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ON PECCEI-QUINN SYMMETRY AND QUARK MASSES IN THE ECONOMICAL 3-3-1 MODEL

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Abstract. *We show that there is an infinite number of $U(1)$ symmetries like Peccei-Quinn symmetry in the 3-3-1 model with minimal scalar sector. Moreover, all of them are completely broken due to the gauge symmetry breaking with the model's scalars. There is no any residual Peccei-Quinn symmetry. Because of the minimal scalar content there are some quarks that are massless at tree-level, but they can get consistent mass contributions at one-loop due to this fact.*

I. INTRODUCTION

There are obvious evidences that we must go beyond the standard model. The leading questions of which are perhaps neutrino oscillation, natural origin of masses and particularly Higgs mechanism, hierarchy problem between weak and Planck scale, and matter-antimatter asymmetry in the universe. In this work we will, however, be interested in alternatives concerning flavor physics. Why are there just three families of fermions? How are the families related? What are the nature of flavor mixings and mass hierarchies? On 3-3-1 models, the gauge symmetry has the form $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (thus named 3-3-1). A fermion content satisfying all the requirements is

$$\begin{aligned}
&\psi_{aL} = (\nu_a, e_a, N_a^c)_L^T \sim (1,3,-1/3)\,, \quad e_{aR} \sim (1,1,-1)\,,\\
&Q_{1L} = (u_1, d_1, U)_L^T \sim (3,3,1/3)\,, \quad Q_{\alpha L} = (d_\alpha, -u_\alpha, D_\alpha)_L^T \sim (3,3^*,0)\,,\\
&u_{aR}, U_R \sim (3,1,2/3)\,, \quad d_{aR}, D_{\alpha R} \sim (3,1,-1/3),
\end{aligned} \tag{1}$$

where $\alpha = \{2,3\}$ and $a = \{1,\alpha\}$ are family indices. The quantum numbers as given in parentheses are respectively based on $(SU(3)_C, SU(3)_L, U(1)_X)$ symmetries. The U and D are exotic quarks, while N_R are right-handed neutrinos. The model is thus named the 3-3-1 model with right-handed neutrinos. If these exotic leptons are not introduced, i.e. instead the third components are now included right-handed charged leptons, we have the minimal 3-3-1 model.

The 3-3-1 gauge symmetry is broken through two stages: $SU(3)_L \otimes U(1)_X \longrightarrow SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{em}$. They are obtained by scalar triplets. One of the weaknesses of the mentioned 3-3-1 models that reduces their predictive possibility is a plenty or complication in the scalar sectors. The attempts on this direction to realize simpler scalar sectors have recently been made. The first one is the 3-3-1 model with right-handed neutrinos and minimal scalar sector—two triplets [?],

$$\begin{aligned} \chi &= \left(\chi_1^0, \chi_2^-, \chi_3^0\right)^T \sim (1,3,-1/3)\,, \\ \phi &= \left(\phi_1^+, \phi_2^0, \phi_3^+\right)^T \sim (1,3,2/3)\,, \end{aligned} \tag{2}$$

with VEV given by

$$\langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \\ \omega \end{pmatrix}, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \tag{3}$$

called the economical 3-3-1 model [?, ?]. The VEV ω is responsible for the first stage of gauge symmetry breaking, while v, u are for the second stage. The minimal 3-3-1 model with minimal scalar sector of two triplets has also been proposed in Ref. [7]. Notice that due to the restricted scalar contents, these models often contain tree-level massless quarks that require corrections. The latter model has provided masses for quarks via high-dimensional effective interactions, whereas the former one has produced quark masses via quantum effects.

II. Peccei-Quinn symmetries in economical 3-3-1 model.

The gauge symmetry of the model is $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. The particle content is defined in equations (1,2). The electric charge operator is given by

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{4}$$

where T_i $(i = 1, 2, 3, ..., 8)$ and X are the charges of $SU(3)_L$ and $U(1)_X$, respectively. The standard model hypercharge operator is thus identified as $Y = -(1/\sqrt{3})T_8 + X$. This model does not contain exotic electric charges, i.e. the exotic quarks have electric charges like ordinary quarks: $Q\,(D) = -1/3$ and $Q\,(U) = 2/3$.

The most general Yukawa interactions are given by

$$\begin{aligned} \mathcal{L}_\mathrm{Y} = \; & h^U \overline{Q}_{1L} \chi U_R + h^D_{\alpha\beta} \overline{Q}_{\alpha L} \chi^* D_{\beta R} \\ & + h^e_{ab} \overline{\psi}_{aL} \phi e_{bR} + h^\nu_{ab} \epsilon_{mnp} (\overline{\psi}^c_{aL})_m (\psi_{bL})_n (\phi)_p \\ & + h^d_a \overline{Q}_{1L} \phi d_{aR} + h^u_{\alpha a} \overline{Q}_{\alpha L} \phi^* u_{aR} \\ & + s^u_a \overline{Q}_{1L} \chi u_{aR} + s^d_{\alpha a} \overline{Q}_{\alpha L} \chi^* d_{aR} \\ & + s^D_\alpha \overline{Q}_{1L} \phi D_{\alpha R} + s^U_\alpha \overline{Q}_{\alpha L} \phi^* U_R + H.c., \end{aligned} \tag{5}$$

where m, n and p stand for $\mathrm{SU}(3)_L$ indices. In [?] we have shown that at the tree level one up-quark and two down-quarks are massless. However, the one-loop corrections can give them consistent masses. In this work we will revisit those corrections by giving a complete calculation when including a realistic mixing of all the three families of quarks as well. We are thus showing that the results in [12] which contrast with ours are not correct.

As the lepton triplets stand, the lepton number in this model does not commute with the gauge symmetry. In fact, it is a residual symmetry of a new-lepton charge $\mathcal{L}$

given by [?]

$$L = \frac{4}{\sqrt{3}}T_8 + \mathcal{L}. \tag{6}$$

The $\mathcal{L}$ charges of the model multiplets can be obtained as

$$\mathcal{L}(\psi_{aL}, Q_{1L}, Q_{\alpha L}, \phi, \chi, e_{aR}, u_{aR}, d_{aR}, U_R, D_{\alpha R}) = \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{4}{3}, 1, 0, 0, -2, 2, \tag{7}$$

respectively. Also, it is easily checked that $L(U) = -L(D) = L(\phi_3) = -L(\chi_{1,2}) = -2$. All the other quarks and scalars have zero lepton-number, $L = 0$. It is worth emphasizing that the residual L is *spontaneously* broken by u due to $L(\chi_1^0) = 2$, which is unlike the standard model. Notice that the Yukawa couplings s's violate $\mathcal{L}$, while the h's do not.

Following [12], we introduce a global $U(1)_H$ symmetry in addition to the gauge symmetry, i.e.

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_H. \tag{8}$$

The condition for this symmetry playing the role like Peccei-Quinn (handedness or chiral) symmetry is anomaly $[SU(3)_C]^2 U(1)_H \neq 0$. On the other hand, since the Yukawa interactions invariant under this symmetry, the relations on the charges of any $U(1)_H$ group are

$$-H_{Q_1} + H_U + H_\chi = 0, \qquad -H_Q + H_D - H_\chi = 0, \tag{9}$$
$$-H_{Q_1} + H_u + H_\chi = 0, \qquad -H_Q + H_d - H_\chi = 0, \tag{10}$$
$$-H_{Q_1} + H_d + H_\phi = 0, \qquad -H_Q + H_u - H_\phi = 0, \tag{11}$$
$$-H_{Q_1} + H_D + H_\phi = 0, \qquad -H_Q + H_U - H_\phi = 0, \tag{12}$$
$$-H_\psi + H_e + H_\phi = 0, \qquad 2H_\psi + H_\phi = 0, \tag{13}$$

where the notation H_Ψ means as the $U(1)_H$ charge of the Ψ multiplet. Notice that all the other terms of Lagrangian are obviously conserved under this symmetry. Using the relations, the anomaly mentioned is rewritten as

$$[SU(3)_C]^2 U(1)_H \sim 2H_\chi + H_\phi \neq 0. \tag{14}$$

In solving equations (9-13,14), we also denote H as a collection of partial solutions H_Ψ in order, and having remarks as follows

(1) The solution is scale invariance, i.e. if H is solution, then cH $(c \neq 0)$ does.
(2) Two solutions called to be different (i.e. linearly independent) if they are not related by scale invariance transformations.
(3) The solutions that contain linearly-independent subsolutions, e.g. $(H_\phi, H_\chi) = (0, 1),\ (1, 0)$, or $(1, 1)$, respectively, are different.
(4) The different solutions will define different Peccei-Quinn like symmetries, respectively.

The charge relations (9-13) yield degenerate equations. Indeed, they can equivalently be rewritten via seven independent equations as follows

$$\begin{aligned} H_u &= H_U, \quad H_d = H_D, \quad H_\psi = -H_\phi/2, \quad H_e = -3H_\phi/2, \\ H_u - H_d &= H_\phi - H_\chi, \quad H_u - H_Q = H_\phi, \quad H_u + H_d = H_Q + H_{Q_1}. \end{aligned} \tag{15}$$

We have 10 variables, while there are 7 equations. Hence, there is an infinite number of solutions (certainly satisfying (14) too). For instant, put $H_\phi = 0$. We have $H_\psi = H_e = 0$, $H_u = H_U = H_Q$, $H_d = H_D = H_{Q_1}$, and $H_u - H_d = -H_\chi$. The solutions of this kind are thus given dependently on two parameters $a \equiv H_\chi \neq 0$ and $b \equiv H_u$ such as

$$H(\phi, \chi, \psi, e, u, U, Q, d, D, Q_1) = (0, a, 0, 0, b, b, b, a + b, a + b, a + b). \tag{16}$$

Since a, b are arbitrary, there are an infinity of different solutions corresponding to whatever pairs (a, b) are linearly independent, for example, $(a, b) = (0, 1),\ (1, 0),\ (1, 1), (1, 2)$ and so on.

In Table 1, we list three of Peccei-Quinn like symmetries in which the first one (second line) is given in [12] that was solely claimed and marked as $U(1)_{PQ}$.

Table 1. Three chiral symmetries taken as examples in the economical 3-3-1 model.

$Q_{\alpha L}$	Q_{1L}	(u_{aR}, U_R)	$(d_{aR}, D_{\alpha R})$	ψ_{aL}	e_{aR}	ϕ	χ
-1	1	0	0	$-1/2$	$-3/2$	1	1
1	2	1	2	0	0	0	1
1	2	2	1	$-1/2$	$-3/2$	1	0

There is no residual symmetry associated with the $U(1)_H$ above after the spontaneous gauge-symmetry breaking which contradicts with [12]. Prove: suppose that there is such one, denoted by $U(1)_{PQ}$. Since it is sevival and conserved after the electroweak symmetry breaking, it has the form as a combination of diagonal generators $PQ = \alpha T_3 + \beta T_8 + \delta X + \gamma H$ $(\gamma \neq 0)$. Also, the charge PQ has to annihilate the vacuums, $PQ(\langle\phi\rangle, \langle\chi\rangle) = 0$. All these are similar to the electric charge operator responsible for electric charge conservation after the electroweak symmetry breaking. We have equations:

$$\begin{aligned} \frac{\alpha}{2} + \frac{\beta}{2\sqrt{3}} + \delta X_\chi + \gamma H_\chi &= 0, \\ -\frac{\alpha}{2} + \frac{\beta}{2\sqrt{3}} + \delta X_\phi + \gamma H_\phi &= 0, \\ -\frac{\beta}{\sqrt{3}} + \delta X_\chi + \gamma H_\chi &= 0. \end{aligned} \tag{17}$$

Combining all three equations, we deduce $\delta(2H_\chi + H_\phi) = 0$. Because $2X_\chi + X_\phi = 0$
If $\delta \neq 0$, then $2H_\chi + H_\phi = 0$ that contradicts to (14), $[SU(3)_C]^2 U(1)_H \sim 2H_\chi + H_\phi \neq 0$. Therefore, there is no residual symmetry of $U(1)_H$. All the Peccei-Quinn like $U(1)_H$ symmetries are completely broken along with the gauge symmetry breaking. If $\delta = 0$ then $\beta = -\alpha/\sqrt{3}$ and $\gamma = \alpha$, Therefore we have $PQ = \alpha Q$ as a solution to finding the electric charge operation (that certainly contradicts to (14) since Q is vectorlike). If one includes baryon number B as well (since $B_\phi = B_\chi = 0$), it results

$$PQ = \alpha Q + \xi B. \tag{18}$$

Only vectorlike symmetries (i.e. non Peccei-Quinn) such as X, B might have surviving residual symmetries after the spontaneous symmetry breaking by the model's scalars.

III. Fermion masses .

In this model, the masses of charged leptons are given at the tree level as usual while the neutrinos can get consistent masses at the one-loop level as explicitly pointed out in Ref. [19]. The implication of the higher-dimensional effective operators responsible for the neutrino masses has also been given therein.

Let us now concentrate on masses of quarks that can be divided into two sectors: up type quarks (u_a, U) with electric charge 2/3 and down type quarks (d_a, D_α) with electric charge $-1/3$. From (5) and (3) we can obtain the mass matrix of the up type quarks (u_1, u_2, u_3, U):

$$M_{\mathrm{up}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -s^u_1 u & -s^u_2 u & -s^u_3 u & -h^U u \\ h^u_{21} v & h^u_{22} v & h^u_{23} v & s^U_2 v \\ h^u_{31} v & h^u_{32} v & h^u_{33} v & s^U_3 v \\ -s^u_1 \omega & -s^u_2 \omega & -s^u_3 \omega & -h^U \omega \end{pmatrix}, \tag{19}$$

and the mass matrix of down type quarks $(d_1, d_2, d_3, D_2, D_3)$:

$$M_{\mathrm{down}} = -\frac{1}{\sqrt{2}} \begin{pmatrix} h^d_1 v & h^d_2 v & h^d_3 v & s^D_2 v & s^D_3 v \\ s^d_{21} u & s^d_{22} u & s^d_{23} u & h^D_{22} u & h^D_{23} u \\ s^d_{31} u & s^d_{32} u & s^d_{33} u & h^D_{32} u & h^D_{33} u \\ s^d_{21} \omega & s^d_{22} \omega & s^d_{23} \omega & h^D_{22} \omega & h^D_{23} \omega \\ s^d_{31} \omega & s^d_{32} \omega & s^d_{33} \omega & h^D_{32} \omega & h^D_{33} \omega \end{pmatrix}. \tag{20}$$

The first and last rows of (19) are proportional. Similarly, the second and fourth rows of (20) are proportional, while the third and last rows of this matrix take the same situation. Hence, in this model the tree level quark spectrum contains three massless eigenstates (one up and two down quarks). So, what are the causes?

There are just two: first all these degeneracies are due to the χ scalar only (with the presence of VEVs u, ω), *not* ϕ; second, the Yukawa couplings of the first and third component of quark triplets/antitriplets to right-handed quarks in those degenerate rows are the same due to $SU(3)_L$ invariance. Obviously, the vanishing quark masses are not a consequence of the $U(1)_H$ symmetry because it actually happens even if we choose $H_\chi = 0$ (in this case the Peccei-Quinn like symmetry resulting from only $H_\phi \neq 0$ does not give any constraint on the massless quark sector). At the one-loop level, all the degeneracies will be separated due to contribution of ϕ as well (see Appendix B of [8]). In such case, the one-loop mass corrections also collectively break the $U(1)_H$ symmetry since both the scalars χ, ϕ are being taken into account, i.e. for those relevant quarks $2H_\chi + H_\phi$ are always nonzero at the one loop level.

In [8], we have already shown that all the tree level massless quarks can get consistent masses at the one-loop level. There, the light quarks and/or mixings of light quarks with exotic quarks got mass contributions. The exotic quark masses are reasonably large and took as a cutoff scale, thus no correction is needed. So why the recalculations as given in

Ref. [12] for the quark masses, that consequently contradict to ours, are incomplete? This is due to the fact that they included even mass corrections for heavy exotic quarks as well. In this case, the cutoff scale of the theory must be larger than the exotic quark masses. As a result, under this cutoff scale all the physics is sensitive. There must be contributions coming from the ϕ scalar as well as ordinary active quarks where the flavor mixing must present. Let us remind that Ref. [12] in this case accounts for the χ contribution only. Thus the masslessness would remain as a result of two points mentioned above.

The above analysis also means that all quarks will get masses if both χ and ϕ contribute so that $2H_\chi + H_\phi \neq 0$ to ensure (14). This can explicitly be understood via an analysis of the effective mass operators [13] responsible for quarks below.

III.1. One-loop corrections

The analysis given below is for the up type quark sector only. That for the down type quark sector can be done similarly and got the same conclusion as the up type quarks. After the one-loop corrections, the mass matrix (19) looks like

$$M_{\mathrm{up}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -s^u_1 u + \Delta_{11} & -s^u_2 u + \Delta_{12} & -s^u_3 u + \Delta_{13} & -h^U u + \Delta_{14} \\ h^u_{21} v + \Delta_{21} & h^u_{22} v + \Delta_{22} & h^u_{23} v + \Delta_{23} & s^U_2 v + \Delta_{24} \\ h^u_{31} v + \Delta_{31} & h^u_{32} v + \Delta_{32} & h^u_{33} v + \Delta_{33} & s^U_3 v + \Delta_{34} \\ -s^u_1 \omega + \Delta_{41} & -s^u_2 \omega + \Delta_{42} & -s^u_3 \omega + \Delta_{43} & -h^U \omega + \Delta_{44} \end{pmatrix}, \tag{21}$$

where Δ_{ij} are all possible one-loop corrections. Obviously this matrix gives all nonzero masses if the first and last rows are not in proportion. To show that tree-level degeneracy separated (i.e. these two rows are now not proportional) it is only necessary to prove the following submatrix:

$$M_{uU} = \frac{1}{\sqrt{2}} \begin{pmatrix} -s^u_1 u + \Delta_{11} & -h^U u + \Delta_{14} \\ -s^u_1 \omega + \Delta_{41} & -h^U \omega + \Delta_{44} \end{pmatrix} \tag{22}$$

having nonzero determinant with general Yukawa couplings and VEVs. Two conditions below should be clarified: (i) The tree-level properties as implemented by the two points above must be broken,

$$\begin{cases} \frac{\Delta_{11}}{\Delta_{14}} \neq \frac{s^u_1}{h^U} \\ \frac{\Delta_{41}}{\Delta_{44}} \neq \frac{s^u_1}{h^U} \end{cases}, \tag{23}$$

$$\begin{cases} \frac{\Delta_{11}}{\Delta_{41}} \neq \frac{u}{\omega} \\ \frac{\Delta_{14}}{\Delta_{44}} \neq \frac{u}{\omega} \end{cases}. \tag{24}$$

Since, by contrast if one of these systems is unsatisfied, which is the case as analyzed in [12], one quark remains massless. (ii) The matrix (22) has nonzero determinant:

$$\det M_{uU} = \frac{1}{2}\left[s^u_1(\omega\Delta_{14} - u\Delta_{44}) + h^U(u\Delta_{41} - \omega\Delta_{11}) + \Delta_{11}\Delta_{44} - \Delta_{41}\Delta_{14}\right] \neq 0 \tag{25}$$

$$\text{or} \quad \frac{1}{2}\left[u(h^U\Delta_{41} - s^u_1\Delta_{44}) + \omega(s^u_1\Delta_{14} - h^U\Delta_{11}) + \Delta_{11}\Delta_{44} - \Delta_{41}\Delta_{14}\right] \neq 0 \tag{26}$$

It is interesting that the first two terms of (25) and (26) mean (24) and (23), respectively.

At the one loop level, there must be similar corrections mediated coming from ordinary quarks, exotic quarks as well as both ordinary and exotic quarks in mediations. We must also include general Yukawa couplings connecting flavors, i.e. $h_{ab} \neq 0,\ s_{ab} \neq 0$ for $a \neq b$ to account for the CKM quark mixing matrix as it should be. It is also remarked that the external scalar lines of those diagrams now consist of ϕ, χ or both χ and ϕ as well. Totally, we have 48 diagrams at the one-loop level (24 for up type quark and 24 for down type quark). See Appendix B of [8] for details. Here, for a convenience let us list all those corrections in terms of the relevant matrix elements as given in Appendix B. All the one-loop corrections are taken into account to yield (22) explicitly

$$\Delta_{11} = \frac{h^u_{\alpha 1}}{s^U_\alpha}\sum_{i=1}^{4}\Delta^i_{14} + \frac{s^u_1}{h^U}\sum_{j=5}^{8}\Delta^j_{14}, \qquad \Delta_{14} = \sum_{k=1}^{8}\Delta^k_{14}, \tag{27}$$

$$\Delta_{41} = \frac{h^u_{\alpha 1}}{s^U_\alpha}\sum_{i=1}^{4}\Delta^i_{44} + \frac{s^u_1}{h^U}\sum_{j=5}^{8}\Delta^j_{44}, \qquad \Delta_{44} = \sum_{k=1}^{8}\Delta^k_{44}. \tag{28}$$

It is easily checked that (23) is satisfied since

$$\frac{h^u_{\alpha 1}}{s^U_\alpha} \neq \frac{s^u_1}{h^U}, \qquad h^u_{\alpha 1} \neq 0, \qquad s^U_\alpha \neq 0, \tag{29}$$

in general. This is due to the contribution of ϕ to the massless quarks (in addition to χ) as well like we can already see from the Yukawa couplings $h^u_{\alpha 1}$ and s^U_α related to this scalar. The system (24) is always correct even we can check that it is also applied for the special case with flavor diagonalization as presented in [8, 12].

Finally let us check (ii). The determinant equals to

$$\det M_{\mathrm{up}} = \frac{1}{2}\left(\frac{h^u_{\alpha 1}}{s^U_\alpha} - \frac{s^u_1}{h^U}\right)\left[h^U\sum_{i=1}^{4}(u\Delta^i_{44} - \omega\Delta^i_{14}) + \sum_{i=1}^{4}\sum_{j=5}^{8}(\Delta^i_{14}\Delta^j_{44} - \Delta^j_{14}\Delta^i_{44})\right], \tag{30}$$

which is always nonzero due to (29). In fact, the last factor $[\cdots]$ can be explicitly given by

$$
\begin{aligned}
& h^U \Big\{[(\omega^2+u^2)\lambda_3+u^2\lambda_4+v^2\lambda_2][u[I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3})+I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_3})]\\
& -\omega[I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_1})+I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_1})]+M^2_{\chi_3}u[B(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2},M^2_{\phi_3})\\
& +B(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2},M^2_{\phi_3})]-M^2_{\chi_1}\omega[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_1})+B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_1})]\\
& -(\omega-u)u\omega[A(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3})+A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi_3})+M^2_{\phi_1}B(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3},M^2_{\phi_1})\\
& +M^2_{\phi_1}B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi_3},M^2_{\phi_1})]]\Big\}+\Big\{u\omega[A(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3})+A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi_3})\\
& +M^2_{\phi_1}B(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3},M^2_{\phi_1})+M^2_{\phi_1}B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi_3},M^2_{\phi_1})]\Big\}[[((\omega^2+u^2)\lambda_1+v^2\lambda_3)\\
& \times[(I(M^2_{Q^{1,3}_1},M^2_{U},M^2_{\chi_1})+I(M^2_{Q^{1,3}_1},M^2_{u_i},M^2_{\chi_1}))-(I(M^2_{Q^{1,3}_1},M^2_{U},M^2_{\chi_3})+I(M^2_{Q^{1,3}_1},M^2_{u_i},M^2_{\chi_3}))]]\\
& +u[A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2})+A(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2})+M^2_{\chi_1}[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_1})\\
& +B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_1})]]-\omega[A(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2})+A(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2})]\\
& +M^2_{\phi_3}[B(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2},M^2_{\phi_3})+B(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2},M^2_{\phi_3})]]]+\big\{u\omega[A(M^2_{U},M^2_{U},M^2_{\chi_3})\\
& +A(M^2_{U},M^2_{u_i},M^2_{\chi_3})+M^2_{\chi_1}B(M^2_{U},M^2_{U},M^2_{\chi_3},M^2_{\chi_1})+M^2_{\chi_1}B(M^2_{U},M^2_{u_i},M^2_{\chi_3},M^2_{\chi_1})]\big\}\\
& \times[[(\omega^2+u^2)\lambda_3+u^2\lambda_4+v^2\lambda_2][(I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_3})+I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_3}))\\
& -(I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_1})+I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_1}))]+\omega[A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2})+A(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2})\\
& +M^2_{\chi_3}[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_3})+B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_3})]]-u[A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2})\\
& +A(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2})+M^2_{\chi_1}[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_1})+B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_1})]]]\\
& +\{[(\omega^2+u^2)\lambda_3+u^2\lambda_4+v^2\lambda_2][I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_1})+I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_1})]\\
& +u\Big[A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2})+A(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2})+M^2_{\chi_1}[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_1})\\
& +B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_1})]\Big]\}\{[(\omega^2+u^2)\lambda_1+v^2\lambda_3][I(M^2_{Q^{1,3}_1},M^2_{U},M^2_{\chi_3})+I(M^2_{Q^{1,3}_1},M^2_{u_i},M^2_{\chi_3})]\\
& +\omega\Big[A(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2})+A(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2})+M^2_{\phi_3}[B(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2},M^2_{\phi_3})\\
& +B(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2},M^2_{\phi_3})]\Big]\}-\{[(\omega^2+u^2)\lambda_1+v^2\lambda_3][I(M^2_{Q^{1,3}_1},M^2_{U},M^2_{\chi_1})+I(M^2_{Q^{1,3}_1},M^2_{u_i},M^2_{\chi_1})]\\
& +u\Big[A(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2})+A(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2})+M^2_{\phi_1}[B(M^2_{Q^2_1},M^2_{d_i},M^2_{\chi_2},M^2_{\phi_1})\\
& +B(M^2_{Q^2_1},M^2_{D_\alpha},M^2_{\chi_2},M^2_{\phi_1})]\Big]\}\{[(\omega^2+u^2)\lambda_3+u^2\lambda_4+v^2\lambda_2][I(M^2_{Q_{\alpha1,3}},M^2_{d_i},M^2_{\phi_1})\\
& +I(M^2_{Q_{\alpha1,3}},M^2_{D_\alpha},M^2_{\phi_1})]+u\Big[A(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2})+A(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2})\\
& +M^2_{\chi_1}[B(M^2_{Q_{\alpha2}},M^2_{u_i},M^2_{\phi^2},M^2_{\chi_1})+B(M^2_{Q_{\alpha2}},M^2_{U},M^2_{\phi^2},M^2_{\chi_1})]\Big]\}, \qquad (31)
\end{aligned}
$$

where the functions I, A and B are defined in Appendix A. We conclude that all the quarks in this model can get nonzero masses at the one-loop level. Although the tree level vanishing masses of quarks is not a consequence of the $U(1)_H$ symmetry, this Peccei-Quinn like symmetry is collectively broken at the one-loop level when the quarks get masses.

III.2. Effective mass operators

As previous section, the $U(1)_H$ symmetry is spontaneously broken via the collective effects at the one-loop level when all the quarks get mass, i.e. $2H_\chi+H_\phi \neq 0$. In this section, we will show that all the quarks can get mass via effective mass operators there the $U(1)_H$ breaking is explicitly recognized. In other words, we will consider effective interactions responsible for fermion masses up to five dimensions. The most general interactions up to five dimensions that lead to fermion masses have the form:

$$\mathcal{L}_Y + \mathcal{L}'_Y, \tag{32}$$

where $\mathcal{L}_Y$ is defined in (5) and $\mathcal{L}'_Y$ (five-dimensional effective mass operators) is given by

$$\begin{aligned}\mathcal{L}'_Y &= \frac{1}{\Lambda}(\overline{Q}_{1L}\phi^*\chi^*)(s'^U U_R + h'^u_a u_{aR})\\ &+\frac{1}{\Lambda}(\overline{Q}_{\alpha L}\phi\chi)(s'^D_{\alpha\beta}D_{\beta R} + h'^d_{\alpha a}d_{aR})\\ &+\frac{1}{\Lambda}s'^\nu_{ab}(\overline{\psi}^c_{aL}\psi_{bL})(\chi\chi)^*\\ &+H.c.\end{aligned} \tag{33}$$

Here, as usual we denote h for $\mathcal{L}$-charge conservation couplings and s for violating ones. Λ is the cutoff scale which can be taken in the same order as ω. It is noteworthy that all the above interactions (as given in $\mathcal{L}'_Y$) are not invariant under $U(1)_H$ since they carry $U(1)_H$ charge proportional to $2H_\chi+H_\phi \neq 0$ like (14). For example, the first interaction has $U(1)_H$ charge: $-H_{Q_1} - H_\phi - H_\chi + H_u = -(2H_\chi + H_\phi)$, with the help of eqs (9-13). All those interactions contain $\phi\chi$ combination. Therefore, the fermion masses are generated if both scalars develop VEV. In this case, the Peccei-Quinn like symmetry $U(1)_H$ is spontaneously broken too.

Substituting VEVs (3) into (32), the mass Lagrangian reads

$$\begin{aligned}\mathcal{L}^{mass}_{fermion} &= -(\overline{u}_{1L}\ \overline{u}_{2L}\ \overline{u}_{3L}\ \overline{U}_L)M_u(u_{1R}\ u_{2R}\ u_{3R}\ U_R)^T\\ &-(\overline{d}_{1L}\ \overline{d}_{2L}\ \overline{d}_{3L}\ \overline{D}_{2L}\ \overline{D}_{3L})M_d(d_{1R}\ d_{2R}\ d_{3R}\ D_{2R}\ D_{3R})^T\\ &-\frac{1}{2}(\overline{\nu}^c_L\ \overline{N}_R)M_\nu(\nu_L\ N^c_R)^T\\ &+H.c.\end{aligned} \tag{34}$$

Here the mass matrices of up type quarks $(u_1\ u_2\ u_3\ U)$, down type quarks $(d_1\ d_2\ d_3\ D_2\ D_3)$ are respectively given by

$$M_{\mathrm{u}} = \frac{1}{\sqrt{2}}\begin{pmatrix} -s^u_1 u - \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^u_1 & -s^u_2 u - \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^u_2 & -s^u_3 u - \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^u_3 & -h^U u - \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}s'^U \\ h^u_{21}v & h^u_{22}v & h^u_{23}v & s^U_2 v \\ h^u_{31}v & h^u_{32}v & h^u_{33}v & s^U_3 v \\ -s^u_1\omega + \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^u_1 & -s^u_2\omega + \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^u_2 & -s^u_3\omega + \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^u_3 & -h^U\omega + \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}s'^U \end{pmatrix}, \tag{35}$$

$$M_{\mathrm{d}} = \frac{-1}{\sqrt{2}}\begin{pmatrix} h^d_1 v & h^d_2 v & h^d_3 v & s^D_2 v & s^D_3 v \\ s^d_{21}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{21} & s^d_{22}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{22} & s^d_{23}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{23} & h^D_{22}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}s'^D_{22} & h^D_{23}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}s'^D_{23} \\ s^d_{31}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{31} & s^d_{32}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{32} & s^d_{33}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}h'^d_{33} & h^D_{32}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}s'^D_{32} & h^D_{33}u + \frac{1}{\sqrt{2}}\frac{v\omega}{\Lambda}s'^D_{33} \\ s^d_{21}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{21} & s^d_{22}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{22} & s^d_{23}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{23} & h^D_{22}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}s'^D_{22} & h^D_{23}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}s'^d_{23} \\ s^d_{31}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{31} & s^d_{32}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{32} & s^d_{33}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}h'^d_{33} & h^D_{32}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}s'^D_{32} & h^D_{33}\omega - \frac{1}{\sqrt{2}}\frac{vu}{\Lambda}s'^D_{33} \end{pmatrix}. \tag{36}$$

Two remarks are in order

(1) Up quarks: If there is no correction, i.e. $s', h' = 0$, the mass matrix (35) has the first line and the fourth line in proportion (degeneracy) that means that one up quark is massless, as mentioned [8]. The presence of corrections, i.e. $s', h' \neq 0$, will separate that degeneracy. Indeed, the first and fourth lines are now in proportion only if $s^u_1/h'^u_1 = s^u_2/h'^u_2 = s^u_3/h'^u_3 = h^U/s'^U$ which is not the case in general. The up quark type mass matrix is now most general that can be diagonalized to obtain the masses of exotic U and ordinary $u_{1,2,3}$.

(2) Down quarks: The second and the fourth lines as well as the third and the fifth lines have the same status as in the up quark type. All these degeneracies are separated. Consequently we have the most general mass matrix for down quark type.

Using the $U(1)_H$ violating triple scalar interactions as mentioned above, those effective mass operators with five dimensions can be explicitly understood as derived from two-loop radiative corrections responsible for the quark masses, with the assumption that $U(1)_H$ was broken in the scalar potential first, in similarity to the radiative Majorana neutrino masses via lepton violating triple scalar potentials in Zee-Babu model [20]. It is noted that the above one-loop corrections can be also translated via the language of effective operators with six dimensions before the $U(1)_H$ breaking happens. A complete calculation of all the corrections presented above as well as obtaining the quark masses and mixing is out of scope of this work. It should be published elsewhere [21].

IV. CONCLUSION

V. Conclusions

As any other 3-3-1 models, the economical 3-3-1 model naturally contains an infinity of $U(1)_H$ symmetries like Peccei-Quinn symmetry with just its scalar content, which is unlike the case of the standard model. In contradiction to other extensions of the standard model including ordinary 3-3-1 models, the economical 3-3-1 model has interesting features as follows

(1) There is no residual symmetry of $U(1)_H$ after the scalars getting VEVs.

(2) The vanishing of quark masses at the tree-level is not a resultant from $U(1)_H$. It is already a consequence of the minimal scalar content under the model gauge symmetry.

(3) All the quarks can get nonzero masses at the one-loop level, there the $U(1)_H$ symmetry is obviously broken.

By this work, it is to emphasis that the economical 3-3-1 model can work with only two scalar triplets. All the fermions can get consistent masses [8, 19]. A further analysis can show observed flavor mixings as indicated by the CKM matrix and PMNS matrix. Also, with the minimal scalar sector the model is very predictive which is worth to be searched for at the current colliders [21].

With the above conclusions, it is to emphasis that the statements in [12] such as unique solution of $U(1)_H$, existence of a residual symmetry of $U(1)_H$, the masslessness of quarks due to that supposed residual symmetry, and one loop corrections to up type quark sector are incorrect or incomplete. The addition of scalars to the economical 3-3-1 model as given in [12] is dynamically not required since this model as seen can work well by itself.

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Appendix A. Integrations

The functions $A(a,b,c)$, $B(a,b,c,d)$ and $I(a,b,c)$ as appeared in the text are given by

$$
\begin{aligned}
A(a,b,c) &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-a)(p^2-b)(p^2-c)} \\
&= \frac{-i}{16\pi^2}\left\{\frac{a\ln a}{(a-b)(a-c)}+\frac{b\ln b}{(b-a)(b-c)}+\frac{c\ln c}{(c-b)(c-a)}\right\}, \qquad (37)\\
B(a,b,c,d) &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-a)(p^2-b)(p^2-c)(p^2-d)} \\
&= \frac{-i}{16\pi^2}\left\{\frac{a\ln a}{(a-b)(a-c)(c-d)}+\frac{b\ln b}{(b-a)(b-c)(b-d)}\right. \\
&\left. +\frac{c\ln c}{(c-b)(c-a)(c-d)}+\frac{d\ln d}{(d-b)(d-a)(d-c)}\right\}, \qquad (38)\\
I(a,b,c) &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2-a)^2(p^2-b)(p^2-c)} \\
&= \frac{-i}{16\pi^2}\left\{\frac{a(2\ln a+1)}{(a-b)(a-c)}-\frac{a^2(2a-b-c)\ln a}{(a-b)^2(a-c)^2}+\frac{b^2\ln b}{(b-a)^2(b-c)}\right. \\
&\left. +\frac{c^2\ln c}{(c-a)^2(c-b)}\right\}. \qquad (39)
\end{aligned}
$$

Appendix B. Corrections

The one-loop corrections to the mass matrix M_{uU} are presented by the diagrams as follows:

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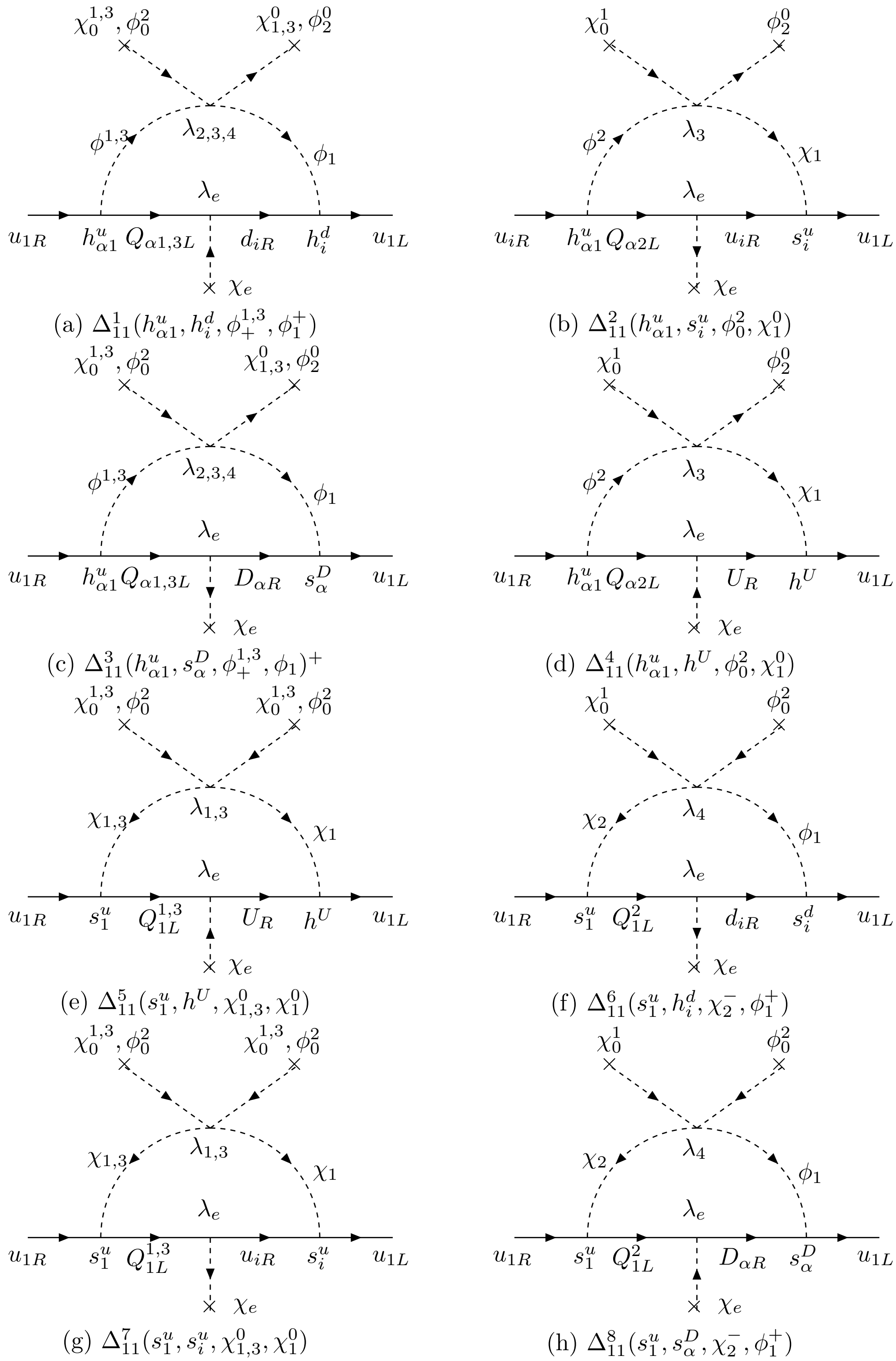


(a) $\Delta_{11}^1(h_{\alpha 1}^u, h_i^d, \phi_+^{1,3}, \phi_1^+)$

(b) $\Delta_{11}^2(h_{\alpha 1}^u, s_i^u, \phi_0^2, \chi_1^0)$

(c) $\Delta_{11}^3(h_{\alpha 1}^u, s_\alpha^D, \phi_+^{1,3}, \phi_1)^+$

(d) $\Delta_{11}^4(h_{\alpha 1}^u, h^U, \phi_0^2, \chi_1^0)$

(e) $\Delta_{11}^5(s_1^u, h^U, \chi_{1,3}^0, \chi_1^0)$

(f) $\Delta_{11}^6(s_1^u, h_i^d, \chi_2^-, \phi_1^+)$

(g) $\Delta_{11}^7(s_1^u, s_i^u, \chi_{1,3}^0, \chi_1^0)$

(h) $\Delta_{11}^8(s_1^u, s_\alpha^D, \chi_2^-, \phi_1^+)$

Fig. 1. Corrections to $(M_{uU})_{11}$

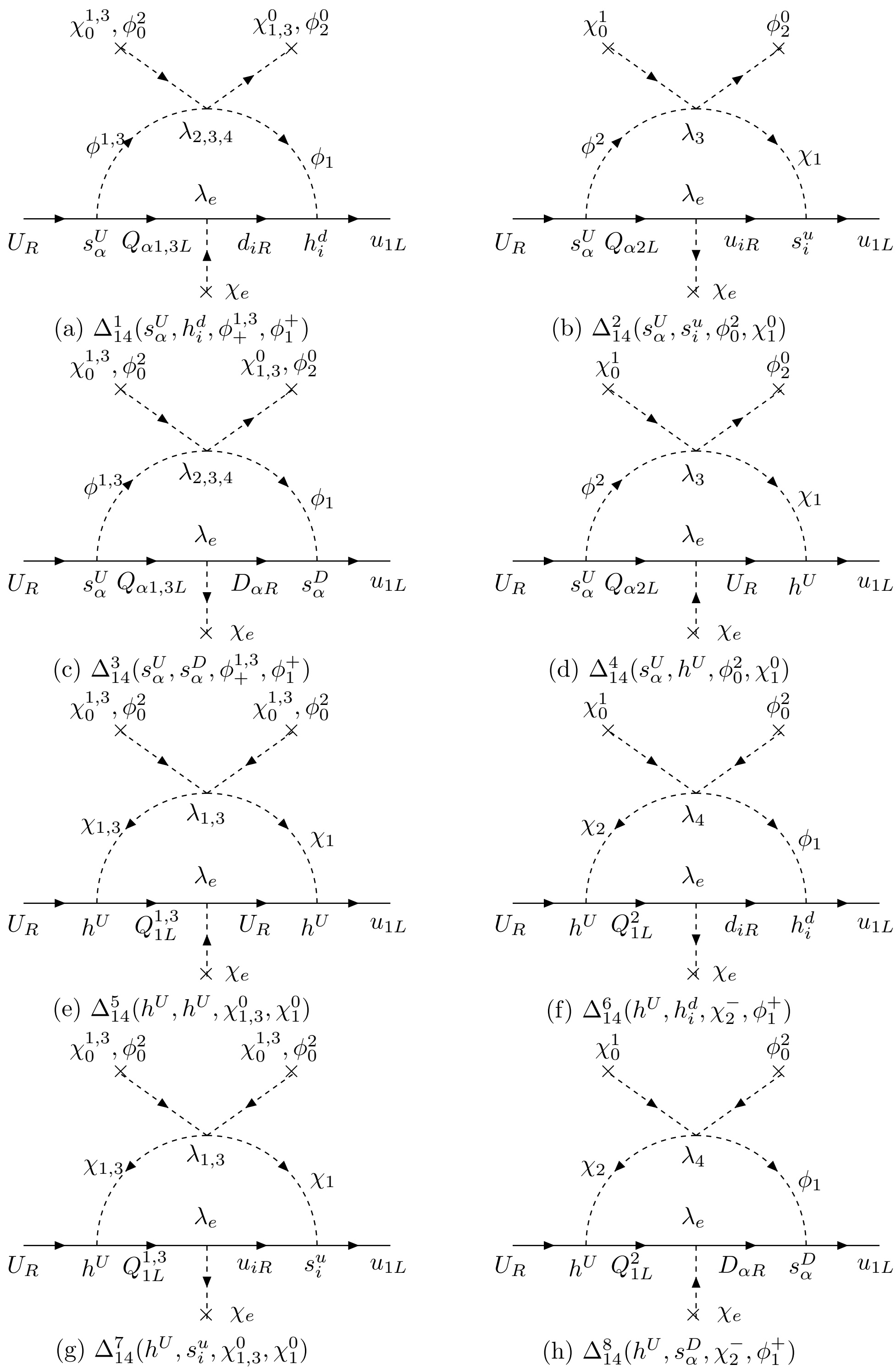
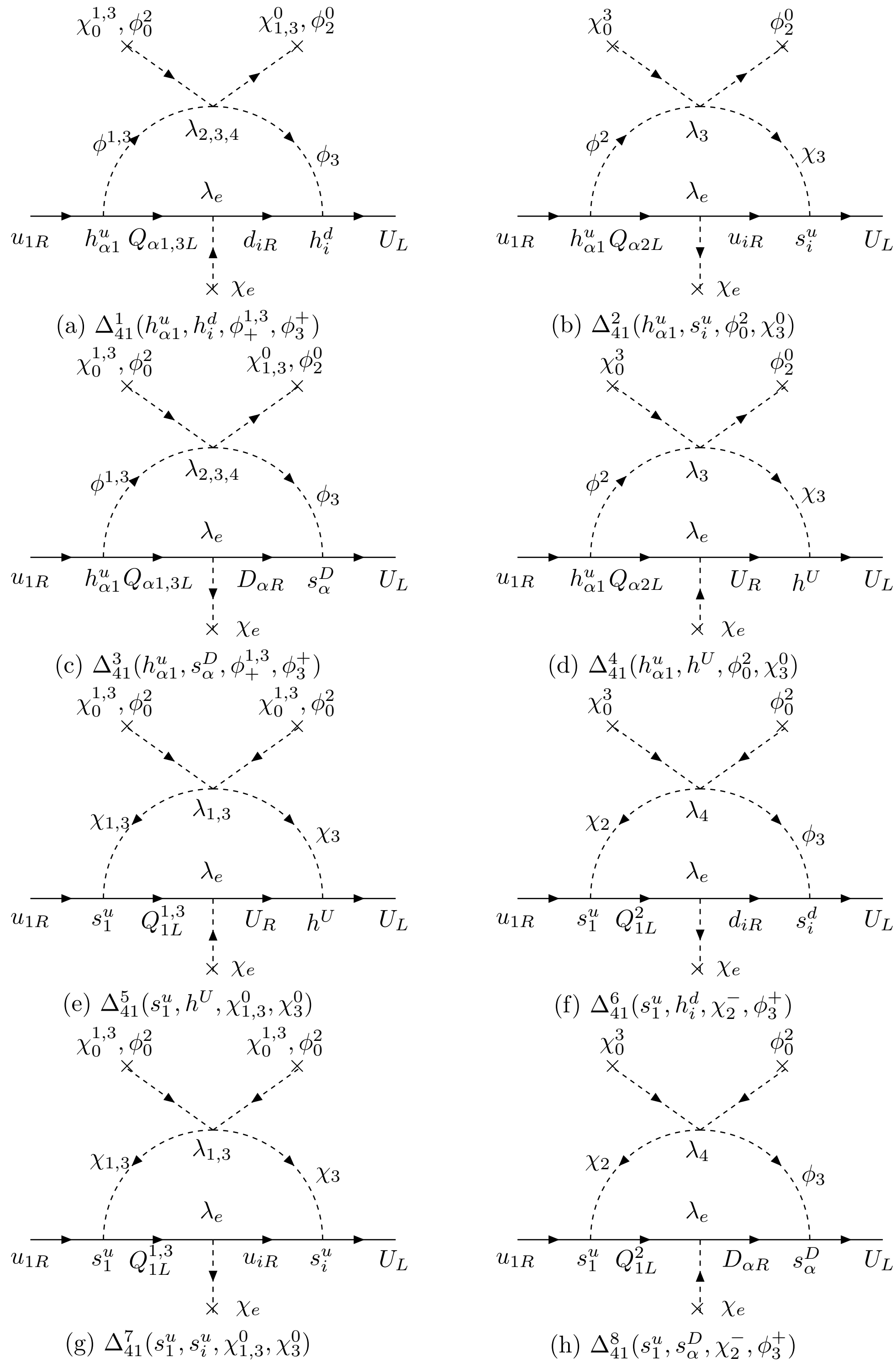
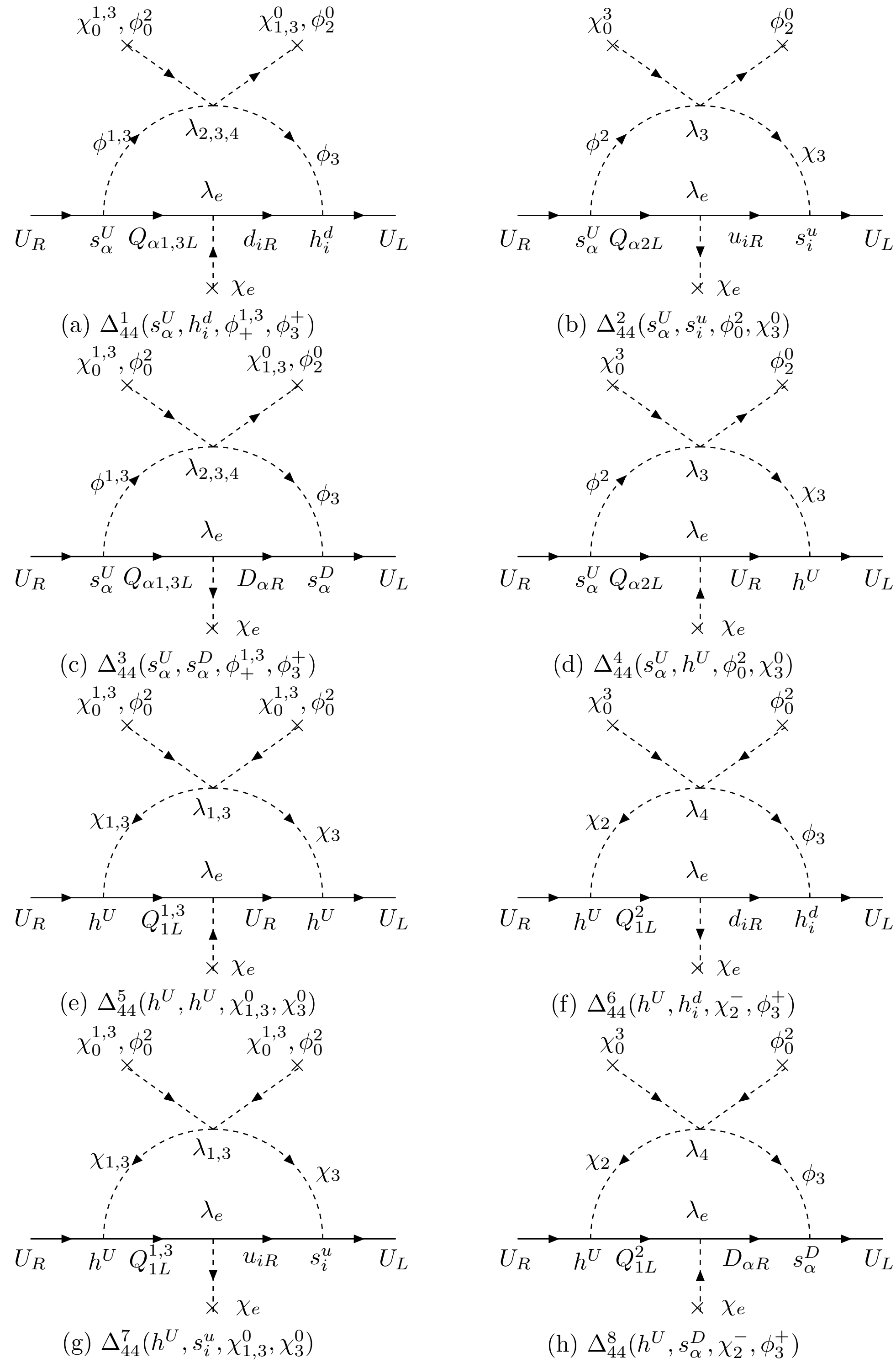


Fig. 2. Corrections to $(M_{uU})_{12}$.



(a) $\Delta_{41}^1(h_{\alpha 1}^u, h_i^d, \phi_+^{1,3}, \phi_3^+)$
(b) $\Delta_{41}^2(h_{\alpha 1}^u, s_i^u, \phi_0^2, \chi_3^0)$
(c) $\Delta_{41}^3(h_{\alpha 1}^u, s_\alpha^D, \phi_+^{1,3}, \phi_3^+)$
(d) $\Delta_{41}^4(h_{\alpha 1}^u, h^U, \phi_0^2, \chi_3^0)$
(e) $\Delta_{41}^5(s_1^u, h^U, \chi_{1,3}^0, \chi_3^0)$
(f) $\Delta_{41}^6(s_1^u, h_i^d, \chi_2^-, \phi_3^+)$
(g) $\Delta_{41}^7(s_1^u, s_i^u, \chi_{1,3}^0, \chi_3^0)$
(h) $\Delta_{41}^8(s_1^u, s_\alpha^D, \chi_2^-, \phi_3^+)$

Fig. 3. Corrections to $(M_{uU})_{21}$.



(a) $\Delta_{44}^1(s_\alpha^U, h_i^d, \phi_+^{1,3}, \phi_3^+)$ (b) $\Delta_{44}^2(s_\alpha^U, s_i^u, \phi_0^2, \chi_3^0)$

(c) $\Delta_{44}^3(s_\alpha^U, s_\alpha^D, \phi_+^{1,3}, \phi_3^+)$ (d) $\Delta_{44}^4(s_\alpha^U, h^U, \phi_0^2, \chi_3^0)$

(e) $\Delta_{44}^5(h^U, h^U, \chi_{1,3}^0, \chi_3^0)$ (f) $\Delta_{44}^6(h^U, h_i^d, \chi_2^-, \phi_3^+)$

(g) $\Delta_{44}^7(h^U, s_i^u, \chi_{1,3}^0, \chi_3^0)$ (h) $\Delta_{44}^8(h^U, s_\alpha^D, \chi_2^-, \phi_3^+)$

Fig. 4. Corrections to $(M_{uU})_{22}$.