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ELECTROPHONON RESONANCE IN COMPOSITIONAL SEMICONDUCTOR SUPERLATTICES

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Abstract. *The electrophonon resonances (EPR) and optical detected electrophonon resonance (ODEPR) effects in compositional semiconductor superlattices (CSSL) are investigated by using the quantum kinetic equation for electrons in the case of electron - longitudinal optical (LO) phonon scattering. General analytic expressions for the absorption power are obtained. We also obtain the photon energy dependence of optical detected electrophonon resonance condition for a specific* $GaAs$-$Ga_{1-x}Al_xAs$ *superlattice. In particular, anomalous behaviors of the ODEPR effect such as the splitting of ODEPR peaks for incident photon energy are discussed. This raises a possibility of detecting experimentally electric subbands in CSSLs by utilizing EPR effects.*

I. INTRODUCTION

Electrophonon resonance (EPR) phenomena arises from an electron scattering due to the absorption and emission of phonons when the energy difference of two electric subbands equals the optical phonon energy. Scattering process with optical phonons is dominant in limiting the mobility of electrons in the polar semiconductors for temperature $T > 50$ K. The EPR was introduced by Bryskin and Firsov [1] who have predicted EPR for nondegenerate semiconductors in a very strong electric field and EPR phenomena in low-dimensional electron gas systems has generated considerable interest in the recent years [2, 3, 4].

The study of EPR effect in the modern quantum devices is very important in understanding transport phenomena in semiconductor. For electron motion in low-dimensional electron systems, the investigation of multi-subband transport effects such as the effective mass, the energy levels, and the electron-phonon interaction has received some attention. The EPR effect is the electrical equivalent of magnetophonon resonance (MPR) [5]. The MPR can be observed directly through a study of the electron cyclotron resonance linewidth and effective mass, i.e., the so-called optically detected magnetophonon resonance (ODMPR) [6, 7], as was demonstrated in 3D semiconductor systems of GaAs by Hai and Peeters [6] and in 2D semiconductor systems of GaAs-Al_xGa_{1-x}As heterojunctions by Barnes [8]. The study of the optically detected electrophonon resonance (ODEPR) effects remains the problem to be solved.

The purpose of the present work is to study EPR and ODEPR effects in compositional semiconductor superlattices (CSSL). Special attention is given to the anomalous behavior of condition for appearance of the EPR and ODEPR peaks from the selection rules. The paper is organized as follows. In the next section, we obtain a general analytical expression for optical absorption coefficient by using the QKE for electrons in CSSLs. Numerical results are presented for a specific CSSL of the GaAs/GaAlAs and anomalous behaviors of the ODEPR effect such as the splitting of ODEPR peaks for incident photon energy are discussed in Sec. III. Finally, conclusions on the possibility of detecting electric subbands in CSSLs experimentally by utilizing EPR effects are given in Sec. IV.

II. GENERAL ANALYTIC EXPRESSION OF ACF IN A CSSL

We use a simple model of CSSLs, in which electron gas is confined by an additional potential along the z-direction and electrons are free on the $(x-y)$ plane. It is well known that the motion of an electron is confined in each layer of the system and that its energy spectrum is quantized into discrete levels in the z-direction. The electron energy on the n miniband $(n = 1, 2, 3, ...)$ of a CSSL depends upon its wave vector $\vec{k}$ through the relation [9]:

$$\varepsilon_n(\vec{k}) = \frac{\hbar^2 \vec{k}_\perp^2}{2m} + \varepsilon_n - \Delta_n \cos(k_z d), \tag{1}$$

where d is the superlattice period, m is the electron effective mass, Δ_n is the half-width of the n-allowed miniband, ε_n is gives the position of that miniband, k_z and $\vec{k}_\perp$ are the wave vector components along and across the superlattice axis.

In the presence of an laser field with electric field vector $\vec{E} = \vec{E}_0 \sin\Omega t$, the Hamiltonian of the electron-optical phonon system in a CSSL in second quantization representation can be written as:

$$\begin{aligned} H(t) &= \sum_{n,\vec{k}_\perp} \varepsilon_n(\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)) a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} \\ &+ \sum_{n,n',\vec{q},\vec{k}_\perp} C_{\vec{q}} I_{n,n'}(q_z) a^+_{n',\vec{k}_\perp+\vec{q}_\perp} a_{n,\vec{k}_\perp} (b_{\vec{q}} + b^+_{-\vec{q}}), \end{aligned} \tag{2}$$

where $\vec{A}(t) = (\vec{E}_0 c/\Omega)\cos\Omega t$ is the vector potential, c being light velocity; $|n, \vec{k}_\perp >$ and $|n', \vec{k}_\perp + \vec{q}_\perp >$ are electron states before and after scattering, $\vec{q} = (\vec{q}_\perp, q_z)$; $a^+_{n,\vec{k}_\perp}$ and $a_{n,\vec{k}_\perp}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (phonon), respectively; $\hbar\omega_{\vec{q}}$ is the energy of optical phonon; $C_{\vec{q}}$ is the electron-phonon interaction constant. For electron-optical phonon interaction, with $\omega_{\vec{q}} \simeq \omega_0$, $C_{\vec{q}}$ is [10]: $|C_{\vec{q}}|^2 = (2\pi e^2 \hbar\omega_0)/(q^2)[(1/\chi_\infty) - (1/\chi_0)]$, where χ_0 and χ_∞ is the static and the high-frequency dielectric constant, respectively, $I_{n,n'}(q_z) =< n|e^{iq_z z}|n' >$ is the form factor of electron

$$I_{n,n'}(q_z) = \sum_{j=1}^{s_0} \int_0^d e^{iq_z d} \Phi_n(z - jd) \Phi_{n'}(z - jd) dz. \tag{3}$$

Here, $\Phi_n(z)$ is the eigenfunction for a single potential well and s_0 is the number of periods of the CSSL.

In order to establish QKE for electrons in CSSLs, we use the general quantum equation for particle number operator [10] or electrons distribution function $f_{n,\vec{k}_\perp}(t) = \left\langle a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} \right\rangle_t$, it takes the form:

$$i\hbar\frac{\partial}{\partial t} f_{n,\vec{k}_\perp}(t) = \left\langle \left[a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp}, H \right] \right\rangle_t, \tag{4}$$

where $\langle\psi\rangle_t$ denotes a statistical average value at the moment t; $\langle\psi\rangle_t = Tr\big(\widehat{W}\widehat{\psi}\big)$ ($\widehat{W}$ is the density matrix operator).

Starting from Hamiltonian (2) and realizing operator algebraic calculations, we obtain the QKE for electrons in the CSSL. It is seen that to obtain the explicit solutions of the equation is very difficult. In this paper we use the first order tautology approximation method to solve this equation. Solving the equation as in Ref. [11], we get the expression for the electron distribution function. The expression of electron distribution function is written as:

$$\begin{aligned} f_{n,\vec{k}_\perp}(t) \;=\; & \frac{1}{\hbar}\sum_{k,\ell=-\infty}^{+\infty} J_{k+\ell}(\frac{\Lambda}{\Omega}) J_k(\frac{\Lambda}{\Omega}) \frac{1}{\ell\Omega} e^{-i\ell\Omega t} \sum_{\vec{q},n'} |C_{\vec{q}}|^2 |I_{n,n'}(q_z)|^2 \\ \times\; & \Bigg\{ \frac{\overline{f}_{n,\vec{k}_\perp} N_q - \overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp}(1+N_q)}{M_1^- + i\delta} + \frac{\overline{f}_{n,\vec{k}_\perp}(1+N_q) - \overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp} N_q}{M_1^+ + i\delta} \\ +\; & \frac{\overline{f}_{n',\vec{k}_\perp-\vec{q}_\perp} N_q - \overline{f}_{n,\vec{k}_\perp}(1+N_q)}{M_2^- + i\delta} + \frac{\overline{f}_{n',\vec{k}_\perp-\vec{q}_\perp}(1+N_q) - \overline{f}_{n,\vec{k}_\perp} N_q}{M_2^+ + i\delta} \Bigg\} \end{aligned} \tag{5}$$

where $\overline{f}_{n,\vec{k}_\perp}$ ($N_{\vec{q}}$) is the time independent component of the distribution function of electrons (phonons), $J_\ell(x)$ is the ℓ^{th} order Bessel function of argument x, $\Lambda = e\vec{E}_0\vec{q}_\perp/(m\Omega)$, $M_1^\mp = \varepsilon_{n'}(\vec{k}_\perp + \vec{q}_\perp) - \varepsilon_n(\vec{k}_\perp) \mp \hbar\omega_q - k\hbar\Omega$, $M_2^\mp = \varepsilon_n(\vec{k}_\perp) - \varepsilon_{n'}(\vec{k}_\perp - \vec{q}_\perp) \mp \hbar\omega_q - k\hbar\Omega$.

Because the motion of electrons is confined along z direction in CSSLs, we only consider the in plane $(x-y)$ current density vector of electrons $\vec{j}_\perp(t)$. The carrier current density formula in a CSSL is taken the form:

$$\vec{j}_\perp(t) = \frac{e\hbar}{m}\sum_{n,\vec{k}_\perp}\left[\vec{k}_\perp - \frac{e}{\hbar c}\vec{A}(t)\right] f_{n,\vec{k}_\perp}(t). \tag{6}$$

Substituting (5) into (6), we find out the expression for current density vector:

$$\vec{j}_\perp(t) = -\frac{e^2\vec{E}_0 n_0}{m\Omega}\cos\Omega t + \vec{j}_1(t), \tag{7}$$

with $\sum_{n,\vec{k}_\perp} f_{n,\vec{k}_\perp}(t) \approx n_0$, and

$$\begin{aligned}\vec{j}_1(t) &= -\frac{e}{m}\sum_{\vec{q},n'}|C_{\vec{q}}|^2|I_{n,n'}(q_z)|^2\sum_{k,\ell=-\infty}^{\infty}J_{k+\ell}(\frac{\Lambda}{\Omega})J_k(\frac{\Lambda}{\Omega})\frac{1}{\ell\Omega}e^{-i\ell\Omega t}\sum_{n,\vec{k}_\perp}N_q\vec{q}\\ &\times\Big\{\frac{\overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp}}{M_1^+ + i\delta}+\frac{\overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp}}{M_1^- + i\delta}+\frac{\overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp}}{M_2^+ + i\delta}+\frac{\overline{f}_{n',\vec{k}_\perp+\vec{q}_\perp}}{M_2^- + i\delta}\Big\}.\end{aligned}\tag{8}$$

By using the electron-optical phonon interaction factor $C_{\vec{q}}$ and the Bessel function, from the expression of current density vector, we obtain the general expression for optical absorption coefficient due to the absorption of k photons in CSSLs:

$$\begin{aligned}\alpha &= \frac{8\pi}{c\sqrt{\chi_\infty}E_0^2}\Big\langle\vec{j}_\perp(t)\vec{E}_0\sin(\Omega t)\Big\rangle_t=\frac{8\pi^2\Omega}{c\sqrt{\chi_\infty}E_0^2}\sum_{\vec{q},n',n}|C_{\vec{q}}|^2|I_{n,n'}(q_z)|^2\\ &\times\sum_{k=-\infty}^{+\infty}kJ_k^2(\frac{\Lambda}{\Omega})N_q\sum_{\vec{k}_\perp}\overline{f}_{n,\vec{k}_\perp}\Big[\delta(M_1^+)+\delta(M_1^-)+\delta(M_2^+)+\delta(M_2^-)\Big].\end{aligned}\tag{9}$$

In this paper, considering the process of two photon absorption ($k=2$), we obtain explicit expression for the optical absorption coefficient

$$\begin{aligned}\alpha &= \frac{e^4}{4\hbar^3\Omega^3c\sqrt{\chi_\infty}}\sqrt{\frac{2\pi}{m\beta^3}}\Big(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\Big)\sum_{n',n}|I_{n,n'}|^2e^{\beta(\varepsilon_F-\varepsilon_n)}\\ &\times\Big\{\sum_{i=1}^{2}\Big(\frac{2mX_i}{\hbar^2}\Big)\exp\Big(\frac{\beta X_i}{2}\Big)K_1\Big(\frac{\beta X_i}{2}\Big)\\ &+\sum_{i=3}^{4}\Big(\frac{2mX_i}{\hbar^2}\Big)\exp\Big(-\frac{\beta X_i}{2}\Big)K_1\Big(\frac{\beta X_i}{2}\Big)\\ &+\frac{1}{2}\Big(\frac{eE_0}{2m\Omega^2}\Big)^2\Big[\sum_{i=1}^{2}\Big(\frac{2mY_i}{\hbar^2}\Big)^2\exp\Big(\frac{\beta Y_i}{2}\Big)K_2\Big(\frac{\beta Y_i}{2}\Big)\\ &+\sum_{i=3}^{4}\Big(\frac{2mY_i}{\hbar^2}\Big)^2\exp\Big(-\frac{\beta Y_i}{2}\Big)K_2\Big(\frac{\beta Y_i}{2}\Big)\Big]\Big\},\end{aligned}\tag{10}$$

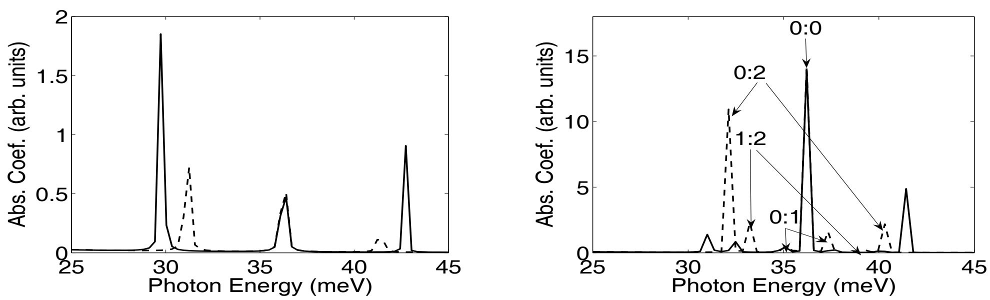
where $\beta = 1/(k_BT)$, k_B being the Boltzmann constant and T - the temperature of the system, ε_F is the Fermi level, and

$$\begin{aligned}I_{n,n'} &= \sum_{j=1}^{s_0}\int_0^d\Phi_n(z-jd)\Phi_{n'}(z-jd)dz\\ X_i &= \varepsilon_n-\varepsilon'_n\mp\hbar\omega_0-\hbar\Omega,\quad Y_i=\varepsilon_n-\varepsilon'_n\mp\hbar\omega_0-2\hbar\Omega\quad(\text{for}\quad i=1,2),\\ X_i &= \varepsilon'_n-\varepsilon_n\mp\hbar\omega_0-\hbar\Omega,\quad Y_i=\varepsilon'_n-\varepsilon_n\mp\hbar\omega_0-2\hbar\Omega\quad(\text{for}\quad i=3,4).\end{aligned}\tag{11}$$

The present result yields a more specific and significant interpretation of the electronic processes for emission and absorption of phonons and photons. The third and fourth terms in (10) are the contributions of the two-photon process. These analytical results appear very involved. However, physical conclusions can be drawn from graphical representations and numerical results, obtained from adequate computational methods.

III. NUMERICAL RESULTS AND DISCUSSION

In order to clarify the mechanism of the EPR effect in CSSLs, in this section, we numerically evaluate, plot and discuss the optical absorption coefficient for the GaAs/GaAlAs with the parameters [12, 13]: $\varepsilon_F = 50$ meV, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $m = 0.067m_0$, m_0 being the mass of free electron, $\hbar\omega_0 = 36.25$ meV, and we put $s_0 = 50$.



(a) (b)

Fig. 1. Absorption coefficient (arb. units) as a function of photon energy $\hbar\Omega$ for $d = 66$ nm (solid lines) and $d = 74$ nm (dashed lines): a) For $n, n' = 2 \div 3$; b) For $n, n' = 0 \div 2$.

In Figs. 1a and 1b, we show the dependence of the absorption coefficient on the photon energy at different separated transitions. We can see very clearly that each curve in Fig. 1a has one central peak and one couple of maximum peaks that are symmetric each together through the central one, while each curve in Fig. 1b has one central peak and two couples of maximum peaks and two peaks of each couple are symmetric each together through the central one. All central peaks are located at $\hbar\omega_0 = 36.25$ meV. It can explained that on each curve, maxima appear at the photon energy of $\hbar\Omega$ satisfying the condition $\hbar\Omega = \hbar\omega_0 \pm \Delta\varepsilon_{n,n'}$. If $\Delta\varepsilon_{n,n'} = 0$, $\hbar\Omega = \hbar\omega_0$ for different intrasubband transitions, therefore, every curve has the central peak at $\hbar\omega_0 = 36.25$ meV. The central peak is contributed from transitions (2-2) and (3-3) in Fig. 1a, from transitions (0-0), (1-1), and (2-2) in Fig. 1b.

If $\Delta\varepsilon_{n,n'} \neq 0$, the maxima are corresponding to intersubband transitions. Because $\varepsilon_n = \frac{\hbar^2\pi^2}{2mL^2}n^2 \simeq \frac{\hbar^2\pi^2}{2md^2}n^2$, so $\Delta\varepsilon_{n,n'} = \varepsilon_{n'} - \varepsilon_n = \frac{\hbar^2\pi^2}{2md^2}(n'^2 - n^2)$ increases with increasing of $|n'^2 - n^2|$. Consequently, the distance between two peaks, $2\Delta\varepsilon_{n,n'}$, increases. The distance depends also on the parameter of the CSSL, the distance decreases with increasing of the period of a CSSL. This result is of significant importance when we use an external electric field to measure the distance between two maxima to determine the energy levels of electrons in CSSLs.

The resonant conditions $\hbar\Omega = \hbar\omega_0 \pm \Delta\varepsilon_{n,n'}$ can be rewritten as

$$\Delta\varepsilon_{n,n'} = \hbar\omega_0 \pm \hbar\Omega. \tag{12}$$

This is the optically detected magnetophonon resonance effect for LO-phonon scattering in a CSSL. When the ODEPR conditions are satisfied, in the course of scattering events, the electrons in the subband levels specified by the level index (n) can make transitions to one of the subband levels (n') by absorbing and/or emitting a photon of energy $\hbar\Omega$ during the absorption of a LO phonon of energy of $\hbar\omega_0$.

IV. CONCLUSION

In this paper, we have obtained a general analytical expression of (nonlinear) ACF of an intensity electromagnetic field in CSSLs. We numerically calculated and plotted the ACF for GaAs-Ga$_{1-x}$Al$_x$As superlattice to clarify the theoretical results. Numerical results for the CSSL present clearly the dependence of the ACF on photon energy $\hbar\Omega$, period of a CSSL.

Computational results show that the dependence of the ACF on the photon energy presents electronphonon resonant peaks with resonant conditions $\varepsilon_{n,n'} = \hbar\omega_0 \pm \hbar\Omega$. This is the optically detected magnetophonon resonance effect for LO-phonon scattering in a CSSL. With $\Delta\varepsilon_{n,n'} \neq 0$ there is the splitting of ODEPR peaks for incident photon energy. Therefore, they can be applied to optically detect the electron spectrum in a CSSL.

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