Proc. Natl. Conf. Theor. Phys. **37** (2012), pp. 59-65

ISOSPIN DEPENDENCE OF THE PRESSURE OF ASYMMETRIC NUCLEAR MATTER

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Abstract. *Isospin dependence of the pressure of asymmetric nuclear matter in the extended Nambu-Jona-Lasinio (ENJL) model is studied by means of the effective potential in the one-loop approximation. The equations of state (EOS) starting from the effective potential is investigated. Our numerical results show that the critical temperature for the phase liquid-gas transition decreases with the increasing neutron excess.*

I. INTRODUCTION

The well-known success of nuclear physics in satisfactorily explaining low energy nuclear phenomena leads to the strong belief that nucleons and mesons are appropriate degrees of freedom. The relativistic treatment of nuclear many-body systems introduced not long ago by Walecka [1] is the QHD-I model based on nucleons and vector and scalar mesons. At mean-field level QHD-I describes nuclear binding and stability of nuclear saturation at normal density ρ_0 as the result of the interplay between a large scalar and a large vector meson condensate. Later on the QHD-II model [2], which incorporated charge vector meson ρ and pseudoscalar meson π into consideration, was developed by Serot and applied to finite nuclei [3]. Along with the success of the Walecka's model, a four-nucleon model of nuclear matter [4], [5] is presented by us which consists of only nucleon degrees of freedom. Through the direct interaction between nucleons forming bound-states as mesons, three types of their condensates, $\langle\bar{\psi}\psi\rangle$, $\langle\bar{\psi}\gamma_\mu\psi\rangle$, $\langle\bar{\psi}\gamma_\mu\vec{\tau}\psi\rangle$ are formed in the nuclear medium. They give the main contribution to the nuclear saturation, liquid-gas phase transition.. In this article we will consider the isospin dependence of the pressure of asymmetric nuclear matter in the extended Nambu-Jona-Lasinio (ENJL) model. The main goal of such studies is to probe the properties of nuclear matter in the region between symmetric nuclear matter and pure neutron matter. This information is important in understanding the explosion mechanism of supernova and the cooling rate of neutron stars.

This paper is organized as follows. In Section II we present the equations of state (EOS) of nuclear matter. Section III deals with numerical calculations. The conclusion and discussion are given in Section IV.

II. THE EQUATIONS OF STATE OF NUCLEAR MATTER

At first let us write down the lagrangian density of the model

$$\pounds \;=\; \bar{\psi}(i\hat{\partial} - M)\psi + \frac{G_s}{2}(\bar{\psi}\psi)^2 - \frac{G_v}{2}(\bar{\psi}\gamma^\mu\psi)^2 - \frac{G_r}{2}(\bar{\psi}\vec{\tau}\gamma^\mu\psi)^2 + \bar{\psi}\gamma^0\mu\psi, \tag{1}$$

where $\boldsymbol{\psi}$ is nucleon field operator, M is the "bare" mass of the nucleon, $\mu = diag(\mu_p, \mu_n)$; $\mu_{p,n} = \mu_B \pm \mu_I/2$ is chemical potential, $\vec{\tau} = \vec{\sigma}/2$ with $\vec{\sigma}$ are the isospin Pauli matrices, γ_μ are Dirac matrices, and $G_{s,v,r}$ are coupling constants.

Bosonizing

$$\check{\sigma} = \bar{\psi}\psi, \quad \check{\omega}_\mu = \bar{\psi}\gamma_\mu\psi, \quad \vec{\check{b}}_\mu = \bar{\psi}\vec{\tau}\gamma_\mu\psi,$$

leads to

$$\begin{aligned}\pounds \;=\;& \bar{\psi}(i\hat{\partial} - M + \gamma^0\mu)\psi + G_s\bar{\psi}\check{\sigma}\psi - G_v\bar{\psi}\gamma^\mu\check{\omega}_\mu\psi - G_r\bar{\psi}\gamma^\mu\vec{\tau}.\vec{\check{b}}_\mu\psi \\ -\;& \frac{G_s}{2}\check{\sigma}^2 + \frac{G_v}{2}\check{\omega}^\mu\check{\omega}_\mu + \frac{G_r}{2}\vec{\check{b}}^\mu\vec{\check{b}}_\mu.\end{aligned} \tag{2}$$

In the mean-field approximation

$$\langle\check{\sigma}\rangle = \sigma, \quad \langle\check{\omega}_\mu\rangle = \omega\delta_{0\mu}, \quad \langle\check{b}_{a\mu}\rangle = b\delta_{3a}\delta_{0\mu}. \tag{3}$$

Inserting (3) into (2) we arrive at

$$\pounds_{MFT} = \bar{\psi}\{i\hat{\partial} - M^* + \gamma^0\mu^*\}\psi - U(\sigma,\omega,b), \tag{4}$$

where

$$M^* = M - G_s\sigma, \tag{5}$$

$$\mu^* = \mu - G_v\omega - G_r\tau^3 b, \tag{6}$$

$$U(\sigma,\omega,b) = \frac{1}{2}[G_s\sigma^2 - G_v\omega^2 - G_r b^2]. \tag{7}$$

The solution M^* of Eq. (5) is the effective mass of the nucleon.
Starting from (4) we get the inverse propagator

$$S^{-1}(k;\sigma,\omega,b) = \begin{pmatrix} (k_0+\mu_p^*)-M^* & -\vec{\sigma}.\vec{k} & 0 & 0 \\ \vec{\sigma}.\vec{k} & -(k_0+\mu_p^*)-M^* & 0 & 0 \\ 0 & 0 & (k_0+\mu_n^*)-M^* & -\vec{\sigma}.\vec{k} \\ 0 & 0 & \vec{\sigma}.\vec{k} & -(k_0+\mu_n^*)-M^* \end{pmatrix}. \tag{8}$$

Thus

$$\det S^{-1}(k;\sigma,\omega,b) = (k_0 + E_+^+)(k_0 - E_-^-)(k_0 + E_+^-)(k_0 - E_-^+), \tag{9}$$

in which

$$E_\mp^\pm = E_k^\pm \mp \left(\mu_B - G_\omega\omega\right), \quad E_k^\pm = E_k \pm (\frac{\mu_I}{2} - \frac{G_\rho}{2}b), \quad E_k = \sqrt{\vec{k}^2 + M^{*2}}. \tag{10}$$

Based on (7) and (8) the effective potential is derived:

$$\Omega = U(\sigma,\omega,b) + \mathrm{iTr}\ln S^{-1} = \frac{1}{2}[G_s\sigma^2 - G_v\omega^2 - G_r b^2] -$$
$$-\frac{T}{\pi^2}\int_0^\infty k^2dk\Big[\ln(1+e^{-E_-^-/T})+\ln(1+e^{-E_+^-/T})+\ln(1+e^{-E_-^+/T})+\ln(1+e^{-E_+^+/T})\Big].\quad(11)$$

The ground state of nuclear matter is determined by the minimum condition

$$\frac{\partial\Omega}{\partial\sigma} = 0, \qquad \frac{\partial\Omega}{\partial\omega} = 0, \qquad \frac{\partial\Omega}{\partial b} = 0. \quad (12)$$

Inserting (11) into (12) we obtain the gap equations

$$\begin{aligned}
\sigma &= \frac{1}{\pi^2}\int_0^\infty k^2dk\frac{M^*}{E_k}\big\{(n_p^- + n_p^+) + (n_n^- + n_n^+)\big\} \equiv \rho_s \\
\omega &= \frac{1}{\pi^2}\int_0^\infty k^2dk\big\{(n_p^- - n_p^+) + (n_n^- - n_n^+)\big\} \equiv \rho_B \\
b &= \frac{1}{2\pi^2}\int_0^\infty k^2dk\big\{(n_p^- - n_p^+) - (n_n^- - n_n^+)\big\} \equiv \rho_I,
\end{aligned} \quad (13)$$

where

$$n_p^- = n_-^-; \quad n_p^+ = n_+^+; \quad n_n^+ = n_+^-; \quad n_n^- = n_-^+; \quad n_\mp^\pm = \big[e^{E_\mp^\pm/T} + 1\big]^{-1},$$

The pressure P is defined as

$$\mathrm{P} = -\Omega|_{\text{taken at minimum}}. \quad (14)$$

Combining Eqs. (11), (13) and (14) together produces the following expression for the pressure

$$\begin{aligned}
\mathrm{P} &= -\frac{G_s}{2}\rho_s^2 + \frac{G_v}{2}\rho_B^2 + \frac{G_r}{2}\rho_I^2 + \frac{T}{\pi^2}\int_0^\infty k^2dk\Big[\ln(1+e^{-E_-^-/T}) \\
&+ \ln(1+e^{-E_+^-/T}) + \ln(1+e^{-E_-^+/T}) + \ln(1+e^{-E_+^+/T})\Big].
\end{aligned} \quad (15)$$

The energy density is obtained by the Legendre transform of P:

$$\begin{aligned}
\varepsilon &= \Omega(\sigma,\omega,b) + T\varsigma + \mu_B\rho_B + \mu_I\rho_I \\
&= \frac{G_s}{2}\rho_s^2 + \frac{G_v}{2}\rho_B^2 + \frac{G_r}{2}\rho_I^2 + \frac{1}{\pi^2}\int_0^\infty k^2dkE_k(n_p^- + n_p^+ + n_n^- + n_n^+).
\end{aligned} \quad (16)$$

with the entropy density defined by

$$\begin{aligned}
\varsigma = \frac{\partial\Omega}{\partial T} = -\frac{1}{\pi^2}\int_0^\infty k^2dk\Big[&n_p^-\ln n_p^- + (1-n_p^-)\ln(1-n_p^-) + n_n^-\ln n_n^- + (1-n_n^-)\ln(1-n_n^-) \\
+ &n_p^+\ln n_p^+ + (1-n_p^+)\ln(1-n_p^+) + n_n^+\ln n_n^+ + (1-n_n^+)\ln(1-n_n^+)\Big].
\end{aligned} \quad (17)$$

Let us introduce the isospin asymmetry α:

$$\alpha = (\rho_n - \rho_p)/\rho_B, \quad (18)$$

in which $\rho_B = \rho_n + \rho_p$ is the baryon density, and ρ_n, ρ_p are the neutron, proton densities, respectively.
Taking into account (5), (13), and (18) together the Eqs. (15), (16) can be rewritten as

$$P = -\frac{(M-M^*)^2}{2G_s} + [\frac{G_v}{2} + \frac{G_r\alpha^2}{8}]\rho_B^2 + \frac{T}{\pi^2}\int_0^\infty k^2dk\big[\ln(1+e^{-\frac{E_k-\mu_p^*}{T}}) + \ln(1+e^{-\frac{E_k+\mu_p^*}{T}}) + \ln(1+e^{-\frac{E_k+\mu_n^*}{T}}) + \ln(1+e^{-\frac{E_k-\mu_n^*}{T}})\big]. \quad (19)$$

$$\varepsilon = \frac{(M-M^*)^2}{2G_s} + [\frac{G_v}{2} + \frac{G_r\alpha^2}{8}]\rho_B^2 + \frac{1}{\pi^2}\int_0^\infty k^2dkE_k(n_p^- + n_p^+ + n_n^- + n_n^+). \quad (20)$$

Eqs. (19) and (20) constitute the equations of state (EOS) governing all thermodynamical processes of nuclear matter.

III. NUMERICAL STUDY.

In order to understand the role of isospin degree of freedom in nuclear matter, let us carry out the numerical study. First we follow the method developed by Walecka [6] to determine the three parameters Gs, Gv, and Gr for symmetric nuclear matter based on the saturation condition: The saturation mechanism requires that at normal density $\rho_B = \rho_0 = 0.17 fm^{-3}$ the binding energy $\varepsilon_{bin} = -M + \varepsilon/\rho_B$ attains its minimum value $(\varepsilon_{bin})_0 \simeq -15,8 MeV$, in which ε is given by (20). It is found that $G_s^2 = 13.62 fm^2$ and $G_v = 0.75G_s$. As to fixing Gr let us employ the expansion of nuclear symmetry energy (NSE) around ρ_0

$$E_{sym} = a_4 + \frac{L}{3}\left(\frac{\rho_B-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho_B-\rho_0}{\rho_0}\right)^2, \quad (21)$$

with a_4 being the bulk symmetry parameter of the Weiszaecker mass formula, experimentally we know $a_4 = 30 - 35 MeV$; L and K_{sym} related respectively to slope and curvature of NSE at ρ_0

$$L = 3\rho_0\left(\frac{\partial E_{sym}}{\partial \rho_B}\right)_{\rho_B=\rho_0},$$

$$K_{sym} = 9\rho_0\left(\frac{\partial^2 E_{sym}}{\partial \rho_B^2}\right)_{\rho_B=\rho_0}.$$

Then G_r is fitted to give $a_4 \simeq 32 MeV$. Its value is $G_r = 0.198G_s$.

Thus, all of the model parameters are fixed, which are in good agreement with those widely expected in the literature [6]. Now we are ready to carry out the numerical computation of the isospin dependence of the pressure of asymmetric nuclear matter.

In Figs.1-2 we obtain a set of isotherms at fixed temperatures. These bear the typical structure of the van der Waals equations of state [7], [8]. As is seen from these figures the critical temperature for the liquid-gas phase transition decreases with increasing neutron excess. This can be understood qualitatively in terms of the increasing contribution

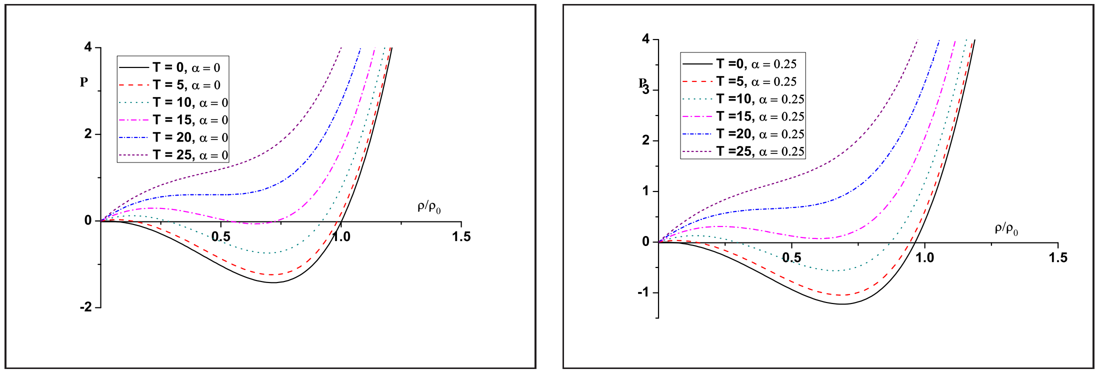

Fig. 1. The pressure for several temperature steps at $\alpha = 0$ and $\alpha = 0.25$.

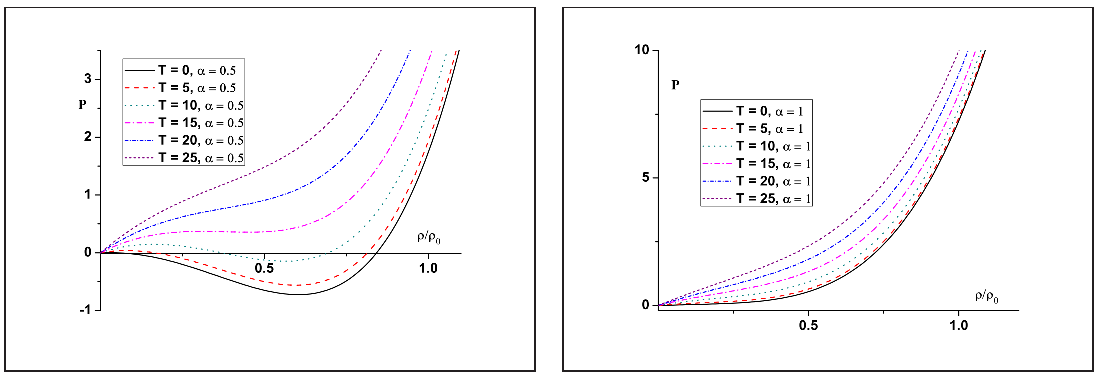

Fig. 2. The pressure for several temperature steps at $\alpha = 0.5$ and $\alpha = 1$.

from the asymmetric pressure. Thus, the liquid-gas phase transition in asymmetric nuclear matter is not only more complex than in symmetric matter but also has new distinct features. This is because they are strongly influenced by the isospin degree of freedom.

In Figs.3-4 we plot the density dependence of $E_{bin}(\rho_B;\alpha)$ at several values of isospin asymmetry α. From these Figures we deduce that for comparison with the results of the chiral approach of nuclear matter [9] the asymmetric nuclear matter in our model is less stiff and the isospin dependence of saturation density is strong enough.

IV. Conclusion and Outlook

In this article we considered isospin dependence of the pressure of asymmetric nuclear matter described by the NJL-type model. Based on the effective potential in the one-loop approximation we reproduced the expression for the pressure determined by the effective potential at minimum. As a consequence, the free energy was obtained straightforwardly. They constitute the equations of state (EOS) of nuclear matter. It was indicated that in asymmetric nuclear matter, the critical temperature for the liquid-gas phase transition decreases with increasing neutron excess and the isospin dependence of saturation

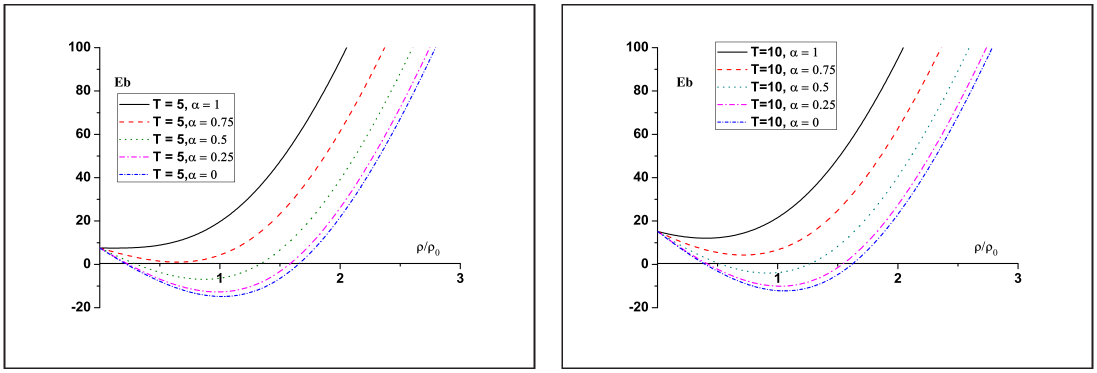

Fig. 3. The density dependence of binding energy at several values of isospin asymmetry and temperature $T = 5MeV$ and $T = 10MeV$.

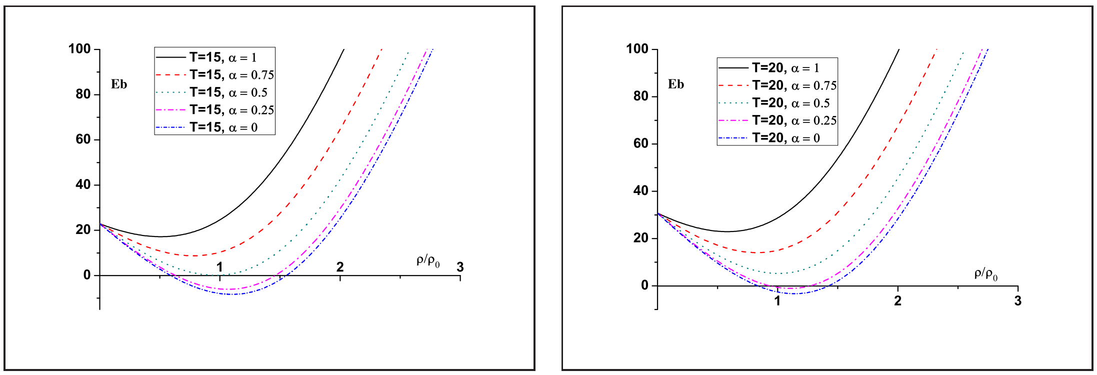

Fig. 4. The density dependence of binding energy at several values of isospin asymmetry and temperature $T = 15MeV$ and $T = 20MeV$.

density is strong enough. This is our major success. In order to understand better the phase structure of asymmetric nuclear matter further study would be carried out by means of numerical computation. This is the objective of our further study.

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Received .