Proc. Natl. Conf. Theor. Phys. **35** (2010), pp. 255-259

NEUTRINO MASSES AND MIXING IN THE STANDARD MODEL WITH A_4-FLAVOR SYMMETRY

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Abstract. *Although the Standard Model is very successful, it also leaves many questions unanswered, one of which is massless of neutrinos. In this talk we introduce A_4 - flavour symmetry into the Standard Model with appropriate extension of scalar representations. As a result, the neutrinos gain naturely small masses in agreement with experiment. The neutrino mixing matrix in terms of tribimaximal form is obtained.*

I. INTRODUCTION

The neutrino experiments imply: masses of neutrinos are small, and tribimaximal mixing neutrinos as proposed by Harrison-Perkins-Scott is given by:

$$U_{\mathrm{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1}$$

The theories of neutrinos have recently been in trying to explain this form [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

II. THE MODEL

II.1. Flavour symmetry A_4

The finite group of the even permutation of four objects,A_4, has 12 elements and 4 equivalence classes, with the number of elements $\underline{1}, \underline{4}, \underline{4}, \underline{3}$, respectively. This means that there are 4 irreducible representations, with dimensions ni, such that $\Sigma_i n_i^2 = 12$. There is only one solution: $n_1 = n_2 = n_3 = 1$ and $n_4 = 3$, and the character table of the 4 representations is shown in Table below:

Table 1. Character table of A_4.

Class	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	n
C_1	1	1	1	3	1
C_2	1	ω	ω^2	0	4
C_3	1	ω^2	ω	0	4
C_4	1	1	1	-1	3

The complex number ω is the cube root of unity, i.e., $e^{2\pi i/3}$. Hence $1+\omega+\omega^2=0$. Calling the 4 irreducible representations, and 3 respectively, we have the decomposition:

$$\begin{aligned}\underline{3}\otimes\underline{3} &= \underline{1}(11+22+33)\oplus\underline{1}'(11+\omega^2 22+\omega 33)\\ &\oplus\underline{1}''(11+\omega 22+\omega^2 33)\oplus\underline{3}(23,31,12)\oplus\underline{3}(32,13,21)\end{aligned} \quad (2)$$

The non-Abelian finite group A4 is also the symmetry group of the regular tetrahedron.

II.2. Lepton mass

Under A4, the fermions and scalars of this model transform as follows:

$$\psi_L=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\sim(2,-1,\underline{3}), \quad (3)$$

$$l_{1R}\sim(1,-2,\underline{1}),\qquad l_{2R}\sim(1,-2,\underline{1}'),\qquad l_{3R}\sim(1,-2,\underline{1}''), \quad (4)$$

$$\phi=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\sim(2,1,\underline{3}). \quad (5)$$

The Yukawa interactions are:

$$\mathcal{L}_Y^{lep} = h_1(\overline{\psi_L}\phi)_{\underline{1}}l_{1R}+h_2(\overline{\psi_L}\phi)_{\underline{1}'}l_{2R}+h_3(\overline{\psi_L}\phi)_{\underline{1}''}l_{3R}+H.c. \quad (6)$$

The vacuum expectation value (VEV) of ϕ is (v_1,v_2,v_3) under A_4. It is now assumed that so that A_4 is broken down to Z_3. The mass Lagrangian for the changed leptons is:

$$\begin{aligned}\mathcal{L}_{mass}^{lep} &= h_1 v(\overline{l_{1L}}+\overline{l_{2L}}+\overline{l_{3L}})l_{1R}\\ &+h_2 v(\overline{l_{1L}}+\omega\overline{l_{2L}}+\omega^2\overline{l_{3L}})l_{2R}\\ &+h_3 v(\overline{l_{1L}}+\omega^2\overline{l_{2L}}+\omega\overline{l_{3L}})l_{3R}\\ &+H.c.\end{aligned} \quad (7)$$

then the mass matrix is then diagonalized:

$$U_L^{-1}M^{lep}U_R=\begin{pmatrix}\sqrt{3}h_1 v & 0 & 0\\ 0 & \sqrt{3}h_2 v & 0\\ 0 & 0 & \sqrt{3}h_3 v\end{pmatrix}=\begin{pmatrix}m_e & 0 & 0\\ 0 & m_\mu & 0\\ 0 & 0 & m_\tau\end{pmatrix}, \quad (8)$$

where

$$U_R=1, U_L=\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}\equiv U_{lL}. \quad (9)$$

II.3. Neutrino sector

To obtain arbitrary Majorana neutrino masses, four Higgs doublets are used:

$$\sigma = \begin{pmatrix} \sigma^0_{11} & \sigma^+_{12} \\ \sigma^+_{12} & \sigma^{++}_{22} \end{pmatrix} \sim (3^*, 2, \underline{1}), \tag{10}$$

$$s = \begin{pmatrix} s^0_{11} & s^+_{12} \\ s^+_{12} & s^{++}_{22} \end{pmatrix} \sim (3^*, 2, \underline{3}). \tag{11}$$

The Yukawa interactions are:

$$\mathcal{L}^\nu_Y = x(\bar{\psi}^c_L \psi_L)_{\underline{1}} \sigma + y(\bar{\psi}^c_L \psi_L)_{\underline{3}} s + H.c. \tag{12}$$

The VEV of σ is:

$$\langle\sigma\rangle = \begin{pmatrix} u & 0 \\ 0 & 0 \end{pmatrix}. \tag{13}$$

The VEV of s is put as:

$$s = (\langle s_1\rangle, \langle s_2\rangle, \langle s_3\rangle) \tag{14}$$

with

$$\langle s_2\rangle = \langle s_3\rangle = 0, \langle s_1\rangle = \begin{pmatrix} t & 0 \\ 0 & 0 \end{pmatrix}, \tag{15}$$

so that it is broken down to Z_2 in neutrino sector.

The mass Lagrangian for the neutrinos is:

$$\mathcal{L}^\nu_Y = x(\overline{\nu^c_{1L}}\nu_{1L} + \overline{\nu^c_{2L}}\nu_{2L} + \overline{\nu^c_{3L}}\nu_{3L})u + y\overline{\nu^c_{2L}}\nu_{3L} t + H.c. \tag{16}$$

The mass matrices are then obtained by

$$M_\nu = \begin{pmatrix} xu & 0 & 0 \\ 0 & xu & \frac{1}{2}yt \\ 0 & \frac{1}{2}yt & xu \end{pmatrix} = U_{\nu L} \begin{pmatrix} xu + \frac{1}{2}yt & 0 & 0 \\ 0 & xu & 0 \\ 0 & 0 & xu - \frac{1}{2}yt \end{pmatrix} U^T_{\nu L}, \tag{17}$$

where

$$U^\nu_L = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{18}$$

The mismatch between U_ν and U_{lL} yields the tribimaximal mixing pattern as proposed by Harrison- Perkins- Scott:

$$U_{HPS} = U^\dagger_{lL} U_\nu = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{19}$$

This is a main result of the paper.

II.4. Scalar Potential

We can separate the general scalar potential into

$$V = V^{\phi} + V^{\sigma s}, \tag{20}$$

where

$$\begin{aligned} V^{\phi} &= \mu_{\phi}^2(\phi^+\phi)_{\underline{1}} + \lambda_1^{\phi}[(\phi^+\phi)_{\underline{1}}(\phi^+\phi)_{\underline{1}}] + \lambda_2^{\phi}[(\phi^+\phi)_{\underline{1'}}(\phi^+\phi)_{\underline{1''}}] \\ &+\lambda_3^{\phi}[(\phi^+\phi)_{\underline{3s}}(\phi^+\phi)_{\underline{3a}}] + \lambda_4^{\phi}[(\phi^+\phi)_{\underline{3s}}(\phi^+\phi)_{\underline{3s}} + H.c.], \end{aligned} \tag{21}$$

$$V^{\sigma s} = V(\sigma) + V(s) + V(\sigma, s) + V(\phi,\sigma) + V(\phi, s) + V(\phi,\sigma,s) + \overline{V}, \tag{22}$$

with

$$V(\sigma) = \mu_{\sigma}^2 Tr(\sigma^+\sigma) + \lambda^{\sigma} Tr(\sigma^+\sigma)^2 + \lambda^{'\sigma}[Tr(\sigma^+\sigma)]^2, \tag{23}$$

$$\begin{aligned} V(s) &= TrV(\phi \to s) + \lambda_1^{'s} Tr(s^+s)_{\underline{1}} Tr(s^+s)_{\underline{1}} + \lambda_2^{'s} Tr(s^+s)_{\underline{1'}} Tr(s^+s)_{\underline{1''}} \\ &+\lambda_3^{'s} Tr(s^+s)_{\underline{3s}} Tr(s^+s)_{\underline{3a}} + \lambda_4^{'s}[Tr(s^+s)_{\underline{3s}} Tr(s^+s)_{\underline{3s}} + H.c.], \end{aligned} \tag{24}$$

$$\begin{aligned} V(\sigma, s) &= \lambda_1^{\sigma s} Tr[(\sigma^+ s)_{\underline{3}}(s^+\sigma)_{\underline{3}}] + \lambda_1^{'\sigma s} Tr(\sigma^+ s)_{\underline{3}} Tr(s^+\sigma)_{\underline{3}} \\ &+\lambda_2^{\sigma s} Tr[(\sigma^+\sigma)_{\underline{1}}(s^+s)_{\underline{1}}] + \lambda_2^{'\sigma s} Tr(\sigma^+\sigma)_{\underline{1}} Tr(s^+s)_{\underline{1}} \\ &+[\lambda_3^{\sigma s} Tr[(s^+s)_{\underline{3s}}(s^+\sigma)_{\underline{3}}] + \lambda_3^{'\sigma s} Tr(s^+s)_{\underline{3s}} Tr(s^+\sigma)_{\underline{3}} \\ &+\lambda_4^{\sigma s} Tr[(s^+s)_{\underline{3a}}(s^+\sigma)_{\underline{3}}] + \lambda_4^{'\sigma s} Tr(s^+s)_{\underline{3a}} Tr(s^+\sigma)_{\underline{3}} + H.c] \\ &+[\lambda_5^{\sigma s} Tr[(s^+\sigma)_{\underline{3}}(s^+\sigma)_{\underline{3}}] + \lambda_5^{'\sigma s} Tr(s^+\sigma)_{\underline{3}} Tr(s^+\sigma)_{\underline{3}} + H.c], \end{aligned} \tag{25}$$

$$\begin{aligned} V(\phi,\sigma) &= \lambda_1^{\phi\sigma} Tr[(\sigma^+\phi)_{\underline{3}}(\phi^+\sigma)_{\underline{3}}] + \lambda_1^{'\phi\sigma} Tr(\sigma^+\phi)_{\underline{3}} Tr(\phi^+\sigma)_{\underline{3}} \\ &+\lambda_2^{\phi\sigma} Tr[(\sigma^+\sigma)_{\underline{1}}(\phi^+\phi)_{\underline{1}}] + \lambda_2^{'\phi\sigma} Tr(\sigma^+\sigma)_{\underline{1}} Tr(\phi^+\phi)_{\underline{1}}, \end{aligned} \tag{26}$$

$$\begin{aligned} V(\phi, s) &= \lambda_{11}^{\phi s} Tr[(\phi^+ s)_{\underline{1}}(s^+\phi)_{\underline{1}}] + \lambda_{11}^{'\phi s} Tr(\phi^+ s)_{\underline{1}} Tr(s^+\phi)_{\underline{1}} \\ &+\lambda_{12}^{\phi s} Tr[(\phi^+ s)_{\underline{1'}}(s^+\phi)_{\underline{1''}}] + \lambda_{12}^{'\phi s} Tr(\phi^+ s)_{\underline{1'}} Tr(s^+\phi)_{\underline{1''}} \\ &+\lambda_{13}^{\phi s} Tr[(\phi^+ s)_{\underline{3s}}(s^+\phi)_{\underline{3a}}] + \lambda_{13}^{'\phi s} Tr(\phi^+ s)_{\underline{3s}} Tr(s^+\phi)_{\underline{3a}} \\ &+[\lambda_{14}^{\phi s} Tr[(\phi^+ s)_{\underline{3s}}(s^+\phi)_{\underline{3s}}] + \lambda_{14}^{'\phi s} Tr(\phi^+ s)_{\underline{3s}} Tr(s^+\phi)_{\underline{3s}} + H.c] \\ &+\lambda_{21}^{\phi s} Tr[(\phi^+\phi)_{\underline{1}}(s^+s)_{\underline{1}}] + \lambda_{21}^{'\phi s} Tr(\phi^+\phi)_{\underline{1}} Tr(s^+s)_{\underline{1}} \\ &+\lambda_{22}^{\phi s} Tr[(\phi^+\phi)_{\underline{1'}}(s^+s)_{\underline{1''}}] + \lambda_{22}^{'\phi s} Tr(\phi^+\phi)_{\underline{1'}} Tr(s^+s)_{\underline{1''}} \\ &+\lambda_{23}^{\phi s} Tr[(\phi^+\phi)_{\underline{3s}}(s^+s)_{\underline{3a}}] + \lambda_{23}^{'\phi s} Tr(\phi^+\phi)_{\underline{3s}} Tr(s^+s)_{\underline{3a}} \\ &+[\lambda_{24}^{\phi s} Tr[(\phi^+\phi)_{\underline{3s}}(s^+s)_{\underline{3s}}] + \lambda_{21}^{'\phi s} Tr(\phi^+\phi)_{\underline{3s}} Tr(s^+s)_{\underline{3s}} + H.c.], \end{aligned} \tag{27}$$

$$V(\phi,\sigma,s) = \mu Tr(\phi^+\sigma^+ s\phi) + H.c., \tag{28}$$

$$\overline{V} = \overline{\mu_0}\phi^T\sigma\phi + \overline{\mu_1}\phi^T s_1\phi. \tag{29}$$

III. CONCLUSION

We have shown the neutrinos gain naturely small masses in agreement with experiment. The neutrino mixing matrix in terms of tribimaximal form is obtained. Based on the flavour symmetry A4, we can understand neutrino experiments [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

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Received 30-09-2010.