Proc. Natl. Conf. Theor. Phys. **36** (2011), pp. 274-277

POLARIZATION VECTOR OF SCATTERING NEUTRONS IN CRYSTAL WITH MAGNETIC HELICOIDAL STRUCTURE

PHAM THI THU HA, NGUYEN DINH DUNG
Department of Physics , College of Science, VNU

Abstract. *In this note, the differential magnetic scattering cross-section of polarized neutrons in crystal with magnetic Helicoidal structure is studied. In order to study this problem the method of nuclear optics of polarized matter have been used. We obtained the differential magnetic scattering cross-section of polarized neutrons and the change of neutron polarization.*

I. INTRODUCTION

In order to study the crystal structure the method of nuclear optics of polarized matter have been used. This method has been used to study polarization of atomic nucleus and correlation function of nuclear spins in works [1,2,3,4,5]. In this note, we study the differential magnetic scattering cross-section of polarized neutrons and polarization vector of scattering neutrons in crystal with magnetic Helicoidal structure.

II. THE DIFFERENTIAL CROSS-SECTION OF MANEGTIC SCATTERING OF POLARIZED NEUTRONS IN CRYSTAL WITH MEGNETIC HELICOIDAL STRUCTURE

Suppose there is a stream of polarized neutron falling on the magnetic Helicoidal structure crystal that has polarized electrons. The differential magnetic scattering cross-section per unit solid angle, per unit energy, is given by [1]:

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{m^2}{(2\pi)^3\hbar^5}\frac{p'}{p}\int\limits_{-\infty}^{+\infty} dte^{\frac{i}{\hbar}(E_{p'}-E_p)t}.Sp\left\{\rho_\sigma\rho_e\left\langle V^+_{p'p}V_{p'p}(t)\right\rangle\right\} \tag{1}$$

where: ρ_σ : density matrix of spin of neutrons; ρ_e: density matrix of spin of electrons; $E_p, E_{p'}$: energy of coming neutrons and scattering neutrons; $\rho_\sigma = \frac{1}{2}(\vec{I}+\vec{p}_0\vec{\sigma})$: polarization vector of neutron.

We only consider potential of magnetic interaction.The matrix element of transition is given by:.

$$V_{p'p} = -\frac{4\pi\hbar^2}{m}r_0\gamma\frac{1}{2}\sum_j F_j(\vec{q})e^{i\vec{q}\vec{R}_j} \times (\vec{S}_j, \vec{\sigma} - (\vec{e}\vec{\sigma})\vec{e}) \tag{2}$$

Where: $\vec{R}_j$ - location vector of nucleus j
$\vec{q} = \vec{p} - \vec{p'}$- scattering vector
$\vec{e} = \frac{\vec{q}}{q}$: unit scattering vector
$\overrightarrow{S_j}$: spin of lattice point j

$\vec{s}$: spin of coming neutron

$F_j(\vec{q}) = \int \psi_j^* \sum\limits_{\upsilon}^{Z_j} \frac{e^{i\vec{q}\vec{r}_\upsilon(\vec{s}_\upsilon \vec{S}_j)}}{S_j(S_j+1)} \psi_j d\tau_j$

ψ_j: wave function of electron in atom j

$\vec{s}_\nu$: spin of electron in atom j

In addition, use the following notations:

$$L_j = (\vec{S}_j, \vec{\sigma} - (\vec{e}\vec{\sigma})\vec{e}) \tag{3}$$

$$\vec{M}_j = (\vec{S}_j - (\vec{e}\vec{S}_j)\vec{e}) \tag{4}$$

We can calculate expression (1) for magnetic Helicoidal crystal structure:

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{m^2}{(2\pi)^3\hbar^5}\frac{p'}{p}\int\limits_{-\infty}^{+\infty} dte^{\frac{i}{\hbar}(E_{p'}-E_p)t}.Sp\left\{\rho_\sigma\rho_e\left\langle A\right\rangle\right\}X_{jj'}\left(\vec{q},t\right) \tag{5}$$

where:

$$A = \left(\frac{2\pi\hbar^2}{m}r_0\gamma\right)^2 \sum_{jj'} F_j(\vec{q})F_{j'}(\vec{q})L_j(0)L_{j'}(t)$$

$$X_{jj'}\left(\vec{q},t\right) = \langle e^{-i\vec{q}\vec{R}_j(0)}e^{i\vec{q}\vec{R}_{j'}(t)}\rangle$$

where we have used:

$$\frac{1}{2}sp\ \left\{(\vec{p}\vec{\sigma})\ L_1L_2\right\} = i\left[\vec{M}_1\times\vec{M}_2\right]\vec{p} \tag{6}$$

$$\frac{1}{2}Sp\left\{L_1L_2\right\} = \left(\vec{M}_1\vec{M}_2\right) \tag{7}$$

We can calculate trace in (5) and obtain:

$$sp\left\{\rho_\sigma\rho_e\left\langle A\right\rangle\right\} = \left(\frac{2\pi\hbar^2}{m}r_0\gamma\right)^2\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)T_1 \tag{8}$$

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{1}{2\pi\hbar}r_0^2\gamma^2\frac{p'}{p}\int\limits_{-\infty}^{+\infty} dte^{\frac{i}{\hbar}(E_{p'}-E_p)t}\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right).T_1.X_{j'j}\left(\vec{q},t\right) \tag{9}$$

where:

$$T_1 = \left\{\left\langle\overrightarrow{M_j(0)}\overrightarrow{M_{j'}}\left(t\right)\right\rangle + i\left\langle\left[\overrightarrow{M_j}(0)\times\overrightarrow{M_{j'}}\left(t\right)\right]\right\rangle\overrightarrow{p_0}\right\}$$

In the magnetic Helicoidal crystal structure, vector of spin of atom can be described by formula:

$$\overrightarrow{S}_j = \frac{1}{2}Se^{-i\vec{k}_0\vec{R}_j}\vec{m}^+ + \frac{1}{2}Se^{i\vec{k}_0\vec{R}_j}\vec{m}^- \tag{10}$$

Where:

$\vec{m}^+ = \vec{m}_x + i\vec{m}_y; \vec{m}^- = \vec{m}_x - i\vec{m}_y$; $\vec{m}_x$ and $\vec{m}_y$ are unit vectors along axis x and y.

$\vec{\mu} = \vec{m} - \left(\vec{e}\vec{m}\right)\vec{e}$

$\vec{\mu}^\pm = \vec{m}^\pm - \left(\vec{e}\vec{m}^\pm\right)\vec{e}$

And $\vec{m}$ is a unit vector along the symmetric axis of the magnetic Helicoidal crystal structure.
Corresponding to (10), $\overrightarrow{M}_j$ can be expanded by the following formula:

$$\overrightarrow{M_j} = \frac{1}{2}S\left[e^{-i\vec{k}_0\vec{R}_j}\vec{\mu}^{+} + e^{i\vec{k}_0\vec{R}_j}\vec{\mu}^{-}\right] \tag{11}$$

Using (8) and (11), we obtain:

$$Sp\left\{\rho_\sigma\rho_e\langle A\rangle\right\} = \frac{\pi^2\hbar^4}{m^2}{r_0}^2\gamma^2S^2\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)T_2 \tag{12}$$

$$T_2 = \left[\left(\vec{\mu}^{+}\vec{\mu}^{-}\right) + i\vec{p}_0\left[\vec{\mu}^{+}\times\vec{\mu}^{-}\right]\right]e^{-i\vec{k}_0\left(\vec{R}_j-\vec{R}_{j'}\right)} + \left[\left(\vec{\mu}^{+}\vec{\mu}^{-}\right) - i\vec{p}_0\left[\vec{\mu}^{+}\times\vec{\mu}^{-}\right]\right]e^{i\vec{k}_0\left(\vec{R}_j-\vec{R}_{j'}\right)}$$

After lengthy calculation we have got the differential scattering cross-section of polarized neutrons in crystal with magnetic Helicoidal structure as follows

$$\frac{d^2\sigma}{d\Omega dE_{p'}} = \frac{1}{8\pi\hbar}r_0^2\gamma^2\frac{p'}{p}\int_{-\infty}^{+\infty}dte^{\frac{i}{\hbar}\left(E_{p'}-E_p\right)t}S^2\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)X_{jj'}\left(\vec{q},t\right)T_2 \tag{13}$$

III. POLARIZATION VECTOR OF SCATTERING NEUTRONS IN CRYSTAL WITH MAGNETIC HELICOIDAL STRUCTURE.

Polarization vector of scattering neutrons in crystal $\overrightarrow{p}$ can be calculated by the following formula [1,2]:

$$\vec{p} = \frac{\int\limits_{-\infty}^{+\infty}dtSp\left\{\rho_\sigma\rho_e\left\langle V^{+}_{p'p}\overrightarrow{\sigma}V_{p'p}(t)\right\rangle\right\}e^{\frac{i}{\hbar}(E_{p'}-E_p)t}}{\int\limits_{-\infty}^{+\infty}dtSp\left\{\rho_\sigma\rho_e\left\langle V^{+}_{p'p}V_{p'p}(t)\right\rangle\right\}e^{\frac{i}{\hbar}(E_{p'}-E_p)t}} \tag{14}$$

where we used:

$$\frac{1}{2}Sp\left\{\left(\overrightarrow{p}\,\overrightarrow{\sigma}\right)L_1\overrightarrow{\sigma}L_2\right\} = \overrightarrow{M_1}\left(\overrightarrow{M_2}\,\overrightarrow{p}\right) + \left(\overrightarrow{M_1}\,\overrightarrow{p}\right)\overrightarrow{M_2} - \overrightarrow{p}\left(\overrightarrow{M_1}\overrightarrow{M_2}\right) \tag{15}$$

$$\frac{1}{2}Sp\left\{L_1\vec{\sigma}L_2\right\} = -i\left[\vec{M}_1\times\vec{M}_2\right] \tag{16}$$

We can calculate the numerator in the formula (14)

$$Sp\left\{\rho_\sigma\rho_e\left\langle V^{+}_{p'_p}\vec{\sigma}V_{p'_p}\right\rangle\right\} = \frac{4\pi^2\hbar^4}{m^2}r_0^2\gamma^2\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)X_{j'j}\left(\vec{q},t\right)T_3 \tag{17}$$

where:

$$T_3 = -i\left\langle\left[\overrightarrow{M_j}\times\overrightarrow{M_{j'}}(t)\right]\right\rangle + \left\langle\overrightarrow{M_j}\right\rangle\left(\left\langle\overrightarrow{M_{j'}}(t)\right\rangle\overrightarrow{p_0}\right) +$$
$$+\left(\left\langle\overrightarrow{M_j}\right\rangle\overrightarrow{p_0}\right)\left\langle\overrightarrow{M_{j'}}(t)\right\rangle - \overrightarrow{p_0}\left\langle\overrightarrow{M_j}\overrightarrow{M_{j'}}(t)\right\rangle$$

Then the polarization vector of scattering neutrons for magnetic Helicoidal crystal structure is given by:

$$Sp\left\{\rho_\sigma\rho_e\left\langle V^+_{p'p}\vec{\sigma}V_{p'p}\right\rangle\right\} = \frac{\pi^2\hbar^4}{m^2}{r_0}^2\gamma^2S^2\sum_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)T_4 \tag{18}$$

where:

$$T_4 = \left\{\vec{\mu}^+\left(\vec{\mu}^-\vec{p}_0\right) - i\left[\vec{\mu}^+\times\vec{\mu}^-\right] + \vec{\mu}^-\left(\vec{\mu}^+\vec{p}_0\right) - \vec{p}_0\left(\vec{\mu}^+\vec{\mu}^-\right)\right\}e^{-i\vec{k}_0\left(\vec{R}_j-\vec{R}_{j'}\right)} +$$
$$+\left\{\vec{\mu}^+\left(\vec{\mu}^-\vec{p}_0\right) - i\left[\vec{\mu}^-\times\vec{\mu}^+\right] + \vec{\mu}^-\left(\vec{\mu}^+\vec{p}_0\right) - \vec{p}_0\left(\vec{\mu}^+\vec{\mu}^-\right)\right\}e^{i\vec{k}_0\left(\vec{R}_j-\vec{R}_{j'}\right)}$$

Using (11),(13) and (14), we can calculate the polarization vector of neutron in magnetic Helicoidal crystal structure, which is given by:

$$\vec{p} = \frac{\int\limits_{-\infty}^{+\infty} dt e^{\frac{i}{\hbar}\left(E_{p'}-E_p\right)t}S^2\sum\limits_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)X_{jj'}\left(\vec{q},t\right)T_4}{\int\limits_{-\infty}^{+\infty} dt e^{\frac{i}{\hbar}\left(E_{p'}-E_p\right)t}S^2\sum\limits_{jj'}F_j\left(\vec{q}\right)F_{j'}\left(\vec{q}\right)X_{jj'}\left(\vec{q},t\right)T_2} \tag{19}$$

IV. CONCLUSION

We obtain the analytical expression for:

i) The differential magnetic scattering cross-section of polarized neutrons in crystal with magnetic Helicoidal structure.

ii) The polarization vector of magnetic scattering in crystal with magnetic Helicoidal structure.

For these expression, one can get useful information of lattice spin correlation function.

In the limit of unpolarized neutrons, we rederived the Idumov-Oderop result[2].

REFERENCES

[1] V. G. Baryshevsky, *Nuclear Optics of Polarized Matter* (1976) Minsk.
[2] I. A. Idumov, R.Oderop, *Magnetic neutron optics* (1966) Science, Moskow.
[3] P. Mazur, D. L. Mills, *Phys. Rev.* **26** (1981) 9.
[4] Nguyen Dinh Dung, *VNU J. Sci.* **1** (1993) 56.
[5] Ly Cong Thanh, Nguyen Thi Khuyen, Nguyen Dinh Dung, *VNU J. Sci Mathematics. Physics* **T.XXII, N.0 AP** (2006) 154156.

Received 30-09-2011.