Proc. Natl. Conf. Theor. Phys. **36** (2011), pp. 278-282

MAGNETIC SCATTERING OF POLARIZED NEUTRONS AND POLARIZATION VECTOR OF SCATTERING NEUTRONS IN FERROMAGNETIC CRYSTALS

NGUYEN THANH NGA, NGUYEN DINH DUNG
Department of physics, college of Science, VNU

Abstract. *In this note, the magnetic scattering of polarized neutrons on ferromagnetic crystals is studied. In order to study this problem the method of nuclear optics of polarized matter has been used. We obtained the analytical expression for the differential magnetic scattering cross-section of polarized neutrons and polarization vector of magnetic scattering neutrons in ferromagnetic crystals.*

I. INTRODUCTION

In order to study of crystal structure, the method of nuclear optics of polarized matter have been used. This method have been used in works [1,2,3,4,5]. In this note, we study the differential cross-section of magnetic scattering of polarized neutrons and polarization vector of magnetic scattering neutrons in ferromagnetic crystal. We showed that, they have important information about correlative function of electron lattice nodes spins

II. THE DIFFERENTIAL MAGNETIC SCATTERING CROSS-SECTION OF POLARIZED NEUTRONS IN FERROMAGNETIC CRYSTALS

Suppose there is a stream of polarized neutrons falling on the ferromagnetic crystals that have polarized electrons. The differential cross-section of magnetic scattering per unit solid angle, per unit energy, is given by:

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{m^2}{(2\pi)^3\hbar^5}\frac{p'}{p}\int\limits_{-\infty}^{+\infty} dt e^{\frac{i}{\hbar}(E_{p'}-E_p)r}.Sp\left\{\rho_\sigma\rho_e\left\langle V^+_{p'p}V_{p'p}(t)\right\rangle\right\} \tag{1}$$

where: ρ_σ - density matrix of spin of neutrons; ρ_e - density matrix of spin of electrons $E_{p'}, E_p$ - energy of coming neutrons and scattering neutrons.

$$\rho_\sigma = \frac{1}{2}(\vec{I} + \vec{p}_0\vec{\sigma})$$

$\vec{p}_0$: polarization vector of neutron.
We consider to magnetic scattering of neutron, therefore we only consider potential of magnetic interaction:

$$V_{p'p} = -\frac{4\pi\hbar^2}{m}r_0\gamma\frac{1}{2}\sum_j F_j(\vec{q})e^{i\vec{q}\vec{R}_j} \times (\vec{S}_j, \vec{\sigma} - (\vec{e}\vec{\sigma})\vec{e}) \tag{2}$$

where: $\vec{R}_j$ - location vector of nucleus j
$\vec{q} = \vec{p} - \vec{p'}$- scattering vector
$\vec{e} = \vec{q}/q$ - unit scattering vector
$\overrightarrow{S_j}$ - spin of lattice point j
$F_j(\vec{q}) = \int \psi_j^* \sum_{\upsilon}^{Z_j} \frac{e^{i\vec{q}\vec{r}_\mu(\vec{s}_\mu \vec{S}_j)}}{S_j(S_j+1)} \psi_j d\tau_j$
ψ_j - wave function of electron in atom j
$\vec{s}_\mu$ - spin of electron μ in atom j
We denote:

$$L_j = (\vec{S}_j, \vec{\sigma} - (\vec{e}\vec{\sigma})\vec{e}) \quad (3)$$

$$\vec{M}_j = (\vec{S}_j - (\vec{e}\vec{S}_j)\vec{e}) \quad (4)$$

Then we have

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{m^2}{(2\pi)^3\hbar^5}\frac{p'}{p}\int_{-\infty}^{+\infty} dte^{\frac{i}{\hbar}(E_{p'}-E_p)t}.Sp\left\{\rho_\sigma \rho_e \left\langle A \right\rangle\right\} X_{jj'}(\vec{q},t) \quad (5)$$

where:

$$A = \left(\frac{4\pi\hbar^2}{m} r_0 \gamma \frac{1}{2}\right)^2 \sum_{jj'} F_j(\vec{q}) F_{j'}(\vec{q}) L_j(0) L_{j'}(t)$$

$$X_{jj'}(\vec{q},t) = \langle e^{-i\vec{q}\vec{R}_j(0)} e^{i\vec{q}\vec{R}_{j'}(t)} \rangle$$

In addition, we have:

$$\frac{1}{2} Sp\left\{L_1 L_2\right\} = \left(\vec{M}_1 \vec{M}_2\right) \quad (6)$$

$$\frac{1}{2} sp\left\{(\vec{p}\vec{\sigma})\; L_1 L_2\right\} = i\left[\vec{M}_1 \times \vec{M}_2\right]\vec{p} \quad (7)$$

Using (3),(4),(6) and (7), we can calculate trace in (5) and obtain:

$$Sp\left\{\rho_\sigma \rho_e \langle A \rangle\right\} = \frac{4\pi^2\hbar^4}{m^2} {r_0}^2 \gamma^2 \sum_{jj'} F_j(\vec{q}) F_{j'}(\vec{q}) \left\{\left\langle \overrightarrow{M_j(0)}\overrightarrow{M_{j'}}(t)\right\rangle + i\left\langle\left[\overrightarrow{M_j}(0) \times \overrightarrow{M_{j'}}(t)\right]\right\rangle \overrightarrow{p_0}\right\} \quad (8)$$

We now calculate expression (8) for ferromagnetic crystal:
In this case, one has:

$$\overrightarrow{S_j} = S_j^z \vec{m} + \frac{1}{2} S_j^+ \vec{m}^- + \frac{1}{2} S_j^- \vec{m}^+ \quad (9)$$

where: $\vec{m}^\pm = \vec{m}_x \pm i\vec{m}_y$; $\vec{m}_x$; $\vec{m}_y$ are unit vector along axis x and axis y

$$\vec{m} = [\vec{m}_x \times \vec{m}_y]$$

Corresponding to (9), $\vec{M}_j$ can be written in the form:

$$\overrightarrow{M_j} = S_j^z \vec{\mu} + \frac{1}{2} S_j^+ \vec{\mu}^- + \frac{1}{2} S_j^- \vec{\mu}^+ \quad (10)$$

where:

$$\vec{\mu} = \vec{m} - (\vec{e}\vec{m})\,\vec{e}; \quad \vec{\mu}^{\pm} = \vec{m}^{\pm} - \left(\vec{e}\vec{m}^{\pm}\right)\vec{e} \tag{11}$$

For the Heisenberg model, we have:

$$\left\langle \overrightarrow{M_j}(0) \right\rangle = \left\langle \vec{M}_j(t) \right\rangle = S_j^z \vec{\mu} \tag{12}$$

For ferromagnetic crystals, cross correlation functions are equal to 0:

$$\left\langle S_j^z(0) S_{j'}^{\pm}(t) \right\rangle = \left\langle S_j^{\pm}(0) S_{j'}^z(t) \right\rangle = \left\langle S_j^{\pm}(0)\, S_{j'}^{\pm}(t) \right\rangle = 0 \tag{13}$$

and:

$$\overrightarrow{\mu}^2 = 1 - (\overrightarrow{e}\,\overrightarrow{m})^2; \left[\overrightarrow{\mu}^+ \times \overrightarrow{\mu}^-\right] = -2i\,(\overrightarrow{e}\,\overrightarrow{m})\,\overrightarrow{e}; \left(\overrightarrow{\mu}^+ \overrightarrow{\mu}^-\right) = 1 + (\overrightarrow{e}\,\overrightarrow{m})^2 \tag{14}$$

Using (10),(13),(14), we obtain:

$$\langle \vec{M}_j(0)\vec{M}_{j'}(t)\rangle = \left\langle S_j^z(0) S_{j'}^z(t) \right\rangle \left[1 - (\vec{e}\vec{m})^2\right] + \tfrac{1}{4} \left\langle S_j^+(0) S_{j'}^-(t) \right\rangle \left[1 + (\vec{e}\vec{m})^2\right] +$$

$$+\frac{1}{4} \left\langle S_j^-(0) S_{j'}^+(t) \right\rangle \left[1 + (\vec{e}\vec{m})^2\right] \tag{15}$$

and:

$$i\langle\left[\vec{M}_j(0) \times \vec{M}_{j'}(t)\right]\rangle\vec{p}_0 = \frac{1}{2}\left\{\left\langle S_j^-(0) S_{j'}^+(t)\right\rangle (\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0}) - \left\langle S_j^+(0) S_{j'}^-(t)\right\rangle (\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0})\right\} \tag{16}$$

Inserting (15) and (16) into (8), we obtain:

$$Sp\left\{\rho_\sigma \rho_e \langle A\rangle\right\} = \tfrac{4\pi^2\hbar^4}{m^2} {r_0}^2 \gamma^2 \sum_{jj'} F_j(\vec{q})\, F_{j'}(\vec{q}) \left\{\left\langle S_j^z(0) S_{j'}^z(t)\right\rangle \left[1 - (\vec{e}\vec{m})^2\right] +\right.$$

$$\tfrac{1}{4}\left\langle S_j^+(0) S_{j'}^-(t)\right\rangle \left[1 + (\vec{e}\vec{m})^2\right] + \tfrac{1}{4}\left\langle S_j^-(0) S_{j'}^+(t)\right\rangle \left[1 + (\vec{e}\vec{m})^2\right] +$$

$$\left.\tfrac{1}{2}\left\langle S_j^-(0) S_{j'}^+(t)\right\rangle (\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0}) - \tfrac{1}{2}\left\langle S_j^+(0) S_{j'}^-(t)\right\rangle (\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0})\right\}$$

Finally,we obtain the differential magnetic scattering cross-section of polarized neutron in ferromagnetic crystal

$$\tfrac{d^2\sigma}{d\Omega dE_{p'}} = \tfrac{1}{2\pi\hbar} r_0^2 \gamma^2 \tfrac{p'}{p} \int\limits_{-\infty}^{+\infty} dt e^{\frac{i}{\hbar}(E_{p'} - E_p)t} \cdot \sum_{jj'} F_j(\vec{q})\, F_{j'}(\vec{q}) \left\{\left\langle S_j^z(0) S_{j'}^z(t)\right\rangle \left(1 - (\vec{e}\vec{m})^2\right)\right.$$

$$+ \tfrac{1}{4}\left\langle S_j^+(0) S_{j'}^-(t)\right\rangle \left[\left(1 + (\vec{e}\vec{m})^2\right) - 2\,(\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0})\right] +$$

$$\left.\tfrac{1}{4}\left\langle S_j^-(0) S_{j'}^+(t)\right\rangle \left(1 + (\vec{e}\vec{m})^2\right) + 2\,(\vec{e}\vec{m})\,(\vec{e}\overrightarrow{p_0})\right\} X_{j'j}(q,t)$$

III. POLARIZATION VECTOR OF MAGNETIC SCATTERING NEUTRON IN FERROMAGNETIC CRYSTAL

Polarization vector of magnetic scattering neutron in crystal is described by formula:

$$\vec{P} = \frac{\int\limits_{-\infty}^{+\infty} dt Sp \left\{ \rho_\sigma \rho_e \left\langle V_{p'p}^{+} \vec{\sigma} V_{p'p}(t) \right\rangle \right\} e^{\frac{i}{\hbar}(E_{p'}-E_p)t}}{\int\limits_{-\infty}^{+\infty} dt Sp \left\{ \rho_\sigma \rho_e \left\langle V_{p'p}^{+} V_{p'p}(t) \right\rangle \right\} e^{\frac{i}{\hbar}(E_{p'}-E_p)t}} \tag{17}$$

Denominator is calculated in section I. We now need to find the numerator of (17). We can show:

$$\frac{1}{2} Sp\{ \, L_1 \vec{\sigma} L_2 \, \} = -i \, [\, \vec{M}_1 \times \vec{M}_2 \,] \tag{18}$$

$$\frac{1}{2} Sp\left\{ \left(\overrightarrow{p} \, \overrightarrow{\sigma} \right) L_1 \, \overrightarrow{\sigma} L_2 \right\} = \overrightarrow{M_1} \left(\overrightarrow{M_2} \overrightarrow{p} \right) + \left(\overrightarrow{M_1} \overrightarrow{p} \right) \overrightarrow{M_2} - \overrightarrow{p} \left(\overrightarrow{M_1} \overrightarrow{M_2} \right) \tag{19}$$

Using (18) and (19), we obtain:

$$Sp\left\{ \rho_\sigma \rho_e V_{p'p}^{+} \vec{\sigma} V_{p'p}(t) \right\} = \frac{4\pi^2\hbar^4}{m^2} {r_0}^2 \gamma^2 \sum_{jj'} F_j\left(\vec{q}\right) F_{j'}\left(\vec{q}\right) \left[-i \left\langle \left[\overrightarrow{M_j}(0) \times \overrightarrow{M_{j'}}\left(t\right) \right] \right\rangle \right.$$
$$\left. + \left\langle \overrightarrow{M_j(0)} \right\rangle \left(\left\langle \overrightarrow{M_{j'}}(t) \right\rangle \overrightarrow{p_0} \right) + \left(\left\langle \overrightarrow{M_j(0)} \right\rangle \overrightarrow{p_0} \right) \left\langle \overrightarrow{M_{j'}}(t) \right\rangle - \overrightarrow{p_0} \left\langle \overrightarrow{M_j}(0) \overrightarrow{M_{j'}}(t) \right\rangle \right] X_{j'j}\left(q,t\right)$$

Using (12),(15) and (16) for ferromagnetic crystal, we obtain:

$$Sp\left\{ \rho_\sigma \rho_e V_{p'p}^{+} V_{p'p}(t) \right\} =$$
$$\frac{4\pi^2\hbar^4}{m^2} {r_0}^2 \gamma^2 \sum_{jj'} F_j\left(\vec{q}\right) F_{j'}\left(\vec{q}\right) \left\{ 2S^2(T) \overrightarrow{\mu} \left(\overrightarrow{\mu} \overrightarrow{p_0} \right) - \left\langle S_j^z(0) S_{j'}^z(t) \right\rangle \left(1 - \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} \right] -$$
$$- \frac{1}{4} \left\langle S_j^{+}(0) S_{j'}^{-}(t) \right\rangle \left[\left(1 + \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} - 2 \left(\overrightarrow{e} \overrightarrow{m} \right) \overrightarrow{e} \right] -$$
$$- \frac{1}{4} \left\langle S_j^{-}(0) S_{j'}^{+}(t) \right\rangle \left[\left(1 + \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} + 2 \left(\overrightarrow{e} \overrightarrow{m} \right) \overrightarrow{e} \right] \right\} X_{j'j}\left(q,t\right)$$

So, polarization vector of magnetic scattering neutron in ferromagnetic crystals can be described by the following formula:

$$\overrightarrow{p} = \frac{\overrightarrow{p_1} + \overrightarrow{p_2}}{\int\limits_{-\infty}^{+\infty} dt Sp \left\{ \rho_\sigma \left\langle V_{p'p}^{+} V_{p'p}(t) \right\rangle \right\} e^{\frac{i}{\hbar}(E_{p'}-E_p)t}} \tag{20}$$

Where:

$$\vec{p}_1 = \frac{1}{2} \int\limits_{-\infty}^{\infty} dt . e^{\frac{i}{\hbar}(E_{p'}-E_p)} \sum_{jj'} F_j\left(\vec{q}\right) F_{j'}\left(\vec{q}\right) \times$$
$$\times \left[\left\langle S_j^{+}(0) S_{j'}^{-}(t) \right\rangle \left(\overrightarrow{e} \overrightarrow{m} \right) \overrightarrow{e} - \left\langle S_j^{-}(0) S_{j'}^{+}(t) \right\rangle \left(\overrightarrow{e} \overrightarrow{m} \right) \overrightarrow{e} \right] X_{j'j}\left(q,t\right)$$

$$\vec{p}_2 = \int\limits_{-\infty}^{\infty} dt . e^{\frac{i}{\hbar}(E_{p'}-E_p)t} \sum_{jj'} F_j\left(\vec{q}\right) F_{j'}\left(\vec{q}\right) \left\{ 2S^2(T) \overrightarrow{\mu} \left(\overrightarrow{\mu} \overrightarrow{p_0} \right) - \left\langle S_j^z(0) S_{j'}^z(t) \right\rangle \left(1 - \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} \right.$$
$$\left. - \frac{1}{4} \left\langle S_j^{+}(0) S_{j'}^{-}(t) \right\rangle \left(1 + \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} - \frac{1}{4} \left\langle S_j^{-}(0) S_{j'}^{+}(t) \right\rangle \left(1 + \left(\overrightarrow{e} \overrightarrow{m} \right)^2 \right) \overrightarrow{p_0} \right\} X_{j'j}\left(q,t\right)$$

IV. CONCLUSION

In this note, we obtain the analytical expressions for:
i) The differential magnetic scattering cross-section of polarized neutron in ferromagnetic crystals.
ii) For the above formulas one can set the information about the lattice spin correlation functions.
In the limit of unpolarized neutron we recover the result of Idumop-Oderop[3]

REFERENCES

[1] V. G. Baryshevsky, *Nuclear Optics of Polarized Matter* (1976) Minsk.
[2] P. Mazur, D. L. Mills, *Phys. Rev. B* **26** (1981) 51-57.
[3] I. A. Idumov, R. Oderop, *Magnetic neutrons optics,* Moskow, Science, (1996)
[4] Nguyen Dinh Dung, *Vestnic BGU* **1** (1987) 61-62.
[5] Luong Minh Tuan, Nguyen Thi Thu Trang, Nguyen Dinh Dung, *VNU. JOURNAL OF SCIENCE: Mathematics-Physics* **T.XXII, N0 2AP** (2006) 178-181.

Received 30-09-2011.