Proc. Natl. Conf. Theor. Phys. **35** (2010), pp. 161-168

RATE OF PHONON EXCITATION AND CONDITIONS FOR PHONON GENERATION IN CYLINDRICAL QUANTUM WIRES

TRAN CONG PHONG, LE THI THU PHUONG, TRAN DINH HIEN
Department of Physics, Hue University's College of Education, 32 Le Loi, Hue, Vietnam

Abstract. *Phonon generation via the Cerenkov effect in cylindrical quantum wires (CQWs) is theoretically studied based on the quantum kinetic equation for phonon population operator. Both electrons and phonons are confined in the CQWs. Analytical expressions for the rate of change of the phonon population and conditions for phonon generation are obtained. Numerical results for GaAs/AlAs cylindrical quantum wires show that the amplitude of the laser field must satisfy additional conditions that are different in comparison with those of the other works. The differences between the generation of bulk LO-phonons and confined LO-phonons are discussed.*

I. INTRODUCTION

Phonon amplification by absorption of laser field energy has been widely investigated in bulk semiconductors [1, 2] and in low-dimensional heterostructures, in which the electron systems are two-dimensional [3, 4, 5, 6, 7] and one-dimensional [8]. These studies provide information on the excitation mechanisms of phonons, their dynamics, electron-phonon interactions, and other important phenomena. The main results of these works are that if the current is due to electron motion in an electric field, phonon amplification (generation) can be achieved via the Cerenkov effect when the electron drift velocity exceeds the phase velocity of the phonons.

High-frequency phonons have been observed for a number of semiconductor materials and heterostructures: GaAs and Ge [9], as well as other materials [10] (see Ref. 5 for a recent review). In quantum semiconductor heterostructures, if the current is due to transitions of carriers between the bound-electron states, generation of phonons can be realized if population inversion of these states occurs. Similar effects for photon generation have been demonstrated in cascade lasers [11]. Intense phonon waves can be exploited for various applications; these include [5, 12] the classical problem of sound amplification by drifting electrons, the modulation of optical signals, the modulation of electric current, phonon active control of electron transport, short-wavelength phonon induced phototransitions in indirect-gap semiconductors, heat removal through stimulated phonon decay, and nondestructive testing of nanostructures (phonon wavelengths can be scaled down to 10 nm).

It is important to noted that both theoretical analysis and experiments [13] have shown that phonon generation by drifting electrons in bulk semiconductor is practically impossible because the rate of phonon generation can not compete with the large rate of phonon losses. However, advanced technology of semiconductor heterostructures has opened new possibilities for applying the Cerenkov effect for phonon generation, especially for acoustic phonon generation, because electron drift velocities and electron densities in low-dimensional systems are much greater than they are in bulk samples. Due to a necessarily strong coupling of electrons and phonons in the same quantum well of quantum wires, we expect phonon generation via the Cerenkov effect to be now capable of competing with phonon losses.

Phonon amplification by absorption of laser field energy can be considered phenomenologically by using the kinetic equation for the phonon population which corresponds to the transition probability per unit time [1, 4, 7, 8] or the Boltzmann equation. However, Herbst *at al.* [14] have shown that carrier-phonon scattering processes on the quantum kinetic level are described by the dynamics of phonon assisted density matrices. Since, in the Boltzmann equation, the scattering rates due to different interaction mechanisms simply add up without interference, all contributions to the quantum kinetic equation resulting from mechanisms other than carrier-phonon interactions have to be neglected. This shows that the quantum kinetic treatment includes a variety of additional phenomena related to the mutual influence of different mechanisms, besides the correct treatment of the short time-and length-scale behavior. In particular, the carrier-phonon scattering dynamic is modified by intraband and interband Fock terms, external fields,... Recently, one of the authors (T. C. Phong) [15] studied phonon amplification in semiconductor superlattices by using the quantum kinetic equation in the case of multi-photon absorption processes.

In this paper, we study the phonon generation rate in a CQW. Starting from the kinetic equation for the phonon, we calculate the rate of phonon population change and find the conditions for phonon generation. Analytical expressions for the rate of phonon excitation and the conditions are obtained for the case of the multiphoton absorption process and a non-degenerate electron gas. The mechanism and the specific characteristics of the phonon generation are illustrated for a realistic quantum wire.

This paper is organized as follows: In Sec. II, we establish the quantum kinetic equation for confined phonons in a CQW under a laser field. In Sec. III, we calculate the rate of change of the phonon population in a CQW. For definiteness, in Sec. IV, we offer numerical results for a specific GaAs/AlAs wire. Conclusions are given in the Sec. V.

II. QUANTUM KINETIC EQUATION FOR A PHONON IN A CQW

In a cylindrical GaAs quantum wire of radius R and length L ($L \gg R$) embedded in AlAs, electrons are confined in GaAs where the potential well develops. Under the infinitely deep well approximation, electron wave functions can be written as

$$\psi_{\ell j,k}(r) = \frac{e^{ikz}}{\sqrt{L}} C_{\ell,j} J_\ell\big(x_{\ell,j}\frac{r}{R}\big) e^{i\ell\phi}, \tag{1}$$

with the corresponding energies

$$\varepsilon_{\ell,j}(k) = \frac{\hbar^2 k^2}{2m_e} + \frac{\hbar^2 (x_{\ell,j})^2}{2m_e R^2}, \tag{2}$$

where $\ell = ..., -1, 0, 1, ...$; $j = 1, 2, 3, ...$; $r = (r, \phi, z)$ are cylindrical coordinates for the system and k denotes the axial wave-vector component. $C_{\ell,j} = 1/(\sqrt{\pi} y_{\ell,j} R)$ is the normalization factor, $x_{\ell,j}$ is the jth zero of the ℓth order Bessel function, i.e., $J_\ell(x_{\ell,j}) = 0$ and $y_{\ell,j} = J_{\ell+1}(x_{\ell,j})$, m_e is the effective mass of electron.

Here, we are interested in the role of phonons. With the confined longitudinal optical (LO) phonon assumption, Hamiltonian of the electron-phonon system in a CQW in the presence of a laser field, $\vec{E} = \vec{E}_0 \sin \Omega t$, is written as

$$H(t) = H_e + H_{ph} + H_{e-ph}, \tag{3}$$

where, the first and second term in Eq. (3) are the Hamiltonian of the electron H_e and the Hamiltonian of the phonon H_{ph}, respectively, they are given by [16, 17]

$$H_e = \sum_{\ell,j,k_z} \varepsilon_{\ell,j}(k_z - \frac{e}{\hbar c}\vec{A}(t))c^+_{\ell,j}(k_z)c_{\ell,j}(k_z), \tag{4}$$

$$H_{ph} = \sum_{m,n,q_z} \hbar\omega_{LO} a^+_{m,n}(q_z)a_{m,n}(q_z), \tag{5}$$

where $c^+_{\ell,j}(k_z)$ and $c_{\ell,j}(k_z)$ ($a^+_{m,n}(q_z)$ and $a_{m,n}(q_z)$) are the creation and the annihilation operators of confined electron (phonon) for state $|\ell, j, k_z >\equiv |\alpha, k_z>$ ($|m, n, q_z>$), respectively; k_z and q_z are the components along the z-axis of confined electron and phonon wave vector.

The one-dimensional Frohlich Hamiltonian in the last term of Eq. (3) describing the interaction between electrons and confined LO-phonon modes can be written as

$$\begin{aligned} H_{e-ph} &= \sum_{\alpha,\alpha'} \sum_{n,k_z,q_z} M_{\alpha,\alpha',n}(q_z) \\ &\quad \times c^+_{\alpha'}(k_z+q_z)c_{\ell,j}(k_z)(a_{m,n}(q_z) + a^+_{m,n}(-q_z)), \end{aligned} \tag{6}$$

$M_{\alpha,\alpha',n}(q_z)$ is the coupling matrix element, it depends on the scattering mechanism [18]

$$M_{\alpha,\alpha',m,n}(q_z) = -e\theta(\frac{\omega_{LO}}{L})^{1/2} C_{\ell-\ell',n} F_{\alpha,\alpha'}(x_{\ell-\ell',n}), \tag{7}$$

with

$$F_{\alpha,\alpha'}(\eta) = 2\int_0^1 \xi d\xi \frac{1}{y^\ell_j y^{\ell'}_{j'}} J_\ell(x^\ell_j \xi) J_{\ell-\ell'}(\eta\xi) J_{\ell'}(x^{\ell'}_{j'}\xi) \tag{8}$$

is the form factor.

The quantum kinetic equation for phonon in a CQW is

$$i\hbar\frac{\partial}{\partial t}\langle a^+_{m,n}(q_z)a_{m,n}(q_z)\rangle_t = i\hbar\frac{\partial N_{m,n,q_z}(t)}{\partial t} = \langle [a^+_{m,n}(q_z)a_{m,n}(q_z), H(t)]\rangle_t, \tag{9}$$

where $< X >_t$ means the usual thermodynamic average of X at moment t. Using $H(t)$ and realizing operator algebraic calculations, we have

$$\begin{aligned} \frac{\partial N_{m,n,q_z}(t)}{\partial t} = \frac{i}{\hbar} \sum_{\alpha,\alpha',k_z} \Big[& M_{\alpha,\alpha',m,n}(q_z) F^{\alpha',k_z+q_z}_{\alpha,k_z}(m,n,q_z,t) \\ & - M_{\alpha,\alpha',m,n}(q_z) F^{\alpha,k_z}_{\alpha',k_z-q_z}(m,n,q_z,t)^* \Big], \end{aligned} \tag{10}$$

where

$$F^{\alpha',\vec{k}'}_{\alpha,\vec{k}}(m,n,q_z,t) \equiv F(t) = \langle c^{+}_{\alpha}(q_z)c_{\alpha'}(k_z-q_z)a_{m,n}(q_z)\rangle_t. \tag{11}$$

We rewrite again the quantum kinetic equation for $F(t)$; then, using the assumption of an adiabatic interaction, $F(t)|_{t\to-\infty}=0$, and then solved by using the method of constant variation, we obtain the quantum kinetic equation for phonons in a CQW

$$\begin{aligned}\frac{\partial N_{m,n,q_z}(t)}{\partial t} = \frac{1}{\hbar^2}\sum_{\alpha,\alpha',k_z}|M_{\alpha,\alpha',m,n}(q_z)|^2\sum_{s=-\infty}^{+\infty}J_s^2(\frac{\Lambda}{\hbar\Omega})\int_{-\infty}^{t}dt' N_{m,n,q_z}(t')\\ \times\Big\{\big[f_{\alpha'}(k_z+q_z)-f_{\alpha}(k_z)\big]e^{\frac{i}{\hbar}(\varepsilon_{\alpha'}(k_z+q_z)-\varepsilon_{\alpha}(k_z)-\hbar\omega_{LO}-s\hbar\Omega)(t-t')}\\ +\big[f_{\alpha}(k_z)-f_{\alpha'}(k_z-q_z)\big]e^{\frac{i}{\hbar}(\varepsilon_{\alpha'}(k_z-q_z)-\varepsilon_{\alpha}(k_z)+\hbar\omega_{LO}+s\hbar\Omega)(t-t')}\Big\},\end{aligned} \tag{12}$$

where $f_{\alpha}(k_z)$ is the distribution function of the electron gas, which is assumed to be in equilibrium, and $\Lambda = e\hbar\vec{q}\vec{E}_0/(m_e\Omega)$ is the field parameter.

III. RATE OF CHANGE THE PHONON POPULATION

Using the Fourier transform technique and the assumption of an adiabatic interaction of the laser field, one obtains the kinetic equation for the phonon population of the q_z mode [1]:

$$\frac{\partial N_{m,n,q_z}(t)}{\partial t} = G_{m,n,q_z}N_{m,n,q_z}(t), \tag{13}$$

where G_{m,n,q_z} is a the parameter that determines the rate of change of the phonon population $N_{m,n,q_z}(t)$ in time due to the interaction with electrons and takes the following form:

$$\begin{aligned}G_{m,n,q_z} &= \frac{2\pi}{\hbar}\sum_{\alpha,\alpha',k_z}|M_{\alpha,\alpha',m,n}(q_z)|^2\sum_{s=-\infty}^{+\infty}J_s^2(\frac{\Lambda}{\hbar\Omega})\big[f_{\alpha'}(k_z+q_z)-f_{\alpha}(k_z)\big]\\ &\times\ \delta\big(\varepsilon_{\alpha'}(k_z+q_z)-\varepsilon_{\alpha}(k_z)-\hbar\omega_{LO}-s\hbar\Omega\big).\end{aligned} \tag{14}$$

The parameter G_{m,n,q_z} has significant meaning. If $G_{m,n,q_z}>0$, the phonon population grows with time (phonon amplification) whereas for $G_{m,n,q_z}<0$, the phonons are damped. In the strong-field limit, $\Lambda >> \hbar\Omega$, the sum over s in Eq. (14) may then be written approximately as [1]

$$\sum_{s=-\infty}^{+\infty}J_s^2(\frac{\Lambda}{\hbar\Omega})\delta(\varepsilon-s\hbar\Omega) = \frac{1}{2}\Big[\delta(\varepsilon-\Lambda)+\delta(\varepsilon+\Lambda)\Big], \tag{15}$$

with $\varepsilon = \varepsilon_{\alpha'}(k_z+q_z)-\varepsilon_{\alpha}(k_z)-\hbar\omega_{LO}$.

The first Delta function in Eq. (15) corresponds to the emission and the second to the absorption of $(\Lambda/\hbar\Omega)$ photons. In other words, in the strong-field, limit only multiphoton processes are dominant, and the electron-phonon interaction takes place with the emission and the absorption of $(\Lambda/\hbar\Omega) >> 1$ photons. Substituting Eq. (15) into Eq. (14), the phonon population change rate becomes $G^{(\pm)}_{m,n,q_z} = G^{(+)}_{m,n,q_z} + G^{(-)}_{m,n,q_z}$, where

$$G^{(\pm)}_{m,n,q_z} = \frac{\pi}{\hbar}\sum_{\alpha,\alpha',k_z}|M_{\alpha,\alpha',m,n}(q_z)|^2\Big[f_{\alpha'}(k_z+q_z) - f_\alpha(k_z)\Big] \times \delta\Big[\varepsilon_{\alpha'}(k_z+q_z) - \varepsilon_\alpha(k_z) - \hbar\omega_{LO} \pm \Lambda\Big]. \quad (16)$$

Transforming the sum over k_z to an integral in k_z space, assuming that the electron gas is non-degenerate, substituting expressions for the energy spectra and the Fermi-Dirac distribution function, $f_\alpha(k_z) = \exp[\beta(\varepsilon_F - \varepsilon_\alpha(k_z)]$, and then carrying out some calculations, we get the expression for the change rate of the phonon population:

$$G^{(\pm)}_{m,n,q_z} = \frac{Lm_e}{2\hbar^3 q_z}\sum_{\alpha,\alpha'}|M_{\alpha,\alpha',m,n}(q_z)|^2 \times \Big\{\exp\Big[\beta\Big(\varepsilon_F - \varepsilon_{\alpha'} - \frac{\hbar^2}{m_e}\Big(\frac{m_e}{\hbar^2 q_z}(\Delta\varepsilon + \hbar\omega_{LO} \mp \Lambda) - q_z\Big)^2\Big)\Big] - \exp\Big[\beta\Big(\varepsilon_F - \varepsilon_\alpha - \frac{m_e}{2\hbar^2 q_z^2}(\Delta\varepsilon + \hbar\omega_{LO} \mp \Lambda)^2\Big)\Big]\Big\}. \quad (17)$$

here, ε_F is the Fermi energy, $\beta = 1/(k_B T)$, k_B is the Boltzmann constant, and T is the temperature of the system, and $\Delta\varepsilon = \varepsilon_{\alpha'} - \varepsilon_\alpha - \hbar^2 q_z^2/(2m_e)$.

From Eq. (17), we can derive the conditions for the phonon generation. It can be seen that only the multi-photon absorption process $(\Lambda/(\hbar\Omega) >> 1)$ corresponding to the signs (+) in the superscript of G_{m,n,q_z} and the signs (-) in front of Λ satisfy the condition $G^{(+)}_{m,n,q_z} > 0$ for phonon generation. In this case, we obtain the condition that laser field must satisfy:

$$\Lambda = \frac{e\hbar\vec{q}\vec{E}_0}{m_e\Omega} > \hbar\omega_{LO}. \quad (18)$$

The condition in Eq. (18) simply means that for a particular phonon wave vector, the necessary condition for the onset of the phonon instability is just the Cerenkov condition $\vec{q}\vec{v}_0 > \omega_{LO}$, where $\vec{v}_0 = e\vec{E}_0/(m_e\Omega)$ and $v_{ph} = \omega_{LO}/q_z$ is the phonon-phase velocity. The above result is formally analogous to the one for the electron-phonon system in the presence of a dc electric field. The difference lies in the fact that in the latter case, $\vec{v}_0$ is replaced by the drift velocity $\vec{v}_d = e\vec{E}_0/(m_e\Omega)$ imposed by the static field [1, 7].

IV. NUMERICAL RESULTS AND DISCUSSION

The following parameters values were used in the calculation of this paper [16, 18]: $\hbar\omega_{LO} = 36.2$ meV, $\chi_{1s} = 12.9$, $\chi_{1\infty} = 10.9$, $m_e = 0.067m_0 = 6.097 \times 10^{-31}$ kg, $L_z = 100$ nm, $\varepsilon_F = 0.5 \times 10^{-18}$ J. The transition included in numerical results is $\ell = 1, \ell' = 0$, $j = j' = 1$.

It can be seen from Fig. 1 that the rate of LO-phonon excitation obtains the both positive (amplification) and negative (absorption) values for confined phonons. This means that phonon amplification can only occur in a narrow range of the wave number. With the conditions given in Fig. 1, only confined phonons with wavenumber in the range of

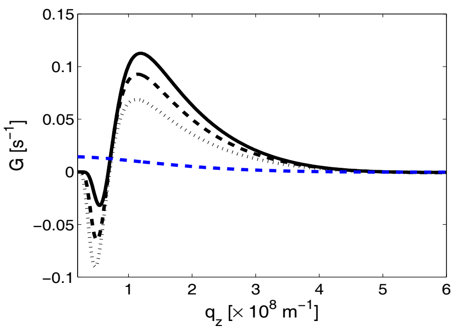

Fig. 1. The dependence of the phonon generation rate on the wave number for two cases: confined LO phonon (at different lattice temperature: the solid, the dashed, and the dotted lines correspond to 150 K, 250 K, and 300 K, respectively) and bulk LO phonon (dashed and dotted line at 200 K). Here, $\Omega = 1.0 \times 10^{13}$ Hz, $R = 16.3$ nm, $E_0 = 10^6$ V/m.

0.7×10^8 m^{-1} and 4.0×10^8 m^{-1} can be created. Phonons with wavenumber lesser than 0.7×10^8 m^{-1} are almost absorbed. In contract, in this range of the wavenumber bulk phonons are always created with the rate smaller than that for confined phonons. The higher temperature is, the lower the rate of phonon generation is and the greater rate of their absorption. Figure 1 also shows that if the phonon wavenumber is greater than 5.0×10^8 m^{-1}, the influence of the phonon confinement nearly disappeared, the external field has no impact on the phonon excitation.

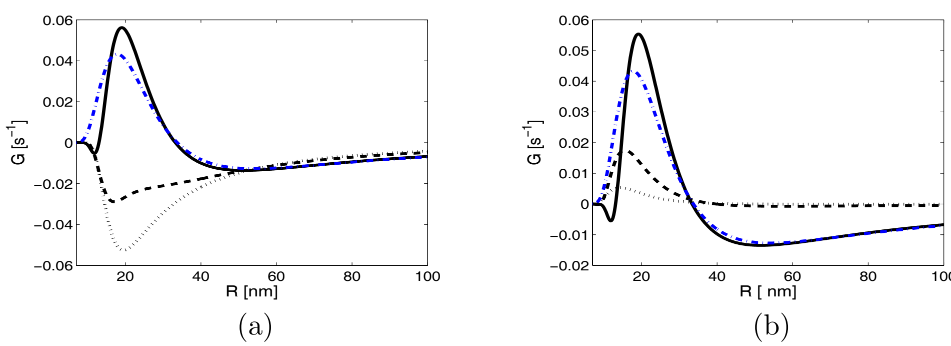

Fig. 2. The dependence of the phonon generation rate on the radius R for two cases: confined LO phonon a) at different laser field frequency: the solid, the dashed, and the dotted lines correspond to 1.0×10^{13} Hz, 2.0×10^{13} Hz, and 3.0×10^{13} Hz, respectively; b) at different wave number: the solid, the dashed, and the dotted lines correspond to 1.0×10^8 m^{-1}, 2.0×10^8 m^{-1}, and 3.0×10^8 m^{-1}, respectively, and bulk LO phonon (dashed and dotted line): a) at $\Omega = 1.0 \times 10^{13}$ Hz; b) at $q_z = 1.0 \times 10^8$ m^{-1}. Here, $T = 250$ K.

Figures 2a and 2b show the dependence of the phonon generation rate on the wire's radius R. We can see that the rate of change of the phonon population for confined phonons possesses both positive and negative value, but for bulk phonon there is only the positive one. when the wire's radius is considerable (greater than 50 nm), the plotted curves nearly coincide. This mean that the phonon confinement no longer takes effect.

V. CONCLUSIONS

In conclusion, we have analytically investigated the possibility of phonon generation by absorption of laser-field energy in a cylindrical quantum wire in the case of a multi-photon absorption process and a non-degenerate electron gas. Starting from the confined phonon assumption and using the quantum kinetic equation for phonons, we have obtained expressions for the rate of change of the phonon population in the cases of confined LO phonons.

The expressions are numerically calculated and plotted for a specific CQW to show the mechanism of phonon generation. The obtained results show that the impact of the phonon confinement is considerable for optical phonon with the sufficient small wavenumber and wire's radius. This impact is insignificant for phonons with large wavenumber as well as wire's radius.

REFERENCES

[1] A. L. Troncini, O. A. C. Nunes, *Phys. Rev. B* **33** (1986) 4125; O. A. C. Nunes, *Phys. Rev. B* **29** (1984) 5679; J. W. Sakai, O. A. C. Nunes, *Sol. Stat. Comm.* **64** (1987) 1396; L. C. Miranda, *J. Phys. C* **9** (1976) 2971.
[2] E. M. Epstein, *Radio in Physics* **18** (1975) 785; *Lett. JEPT* **13** (1971) 511.
[3] J. W. Sakai, O. A. C. Nunes, *Sol. Stat. Comm.* **74** (1990) 397.
[4] P. Zhao, *Phys. Rev. B* **49** (1994) 13589.
[5] S. M. Komirenko, K. W. Kim, A. A. Dimidenko, V. A. Kochelap, M. A. Stroscio, *Phys. Rev. B* **62** (2000) 7459; S. M. Komirenko, K. W. Kim, V. A. Kochelap, I. Fedorov, M. A. Stroscio, *Phys. Rev. B* **63** (2001) 165308; B. A. Glavin, V. A. Kochelap, T. L. Linnik, K. W. Kim, M. A. Stroscio, *Phys. Rev. B* **65** (2002) 085303; S. M. Komirenko, K. W. Kim, A.A. Dimidenko, V. A. Kochelap, M. A. Stroscio, *Physica B* **316-317** (2002) 356.
[6] B. A. Glavin, V. A. Kochelap, T. L. Linnik, *Appl. Phys. Lett.* **74** (1999) 3525; S. M. Komirenko, K. W. Kim, A. A. Dimidenko, V. A. Kochelap, M. A. Stroscio, *Appl. Phys. Lett.* **76** (2000) 1869; *J. Appl. Phys.* **90** (2001) 3934.
[7] F. Peng, *Phys. Rev. B* **49** (1994) 4646.
[8] F. Peng, *J. Phys.: Condens. Matt.* **11** (1999) 4039; H. Totland, Y. M. Galperin, V. L. Gurevich, *Physica Scripta* **79** (1999) 83.
[9] W. A. Kutt, W. Albrecht, H. Kurz, *IEEE J. Quantum Electron* **28** (1992) 2434.
[10] R. Merlin, *Solid State Commun.* **102** (1997) 207; T. Dekorsy, G. C. Cho, H. Kurz, in *Light Scattering in Solids VIII*, edited by M. Cardona and G. Gutherodt, 2000 Springer, Berlin.
[11] J. Faist, F. Capasso, D. L. Silco, C. Sirtori, A. L. Hutchinson, A. Y. Cho, *Science* **264** (1994) 553.
[12] D. N. Hill, S. A. Cavill, A. V. Akimov, F. F. Ouali, E. S. Moskalenko, L. J. Challis, A. J. Kent, F. W. Sheard, P. Kral, M. Henini, *Phys. Status Solidi B* **204** (1997) 431; S. A. Cavill, A. V. Akimov, F. F. Ouali, L. J. Challis, A. J. Kent, M. Henini, *Physica B* **263-264** (1999) 537; X. Hu, F. Nori, *Physica B* **263** (1999) 16.
[13] Ü. Özgür, Chang-Won Lee, H. O. Everitt, *Phys. Rev. Lett.* **86** (2001) 5604.
[14] M. Herbst, M. Glanemann, V. M. Axt, T. Kuhn, *Phys. Rev. B* **67** (2003) 195305.
[15] T. C. Phong, N. T. Dung, in *Proceedings of 7th Vietnamese-German Seminar on Physics and Engineering*, March 28 to April 2, 2004, Halong, Vietnam, p. 76.
[16] YU You-Bin, *Commun. Theor. Phys.* **49** (2008) 1615.

[17] N. L. Kang, Y. J. Lee, S. D. Choi, *J. Korean Phys. Soc.* **44** (2004) 1535.
[18] X. F. Wang, X. L. Lei, *Phys. Rev. B* **49** (1994) 4780.

Received 10-10-2010.