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GLAUBER TYPE REPRESENTATION FOR THE SCATTERING AMPLITUDE OF HIGH-ENERGY DIRAC PARTICLES ON THE SMOOTH POTENTIAL

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Abstract. *By the functional integration method the deduction of the Glauber-type representation for scattering amplitude of the spin of particles on the smooth potential is performed. This method generalizing the usual operation of taking the square of the Dirac equation is proposed for obtaining the functional integral representation of the amplitude. It has shown that the scattering amplitudes, which have obtained by moving to shell mass, of normal Green function and quadratic Green function, are the same types, and taking into account of spin of particles leads to new term which is responsible for the spin flip in the scattering processes.*

I. INTRODUCTION

The eikonal approximation for the potential scattering amplitude of high-energy and small scattering angles has been investigated by many authors /1-5/. In fact these investigations do not take into account the spin structure of the scattered processes. It has become however well known from experiment that spin effects are of great interest in many processes /5, 6/. Therefore, in this paper we consider the problems generalizing the eikonal approximation with the view to taking into account spin effects, namely we consider the scattering of the Dirac particles on the smooth potential. The exposition proceeds as follows: In the second section we present a method to find exact closed form expression for the Dirac particle Green function in a smooth potential in the formwork of the functional approach. Then by transitioning to the mass shell of the external one particle Green's function we obtain a closed representation for the potential scattering amplitude expressed in the form of functional integrals. For estimating the functional integrals we used the straight line path approximation based on the idea of rectilinear paths of the interacting particles at asymptotically high-energy and small scattering angles. Section 3 is devoted to investigating the asymptotic behavior of this amplitude in high energy and small scattering angles. Finally we discuss the obtained results and possible generalizations of the approach into quantum gravity /6-11/.

II. CLOSED EXPRESSION FOR SCATTERING AMPLITUDE IN THE FORM OF THE FUNCTIONAL FORM

Following /1, 6/ the scattering amplitude of the Dirac particles in the external smooth arbitrary potential , is defined by the formula

$$F\left(p,q|\tilde{A}\right)=\frac{1}{2m}\bar{u}(p)\left[\lim_{p^2,q^2\to m^2}\left(p^2-m^2\right)\left(q^2-m^2\right)G_\lambda\left(q,p|\tilde{A}\right)\right]u(q), \qquad (1)$$

where spinors $\bar{u}(p)$ and $u(p)$ on the mass shell $\left(p^2-m^2\right)$ and $\left(q^2-m^2\right)$, satisfied the Dirac equation and the normalization condition $\bar{u}(p)u(p) = 2m$;

$$G\left(x, y|\tilde{A}\right) = \left[i\gamma_\mu\partial_\mu + m + \lambda\tilde{A}(x)\right] G_\lambda\left(x, y|\tilde{A}\right),$$

the Green function $G_\lambda(x, y|\tilde{A})$ of particle in the external smooth arbitrary potential , and satisfied the quadratic Dirac equation,

$$\left[\left(i\gamma_\mu\partial_\mu\right)^2 + 2\tilde{a}_\mu^{(\lambda)} i\partial_\mu + \tilde{\Phi}_\lambda - m^2\right] G_\lambda\left(x, y|\tilde{A}\right) = -\delta^{(4)}(x-y), \tag{2}$$

$\tilde{\Phi}_\lambda = \left[m(1-\lambda) + \lambda i\gamma_\mu\partial_\mu\right]\tilde{A} + \lambda\tilde{A}^2$, $\tilde{a}_\mu^{(\lambda)} = \frac{1}{2}\left[\tilde{A}\gamma_\mu + \lambda\gamma_\mu\tilde{A}\right]$, where λ is an arbitrary number, the Green function $G(x, y|A)$ satisfies the Dirac equation

$$\left[i\gamma_\mu\partial_\mu - m + \tilde{A}(x)\right] G\left(x, y|\tilde{A}\right) = -\delta^{(4)}(x-y). \tag{3}$$

Note that the choice of the parameter λ is arbitrary and does not effect the physical results, and can be set at any convenient values. In appendix A we show the expression $\int d^4x d^4y e^{ipx-iqy}\tilde{A}(x)G_\lambda\left(x, y|\,\tilde{A}\right) = 0$ has no poles at $p^2 = m^2$. The Green function of the equation (2) $G_\lambda(x, y|\tilde{A})$ and the Green function of the equation (3) $G(x, y|\tilde{A})$ in the mass shell $\left(p^2-m^2\right)$ and $\left(q^2-m^2\right)$, respectively, as it is shown, give the same scattering amplitude. The equality of expressions for these above amplitudes by mean of the perturbation theory is proven in appendix B. Using the closed expressions for Green functions $G_\lambda(x, y|\tilde{A})$ obtained in form functional integrals, we separate out from the Green function $G_\lambda(x, y|\tilde{A})$ with the poles $\left(p^2-m^2\right)^{-1}$ and $\left(q^2-m^2\right)^{-1}$ which cancel against the factors in Eq (1) as we have done in /1, 5, 6/, for the scattering amplitude. We find the following expressions:

$$F\left(p, q|\tilde{A}\right) = \frac{1}{2m}\bar{u}(p)\int d^4x e^{i(p-q)x}\int\left[\delta^4\nu\right]_{-\infty}^{\infty} T_\gamma\left[M(x|0)\int_0^t d\alpha e^{i\alpha\int\limits_{-\infty}^{\infty} M(x|\xi)}\right] u(q), \tag{4}$$

$$M(x|\xi) = \left[2\nu(\xi) + a(\xi)\right]\tilde{a}^{(\lambda)}(x_\xi) + \tilde{\Phi}_\lambda(x_\xi) - \tilde{a}^{(\lambda)2}(x_\xi) - i\partial\tilde{a}^{(\lambda)}(x_\xi),$$

$$x_\xi = x + \left[2p\theta(\xi) + 2q\theta(-\xi)\right]\xi + 2\int\limits_0^\xi \nu(\eta)d\eta; \tag{5}$$

$$\theta(\xi) = \begin{cases} 1, & \xi > 0, \\ 1/2, & \xi = 0, \\ 0, & \xi < 0, \end{cases} \qquad \left[\delta\nu\right]_{-\infty}^{\infty} = \frac{\delta\nu \exp\left\{-i\int\limits_{-\infty}^{\infty}\nu^2(\xi)d\xi\right\}}{\int\delta\nu\exp\left\{-i\int\limits_{-\infty}^{\infty}\nu^2(\xi)d\xi\right\}} \tag{6}$$

T_γ is the symbol of ordering the gamma matrices with respect to the variable ξ.

III. ASYMPTOTIC BEHAVIOR OF THE SCATTERING AMPLITUDE AT HIGH ENERGY

For the calculation of the functional integrals (4) we use the straight line path approximation, i.e. we assume that in the high energy particle scattering on the smooth potential and small scattering angles, one can neglect the dependence on the functional variables $\nu(\eta)$. In other words, it is considered that the main contribution to the functional integrals (4) comes from a trajectory particle moving freely from the momentum $\tilde{p}$ with $\xi > 0$ and momentum $\tilde{q}$ with $\xi < 0$, and passing via the point x with $\xi = 0$.

$$\bar{u}(p) = \bar{\Psi}_p\,(1,\tilde{\sigma}\tilde{p}/|\tilde{p}|)\,\sqrt{m}; \quad u(q) = (1,\,\tilde{\sigma}\tilde{p}/|\tilde{p}|)\,\Psi_q\sqrt{m},\, |\tilde{p}| \approx |\tilde{q}|\,, \tag{7}$$

where $\bar{\Psi}_p$ and Ψ_q are ordinary two-component spinors. To choose the symbol $x_\mu = (t, r)$; $r = (z, x_\perp)$ then the z- axis is chosen along the vector p. Note that the approximate results above shown the scattering amplitude we can notify the eikonal representation of amplitude

$$F(p,q) = 2\pi\delta(p_0 - q_o) f(p,q)$$

$$f(p,q) = -ip_z\bar{\Psi}_p \int d^2x_\perp e^{ix_\perp(p-q)}\left[\bar{\Gamma}(x_\perp) - 1\right]\Psi_q \tag{8}$$

$$\bar{\Gamma}(x_\perp) = \frac{1}{2}(1,\sigma_z)T_\gamma\left[e^{i\hat{X}_\gamma}\right]\begin{bmatrix}1\\ \sigma_z\end{bmatrix} \tag{9}$$

$$\hat{X}_\gamma\,(\tilde{x}_\perp) = \int_{-\infty}^{\infty}\frac{dz}{2p_z}\left[2p\tilde{a}^{(\lambda)}(r) + \left(\Phi_\lambda(r) + i\partial\tilde{a}^{(\lambda)}(r)\right)\right] \tag{10}$$

where $\tilde{a}^{(\lambda)}, \tilde{\Phi}_\lambda$ is defined by Eq (2). When $p_o \to \infty$ to $p\tilde{a}^{(\lambda)} \approx p_0$, all terms, which do not depend on energy in square brackets of the expression (10), can be ignored we obtain

$$\hat{X}_\gamma\,(\tilde{x}_\perp) = \int_{-\infty}^{\infty}\frac{dz}{p_z}p\tilde{a}^{(\lambda)}(r) \tag{11}$$

When accepting the formula (8,9,10) in the Eq. (4) we have applied a replacement $2p_z\xi = z$, assuming the matrix in $\hat{X}_\gamma\,(\tilde{x}_\perp)$ which will depend on z in the ordering variables. Using the arbitrariness in the value parameter squared λ, for simplicity of the expression $\tilde{a}_\mu^\lambda$ it is convenient to choice $\lambda = 1,\ \lambda = -1$. Other choice λ only complicates the calculations. Let us consider a now some concrete potential; in first case we consider the vector potential $\tilde{a}_\mu^{(+)} = A_\mu(x)$ whose 4-components satisfy the condition $pA \approx 0\,(p_0)$. For phase χ there is the right formula (11). In scattering on vector potential we have,

$$\Gamma\,(\tilde{x}_\perp) = e^{i\chi(\rho)}, \chi\,(\tilde{x}_\perp) = \int_{-\infty}^{\infty} dz\frac{pA\,(r)}{p_z} \tag{12}$$

in asymptotic the scattering amplitude fully do not depends the spin. This expression is agreed with result obtained in / 8/. In second case we consider the scattering of particle with anomaly magnetic moment in external electromagnetic field $\tilde{\mathrm{A}}=\sigma_{\mu\nu}\mathrm{F}_{\mu\nu}$.

$$\mathrm{F}_{\mu\nu}=\frac{1}{2}\left(\partial_\mu \mathrm{A}_\nu-\partial_\nu \mathrm{A}_\mu\right),\tilde{\mathrm{a}}_\mu^{(-)}=\frac{1}{2}\left[\tilde{\mathrm{A}}(\mathrm{x})\gamma_\mu-\gamma_\mu\tilde{\mathrm{A}}(\mathrm{x})\right],\mathrm{A}_\mu=(0,0,0,-\mathrm{U}) \tag{13}$$

The arrangement of the γ- matrix in the above equation (9) and (10) becomes impossible. For $\hat{\Gamma}$ -matrix we find

$$\bar{\Gamma}(\mathrm{x}_\perp)=\frac{1}{2}\left(1,\sigma_\mathrm{z}\right)\mathrm{T}_\gamma\exp\left\{\mathrm{i}\frac{\mathrm{p}_0}{\mathrm{p}_\mathrm{z}}\int\limits_{-\infty}^{\infty}\mathrm{dz}\left[\gamma(\mathrm{z})\mathrm{n}\,\partial_\rho\mathrm{U}(\mathrm{r})+\left(\gamma_3-\gamma_0\frac{\mathrm{p}_\mathrm{z}}{\mathrm{p}_0}\right)\partial_\mathrm{z}\mathrm{U}(\mathrm{r})\right]\right\}\begin{pmatrix}1\\ \sigma_\mathrm{z}\end{pmatrix} \tag{14}$$

Note that, the expansion of the last expression (14) in a series in powers of $\left(\gamma_3-\gamma_0\frac{\mathrm{p}_\mathrm{z}}{\mathrm{p}_0}\right)$ is actually respect to $\left(\gamma_3-\gamma_0\frac{\mathrm{p}_\mathrm{z}}{\mathrm{p}_0}\right)^2=-\frac{\mathrm{m}^2}{\mathrm{p}_\mathrm{z}^2}$ since

$$(1,\sigma_\mathrm{z})\left(\gamma_3-\gamma_0\frac{\mathrm{p}_\mathrm{z}}{\mathrm{p}_0}\right)\begin{pmatrix}1\\ \sigma_\mathrm{z}\end{pmatrix}=0.$$

Therefore, the second term in the argument of the exponent in (14) can be ignored altogether. Then for $\hat{\Gamma}$ we have

$$\hat{\Gamma}(\mathrm{x}_\perp)=\mathrm{ch}\chi(\rho)+\left[\mathrm{n}\times\sigma\right]_\mathrm{z}\mathrm{sh}\,\chi,\ \ \chi(\rho)=\frac{2\mathrm{p}_0^2}{\mathrm{p}_\mathrm{z}}\int\limits_{-\infty}^{\infty}\mathrm{dz}\partial_\rho\varphi(\mathrm{r}) \tag{15}$$

After substitution of (14) into (8) and integration in (8) by angle variable can be readily performed, and for the scattering amplitude we find

$$\mathrm{f}(\mathrm{p},\mathrm{q})=\bar{\Psi}_\mathrm{p}\left[\mathrm{f}_0(|\mathrm{p}-\mathrm{q}|)+(\sigma\cdot\mathrm{m})\mathrm{f}_1(|\mathrm{p}-\mathrm{q}|)\right]\Psi_\mathrm{q};m=\frac{p\times q}{|p\times q|}, \tag{16}$$

where $f_0\left(|\mathrm{p}-\mathrm{q}|\right)=f_0\left(\Delta\right)$ and $f_1\left(|\mathrm{p}-\mathrm{q}|\right)=f_1\left(\Delta\right)$ describe processes without and with spin flip, respectively, and they are given by,

$$f_0(\Delta)=\frac{\mathrm{p}_\mathrm{z}}{2\pi\mathrm{i}}\int\limits_0^{\infty}\rho\mathrm{d}\rho\mathrm{J}_0(\rho\Delta)\left[\mathrm{ch}\,\chi(\rho)-1\right], \tag{17}$$

$$f_1(\Delta)=\frac{\mathrm{p}_\mathrm{z}}{2\pi}\int\limits_0^{\infty}\rho\mathrm{d}\rho\mathrm{J}_1(\rho\Delta)\mathrm{sh}\,\chi(\rho). \tag{18}$$

Comparing the results (15,16,17,18) that have been obtained with those of other authors, those resulted in our work has shown that spin of particles leads to new term which is responsible for spin flip in the scattering processes. They have been outstanding. The results obtained here can be applied in quantum gravity, as they here are similar to those we see in the scattering of Dirac particle in the gravitational potential /6-11/ in near future.

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APPENDIX A

In this section, we will prove that the expression $\int \mathrm{d^4xd^4ye^{ipx-iqy}}\tilde{\mathrm{A}}(\mathrm{x})\mathrm{G}_\lambda\left(\mathrm{x,y}|\,\tilde{\mathrm{A}}\right)$ has no poles at $\mathrm{p^2= m^2}$, it means,

$$\mathrm{J} = \lim_{\mathrm{p^2,q^2\to m^2}} \left(\mathrm{p^2 - m^2}\right) \int \mathrm{d^4xd^4ye^{ipx-iqy}}\tilde{\mathrm{A}}(\mathrm{x})\mathrm{G}_\lambda\left(\mathrm{x,y}|\,\tilde{\mathrm{A}}\right) = 0 \tag{19}$$

To eliminate pole $\mathrm{p^2= m^2}$ we use the functional integral representation in quadratic Green function:

$$\mathrm{G}_\lambda\left(\mathrm{x,y}|\,\tilde{\mathrm{A}}\right) = \mathrm{i}\int_0^\infty \mathrm{dse}^{-\mathrm{im}^2 s} \int [\delta\nu]_0^{\mathrm{s}} \mathrm{C}\,[\nu]_0^{\mathrm{s}}\, \delta^4\left(\mathrm{x - y} - 2\int_0^{\mathrm{s}} \nu\,(\eta)\,\mathrm{d}\eta\right) \tag{20}$$

where, $\mathrm{C}\,[\nu]_0^{\mathrm{s}} = T_\gamma \exp\left\{\mathrm{i}\int_0^{\mathrm{s}} \mathrm{d}\xi\left[2\nu\tilde{\mathrm{a}}\,(\mathrm{x}_\xi) + \tilde{\Phi}\,(\mathrm{x}_\xi) - \tilde{\mathrm{a}}^2\,(\mathrm{x}_\xi) - \mathrm{i}\partial\tilde{\mathrm{a}}\,(\mathrm{x}_\xi)\right]\right\}$ with $\mathrm{x}_\xi = \mathrm{x} + 2\int_0^{\xi} \nu\,(\eta)\,\mathrm{d}\eta$. We have calculated previous integral by Delta-function over x-variable, and subtituted $\nu(\xi) \to \nu(\xi) + \mathrm{p}$.

Because of $\int_0^{\mathrm{s}} (\nu\,(\eta) + \mathrm{p})\,\mathrm{d}\eta = \mathrm{ps} + \int_0^{\mathrm{s}} \nu\,(\eta)\,\mathrm{d}\eta$

and by using formula $\lim_{\varepsilon\to 0,\ \mathrm{p^2\to m^2}} \left(\mathrm{p^2 - m^2}\right)\mathrm{i}\int_0^\infty \mathrm{dse}^{\mathrm{is(p^2-m^2)}-\varepsilon \mathrm{s}}\mathrm{f\,(s) = f\,(\infty)}$, with $\int [\delta\nu]_0^{\mathrm{s}} \int \mathrm{d}^4\mathrm{y}\mathrm{e}^{\mathrm{i}(\mathrm{p}-\mathrm{q})\mathrm{y}} \tilde{\mathrm{A}} \left(\mathrm{y} + 2\mathrm{ps} + 2\int\limits_0^{\mathrm{s}} \nu(\eta)\,\mathrm{d}\eta\right) \mathrm{e}^{\mathrm{ip}2\int\limits_0^{\mathrm{s}} \nu(\eta)\mathrm{d}\eta} \mathrm{C}[\nu + \mathrm{p}]_0^{\mathrm{s}}$, where $f(\mathrm{s})$ is a limited function and limit $\lim \tilde{\mathrm{A}} \left(\mathrm{y} + 2\mathrm{ps} + 2\int\limits_0^{\mathrm{s}} \nu(\eta)\,\mathrm{d}\eta\right) = 0$ as in the scattering problem $\tilde{\mathrm{A}}(\infty) = 0$. So that (19) is equal to zero.

APPENDIX B

We will prove that the scattering amplitudes, which have obtained by moving to mass shell, of ordinary and quadratic Green functions are the same types by perturbation theory in this section Let consider the third term of (1) which can be performed:

$$2\mathrm{mF}^{(3)}\left(\mathrm{p},\mathrm{q}|\,\tilde{\mathrm{A}}\right) = \bar{\mathrm{u}}(\mathrm{p}) \int \mathrm{d}^4\mathrm{k}_1\mathrm{d}^4\mathrm{k}_2 \left[\mathrm{L}^{(3)} + \mathrm{K}^{(3)}\right]\mathrm{u}(\mathrm{q}), \tag{21}$$

where, $L^{(3)}$ is corresponding with the linear interaction in potential

$$\left[2\tilde{\mathrm{a}}_\mu \mathrm{i}\partial_\mu + \tilde{\Phi}\right] = \left[\mathrm{m}(1-\lambda) + \lambda \mathrm{i}\tilde{\partial}_{\mathrm{x}}\right]\tilde{\mathrm{A}} + \mathrm{i}\tilde{\mathrm{A}}\hat{\partial}_{\mathrm{x}}$$

and $K^{(3)}$) is contribution of quadratic potential $\lambda\tilde{\mathrm{A}}^2$. The expression of linear potential $L^{(3)}$ is:

$$\mathrm{L}^{(3)} = \frac{\left[\tilde{\mathrm{A}}_1\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right) + \lambda(\hat{\mathrm{p}} - \mathrm{m})\tilde{\mathrm{A}}_1\right]}{\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1)^2\right]} \left[\tilde{\mathrm{h}}_2\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right) - \right.$$

$$\left. -\lambda\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right)\tilde{\mathrm{A}}_2\right] \times \frac{\left[\tilde{\mathrm{A}}_3(\mathrm{m} + \hat{\mathrm{q}}) - \lambda\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right)\tilde{\mathrm{A}}_3\right]}{\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1+\mathrm{k}_2)^2\right]}$$

with $\tilde{\mathrm{A}}_{1,2} = \tilde{\mathrm{A}}(\mathrm{k}_{1,2})$; $\tilde{\mathrm{A}}(\mathrm{k}) = \int \mathrm{d}^4\mathrm{x}\mathrm{e}^{\mathrm{ikx}}\tilde{\mathrm{A}}(\mathrm{x})$; $\tilde{\mathrm{A}}_3 = \tilde{\mathrm{A}}(-\mathrm{k}_1 - \mathrm{k}_2 - \mathrm{p} + \mathrm{q})$ and $\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right)^2 = \mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1)^2$; $\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right)^2 = \mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1+\mathrm{k}_2)^2$, so we have

$$\mathrm{L}^{(3)} = \frac{2\mathrm{m}\tilde{\mathrm{A}}_1\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right)\tilde{\mathrm{A}}_2\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right)\tilde{\mathrm{A}}_3}{\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1)^2\right]\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1+\mathrm{k}_2)^2\right]} - \frac{\lambda\tilde{\mathrm{A}}_1\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right)\tilde{\mathrm{A}}_2\tilde{\mathrm{A}}_3}{\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1)^2\right]} -$$

$$-\lambda\tilde{\mathrm{A}}_1\tilde{\mathrm{A}}_2\frac{\left[2\mathrm{m} - \lambda\left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right)\right]\tilde{\mathrm{A}}_3}{\left[\mathrm{m}^2 - (\mathrm{p}+\mathrm{k}_1+\mathrm{k}_2)^2\right]} \tag{22}$$

Let us consider the expression of $K^{(3)}$):

$$K^{(3)} = \lambda\frac{\tilde{A}_1\left(\hat{p} + \hat{k}_1 + m\right)\tilde{A}_2\tilde{A}_3}{\left[m^2 - (p+k_1)^2\right]} + \lambda\frac{2m\tilde{A}_1\tilde{A}_2\tilde{A}_3}{\left[m^2 - (p+k_1+k_2)^2\right]} -$$

$$-\lambda^2 \frac{\tilde{A}_1 \tilde{A}_2 \left(\hat{p} + \hat{k}_1 + \hat{k}_2 + m\right) \tilde{A}_3}{\left[m^2 - (p + k_1 + k_2)^2\right]}$$

It is easy to realize that two latter term of (22) are equal to $K^{(3)})$ with opposite sign. So (21) becomes:

$$\mathrm{F}^{(3)} \left(\mathrm{p}, \mathrm{q} | \tilde{\mathrm{A}}\right) = \bar{\mathrm{u}}(\mathrm{p}) \int \mathrm{d}^4\mathrm{k}_1 \mathrm{d}^4\mathrm{k}_2 \frac{\tilde{\mathrm{A}}_1 \left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \mathrm{m}\right) \tilde{\mathrm{A}}_2 \left(\hat{\mathrm{p}} + \hat{\mathrm{k}}_1 + \hat{\mathrm{k}}_2 + \mathrm{m}\right) \tilde{\mathrm{A}}_3}{\left[\mathrm{m}^2 - (\mathrm{p} + \mathrm{k}_1)^2\right] \left[\mathrm{m}^2 - (\mathrm{p} + \mathrm{k}_1 + \mathrm{k}_2)^2\right]} \mathrm{u}(\mathrm{q})$$

And it coincides with the third term of the expression of scattering amplitude, which has obtained by moving to mass shell of ordinary Green function.

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