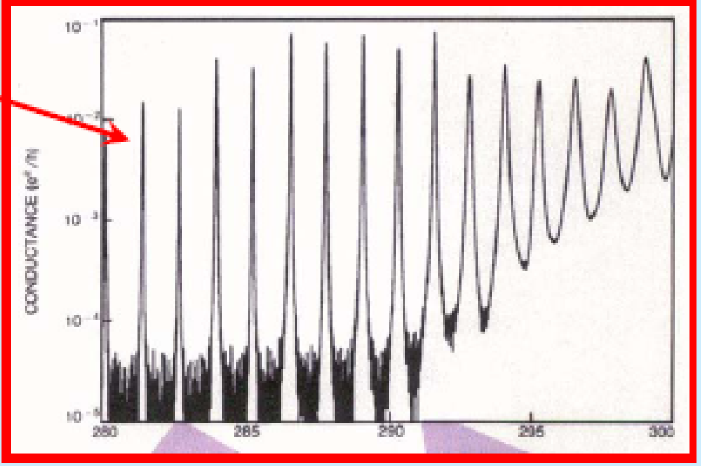
$$Q = -Ne$$



At

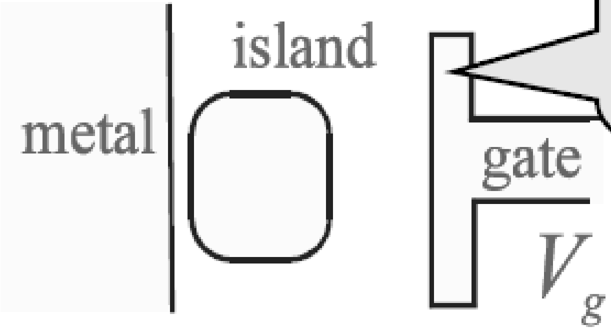
$$V_g = -\left(N + \frac{1}{2}\right)\frac{e}{C}$$

the energy cost vanishes !



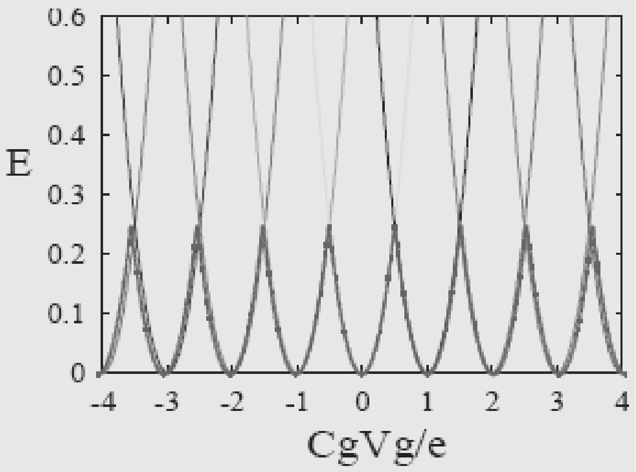
Single-electron transistor (SET)

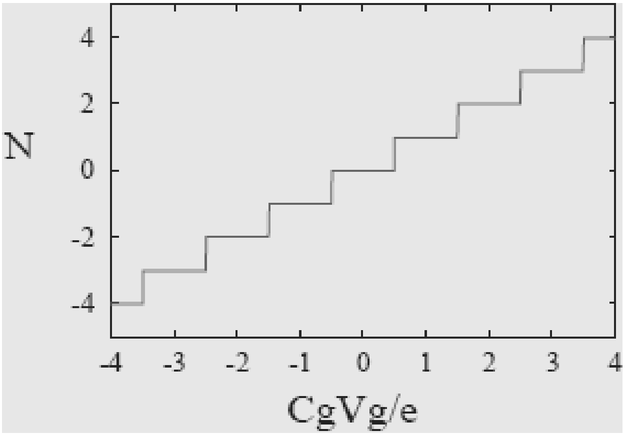
How many electrons?



Shifts potential
= induces charge
CgVg

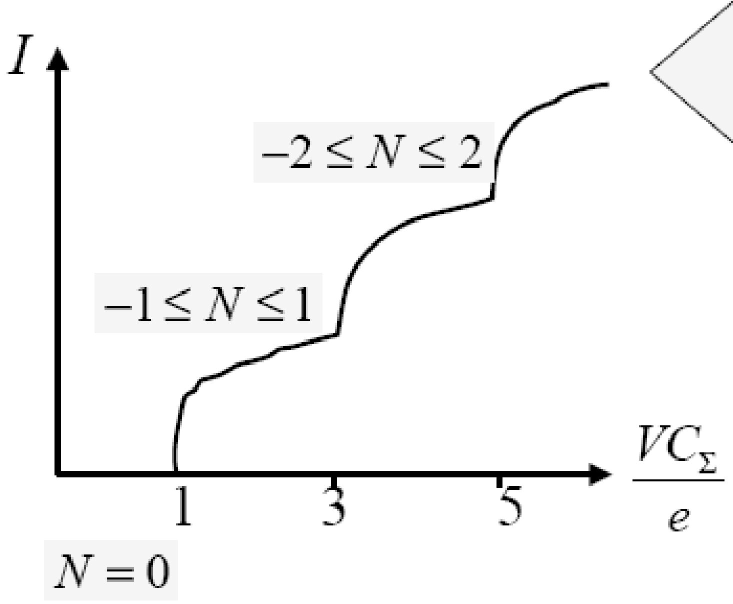
$$E = E_C\,(N-\frac{C_gV_g}{e})^2 \Rightarrow N = \left[\frac{C_gV_g}{e}+\frac{1}{2}\right]$$





Coulomb staircase

Nonlinear transport:
upon increasing V more charging states become available



Ohm's law: I=GV

$V_{th,n} = (2n+1)e / C_\Sigma$

$Q_g = 0$

LATERAL QDs : possible parameters

Temperature < 1 K (as low as 10-30 mK)

elastic mean free path $\approx$ 1- 150 μm

$n_s \approx 10^{11}$-10^{12} cm^{-2}

$E_F \approx$ 10 - 20 meV

$\lambda_F \approx$ 50 nm

electrons: 0 - hundreds

single particle level spacing=Δ ~ (0.01 meV ~ 0.1 K)

Thouless energy = E_{th} ~ (0.3 meV ~ 3 K)

charging energy = E_c ~ (1 meV ~ 10K)

THE “UNIVERSAL” HAMILTONIAN

$$H = H_{sp} + H_{\text{int}}$$

$$H_{\text{int}} = \frac{1}{2}\sum_{\alpha,\beta,\gamma,\delta} H_{\alpha\beta\gamma\delta}\,\widehat{\Psi}^{\dagger}{}_{\alpha\sigma_1}\,\widehat{\Psi}^{\dagger}{}_{\beta\sigma_2}\,\widehat{\Psi}_{\gamma\sigma_2}\,\widehat{\Psi}_{\delta\sigma_1}$$

$$H_{\alpha\beta\gamma\delta} = \int d\mathbf{r}_1 d\mathbf{r}_2 V(\mathbf{r}_1 - \mathbf{r}_2)\phi_\alpha(\mathbf{r}_1)\phi_\beta(\mathbf{r}_2)\phi^{*}{}_{\gamma}(\mathbf{r}_2)\phi^{*}{}_{\delta}(\mathbf{r}_1)$$

Note: only orbital indices (no spin-orbit)

$$H_{\text{int}} = \underset{\Uparrow}{H}{}^{(0)}{}_{\text{int}} + \underset{\Uparrow}{H}{}^{(1/g)}{}_{\text{int}}$$

universal **non-universal, fluctuating**

CHARGING HAMILTONIAN

$$H = H_{sp} + H_{\text{int}}$$

$$H_{\text{int}} = H^{(0)}{}_{\text{int}} + H^{(1/g)}{}_{\text{int}}$$

$$H^{(0)}{}_{\text{int}} \Rightarrow E_c(\hat{n} - N_0)^2$$

$$\rightarrow E_c(\sum \Psi^{\dagger}_{\alpha}\Psi_{\alpha}) \cdot (\sum \Psi^{\dagger}_{\beta}\Psi_{\beta}) - 2E_c N_0 + E_c N_0{}^2$$

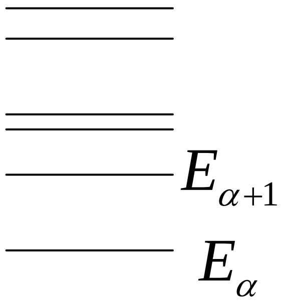
interaction **external gate V** **constant**

Metallic Quantum Dot: Universal Hamiltonian

Metallic grain or small island of electron gas

Quantum Dot

Electron-electron interactions in isolated metallic grains



Mean-level spacing $\Delta = \langle E_{\alpha+1} - E_{\alpha} \rangle$ (kinetic energy)

Thouless energy $E_T \sim D \cdot L^{-2}$ diffusive regime

$E_T \sim v_F L^{-1}$ ballistic regime

$$g = E_T / \Delta \gg 1$$ **metallic grain**

GUE

$$H_0 = \sum_{\alpha} E_{\alpha} n_{\alpha}$$

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{\hat{S}})^2 - \cancel{\lambda_{BCS} \hat{T}^{+} \hat{T}}$$

$$\hat{n} = \sum_{\alpha,\sigma} d^{+}_{\alpha\sigma} d_{\alpha\sigma} \qquad S^{\gamma} = \frac{1}{2} \sum_{\alpha,\sigma,\sigma'} d^{+}_{\alpha\sigma} \sigma^{\gamma}_{\sigma\sigma'} d_{\alpha\sigma'} \qquad T = \sum_{\alpha} d_{\alpha\uparrow} d_{\alpha\downarrow}$$

charge spin superconducting

$$E_c = \frac{e^2}{2C}$$

Coulomb blockade

Scaling:

Short-range interaction $E_c = 4|J| \sim \Delta$

Coulomb interaction $E_c = r_s (k_F L)^{d-1} \Delta \gg |J|$

Kurland, Aleiner, Altshuler (2000)
Aleiner, Brouwer, Glazman (2002)

TRANSPORT THROUGH a QD: thermally activated conduction

$$H = H_{sp} + E_c(\hat{n} - N_0)^2 + H_{leads} + H_{tunneling}$$

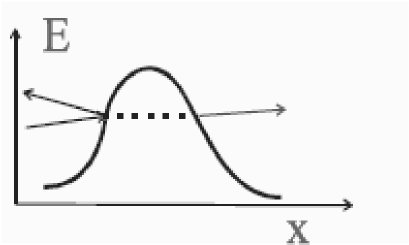
$$H_{leads} = \sum \xi_k c_k^\dagger c_k \quad \text{for each lead}$$

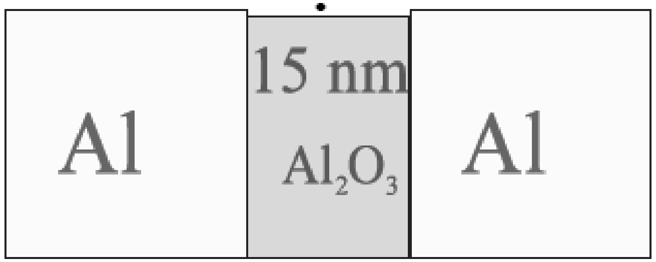
$$H_{tunneling} = \sum_{\alpha,k,n,s} t_{\alpha n} c_{\alpha ks}^\dagger d_{ks} + h.c.$$

lead *lead energy* *dot level* *spin*

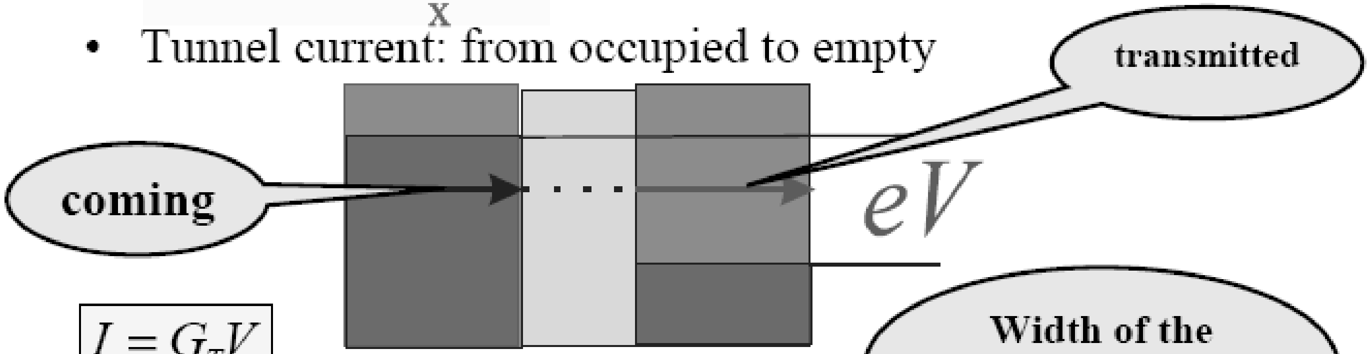
Tunneling in metals (No CB)

- Tunnel barrier





- Tunnel current: from occupied to empty



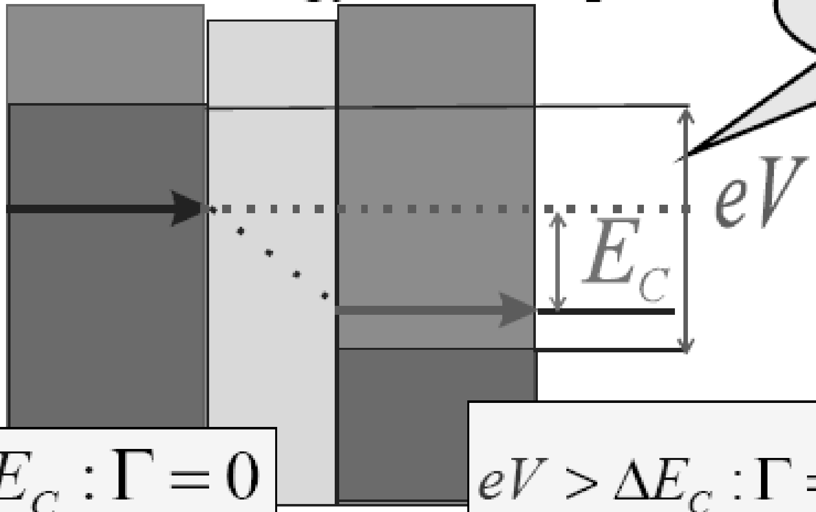
$$\boxed{I = G_T V}$$

- Rate: $\boxed{\Gamma = \frac{I}{e} = \frac{G_T}{e^2} eV}$

Width of the available energy strip

Tunneling and Coulomb blockade

- Now the same with Coulomb blockade
- Electrostatic energy must be paid



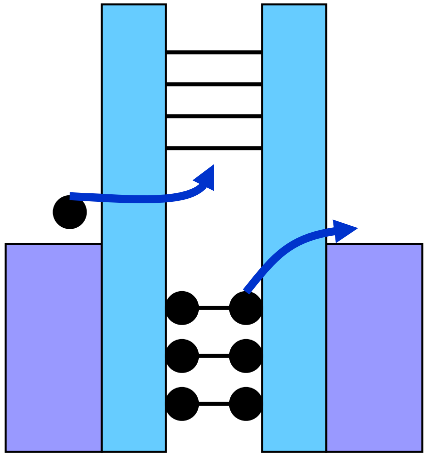
Width is reduced!

$$eV < \Delta E_C : \Gamma = 0$$

$$eV > \Delta E_C : \Gamma = \frac{G_T}{e^2}(eV - \Delta E_C)$$

- blockade

Inelastic cotunneling



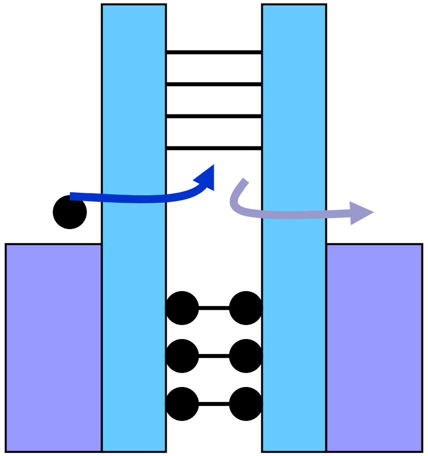
inelastic cotunneling

(excitations left behind)

state k on Left →

state k' on Right + Dot (n filled; m empty)

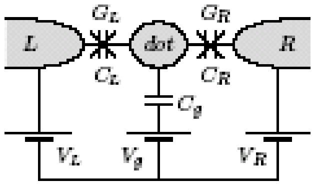
Elastic cotunneling

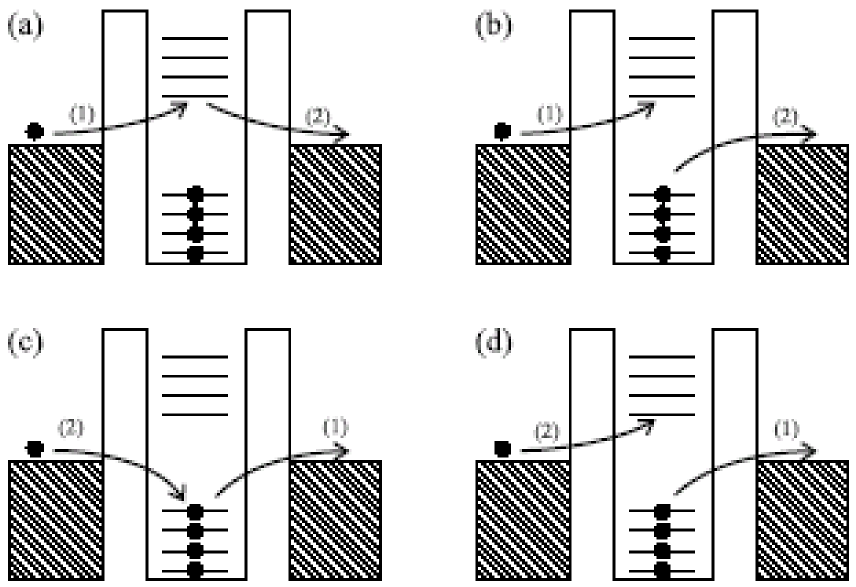


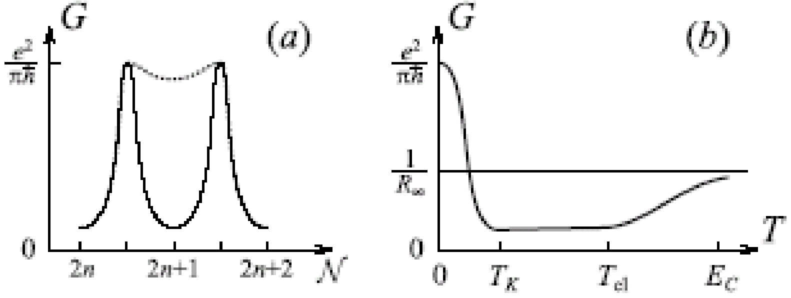
elastic   cotunneling

(no excitations left behind)

Tunneling and co-tunneling (summary)



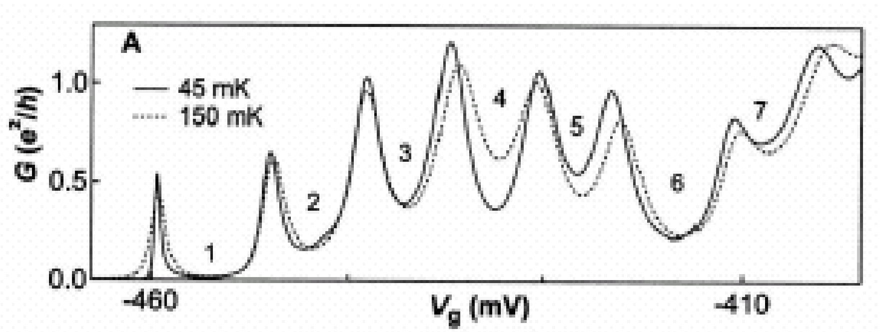




Electron-like

co-tunneling

Hole-like



Elastic

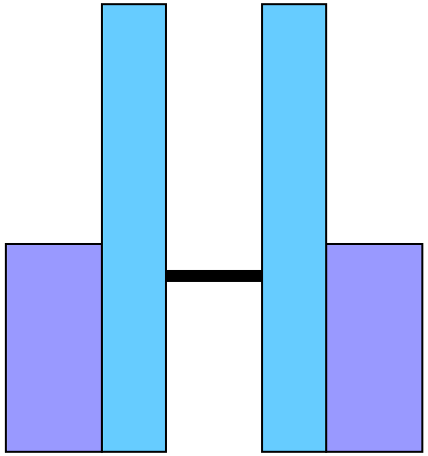
Inelastic

DISCRETE SPECTRUM QD:
coherent vs. Incoherent transport

$$H = H_L + H_R + H_{dot} + H_{tun}$$

$$H_{dot} = \varepsilon \sum_{\sigma} n_{\sigma} + E_c n_{\uparrow} n_{\downarrow}$$

$$H_{tun} = t_{\alpha} c^{+}_{\alpha k \sigma} d_{\sigma} + h.c.$$



a single orbital level
spin ↑ or ↓

This is a story for next lecture!