Proc. Natl. Conf. Theor. Phys. **36** (2011), pp. 234-238

SIMPLE MODEL FOR MARKET RETURNS DISTRIBUTION

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Abstract. *It has been observed that the distribution of stock market returns are convergent from Boltzmann distribution to Gaussian asymptotic one. We proposed a new and simple dynamic model to describes this convergence by time parameter, with the introduction of the relaxation time concept for a market. The results showed that our model fits well with the financial market data. Relaxation time value is little in the stable period, and big in the crisis period of the market.*

I. INTRODUCTION

It has been sixteen years[1] since the first time the name "econophysics" was introduced. Sixteen years is quite a long time for a man, but a short time for a oak, as well as for a new research branch. Although that, there has been more and more interests of physicist in to econophysics. There are also considerable successes of econophysicist[2], from both physics point of view and economic point of view.

The main method used in econophysics is statistical one. Both statistical mechanics and economics study big ensembles: collections of states of a system or market returns[3]. The fundamental law of equilibrium statistical mechanics is Boltzmann one, which measures probability for the distribution of states of a system. At a well-defined temperature T, it gives the probability that the system is in the specified state. Temperature plays an important role, the higher temperature is, the crowder state the system is in. But when looking at a stable system, temperature seems to have no effect on the distribution of the states. The mathematician always believe in an Gaussian distribution of a stable system.

Make an observation in the distribution of returns on stock markets, econophysicists realized the transition from Boltzmann distribution to Gaussian one[4]. For a very short time period, the distribution of returns can be described by Boltzmann distribution with high kurtosis. For a longer time period, this distribution tends to converge to the Gaussian one with smoother peak and the probability to have big returns closes to zero.

Some models had been proposed based on the mathematical calculation[4], trying to describe theses distributions. But there still are some missing links between the Boltzmann phase and the Gaussian one. That's why we propose in this paper a new simple model which traces out the transition of market returns distribution. The model has been built based on the NASDAQ database, describes the distribution of market returns by time parameter, with the introduction of the relaxation time concept for a market.

Some modeling results showed that the model is simple, and promise great advantage of application while it fits well with the real data.

II. TIME EVOLUTION OF MARKET RETURNS DISTRIBUTION

Returns r, known as the difference of price of a financial asset over a period of time, is one of the most used values in financial studies. Look at the volatility of Dow Jones Industrial Average (DJIA) returns (2006/2011), it could be easily remarked the relationship between returns fluctuation and financial crises

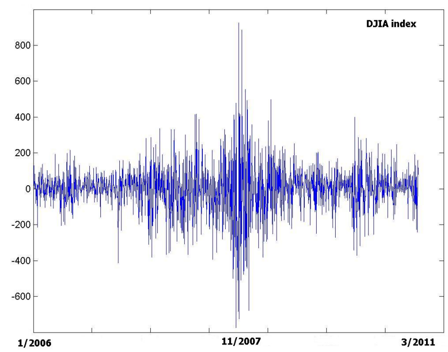

Fig. 1. Volatility of DJIA index returns from 2006 to 2011

That's why the transition of returns distribution is the objects of many econophysics studies[3, 5, 6, 7], including this paper.

In probability theory, the Gaussian distribution is a continuous probability distribution that is often used as a first approximation to describe real-valued random variables that tend to cluster around a single mean value. Theoretically, Gaussian distribution could be used to describe returns when study the market for long time enough, also means, when the returns set is big enough. For a short time study, means for little set of returns, this set of values could be considered as discontinuous one.

Boltzmann distribution is a certain distribution function or probability measure for the distribution of the states of a system. The shape of Boltzmann distribution is antisymmetric, the maximum of probability is usually found at a non-zero value. There are some similar behaviors between returns distribution and Boltzmann one. Considering the returns in short time period, its values can be considered as discontinuous, and the most possible return value must difference from zero (if not, there is no gain no lost for every trading). For long time period study, return values set is much bigger, the values are continuous, the market tends to stable state, the return varies around an average value. The distribution of market returns tends to converge to Gaussian distribution.

With real values data from DJIA and NASDAQ, distribution of returns has been shown as normal distribution.

DJIA index returns from 2007 to 2011 respects well Gaussian law

Returns in one day of Alcoa (Aluminum Company of America) evolves from Boltzmann distribution (30 minutes) to Gaussian one (4 hours).

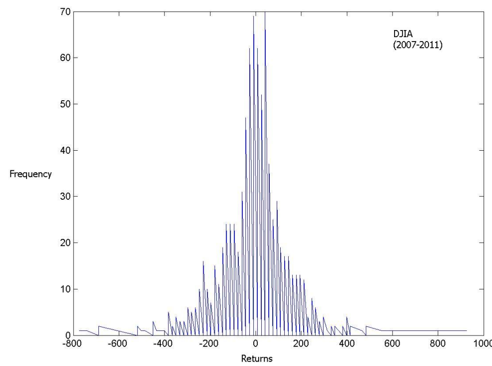

Fig. 2. DJIA index returns from 2007 to 2011

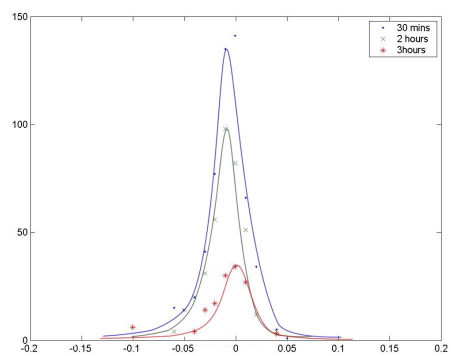

Fig. 3. Distribution of Alcoa returns in one day (2011)

This phenomenon is well observed in every trading studies. And in the next steps, we propose a simple model to describe it mathematically.

III. SIMPLE MODEL FOR MARKET RETURNS DISTRIBUTION

For short time period, the return distributes following Boltzmann law

$$P_B(r) = C_B e^{-r/T}, \tag{1}$$

where P_B is probability to have the return r in the asset of returns; T is an effective temperature; C_B is normalizing constant. By definition, $\int_{-\infty}^{+\infty} P_B(r)dr = 1$.

After several times the returns re-distributes following the Gaussian distribution

$$P_G(r) = C_G e^{-r^2/\sigma^2}, \tag{2}$$

where P_G is probability to have the return r in the asset of returns; σ^2 is variance; C_G is normalizing constant. By definition, $\int_{-\infty}^{+\infty} P_G(r)dr = 1$.

When time t closes to 0, returns distributes following Boltzmann law. When t get bigger the distribution of returns tends to converge to Gaussian one. So we propose the model

$$P(r,t) = P_B u(t) + P_G v(t), \tag{3}$$

Normalizing condition for the distribution is that $\int_{-\infty}^{+\infty} P(r,t) dr dt = 1$.

$u(t)$ and $v(t)$ describe time dependence of probability function P, and are chosen as

$$\begin{cases} t = 0 & : u(0) = 1, v(0) = 0 \\ t \to \infty & : u(\infty) = 0, v(\infty) = 1 \end{cases}. \tag{4}$$

The simplest solution is that

$$\begin{cases} u(t) = e^{-t/\tau} \\ v(t) = 1 - e^{-t/\tau} \end{cases}, \tag{5}$$

with τ the relaxation time. After τ time the returns distribution evolves from Boltzmann distribution to Gaussian one.

$$P(r,t) = C_B e^{-r/T} e^{-t/\tau} + C_G e^{-r^2/\sigma^2} (1 - e^{-t/\tau}). \tag{6}$$

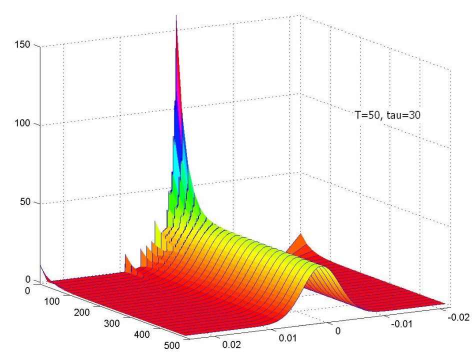

Fig. 4. Distribution of DJIA returns in 1801 days (2009-2011)

When $t = 0$, $P(r,0) \approx P_B(r)$, when $t \to \infty$, $P(r,\infty) \approx P_G(r)$

IV. CONCLUSION AND DISCUSSION

The model has been built and verified based on the database of NASDAQ and Dow Jones Industrial Average[10, 11]. The modeling result fits well with real values distribution.

There has been introduced three measurable parameters: effective temperature T, variance σ and relaxation time τ which help to quantify the chaotic state of the market. Variance σ can be also considered as the market stability factor. The model will provide a simple and useful tool for the financial analyst.

This paper has just presented the simplest model to describe market fluctuation. In fact financial return are known to be non-gaussian and exhibit fat-tailed distribution[8, 9]. In the next publication, we will evaluate the model and built the theory for it to have a better description on market fluctuation.

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Received 30-09-2011.