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AMPLITUDE AND PHASE DYNAMICS OF SPIN-DEGENERATED POLARITON CONDENSATE IN SEMICONDUCTOR MICROCAVITIES

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Abstract. *The complex Gross-Pitaevskii equations for amplitude and phase of spin-degenerated polaritons condensate in semiconductor microcavities are built within spinor polariton model. These equations have been obtained by adiabatic elimination the corresponding three-point polarizations with p-p scattering as the dominant mechanism. If neglecting scattering terms, our results is agreement with calculation of M.O.Borgh et al* [9]. *Together with the solutions the Boltzmann equations for the excited states, our result will form the basis for a complete evaluation of the linewidth and the second-order correlation function.*

I. INTRODUCTION

Bose-Einstein condensation of polaritons in semiconductor microcavities have been recently shown in many experiment [1], [2]. Many theoretical calculations have been caried out to confirm existence of the condensated state of microcavity polariton. The result by H.Haug et al for polariton distribution function and chemical potential by using the semiclassical Boltzmann equation [3] is good quantitative agreement with experimental observation by Yamamoto et al [1]. Recently, the interest in the condensed microcavity polaritons shifted toward detailed studies of the dynamics of their coherence properties. The coherence has been found by a Schawlow-Townes decrease in the emission linewidth, but only at slightly higher values of the condensate populations, the linewidth is observed to go through a sharp minimum, after that, the linewidth increase again. The analytical formula and numerical results for linewidth have been evaluated completely by H.Haug et al by using quantum Langevin equation for the coherent condensate amplitude [4]. Another important test of the coherence properties of the condensate is the second-order correlation function. This function have been measured by Deng et al for GaAs mc's [7] and Kasprazk et al in CdTe mc's [8], and have been calculated by many different methods [5], [4], [6]. Coherence of condensed microcavity polaritons has been continued studying for spinor polariton model. A method to derive analytical formula for the linewidth and the second-order correlation function is building condensed amplitude and phase equations.

In this paper, spinor polariton model is studied, with p-p scattering as the dominant mechanism. We build the complex Gross-Piteavskii equations for the ground-state operators. The corresponding three-point polarizations have been eliminated adiabatically to obtain the complex Gross-Pitaevskii equations for amplitude and phase of spin-degenerated polaritons condensate in semiconductor microcavities.

II. DERIVATION OF THE COMPLEX GROSS-PITEAVSKII FOR THE GROUND-STATE OPERATORS

Hamiltonian of spinor polariton in semiconductor microcavity:

$$\begin{aligned} H &= \sum_{\vec{k},s} e_{\vec{k}} b^{+}_{\vec{k},s} b_{\vec{k},s} + \frac{1}{2}\sum_{\vec{k}} \left(\Omega_{\vec{k}} b^{+}_{\vec{k},1} b_{\vec{k},2} + H.c\right) \\ &+\frac{1}{4}\Bigg[\sum_{\vec{k},\vec{k'},\vec{q},s} V(\vec{k}+\vec{q},\vec{k'}-\vec{q},\vec{k},\vec{k'})\left(b^{+}_{\vec{k},s} b^{+}_{\vec{k'},s} b_{\vec{k'}-\vec{q},s} b_{\vec{k}+\vec{q},s}\right) \\ &+ \sum_{\vec{k},\vec{k'},\vec{q},s\neq s'} U(\vec{k}+\vec{q},\vec{k'}-\vec{q},\vec{k},\vec{k'})\left(b^{+}_{\vec{k},s} b^{+}_{\vec{k'},s'} b_{\vec{k'}-\vec{q},s'} b_{\vec{k}+\vec{q},s} + b^{+}_{\vec{k},s} b^{+}_{\vec{k'},s'} b_{\vec{k'}-\vec{q},s} b_{\vec{k}+\vec{q},s'}\right) + H.c\Bigg] \end{aligned}$$

Where $e_{\vec{k}}$ is energy of the lower p branch. $\Omega_{\vec{k}}$ is complex energy, related to the TE-TM splitting. The matrix elements $V\left(\vec{k}+\vec{q},\vec{k'}-\vec{q},\vec{k},\vec{k'}\right)$, $U\left(\vec{k}+\vec{q},\vec{k'}-\vec{q},\vec{k},\vec{k'}\right)$ describe scattering of ps in the relative triplet and singlet configuration, respectively. They are given by:

$$\begin{aligned} V\left(\vec{k},\vec{k'},\vec{k'}-\vec{q},\vec{k}+\vec{q}\right) &= 6E_B a_B^2 u_{\vec{k}+\vec{q}} u_{\vec{k'}-\vec{q}} u_{\vec{k'}} u_{\vec{k}} \\ U\left(\vec{k},\vec{k'},\vec{k'}-\vec{q},\vec{k}+\vec{q}\right) &= -\alpha V\left(\vec{k},\vec{k'},\vec{k'}-\vec{q},\vec{k}+\vec{q}\right), \qquad \alpha > 0 \end{aligned}$$

Where E_B and a_b are the binding energy and Borh radius of exciton in 2D, respectively. The Heisenberg equations for the ground-state operators (with i, j are spin indexes ($i \neq j$))

$$\begin{aligned} \frac{db_{0,i}}{dt} &= -\frac{i}{\hbar} e_0 b_{0,i} - \frac{i}{2\hbar}\Omega_0 b_{0,j} \\ &-\frac{i}{\hbar}\sum_{\vec{k}} V(\vec{k},0,\vec{k},0) n_{\vec{k},i} b_{0,i} - \frac{i}{2\hbar}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0} V(\vec{k}-\vec{q},\vec{q},\vec{k},0) b^{+}_{\vec{k},i} b_{\vec{k}-\vec{q},i} b_{\vec{q},i} \\ &-\frac{i}{\hbar}\sum_{\vec{k}} U(\vec{k},0,\vec{k},0) n_{\vec{k},j} b_{0,i} - \frac{i}{\hbar}\sum_{\vec{k},\vec{q}\neq 0} U(\vec{k}-\vec{q},\vec{k},\vec{q},0) b^{+}_{\vec{k},j} b_{\vec{k}-\vec{q},j} b_{\vec{q},i} \end{aligned} \tag{1}$$

Adiabatic elimination the corresponding three-point polarizations, we have the complex Gross-Piteavskii for the ground-state operators:

$$\begin{aligned} \frac{db_{0,i}}{dt} &= -\frac{i}{\hbar} e_0 b_{0,i} - \frac{i}{2\hbar}\Omega_0 b_{0,j} \\ &-\frac{i}{\hbar}\sum_{\vec{k}} V(\vec{k},0,\vec{k},0) n_{\vec{k},i} b_{0,i} - \frac{i}{\hbar}\sum_{\vec{k}} U(\vec{k},0,\vec{k},0) n_{\vec{k},j} b_{0,i} \\ &+\frac{i}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0} \mid V(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2 \\ &\qquad \left(n_{\vec{k}-\vec{q},i} n_{\vec{q},i} - n_{\vec{k},i}\left(1+n_{\vec{k}-\vec{q},i}\right) - n_{\vec{k},i} n_{\vec{q},i}\right)\left(\frac{P}{\Omega_i}\right) b_{0,i} \end{aligned}$$

$$+\frac{\pi}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0}\mid V(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2$$
$$\left(n_{\vec{k}-\vec{q},i}n_{\vec{q},i}-n_{\vec{k},i}\left(1+n_{\vec{k}-\vec{q},i}\right)-n_{\vec{k},i}n_{\vec{q},i}\right)\delta\left(\Omega_i\right)b_{0,i}$$
$$+\frac{i}{\hbar^2}\sum_{\vec{q}\neq 0}\mid U(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2$$
$$\left(n_{\vec{k}-\vec{q},j}n_{\vec{q},i}-n_{\vec{k},j}\left(1+n_{\vec{k}-\vec{q},j}\right)-n_{\vec{k},j}n_{\vec{q},i}\right)\left(\frac{P}{\Omega_i'}\right)b_{0,i}$$
$$+\frac{\pi}{\hbar^2}\sum_{\vec{q}\neq 0}\mid U(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2$$
$$\left(n_{\vec{k}-\vec{q},j}n_{\vec{q},i}-n_{\vec{k},j}\left(1+n_{\vec{k}-\vec{q},j}\right)-n_{\vec{k},j}n_{\vec{q},i}\right)\delta\left(\Omega_i'\right)b_{0,i} \tag{2}$$

where

$$\Omega_i = \frac{1}{\hbar}\Bigg[e_{\vec{k}}-e_{\vec{q}}-e_{\vec{k}-\vec{q}}+e_0 + \sum_{\vec{k}'}\left[U(\vec{k},\vec{k}',\vec{k},\vec{k}')-U(\vec{q},\vec{k}',\vec{q},\vec{k}')-U(\vec{k}-\vec{q},\vec{k}',\vec{k}-\vec{q},\vec{k}')\right]n_{\vec{k}',j}\Bigg] \tag{3}$$

$$\Omega_i' = \frac{1}{\hbar}\Bigg[e_{\vec{k}}-e_{\vec{q}}-e_{\vec{k}-\vec{q}}+e_0 + \sum_{\vec{k}'}\left[\left(V(\vec{k},\vec{k}',\vec{k},\vec{k}')-V(\vec{k}-\vec{q},\vec{k}',\vec{k}-\vec{q},\vec{k}')\right)n_{\vec{k}',j}-V(\vec{q},\vec{k}',\vec{q},\vec{k}')n_{\vec{k}'i}\right]\Bigg] \tag{4}$$

III. AMPLITUDE AND PHASE DYNAMICS OF SPIN-DEGENERATED POLARITON CONDENSATE

Use the decomposition of the condensate amplitude:

$$b_{0,1}=\rho_{0,1}e^{-i\left(\phi+\frac{\theta}{2}\right)};b_{0,2}=\rho_{0,1}e^{-i\left(\phi-\frac{\theta}{2}\right)} \tag{5}$$

To discuss the dynamics, we using:

$$R=\frac{\rho_{0,1}^2+\rho_{0,2}^2}{2};z=\frac{\rho_{0,1}^2-\rho_{0,2}^2}{2} \tag{6}$$

We take amplitude and phase equations of spin-degenerated polariton condensate:

$$\dot{\theta} = -\frac{\Omega_0 z\cos\theta}{\hbar\sqrt{(R^2-z^2)}}+\frac{2}{\hbar}V(0,0,0,0)\left(1+\alpha\right)z$$
$$+\frac{1}{\hbar}\sum_{\vec{k}\neq 0}V(\vec{k},0,\vec{k},0)\left(1+\alpha\right)\left(n_{\vec{k},1}-n_{\vec{k},2}\right)$$

$$
\begin{aligned}
& -\frac{1}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},0,\vec{q},0)\mid^2 \left(n_{\vec{q},1}^2\left(\frac{P}{\Omega_1}\right) - n_{\vec{q},2}^2\left(\frac{P}{\Omega_2}\right)\right) \\
& +\frac{1}{2\hbar^2}\sum_{\vec{k}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left[\left(\frac{P}{\Omega_1}\right) + \left(\frac{P}{\Omega_1}\right) + 2\left(n_{\vec{q},1}\left(\frac{P}{\Omega_1}\right) + n_{\vec{q},2}\left(\frac{P}{\Omega_2}\right)\right)\right] z \\
& +\frac{1}{2\hbar^2}\sum_{\vec{k}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left[\left(\frac{P}{\Omega_1}\right) - \left(\frac{P}{\Omega_1}\right) + 2\left(n_{\vec{q},1}\left(\frac{P}{\Omega_1}\right) - n_{\vec{q},2}\left(\frac{P}{\Omega_2}\right)\right)\right] R \\
& -\frac{1}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0,\vec{k}\neq 0} \mid V(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2 \\
& \qquad \left[\left(n_{\vec{k}-\vec{q},1}n_{\vec{q},1} - n_{\vec{k},1}\left(1+n_{\vec{k}-\vec{q},1}\right) - n_{\vec{k},1}n_{\vec{q},1}\right)\left(\frac{P}{\Omega_1}\right)\right. \\
& \qquad \left. -\left(n_{\vec{k}-\vec{q},2}n_{\vec{q},2} - n_{\vec{k},2}\left(1+n_{\vec{k}-\vec{q},2}\right) - n_{\vec{k},2}n_{\vec{q},2}\right)\left(\frac{P}{\Omega_2}\right)\right] \\
& -\frac{1}{\hbar^2}\sum_{\vec{q}\neq 0} \mid U(\vec{k},\vec{k}-\vec{q},\vec{q},0)\mid^2 \\
& \qquad \left[\left(n_{\vec{k}-\vec{q},2}n_{\vec{q},1} - n_{\vec{k},2}\left(1+n_{\vec{k}-\vec{q},2}\right) - n_{\vec{k},2}n_{\vec{q},1}\right)\left(\frac{P}{\Omega_1'}\right)\right. \\
& \qquad \left. -\left(n_{\vec{k}-\vec{q},1}n_{\vec{q},2} - n_{\vec{k},1}\left(1+n_{\vec{k}-\vec{q},1}\right) - n_{\vec{k},1}n_{\vec{q},2}\right)\left(\frac{P}{\Omega_2'}\right)\right]
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\dot{z} \;=\; & \frac{\Omega_0}{\hbar}\sqrt{(R^2-z^2)}\sin\theta \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \\
& \qquad \Bigg[\left(n_{\vec{q},1}^2\delta\left(\Omega_1\right) + n_{\vec{q},2}^2\delta\left(\Omega_2\right)\right) \\
& \qquad\quad -2\Big[\left(\delta\left(\Omega_1\right)+\delta\left(\Omega_2\right)\right) + 2\left(n_{\vec{q},2}\delta\left(\Omega_2\right) + n_{\vec{q},1}\delta\left(\Omega_1\right)\right)\Big] R\Bigg] z \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left(n_{\vec{q},1}^2\delta\left(\Omega_1\right) - n_{\vec{q},2}^2\left(\Omega_2\right)\right) R \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left[2\left(n_{\vec{q},2}\delta\left(\Omega_2\right) - n_{\vec{q},1}\delta\left(\Omega_1\right)\right) + \left(\delta\left(\Omega_2\right) - \delta\left(\Omega_1\right)\right)\right] R^2 \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left[2\left(n_{\vec{q},2}\delta\left(\Omega_2\right) - n_{\vec{q},1}\delta\left(\Omega_1\right)\right) + \left(\delta\left(\Omega_2\right) - \delta\left(\Omega_1\right)\right)\right] z^2 \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0,\vec{k}\neq 0} \mid V(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \left[\left(n_{\vec{k}-\vec{q},1}, n_{\vec{q},1} - n_{\vec{k},1}\left(1+n_{\vec{k}-\vec{q},1}\right) - n_{\vec{k},1}n_{\vec{q},1}\right)\delta\left(\Omega_1\right)\right. \\
& \qquad \left. -\left(n_{\vec{k}-\vec{q},2}, n_{\vec{q},2} - n_{\vec{k},2}\left(1+n_{\vec{k}-\vec{q},2}\right) - n_{\vec{k},2}n_{\vec{q},2}\right)\delta\left(\Omega_2\right)\right] R
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\pi}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0,\vec{k}\neq 0} \mid V(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},1}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},1}\Big)\delta\left(\Omega_1\right) \\
& \qquad +\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},2}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},2}\Big)\delta\left(\Omega_2\right)\Big]z \\
& +\frac{\pi}{\hbar^2}\sum_{\vec{q}\neq 0} \mid U(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},1}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},1}\Big)\delta\left(\Omega_1'\right) \\
& \qquad -\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},2}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},2}\Big)\delta\left(\Omega_2'\right)\Big]R \\
& +\frac{\pi}{\hbar^2}\sum_{\vec{q}\neq 0} \mid U(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},1}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},1}\Big)\delta\left(\Omega_1'\right) \\
& \qquad +\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},2}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},2}\Big)\delta\left(\Omega_2'\right)\Big]z \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\dot{R} \;=\; & \frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \Big[\delta\left(\Omega_1\right)+\delta\left(\Omega_2\right)+2\left(n_{\vec{q},1}\delta\left(\Omega_1\right)+n_{\vec{q},2}\left(\Omega_2\right)\right)\Big] \\
& \qquad \left[\frac{\Big(n_{\vec{q},1}^2\delta\left(\Omega_1\right)+n_{\vec{q},2}^2\delta\left(\Omega_2\right)\Big)}{\Big[\delta\left(\Omega_1\right)+\delta\left(\Omega_2\right)+2\left(n_{\vec{q},1}\delta\left(\Omega_1\right)+n_{\vec{q},2}\left(\Omega_2\right)\right)\Big]}R-R^2-z^2\right] \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \Big[2\left(\delta\left(\Omega_2\right)-\delta\left(\Omega_1\right)\right)+4\left(n_{\vec{q},2}\delta\left(\Omega_2\right)-n_{\vec{q},1}\delta\left(\Omega_1\right)\right)\Big]Rz \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{q}\neq 0} \mid V(\vec{q},\vec{q},0,0)\mid^2 \left(n_{\vec{q},1}^2\delta\left(\Omega_1\right)-n_{\vec{q},2}^2\delta\left(\Omega_2\right)\right)z \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0,\vec{k}\neq 0} \mid V(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},1}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},1}\Big)\delta\left(\Omega_1\right) \\
& \qquad +\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},2}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},2}\Big)\delta\left(\Omega_2\right)\Big]R \\
& +\frac{\pi}{2\hbar^2}\sum_{\vec{k}\neq\vec{q},\vec{q}\neq 0,\vec{k}\neq 0} \mid V(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},1}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},1}\Big)\delta\left(\Omega_1\right) \\
& \qquad -\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},2}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},2}\Big)\delta\left(\Omega_2\right)\Big]z \\
& +\frac{\pi}{\hbar^2}\sum_{\vec{q}\neq 0} \mid U(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2 \\
& \qquad \Big[\Big(n_{\vec{k}-\vec{q},2},n_{\vec{q},1}-n_{\vec{k},2}\Big(1+n_{\vec{k}-\vec{q},2}\Big)-n_{\vec{k},2}n_{\vec{q},1}\Big)\delta\left(\Omega_1'\right) \\
& \qquad +\Big(n_{\vec{k}-\vec{q},1},n_{\vec{q},2}-n_{\vec{k},1}\Big(1+n_{\vec{k}-\vec{q},1}\Big)-n_{\vec{k},1}n_{\vec{q},2}\Big)\delta\left(\Omega_2'\right)\Big]R
\end{aligned}
$$

$$+\frac{\pi}{\hbar^2}\sum_{\vec{q}\neq 0}\mid U(\vec{k}-\vec{q},\vec{k},\vec{q},0)\mid^2$$

$$\left[\left(n_{\vec{k}-\vec{q},2},n_{\vec{q},1}-n_{\vec{k},2}\left(1+n_{\vec{k}-\vec{q},2}\right)-n_{\vec{k},2}n_{\vec{q},1}\right)\delta\left(\Omega_1'\right)\right.$$
$$\left.-\left(n_{\vec{k}-\vec{q},1},n_{\vec{q},2}-n_{\vec{k},1}\left(1+n_{\vec{k}-\vec{q},1}\right)-n_{\vec{k},1}n_{\vec{q},2}\right)\delta\left(\Omega_2'\right)\right]z \qquad (9)$$

IV. CONCLUSION

Although we haven't take solutions for the complex Gross-Pitaevskii equations, our result, if neglecting scattering terms, is agreement with caculation of M. O. Borgh et al [9]. Our result, together with the solutions the Boltzmann equations for the excited states, will form the basis for a complete evaluation of the linewidth and the second-order correlation function.

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