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ON THE HALL EFFECT IN PARABOLIC QUANTUM WELLS WITH AN IN-PLANE MAGNETIC FIELD IN THE PRESENCE OF A STRONG ELECTROMAGNETIC WAVE (LASER RADIATION)

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Abstract. *The Hall effect in a quantum well (QW) with a parabolic potential* $V(z) = m\omega_z^2 z^2/2$ *(where* m *and* ω_z *are the effective mass of electron and the confinement frequency of QW, respectively), subjected to a crossed dc electric field* $\vec{E}_1 = (0, 0, E_1)$ *and magnetic field* $\vec{B} = (0, B, 0)$ *(*$\vec{B}$ *is in-plane of the plane of free motion of electrons), in the presence of a strong electromagnetic wave (EMW) characterized by electric field* $\vec{E} = (E_0 \sin(\Omega t), 0, 0)$ *(where* E_0 *and* Ω *are the amplitude and the frequency of EMW, respectively), is studied theoretically utilizing quantum kinetic equation for electrons. By considering the electron - acoustic phonon interaction, we obtain analytic expressions for the components* σ_{zz} *and* σ_{xz} *of the Hall conductivity as well as the Hall coefficient (HC) with a dependence on* B, E_1, E_0, Ω, *temperature* T *of the system and the characteristic parameters of QW. These expressions are fairly different in comparison to those obtained for bulk semiconductors. The results are numerically evaluated and graphed for a specific quantum well, GaAs/AlGaAs, to show clearly the dependence of the Hall conductivity and the HC on above parameters. The dependence of the HC on the magnetic field shows the resonant peaks satisfying condition* $\hbar\sqrt{\omega_c^2 + \omega_z^2} = \hbar\Omega$ *where* ω_c *is the cyclotron frequency. The HC is nonlinear dependent on the amplitude of EMW. Furthermore, the HC is always positive whereas it has both negative and positive values in the case of electron - optical phonon interaction.*
Keywords*: Hall effect, quantum kinetic equation, parabolic quantum wells, electron - phonon interaction.*

I. INTRODUCTION

It is well-known that the confinement of electrons in low-dimensional systems considerably enhances the electron mobility and leads to their unusual behaviors under external stimuli. As the result, the properties of low dimensional systems, especially electrical and optical properties are very different in comparison with normal semiconductors [1, 2]. This brings a vast possibility in application to design optoelectronics devices. In the past few years, there have been many papers dealing with problems related to the incidence of electromagnetic wave (EMW) in low-dimensional semiconductor systems. The linear absorption of a weak EMW caused by confined electrons in low dimensional systems has been investigated by using Kubo - Mori method [3, 4]. Calculations of the nonlinear absorption coefficients of a strong EMW (laser radiation) by using the quantum kinetic equation for electrons in bulk semiconductors [5, 6], in compositional semiconductor superlattices [7] and in quantum wires [8] have also been reported. Also, the Hall effect in bulk semiconductors in the presence of EMW has been studied in much details by using quantum kinetic equation method [9-13]. In [9, 10] the odd magnetoresistance was calculated when the nonlinear semiconductors are subjected to a magnetic field and an EMW with low frequency, the nonlinearity is resulted from the nonparabolicity of distribution functions of carriers. In [11, 12], the magnetoresistance was derived in the presence of a strong EMW (laser field) for two cases: the magnetic field and the electric field of the EMW are perpendicular [11], and are parallel [12]. The existence of the odd magnetoresistance was explained by the effect of the strong EMW on the probability of collision, i.e., the collision integral depends on the amplitude and frequency of the EMW. This problem is also studied in the presence of both low frequency and high frequency EMW [13]. Moreover, the dependence of magnetoresistance as well as magnetoconductivity on the relative angle of applied fields has been considered carefully [9-13]. The behaviors of this effect are much more interesting in low-dimensional systems, especially the two-dimensional electron gas (2DEG) system.

One of the 2DEG models is the parabolic quantum well (PQW) which has attracted many interests in recent years. One of the most interesting problems in 2DEG is the Hall effect. However, most of previous works only considered the case when the EMW was absent, the magnetic field was perpendicular to the plane of free motion of electrons and low temperatures. To our knowledge, the Hall effect in PQW in the presence of a strong EMW (laser radiation) remains as problem to study. So, in a recent work [14] we have studied the Hall effect in a PQW subjected to a crossed dc electric field (EF) $\vec{E}_1 = (0, 0, E_1)$ and magnetic field $\vec{B} = (0, B, 0)$ in the presence of a strong electromagnetic wave (laser radiation) characterized by electric field $\vec{E} = (E_0 \sin \Omega t, 0, 0)$, the confinement potential is assumed to be $V(z) = m\omega_z^2 z^2/2$, the magnetic field is oriented in the plane of free motion of electrons (in-plane magnetic field), the electron - optical phonon interaction has been taken into account and the influence of a strong EMW has been considered in details. To make a comparison of the effect of different scattering mechanisms, in this work, we study this model for the case of electron - acoustic phonon interaction. This type of interaction is dominant at low temperatures and electron gas is then degenerate. We derive analytical expressions for the electrical conductivity tensor and the Hall coefficient (HC) taking account of arbitrary transitions between the Landau levels. The analytical result is numerically evaluated and graphed for a specific quantum well, GaAs/AlGaAs, to show clearly the dependence of the Hall coefficient on above parameters. The present paper is organized as follows. In the next section, we describe the simple model of a parabolic quantum well and present briefly the basic formulas for the calculation. Numerical results and discussion are given in Sec. **III**. Finally, remarks and conclusions are shown briefly in Sec. **IV**.

II. HALL EFFECT IN PARABOLIC QUANTUM WELLS IN THE PRESENCE OF A LASER RADIATION

II.1. Electronic structure in a parabolic quantum well

Consider a perfect PQW structure subjected to a crossed electric field $\vec{E}_1 = (0, 0, E_1)$ and magnetic field $\vec{B} = (0, B, 0)$ and choose a vector potential $\vec{A} = (zB, 0, 0)$ to describe the applied dc magnetic field. If the confinement potential is assumed to take the form $V(z) = m\omega_z^2 z^2/2$, then the single-particle wave function and its eigenenergy are given by

[15]:

$$\Psi(\vec{r}) = \frac{1}{2\pi} e^{i\vec{k}_\perp \vec{r}} \phi_N(z - z_0), \quad (1)$$

$$\varepsilon_N(k_x) = \hbar\omega_p\left(N + \frac{1}{2}\right) + \frac{1}{2m}\left[\hbar^2 k_x^2 - \left(\frac{\hbar k_x \omega_c + eE_1}{\omega_p}\right)^2\right],$$

$$N = 0, 1, 2, \ldots, \quad (2)$$

where m and e are the effective mass and the charge of a conduction electron, respectively, $\vec{k}_\perp = (k_x, k_y)$ is its wave vector in the (x, y) plan; $z_0 = (\hbar k_x \omega_c + eE_1)/m\omega_p^2$; $\omega_p^2 = \omega_z^2 + \omega_c^2$, ω_z and $\omega_c = eB/m$ are the confinement and the cyclotron frequencies, respectively, and

$$\phi_N(z - z_0) = H_N(z - z_0)\exp\left[-(z - z_0)^2/2\right], \quad (3)$$

with $H_N(z)$ being the Hermite polynomial of N^{th} order. In the following, we will use Eqs. (1)-(3) to derive the expression for the Hall conductivity as well as the Hall coefficient utilizing the quantum kinetic equation method in the presence of a strong EMW.

II.2. Expressions for the Hall conductivity and the Hall coefficient

In the presence of a strong EMW with electric field vector $\vec{E} = (E_0 \sin\Omega t, 0, 0)$, the Hamiltonian of the electron-acoustic phonon system in the above-mentioned PQW in the second quantization representation can be written as:

$$H = H_0 + U, \quad (4)$$

$$H_0 = \sum_{N,\vec{k}_x} \varepsilon_N\left(\vec{k}_x - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{N,\vec{k}_x} a_{N,\vec{k}_x} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} \quad (5)$$

$$U = \sum_{N,N'} \sum_{\vec{q},\vec{k}_x} D_{N,N'}(\vec{q}) a^+_{N',\vec{k}_x+\vec{q}_x} a_{N,\vec{k}_x}(b_{\vec{q}} + b^+_{-\vec{q}}) \quad (6)$$

where, $\vec{k}_x = (k_x, 0, 0)$, $|N, \vec{k}_x>$ and $|N', \vec{k}_x + \vec{q}_\perp>$ are electron states before and after scattering; $\hbar\omega_{\vec{q}}$ is the energy of an acoustic phonon with the wave vector $\vec{q} = (\vec{q}_\perp, q_z)$; $a^+_{N,\vec{k}_x}$ and $a_{N,\vec{k}_x}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (phonon), respectively; $\vec{A}(t)$ is the vector potential of laser field; $D_{N,N'}(\vec{q}) = C_{\vec{q}} I_{N,N'}(q_z)$, where $C_{\vec{q}}$ is the electron - acoustic phonon interaction constant, and $I_{N,N'}(q_z) =< N|e^{iq_z z}|N'>$ is the form factor of electron.

By using Hamiltonian (4) and the procedures as in the works [9-14], we obtain the quantum kinetic equation for electrons in the single (constant) scattering time approximation. Then utilizing the similar way as in Ref. [14] and performing the analytical calculation for the total current density we have the expression for the conductivity tensor σ_{im}. After some manipulation, we find out:

$$\sigma_{im} = \frac{e\tau}{m} \frac{b\tau}{(1 + \omega_c^2\tau^2)^2}\left[\delta_{ij} - \omega_c\tau\epsilon_{ijk}h_k + \omega_c^2\tau^2 h_i h_j\right]\delta_{jl} \times \left[\delta_{lm} - \omega_c\tau\epsilon_{lmp}h_p + \omega_c^2\tau^2 h_l h_m\right], \quad (7)$$

where τ is the momentum relaxation time; δ_{ij} is the Kronecker delta; ϵ_{ijk} being the anti-symmetric Levi - Civita tensor; the Latin symbols i, j, k, l, m, p stand for the components x, y, z of the Cartesian coordinates,

$$b = \frac{4\pi e}{m\hbar} \sum_{N,N'} \{b_1 + b_2 + b_3 + b_4\}, \tag{8}$$

$$\begin{aligned} b_1 &= \frac{AL_x I(N,N')}{2\sqrt{2}\pi^3\sqrt{\Delta^k \Delta_1^q}} \Big\{ k_0^+ \Big[\left(q_{1(+)}^+\right)^3 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(+)}^+\right) - 1\right]^{-1}, \\ &+ \left(q_{1(+)}^-\right)^3 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(+)}^-\right) - 1\right]^{-1} \Big] + k_0^- \Big[\left(q_{1(-)}^+\right)^3 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(-)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{1(-)}^-\right)^3 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(-)}^-\right) - 1\right]^{-1} \Big] \Big\}, \end{aligned} \tag{9}$$

$$\begin{aligned} b_2 &= -\frac{\theta AL_x I(N,N')}{4\sqrt{2}\pi^3\sqrt{\Delta^k \Delta_1^q}} \Big\{ k_0^+ \Big[\left(q_{1(+)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(+)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{1(+)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(+)}^-\right) - 1\right]^{-1} \Big] + k_0^- \Big[\left(q_{1(-)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(-)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{1(-)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{1(-)}^-\right) - 1\right]^{-1} \Big] \Big\}, \end{aligned} \tag{10}$$

$$\begin{aligned} b_3 &= \frac{\theta AL_x I(N,N')}{8\sqrt{2}\pi^3\sqrt{\Delta^k \Delta_2^q}} \Big\{ k_0^+ \Big[\left(q_{2(+)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{2(+)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{2(+)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{2(+)}^-\right) - 1\right]^{-1} \Big] + k_0^- \Big[\left(q_{2(-)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{2(-)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{2(-)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{2(-)}^-\right) - 1\right]^{-1} \Big] \Big\}, \end{aligned} \tag{11}$$

$$\begin{aligned} b_4 &= \frac{\theta AL_x I(N,N')}{8\sqrt{2}\pi^3\sqrt{\Delta^k \Delta_3^q}} \Big\{ k_0^+ \Big[\left(q_{3(+)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{3(+)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{3(+)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{3(+)}^-\right) - 1\right]^{-1} \Big] + k_0^- \Big[\left(q_{3(-)}^+\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{3(-)}^+\right) - 1\right]^{-1} \\ &+ \left(q_{3(-)}^-\right)^5 \left[\exp\left(\sqrt{2}\beta\hbar v_s q_{3(-)}^-\right) - 1\right]^{-1} \Big] \Big\}, \end{aligned} \tag{12}$$

where $\beta = 1/(k_B T)$, $\theta = e^2 E_0^2(1 - \omega_c^2/\omega_p^2)/m^2\Omega^4$, $\Delta^k = \gamma^2 - 4\alpha\delta$, $\Delta_1^q = \Delta^k - 4\alpha C_1$, $\Delta_2^q = \Delta^k - 4\alpha C_2$, $\Delta_3^q = \Delta^k - 4\alpha C_3$, $\alpha = (\hbar^2/2m)(1 - \omega_c^2/\omega_p^2)$, $\gamma = eE_1\hbar\omega_c/m\omega_p^2$, $\delta = (N+1/2)\hbar\omega_p - e^2E_1^2/(2m\omega_p^2) - \varepsilon_F$, $C_1 = (N' - N)\hbar\omega_p - \hbar\omega_0$, $C_2 = C_1 + \hbar\Omega$, $C_3 = C_1 - \hbar\Omega$,

$$k_0^\pm = \frac{\gamma \pm \sqrt{\Delta^k}}{2\alpha}, \quad q_{\ell(+)}^\pm = \frac{-\sqrt{\Delta^k} \pm \sqrt{\Delta_\ell^q}}{2\alpha}, \quad q_{\ell(-)}^\pm = \frac{\sqrt{\Delta^k} \pm \sqrt{\Delta_\ell^q}}{2\alpha}, \quad \ell = 1, 2, 3; \tag{13}$$

$A = \xi^2/2\rho v_s$ with v_s, ξ and ρ are the sound velocity, the deformation potential constant and the mass density, respectively; k_{B} is the Boltzmann constant; L_x and ε_{F} are the normalization length in x direction and the Fermi level, respectively; and

$$I(N, N') = \int_{-\infty}^{\infty} |I_{N,N'}(q_z)|^2 dq_z. \tag{14}$$

The HC is given by the formula [17]

$$R_H = \frac{\rho_{xz}}{\mathrm{B}} = -\frac{1}{\mathrm{B}} \frac{\sigma_{xz}}{\sigma_{xz}^2 + \sigma_{zz}^2}, \tag{15}$$

where σ_{xz} and σ_{zz} are given by Eq. (7). Equation (15) shows the dependence of the HC on the external fields, including the EMW. It is obtained for arbitrary values of the indices N and N'. We can see that the analytical result appears very involved. In the next section, we will give a deeper insight into this dependence by carrying out a numerical evaluation with the help of computer programm.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present detailed numerical calculations of the HC in a PQW subjected to uniform crossed magnetic and electric fields in the presence of an EMW. For the numerical evaluation, we consider the model of a PQW of GaAs/AlGaAs with the following parameters [8, 15, 16]: $\varepsilon_{\mathrm{F}} = 50meV$, $\xi = 13.5eV$, $\rho = 5.32g.cm^{-3}$, $v_s = 5378m.s^{-1}$, $m = 0.067 \times m_0$ (m_0 is the mass of free electron) and for the sake of simplicity we choose $\tau = 10^{-12}s$, $L_x = 10^{-9}m$, also only consider the transition $N = 0$, $N' = 1$.

The HC is plotted as function of the magnetic field at different values of the confinement frequency in Fig. 1. It is seen that the HC is positive and varies strongly with increasing the magnetic field. Each curve has one maximum peak and the values of the HC at the maxima are much larger than other values. By using the computational program we easily determine the position of the peak in each curve. All the peaks correspond to the values of magnetic field satisfying the resonant condition $\hbar\omega_p = \hbar\Omega$ or $\hbar\sqrt{\omega_{\mathrm{c}}^2 + \omega_z^2} = \hbar\Omega$. Evidently, when the confinement frequency ω_z increases, the value of ω_{c} (the magnetic field) satisfying this condition decreases. So the peak shifts to the left (the region of small magnetic field) as ω_z increases as we see in the figure. Moreover, when the confinement frequency tends to zero the resonant condition becomes $\hbar\omega_{\mathrm{c}} = \hbar\Omega$. This is actually the usual cyclotron resonance condition has been obtained in bulk semiconductors.

In Fig. 2 and Fig. 3, we show the dependence of the HC on the amplitude of EMW at different values of the confinement frequency and on the temperature at different values of the dc electric field E_1, respectively; the necessary parameters involved in the computation are the same as those in Fig. 1. In Fig. 2 we can see that the dependence of the HC on the amplitude E_0 is nonlinear. The HC parabolically increases with increasing amplitude E_0, also this dependence is stronger at small value of the confinement frequency. In Fig. 3 the HC does not change at low temperatures and increases very weakly when the temperature increases at large region. The most interesting behavior is that the HC has the same value for different values of the dc electric field at a specific value of the temperature ($\sim 6K$ in this figure). This means that there is a specific value of the temperature at which the HC

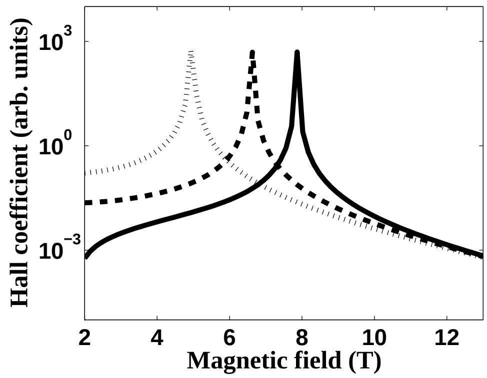
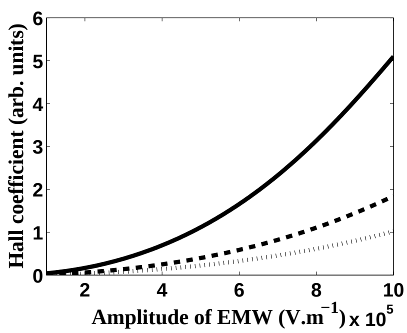


Fig. 1. Hall coefficients (arb. units) as functions of the magnetic field at different values of the confinement frequency: $\omega_z = 3.0\times10^{13}$ s^{-1} (solid line), $\omega_z = 3.2\times10^{13}$ s^{-1} (dashed line), and $\omega_z = 3.4\times10^{13}$ s^{-1} (dotted line). Here, $T = 2$ K, $E = 5\times10^3$ V.m^{-1}, $E_0 = 10^5$ V.m^{-1}, and $\Omega = 5 \times 10^{13}$ s^{-1}.



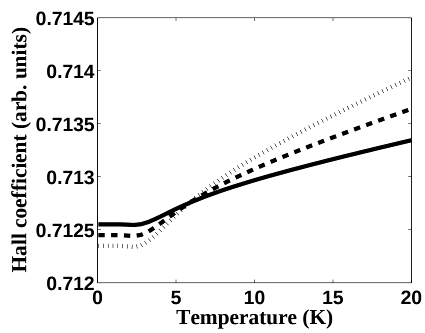


Fig. 2. Hall coefficients (arb. units) as functions of the amplitude of the EMW E_0 at different values of the confinement frequency: $\omega_z = 3.0 \times 10^{13}$ s^{-1} (solid line), $\omega_z = 3.2 \times 10^{13}$ s^{-1} (dashed line), and $\omega_z = 3.4 \times 10^{13}$ s^{-1} (dotted line). Here, $T = 2$ K, $B = 9$ T, $E = 5 \times 10^3$ V.m^{-1}, and $\Omega = 5 \times 10^{13}$ s^{-1}.

Fig. 3. Hall coefficients (arb. units) as functions of the temperature at the dc electric field of 2×10^3 V.m^{-1} (solid line), 3×10^3 V.m^{-1} (dashed line), and 4×10^3 V.m^{-1} (dotted line). Here, $\omega_z = 3.5 \times 10^{13}$ s^{-1}, $B = 9$ T, $E_0 = 10^5$ V.m^{-1}, and $\Omega = 5 \times 10^{13}$ s^{-1}.

does not depend on the dc electric field. Moreover, the HC in this study is always positive whereas it has both negative and positive values in the case of electron - optical phonon interaction [14].

IV. CONCLUSIONS

In this work, we have studied the Hall effect in quantum wells with parabolic potential subjected to a crossed dc electric and magnetic fields in the presence of a strong EMW (laser radiation). The electron - acoustic phonon interaction is taken into account at low temperature and electron gas is degenerate. We obtain the expressions of the Hall conductivity as well as the HC. The influence of EMW is interpreted by the dependence of the Hall conductivity and the HC on the amplitude E_0 and the frequency Ω of the EMW besides the dependence on the magnetic B and the dc electric field E_1 as in the ordinary Hall effect. The analytical results are numerically evaluated and plotted for a specific quantum well GaAs/AlGaAs to show clearly the dependence of HC on the external fields and parameters of system. The dependence of the HC on the magnetic field shows the resonant peaks satisfying condition $\hbar\sqrt{\omega_c^2 + \omega_z^2} = \hbar\Omega$. The HC depends nonlinearly on the amplitude of EMW and weakly depends on the temperature. Furthermore, the HC is always positive whereas it has both negative and positive values in the case of electron - optical phonon interaction.

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