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CALCULATIONS OF THE ACOUSTOELECTRIC CURRENT IN A RECTANGULAR QUANTUM WIRE

NGUYEN VAN NGHIA[1], DINH QUOC VUONG[2]
[1]*Department of Physics, Water Resources University*
NGUYEN QUANG BAU[2]
[2]*Department of Physics, College of Natural Sciences, Hanoi National University*

Abstract. *The acoustoelectric current in a rectangular quantum wire (RQW) with an infinite potential is calculated by using the quantum kinetic equation for the distribution function electrons interacting with internal and external phonons. The analytic expression for the acoustoelectric current in the RQW with an infinite potential is obtained. The dependence of the acoustoelectric current on the temperature of the system T, the acoustic wave number q and the parameters of the RQW with an infinite potential are obtained. The theoretical results are numerically evaluated, plotted and discussed for the specific RQW with an infinite potential GaAs. The results are compared with those for normal bulk semiconductors and quantum well to show the differences.*

I. INTRODUCTION

Recent, there have been more and more interests in studying the behavior of low-dimensional system, such as superlattices, quantum wells, quantum wires and quantum dots. In particular, in quantum wires, the motion of electrons is restricted in two dimensions, so that they can flow freely in one dimension. The confinement of electron in these systems changes the electron mobility remarkably. This results in a number of new phenomena, which concern a reduction of sample dimensions. In particular differ considerably from those in the bulk semiconductor, electron-phonon interaction and scattering rates [1], acoustic-electromagnetic wave interaction [2].

It is well known that the propagation of the acoustic wave in conductors is accompanied by the transfer of the energy and momentum to conduction electrons which may give rise to a current usually called the acoustoelectric current, in case of an open circuit called acoustoelectric effect. This leads to the emergence of a longitudinal acoustoelectric effect, i.e., a stationary electric current running in a sample in the direction opposite to that of the wave. The study of acoustoelectric effect in bulk materials has received a lot of attention [3-5]. Recently, there have been growing interests in investigating this effect in mesoscopic structures [6, 7]. Especially, in recent time the acoustoelectric effect was studied in both a one dimensional channel [8] and in a finite-length ballistic quantum channel [9, 10]. In addition, the acoustoelectric effect was measured by an experiment in a submicron-separated quantum wire [11], in a carbon nanotube [12] and this effect was also studied in the cylindrical quantum wire (CQW) with an infinite potential [13].

However, the acoustoelectric current in the RQW with an infinite potential has not been studied yet. Therefore, the purpose of this work is to examine this current in the RQW with an infinite potential. In this paper, we present a calculation of the acoustoelectric current in a RQW with an infinite potential by using the quantum kinetic equation for the distribution function of electrons interacting with internal and external phonons. We assume the deformation mechanism of electron-acoustic phonon interaction. We have obtained the acoustoelectric current in the RQW with an infinite potential. The dependence of the expression for the acoustoelectric current on acoustic wave numbers, the temperature and the width of the RQW has been shown. Numerical calculations are carried out for $GaAs$ RWQ to clarify our results.

II. THE QUANTUM KINETIC EQUATION FOR ELECTRONS IN THE PRESENCE OF AN ULTRASOUND

Let us suppose that the acoustic wave of frequency ω_q is propagating along the RQW with an infinite potential axis (Oz) and the magnetic field is oriented along the Ox axis. We consider the most realistic case from the point of view of a low-temperature experiment, when $\omega_q/\eta = v_s|q|/\eta \ll 1$ and $ql \gg 1$, where v_s is the velocity of the acoustic wave, q is the acoustic wave number and l is the electron mean free path. The compatibility of these conditions is provided by the smallness of the sound velocity in comparison with the characteristic Fermi velocity of electrons.

When the magnetic field is applied in the x-direction, in case the vector potential is chose $A = -zH$, the eigenfunction of an unperturbed electron is expressed as follows:

$$\psi_{n,N,\vec{p}}(\vec{r}) = \frac{2}{\sqrt{abL}} sin\left(\frac{n\pi^2}{a}x\right) sin\left(\frac{N\pi^2}{b}y\right) \exp\left(i\frac{p_z}{\hbar}z\right), \tag{1}$$

where a and b are, respectively, the cross-sectional dimensions along x- and y-directions, n, N are the subband indexes, L is the length of the wire, and $\vec{p} = (0, 0, p_z)$ is the electron momentum vector along z-direction. The electron energy spectrum takes the form:

$$\varepsilon_{\vec{p}_z}^{n,N} = \frac{\vec{p}_z^2}{2m} + \frac{\pi^2\hbar^2}{2m}\left(\frac{n^2}{a^2} + \frac{N^2}{b^2}\right). \tag{2}$$

If the conditions $\omega_q/\eta = v_s|q|/\eta \ll 1$ and $ql \gg 1$ are satisfied, a macroscopic approach to the description of the acoustoelectric effect is inapplicable and the problem should be treated by using quantum mechanical methods. We also consider the acoustic wave as a packet of coherent phonons. Therefore, first we have to first find the Hamiltonian describing the interaction of the electron-phonon system in the RQW with an infinite potential, which can be written in the secondary quantization representation as follows:

$$\begin{aligned} H = &\sum_{n,N,\vec{p}_z} \varepsilon_{\vec{p}_z}^{n,N} a_{n,N,\vec{p}_z}^{+} a_{n,N,\vec{p}_z} + \sum_{n,N,n',N',\vec{k}} C_{\vec{k}} I_{n,N,n,N'} a_{n',N',\vec{p}_z+\vec{k}}^{+} a_{n',N',\vec{p}_z} (b_{\vec{k}} + b_{-\vec{k}}^{+}) \\ &+ \sum_{\vec{k}} \hbar\omega_{\vec{k}} b_{\vec{k}}^{+} b_{\vec{k}} + \sum_{n,N,n',N',\vec{q}} C_{\vec{q}} U_{n,N,n,N'} a_{n',N',\vec{p}_z+\vec{q}}^{+} a_{n',N',\vec{p}_z} b_{\vec{q}} \exp(-i\omega_q t), \end{aligned} \tag{3}$$

where $C_{\vec{k}} = \frac{|\Lambda|^2 q}{2\rho v_s SL}$ is the electron-internal phonon interaction factor, $C_{\vec{q}} = \frac{|\Lambda|^2 v_l^4 \hbar\omega_{\vec{q}}^3}{2\rho FS}$ is the electron-external phonon interaction factor, with $F = q\left[\frac{1+\sigma_l^2}{2\sigma_t} + \left(\frac{\sigma_l}{\sigma_t} - 2\right)\frac{1+\sigma_t^2}{2\sigma_t}\right]$, $\sigma_l = \sqrt{1-(v_s/v_l)^2}$, $\sigma_t = \sqrt{1-(v_s/v_t)^2}$, v_l (v_t) is the velocitie of the longitudinal (transverse) bulk acoustic wave, ρ is the mass density of the medium, $S = ab$ is the surface area, Λ is the deformation potential constant, $a^+_{n,N,\vec{p}_z}$ ($a_{n,N,\vec{p}_z}$) is the creation (annihilation) operator of the electron, respectively, and $b_{\vec{q}}$ is the annihilation operator of the external phonon. $|n, \vec{k}\rangle$ ($|n', \vec{k} + \vec{q}\rangle$) is electron state before (after) interaction, $U_{n,N,n,N}$ is the matrix element of the operator $U = exp(iqy - k_l z)$, $k_l = \sqrt{q^2 - (\omega_q/v_l)^2}$ is the spatial attenuation factor of the potential part of the displacement field. Using Eq.(1) it is straightforward to evaluate the matrix elements of the operator U. The result is $U_{n,N,n,N} = 4exp(-k_l L)/L$.

The electronic form factor, $I_{n,N,n,N}$, is written as [15] follows:

$$I_{n,N,n',N'} = \frac{32\pi^4(q_x a n n')^2[1-(-1)^{n+n'}cos(q_x a)]}{[(q_x a)^4 - 2\pi^2(q_x a)^2(n^2+n'^2) + \pi^4(n^2-n'^2)^2]^2} \times \\ \times \frac{32\pi^4(q_y b N N')^2[1-(-1)^{N+N'}cos(q_y b)]}{[(q_y b)^4 - 2\pi^2(q_y b)^2(N^2+N'^2) + \pi^4(N^2-N'^2)^2]^2}, \quad (4)$$

here n, n' (N, N') is the position (radial) quantum number, q_x, q_y is wave vector.

In order to establish the quantum kinetic equation for electrons in the presence of an ultrasound, we use equation of motion of statistical average value for electrons $i\hbar\frac{\partial\langle f_{n,N,\vec{p}_z}(t)\rangle_t}{\partial t} = \langle f_{n,N,\vec{p}_z}(t), H\rangle_t$, where $\langle X\rangle_t$ means the usual thermodynamic average of operator X and $f_{n,N,\vec{p}_z}(t) = \langle a^+_{n,N,\vec{p}_z} a_{n,N,\vec{p}_z}\rangle_t$ is the particle number operator or the electron distribution function.

Using Hamiltonian in Eq.(3) and performing operator algebraic calculations, we obtain a quantum kinetic equation for the electron. This can be formulated as follows:

$$\frac{\partial f_{n,N,\vec{p}_z}(t)}{\partial t} = -\frac{1}{\hbar}\sum_{n',N',\vec{q}} |C_q|^2 |I_{n,N,n',N'}|^2 N_q \times \\ \times \int_{-\infty}^{t} dt' \{[f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{q}}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n,N}_{\vec{p}_z+\vec{q}} - \varepsilon^{n',N'}_{\vec{p}_z} - \hbar\omega_q + i\delta\right)(t-t')\right] \\ + [f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{q}}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n',N'}_{\vec{p}_z+\vec{q}} - \varepsilon^{n,N}_{\vec{p}_z} + \hbar\omega_q + i\delta\right)(t-t')\right] \\ - [f_{n',N',\vec{p}_z-\vec{q}} - f_{n,N,\vec{p}_z}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n,N}_{\vec{p}_z} - \varepsilon^{n',N'}_{\vec{p}_z-\vec{q}} - \hbar\omega_q + i\delta\right)(t-t')\right] \\ - [f_{n',N',\vec{p}_z-\vec{q}} - f_{n,N,\vec{p}_z}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n,N}_{\vec{p}_z} - \varepsilon^{n',N'}_{\vec{p}_z-\vec{q}} + \hbar\omega_q + i\delta\right)(t-t')\right]\} \\ - \frac{1}{\hbar^2}\sum_{n',N',\vec{k}} |C_k|^2 |U_{n,N,n',N'}|^2 N_k \times \\ \times \int_{-\infty}^{t} dt' \{[f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{k}}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n',N'}_{\vec{p}_z+\vec{k}} - \varepsilon^{n,N}_{\vec{p}_z} + \hbar\omega_q - \hbar\omega_k + i\delta\right)(t-t')\right] \\ - [f_{n',N',\vec{p}_z-\vec{k}} - f_{n,N,\vec{p}_z}] exp\left[\frac{i}{\hbar}\left(\varepsilon^{n,N}_{\vec{p}_z} - \varepsilon^{n',N'}_{\vec{p}_z-\vec{k}} + \hbar\omega_q - \hbar\omega_k + i\delta\right)(t-t')\right]\}, \quad (5)$$

with N_q being the external phonon number, N_k is the internal phonon number and δ is the Kronecker delta symbol.

Solving the Eq.(5), we find

$$f_{n,N,\vec{p}_z}(t) = \frac{2\pi\tau}{\hbar^2}\sum_{n',N',\vec{q}}|C_q|^2|I_{n,N,n',N'}|^2 N_q\{[f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{q}}]\delta(\varepsilon^{n',N'}_{\vec{p}_z+\vec{q}} - \varepsilon^{n,N}_{\vec{p}_z} - \hbar\omega_q)$$
$$+ [f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{q}}]\delta(\varepsilon^{n',N'}_{\vec{p}_z+\vec{q}} - \varepsilon^{n,N}_{\vec{p}_z} + \hbar\omega_q)\}$$
$$+ \frac{\pi\tau}{\hbar^2}\sum_{n',N',\vec{k}}|C_k|^2|U_{n,N,n',N'}|^2 N_k\{[f_{n,N,\vec{p}_z} - f_{n',N',\vec{p}_z+\vec{k}}]\delta(\varepsilon^{n',N'}_{\vec{p}_z+\vec{k}} - \varepsilon^{n,N}_{\vec{p}_z} + \hbar\omega_q - \hbar\omega_k)$$
$$- [f_{n',N',\vec{p}_z-\vec{k}} - f_{n,N,\vec{p}_z}]\delta(\varepsilon^{n',N'}_{\vec{p}_z-\vec{k}} - \varepsilon^{n,N}_{\vec{p}_z} - \hbar\omega_q + \hbar\omega_k)\}, \tag{6}$$

where τ is momentum relaxation time.

III. ANALYTICAL EXPRESSION FOR THE ACOUSTOELECTRIC CURRENT

The density of the acoustoelectric current is generally given by:

$$j = \frac{e}{\pi\hbar}\sum_{n',N'}\int V_{\vec{p}_z} f_{n,N,\vec{p}_z} dp_z, \tag{7}$$

here $V_{\vec{p}_z}$ is the average drift velocity of the moving charges.

Substituting Eq.(6) into Eq.(7) and we linearize the equation by replacing $f_{n,N}$ by f_F. With $f_F = [1 - exp(\beta(\varepsilon - \varepsilon_F))]^{-1}$ is the Fermi-Dirac distribution function, $\beta = 1/k_B T$, k_B is Boltzmann constant, T is the temperature of the system and ε_F is the Fermi energy. By carrying out manipulations we obtained an expression for the acoustoelectric current as follows

$$j = \frac{e\tau|\Lambda|^2 m^2}{\pi^2\hbar^6\rho v_s\omega_q\beta}e^{\beta\varepsilon_F}\sum_{n,N,n',N'}|I_{n,N,n',N'}|^2 exp\left(-\frac{\beta\pi^2\hbar^2}{2m}\left(\frac{n^2}{a^2}+\frac{N^2}{b^2}\right)\right)\times$$
$$\times\{\xi_1 e^{-\xi_1}\left[\xi_1 K_0(\xi_1) + 3\left(\frac{2\xi_1}{\hbar\beta}\right)^2(K_1(\xi_1)+K_2(\xi_1)) + 8\left(\frac{2\xi_1}{\hbar\beta}\right)^5 K_3(\xi_1)\right]$$
$$+ \xi_2 e^{-\xi_2}\left[\xi_2 K_0(\xi_2) + 3\left(\frac{2\xi_2}{\hbar\beta}\right)^2(K_1(\xi_2)+K_2(\xi_2)) + 8\left(\frac{2\xi_2}{\hbar\beta}\right)^5 K_3(\xi_2)\right]\}$$
$$+ \frac{32e\tau|\Lambda|^2 v_l^2\omega_q^2 W\pi}{\hbar^6\rho F S L^2 v_s}\left(\frac{m}{\beta}\right)^{\frac{3}{2}} e^{\beta\varepsilon_F}\sum_{n,N,n',N'} exp\left(-\frac{\beta\pi^2\hbar^2}{2m}\left(\frac{n^2}{a^2}+\frac{N^2}{b^2}\right) - 2L\sqrt{q^2 - \frac{\omega_q^2}{v_l^2}}\right)\times$$
$$\times\{\chi_1^{\frac{5}{2}}e^{-\chi_1}\left[K_{\frac{5}{2}}(\chi_1) + 3K_{\frac{3}{2}}(\chi_1) + 3K_{\frac{1}{2}}(\chi_1) + K_{-\frac{1}{2}}(\chi_1)\right]$$
$$- \chi_2^{\frac{5}{2}}e^{-\chi_2}\left[K_{\frac{5}{2}}(\chi_2) + 3K_{\frac{3}{2}}(\chi_2) + 3K_{\frac{1}{2}}(\chi_2) + K_{-\frac{1}{2}}(\chi_2)\right]\}, \tag{8}$$

with

$$\xi_1 = \frac{\hbar\beta}{2}\left(\frac{\Delta_{n',N',n,N}}{\hbar} - \omega_q\right); \quad \xi_2 = \frac{\hbar\beta}{2}\left(\frac{\Delta_{n',N',n,N}}{\hbar} + \omega_q\right), \tag{9}$$

$$\chi_1 = \xi_1 + \frac{\hbar\beta\omega_k}{2}; \quad \chi_2 = \xi_2 - \frac{\hbar\beta\omega_k}{2}; \quad \Delta_{n',N',n,N} = \frac{\pi^2\hbar^2}{2ma^2}(n'^2 - n^2) + \frac{\pi^2\hbar^2}{2mb^2}(N'^2 - N^2). \tag{10}$$

The Eq.(8) is the acoustoelectric current in a RQW with an infinite potential. We can see that the dependence on the frequency $\omega_{\vec{q}}$ is nonlinear. These results are different from those obtained in bulk semiconductor [5] and in the CQW with an infinite potential [13].

IV. NUMERICAL RESULTS AND DISCUSSIONS

To clarify the results that have been obtained, in this section, we consider the acoustoelectric current in a $GaAs$ RQW with an infinite potential. This quantity is considered as a function the frequency $\omega_{\vec{q}}$ of ultrasound, the temperature of system T, and the parameters of the RQW with an infinite potential. The parameters used in the numerical calculations are as follow: $\tau = 10^{-12}s; \Lambda = 13.5eV; a = b = 100\mathring{A}; W = 10^4 Wm^{-2}; \rho = 5320kgm^{-3}; v_s = 5370ms^{-1}; \omega_{\vec{q}_z} = 10^9 s^{-1}; v_l = 2\times 10^3 ms^{-1}, v_t = 18\times 10^2 ms^{-1}; m = 0.067m_e$, m_e being the mass of free electron.

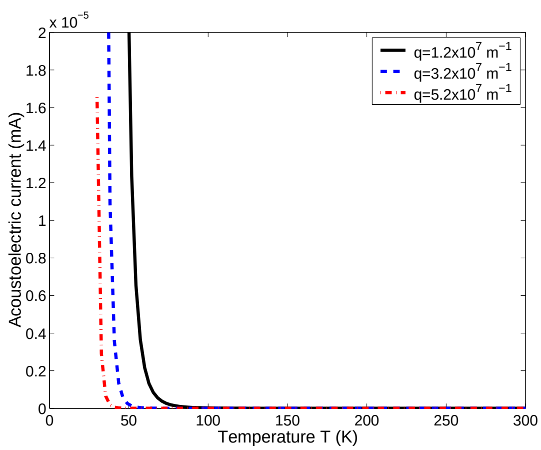

Figure 1: *The dependence of the acoustoelectric current on the temperature T.*

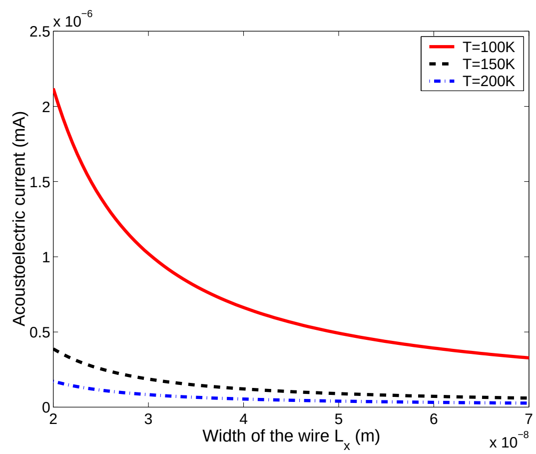

Figure 2: *The dependence of the acoustoelectric current on the width of the RQW.*

Figure 1 gives the dependence of the acoustoelectric current on the temperature T of the RQW with the acoustic wave number $q = 1.20\times 10^7 m^{-1}$, $q = 3.20\times 10^7 m^{-1}$ and $q = 5.20\times 10^6 m^{-1}$. The result shows the different behavior from results in the quantum well [14]. As in the quantum well, in the RQW with an infinite potential the acoustoelectric current is non-linear, but in the RQW the value of the acoustoelectric current strongly decreases with the temperature in a small value range.

In the figure 2, we show the dependence of the acoustoelectric current on the width of the RQW with the temperature $T = 100K$, $T = 150K$ and $T = 200K$. The value of the acoustoelectric current decreases as the width of the RQW increases.

V. CONCLUSIONS

In this paper, we have theoretically investigated the possibility of the acoustoelectric current in the RQW with an infinite potential. We have obtained analytical expressions for the acoustoelectric current in the RQW for the quantum limit case. We find strong dependences of acoustoelectric current on the acoustic wave number q, the temperature T and the width of the RQW. The result shows that the cause of the acoustoelectric current is the existence of partial current generated by the different energy groups of electrons, and the dependence of the electron energy due to momentum relaxation time.

The numerical result obtained for $GaAs$ RQW with an infinite potential shows the dependence of the acoustoelectric current on the width of the RQW is reduced in the low temperature region. The dependence of the acoustoelectric current on the temperature of the system is significantly reduced in the low temperature region and the current is approximately constant in the high temperature region. This dependence is different in comparison with that in quantum well [14]. The results show a geometrical dependence of the acoustoelectric current due to the confinement of electrons in RQW with an infinite potential.

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