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# PARAMETRIC RESONANCE OF CONFINED ACOUSTIC AND OPTICAL PHONONS IN RECTANGULAR QUANTUM WIRE

LE THI THU PHUONG
*Hue University's College of Education, 34 Le Loi, Hue, Vietnam*
NGUYEN VU NHAN, NGUYEN THUY LINH, NGUYEN QUANG BAU
*Hanoi National University, 334 Nguyen Trai, Hanoi, Vietnam*

**Abstract.** *The effect of confined phonons on parametric resonance of acoustic and optical phonons in rectangular quantum wires (RQWs) in the presence of an external electromagnetic field is theoretically studied by using a set of quantum kinetic equations for phonons. The analytical expression of intensity of threshold field $E_{th}$ of the parametric resonance of confined acoustic and optical phonons in RQWs is obtained. Numerical results for the the threshold field intensity in a GaAs/GaAsAl quantum wire are presented for both models of phonon confinement for a range of values of the relevant parameter. Numerical results are compared with those for the unconfined phonons to clarify the difference and show that the confinement of the phonons effect significantly on the threshold amplitude of the field.*

## I. INTRODUCTION

In the presence of an external electromagnetic field (EEF), an electron gas is well known to become non-stationary. When the conditions of the parametric resonance (PR) are satisfied, parametric interactions and transformations (PIT) of the same kind of excitations, such as phonon-phonon and plasmon-plasmon excitations, or of different kinds of excitations, such as plasmon-phonon excitations, will arise; i.e., energy exchanges processes between these excitations will occur [1]. The physical picture can be described as follows: Due to the electron-phonon interaction, propagation of an acoustic phonon with a frequency $\omega_{\vec{q}}$ is accompanied by a density wave with the same frequency. When an EEF with a frequency $\Omega$ is presented, a charge density waves (CDW) with a combination frequency $\omega_{\vec{q}} \pm L\Omega$ $(L = 1, 2, 3...)$ will appear. If among the CDW there exists a certain wave having a frequency that coincides, or approximately coincides, with the frequency of the optical phonon, $\upsilon_{\vec{q}}$, optical phonons will appear. These optical phonons cause a CDW with a combination frequency of $\upsilon_{\vec{q}} \pm L\Omega$, and when $\upsilon_{\vec{q}} \pm L\Omega \cong \omega_{\vec{q}}$, a certain CDW causes the acoustic phonons mentioned above. The result of the study shows that the PIT can speed up the damping process for one excitation and the amplification process for another excitation; namely, acoustic phonons are amplified while optical phonon are decreased or vice versa. For low-dimensional semiconductors, there have been several works on the generation and amplification of acoustic phonons [2, 3] but just for unconfined phonons. In order to continue these ideas, the purpose of this paper is to also study the parametric resonance of acoustic and optical phonons, but in a RQW and phonons are confined. The electron gas is assumed to be non-degenerate. In Sec. II, we introduce the interaction Hamiltonian of the parametric interaction model and the dispersion equation obtained from the quantum transport equations for phonons. In Sec. III, we present the results of an analytical approximation for the resonant acoustic phonon frequency and the threshold amplitude of the field for parametric amplification of acoustic phonons. Numerical results and Conclusions are shown in Sec. IV and V.

## II. MODEL AND DISPERSION EQUATION

Let us consider a quantum wire of rectangular cross section along the $z$ axis, with finite $x$ and $y$ dimensions, given, respectively, by $L_x$ and $L_y$. We assume that the walls are impenetrable; that is, the confining potential well is an infinite square well. In this case, the state and the electron energy spectra have the form [4]

$$|\alpha, k_z\rangle \equiv |\ell, j, k_z\rangle = \frac{e^{ik_z z}}{\sqrt{L_z}} \frac{2}{\sqrt{L_x L_y}} \cos\Big(\frac{\pi \ell x}{L_x}\Big) \cos\Big(\frac{\pi j y}{L_y}\Big), \tag{1}$$

$$\varepsilon_\alpha(k_z) = \frac{\hbar^2 k_z^2}{2m_e} + \frac{\pi^2 \hbar^2}{2m_e}\Big(\frac{\ell^2}{L_x^2} + \frac{j^2}{L_y^2}\Big) = \frac{\hbar^2 k_z^2}{2m_e} + \varepsilon_\alpha, \tag{2}$$

where $k_z$ is the electron wave vector along the wire's $z$ axis and $m_e$ is the electron effective mass, $L_z$ is the length of the RQW, $-L_x/2 \le x \le L_x/2$ and $-L_y/2 \le y \le L_y/2$.

Here, we are interested in the role of phonons. With the confined-phonon assumption, the total Hamiltonian of the electron-phonon system in a RQW in the presence of a laser field, $\vec{E} = \vec{E_0} \sin \Omega t$, can be partitioned as

$$H = H_e + H_{ph} + H_{e-ph} \tag{3}$$

The first term in Eq. (3) is the Hamiltonian of the electron [5]

$$H_e = \sum_{\alpha, k_z} \varepsilon_\alpha (k_z - \frac{e}{\hbar c} \vec{A}(t)) c_\alpha^+(k_z) c_\alpha(k_z), \tag{4}$$

$H_{ph}$ is the Hamiltonian of phonons. Here, two kinds of phonon modes, confined acoutic phonon and optical phonon, are considered. So $H_{ph}$ takes the form [6]:

$$H_{ph} = \sum_{m,n,q_z} \hbar\omega_{m,n,q_z} a_{m,n}^+(q_z) a_{m,n}(q_z) + \sum_{m,n,q_z} \hbar\upsilon_{m,n,q_z} b_{m,n}^+(q_z) b_{m,n}(q_z) \tag{5}$$

The one-dimensional Frohlich Hamiltonian in the last term of Eq. (3) describing the interaction between an electron and two different confined phonon modes can be written as [7]:

$$\begin{aligned} H_{e-ph} = &\sum_{\alpha,\alpha'} \sum_{m,n,k_z,q_z} M_{\alpha,\alpha',m,n}(q_z) c_{\alpha'}^+(k_z+q_z) c_\alpha(k_z)(a_{m,n}(q_z) + a_{m,n}^+(-q_z)) \\ &+ \sum_{\alpha,\alpha'} \sum_{m,n,k_z,q_z} G_{\alpha,\alpha',m,n}(q_z) c_{\alpha'}^+(k_z+q_z) c_\alpha(k_z)(b_{m,n}(q_z) + b_{m,n}^+(-q_z)) \end{aligned} \tag{6}$$

with $\varepsilon_\alpha$ and $c_\alpha(k_z)^+(c_\alpha(k_z))$ being the energy spectrum and the creation (annihilation) operator of an electron for state $|\alpha, k_z\rangle$, and $a_{m,n}^+(q_z)$ $(b_{m,n}^+(q_z))$ is the creation operator of an acoustic (optical) phonon for energy $\hbar\omega_{m,n,q_z}(\hbar\upsilon_{m,n,q_z})$. In this paper, we will deal with confined phonons; therefore the electron-phonon interaction matrix element take the forms $M_{\alpha,\alpha',m,n}(q_z) = C_{m,n}(q_z) I^{m,n}_{\alpha,\alpha'}(q_{m,n})$ and $G_{\alpha,\alpha',m,n}(q_z) = D_{m,n}(q_z) I^{m,n}_{\alpha,\alpha'}(q_{m,n})$ where [8]

$$\begin{aligned} I^{m,n}_{\alpha,\alpha'}(q_{m,n}) &= \frac{4}{L_x L_y}\int_{-L_x/2}^{L_x/2} dx \int_{-L_y/2}^{L_y/2} dy \cos\big(\frac{\pi \ell x}{L_x}\big)\cos\big(\frac{\pi j y}{L_y}\big)\cos\big(\frac{\pi \ell' x}{L_x}\big)\cos\big(\frac{\pi j' y}{L_y}\big) \\ &\times \quad \zeta_m(x)\zeta_n(y) \end{aligned} \tag{7}$$

The function $\zeta_m$, which is determined by the confinement mode, is defined as follows

$$\zeta_m(x) = \cos\big(\frac{m\pi x}{L_x} + \frac{\pi}{2}\delta_m\big) \tag{8}$$

The confined electron- acoustic and -optical phonon interaction constants take the forms [9]

$$\big|C_{m,n}(q_z)\big|^2 = \frac{\hbar \xi^2}{2\rho v_a V}\sqrt{\big(\frac{m\pi}{L_x}\big)^2 + \big(\frac{n\pi}{L_y}\big)^2 + q_z^2}, \tag{9}$$

$$\big|D_{m,n}(q_z)\big|^2 = \frac{e^2\hbar\omega_0}{2V\chi}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{\big(\frac{m\pi}{L_x}\big)^2 + \big(\frac{n\pi}{L_y}\big)^2 + q_z^2}, \tag{10}$$

In order to establish a set of quantum transport equations for confined -acoustic and - optical phonons, we use the general quantum distribution functions for the confined phonons, $< a_{m,n}(q_z) >_t$ and $< b_{m,n}(q_z) >_t$, where $< \psi >_t$ denotes a statistical average at the moment $t$: $< \psi >_t = Tr(\hat{W}\hat{\psi})$, where $\hat{W}$ is the density matrix operator and $Tr$ denotes the trace. Using the Hamiltonian $H$ and realizing the operator algebraic calculations, we obtain a set of coupled quantum transport equations for the confined acoustic phonons.

From that, we can obtain an equation for the Fourier transformation $A_{m,n}(q_z,\omega)$ of $< a_{m,n}(q_z) >_t$

$$\begin{aligned} (\omega - \omega_{m,n,q_z})\, A_{m,n}(q_z,\omega) = \frac{2}{\hbar}\sum_{\alpha,\alpha'}\sum_{m,n,q_z}\sum_{L=-\infty}^{+\infty}\Big\{\big\{|C_{m,n}(q_z)|^2\big|I^{m,n}_{\alpha,\alpha'}\big|^2 \\ \times\frac{\omega_{m,n,q_z} A_{m,n}(q_z,\omega - L\Omega)}{\omega - L\Omega + \omega_{m,n,q_z}} + C_{m,n}(q_z)D_{m,n}(q_z)\big|I^{m,n}_{\alpha,\alpha'}\big|^2 \upsilon_{m,n,q_z} \\ \times\frac{B_{m,n}(q_z,\omega - L\Omega)}{\omega - L\Omega + \upsilon_{m,n,q_z}}\big\}\Pi_L\big(m,n,q_z,\omega\big)\Big\} \end{aligned} \tag{11}$$

where we have set

$$\Pi_L\big(m,n,q_z,\omega\big) = \sum_{s=-\infty}^{+\infty} J_s\big(\frac{\lambda}{\Omega}\big)J_{s+L}\big(\frac{\lambda}{\Omega}\big)\Gamma_{m,n,q_z}(\omega + s\Omega), \tag{12}$$

$$\Gamma_{m,n,q_z}\big(\omega + s\Omega\big) = \sum_{k_z}\frac{f_\alpha(k_z) - f_{\alpha'}(k_z - q_z)}{\big(\varepsilon_\alpha(k_z) - \varepsilon_{\alpha'}(k_z - q_z) - \hbar s\Omega - \hbar\omega - i\hbar\delta\big)}. \tag{13}$$

It can be noted that $\Gamma_{m,n,q_z}(\omega + s\Omega)$ is the polarization operator of the electron distribution function and that the quantity $\delta$ is infinitesimal and appears due to the assumption of an adiabatic interaction of the EEF, $f_\alpha(k_z)$ is the distribution function of electrons in the state $|\alpha, k_z>$, $J_s(\lambda/\Omega)$ is the Bessel function, and $\lambda = eq_zE_0/m_e\Omega$ ($m_e$ is the electron mass).

Repeating the above processes, we can also obtain an equation for the Fourier transformation $A^+_{m,n}(-q_z,\omega)$ of $< a^+_{m,n}(-q_z) >_t$ and the relative expression between $A_{m,n,q_z}(\omega)$ and $A^+_{m,n}(-q_z,\omega)$. In the same way, but for confined optical phonons, we obtain a similar equation in which $\omega_{m,n,q_z}$, $A_{m,n}(q_z,\omega)$, $A_{m,n}(q_z,\omega-L\Omega)$, $C_{m,n}(q_z)$, $D_{m,n}(q_z)$, and $\upsilon_{m,p,q_z}$ are replaced with $\upsilon_{m,n,q_z}$, $B_{m,n}(q_z,\omega)$, $B_{m,n}(q_z,\omega-L\Omega)$, $D_{m,n}(q_z)$, $C_{m,n}(q_z)$, and $\omega_{m,n,q_z}$, respectively. In the equations, $A_{m,n}(q_z,\omega)$ and $B_{m,n}(q_z,\omega)$ are the Fourier transformations of $< a_{m,n}(q_z) >_t$ and $< b_{m,n}(q_z) >_t$ respectively. In these coupled equations, the first terms describe the interactions between phonons that belong to the same kind (acoustic-acoustic or optical-optical phonons) while the second terms describe interactions between phonons that belong to different kinds (acoustic-optical phonon). We can put $L = 0$ in the first terms of the coupled equations because we are now focusing on the PIT of the confined acoustic and the confined optical phonons. Solving the set, we obtain a general dispersion equation for the PIT of the acoustic and the optical phonons

$$\begin{aligned}
&\big[\omega^2-\omega^2_{m,n,q_z}-\frac{2}{\hbar}\sum_{\alpha,\alpha'}\sum_{m,n,q_z}C^2_{m,n}(q_z)|I^{m,n}_{\alpha,\alpha'}(q_{m,n})|^2\omega_{m,n,q_z}\Pi_0\big(m,n,q_z,\omega\big)\big]\big[\big(\omega-L\Omega\big)^2\\
&-\upsilon^2_{m,n,q_z}-\frac{2}{\hbar}\sum_{\alpha,\alpha'}\sum_{m,n,q_z}D^2_{m,n}(q_z)|I^{m,n}_{\alpha,\alpha'}(q_{m,n})|^2\Pi_0(m,n,q_z,\omega-L\Omega)\upsilon_{m,n,q_z}\big]\\
&=\frac{4}{\hbar^2}\sum_{\alpha,\alpha'}\sum_{m,n,q_z}\sum_{L=-\infty}^{+\infty}C^2_{m,n}(q_z)D^2_{m,n}(q_z)\big|I^{m,n}_{\alpha,\alpha'}\big|^4\omega_{m,n,q_z}\upsilon_{m,n,q_z}\\
&\times\Pi_L\big(m,n,q_z,\omega\big)\Pi_L\big(m,n,q_z,\omega-L\Omega\big). \qquad (14)
\end{aligned}$$

## III. CONDITION FOR PARAMETRIC AMPLIFICATION

The solution to the general dispersion equation, Eq. (14), is complicated; therefore, we limit our calculation to the case of the first-order resonance ($L = 1$), in which $\omega_{m,n,q_z} \pm \upsilon_{m,n,q_z} = \Omega$. We also assume that the electron-phonon interactions satisfy the condition $\left|C_{m,n}(q_z)I^{m,n}_{\alpha,\alpha'}(q_{m,n})\right|^2\left|D_{m,n}(q_z)I^{m,n}_{\alpha,\alpha'}(q_{m,n})\right|^2 \ll 1$. With these limitations, if we write the dispersion relations for acoustic and optical phonons as $\omega_{ac}(m,n,q_z) = \omega_a + i\tau_a$ and $\omega_{op}(m,n,q_z) = \omega_0 + i\tau_0$, we obtain the resonant acoustic phonon modes [10]

$$\omega^{(\pm)}_{\pm} = \omega_a + \frac{1}{2}\left[(\upsilon_a \pm \upsilon_0)\Delta q_z - i(\tau_a+\tau_0)\right] \pm \sqrt{\left[(\upsilon_a \mp \upsilon_0)\Delta q_z - i(\tau_a-\tau_0)\right]^2 \pm \Lambda^2} \qquad (15)$$

where $\upsilon_a$ and $\omega_a$ ($\upsilon_0$ and $\omega_0$) are the group velocity and the renormalization (by the electron -phonon interaction) frequency of the acoustic (optical) phonon, respectively, $\Delta q_z = q_z - q_0$, $q_0$ being the wave number for which the resonance is maximal,

$$\Lambda = \frac{2}{\hbar\Omega}\sum_{\alpha,\alpha',m,n}C_{m,n}(q_z)D_{m,n}(q_z)\left|I^{m,n}_{\alpha,\alpha'}(q_{m,n})\right|^2.\Pi_1\left(q_z,\omega_{m,n,q_z}\right) \qquad (16)$$

In Eq. (15), the signs ($\pm$) in the subscript of $\omega_{\pm}^{(\pm)}$ correspond to the signs ($\pm$) in front of the root, and the signs ($\pm$) in the superscript of $\omega_{\pm}^{(\pm)}$ correspond to the other sign pairs. These signs depend on the resonance condition $\omega_{m,n,q_z} \pm \upsilon_{m,n,q_z} = \Omega$. For instance, the existence of a positive imaginary part of $\omega_{+}^{(-)}$ implies a parametric amplification of the acoustic phonon. In such cases that $\lambda/\Omega \ll 1$, the maximal resonance, and $q = q_z$, we obtain

$$F = Im\left[\omega_{+}^{(-)}\right] = \frac{1}{2}\left[-(\tau_a+\tau_0)+\sqrt{(\tau_a-\tau_0)^2+|\Lambda|^2}\right] \tag{17}$$

where $\tau_a$ and $\tau_0$ are the imaginary parts of the frequencies of the acoustic and the optical phonons and take the forms

$$\tau_a = -\frac{1}{\hbar}\sum_{\alpha,\alpha',m,n}\left|C_{m,n}(q_z)I_{\alpha,\alpha'}^{m,n}(q_{m,n})\right|^2 \xi\left(\omega_{m,n,q_z}\right), \tag{18}$$

$$\tau_0 = -\frac{1}{\hbar}\sum_{\alpha,\alpha',m,n}\left|D_{m,n}(q_z)I_{\alpha,\alpha'}^{m,n}(q_{m,n})\right|^2 \xi\left(\upsilon_{m,n,q_z}\right), \tag{19}$$

and

$$\begin{aligned} |\Lambda| &= \frac{\lambda}{\hbar\Omega}\sum_{\alpha,\alpha',m,n} C_{m,n}(q_z)D_{m,n}(q_z)\left|I_{\alpha,\alpha'}^{m,n}(q_{m,n})\right|^2 \\ &\times \left\{\left[\theta\left(\omega_{m,n,q_z}\right)-\theta\left(\omega_{m,n,q_z}-\Omega\right)\right]^2+\left[\xi\left(\omega_{m,n,q_z}\right)-\xi\left(\omega_{m,n,q_z}-\Omega\right)\right]^2\right\}^{1/2}, \end{aligned} \tag{20}$$

with

$$\begin{aligned} \xi\left(\omega_{m,n,q_z}\right) &= \frac{m_e L_z}{2\hbar^2 q_z}\exp\left[\beta\left(\varepsilon_F-\varepsilon_\alpha\right)\right]\exp\left[-\frac{m\beta\varepsilon_{\alpha,\alpha'}^2\left(\omega_{m,n,q_z}\right)}{2\hbar^2 q_z^2}\right] \\ &\times\left\{1-\exp\left(\beta\hbar\omega_{m,n,q_z}\right)\right\}, \end{aligned} \tag{21}$$

$$\theta\left(\omega_{m,n,q_z}\right) = \frac{L_z\sqrt{m_e}}{\hbar\sqrt{2\pi\beta}}e^{\beta\varepsilon_F}\left\{\exp(-\beta\varepsilon_\alpha)-\exp(-\beta\varepsilon_{\alpha'})\right\}\times\frac{1}{\varepsilon_{\alpha,\alpha'}\left(\omega_{m,n,q_z}\right)}, \tag{22}$$

with $\beta = \frac{1}{k_B T}$, $k_B$ being the Boltzmann constant and $T$ the temperature of the system. $\varepsilon_F$ is the Fermi level, and

$$\varepsilon_{\alpha,\alpha'}(\omega_{m,n,q_z}) = \varepsilon_\alpha - \varepsilon_{\alpha'} - \hbar\omega_{m,n,q_z} - \frac{\hbar^2 q_z^2}{2m} \tag{23}$$

From Eq. (17), the condition for the resonant acoustic phonon modes to have a positive imaginary part leads to $|\Lambda|^2 > 4\tau_a\tau_0$. Using these conditions and Eqs. (18)-(20) yields the threshold amplitude for the EEF for a non-degenerate electron gas:

$$E_0 > E_{th} = \frac{2m_e\Omega^2}{eq_z}\frac{\sqrt{\xi\left(\omega_{m,n,q_z}\right)\xi\left(\upsilon_{m,n,q_z}\right)}}{\sqrt{\left[\theta\left(\omega_{m,n,q_z}\right)-\theta\left(\omega_{m,n,q_z}-\Omega\right)\right]^2+\left[\xi\left(\omega_{m,n,q_z}\right)-\xi\left(\omega_{m,n,q_z}-\Omega\right)\right]^2}}. \tag{24}$$

Equation (24) means that the parametric amplification of the acoustic phonons is achieved when the amplitude of the EEF is higher than some threshold amplitude.

## IV. NUMERICAL RESULTS AND DISCUSSION

To numerically estimate the threshold amplitude, $E_{th}$, and the parametric amplification coefficient, $F$ for parametric amplification of confined acoustic phonons, we use specific GaAs/GaAsAl RQW with the parameters [6, 11, 12]: $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $\xi = 13.5$ eV, $\rho = 5.32$ gcm$^{-3}$, $v_a = 5370$ ms$^{-1}$, $\varepsilon_F = 50$ meV, $L_y = 10\mathring{A}$, $L_z = 100\mathring{A}$, $m = 0.067m_0$, $m_0$ being the mass of free electron, and $\hbar\nu_{m,n,q_z} = 36.25$ meV, $T = 77$ K, $\Omega = 4\times10^{13}$ Hz, the set of transition is $n = 2, \ell = 1, n' = 2, \ell' = 1, m = 1, n = 1$.

In Fig. 1, 2, we show $E_{th}$ as a function of the reduced wave number $q_z$, and the length $L_x$ of RQW at temperature of 77K. The figures show that the shape of the curves are total similar each other, the curves have maximal values and are non-symmetric around the maxima. This is due to the fact that a fixed EEF, with an amplitude greater than the corresponding threshold amplitude, can induce parametric amplification for confined acoustic phonons in two regions of the wave number corresponding to the two sign in $\omega_{m,n,q_z} \pm \nu_{m,n,q_z} = \Omega$. $E_{th}$ strongly depends on temperature $T$, as the temperature drops, the maximum value of $E_{th}$ decreases significantly, and its peak shifts to larger wave numbers. However, in Fig. 2, when the length $L_x$ of RQW increases, the value of $E_{th}$ increases and reaches a saturated value. We can see that in the case of confined phonons the amplitudes are greater than the corresponding threshold amplitude in the case of unconfined phonons.

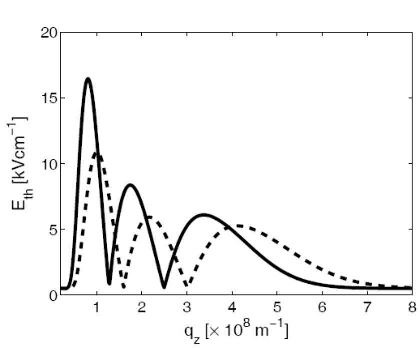


**Fig. 1.** $E_{th}$ (kVcm$^{-1}$) as a function of $q_z$ in two cases: confined-phonon (solid line) and bulk-phonon (dashed line). Here, $L_x = 40\mathring{A}$

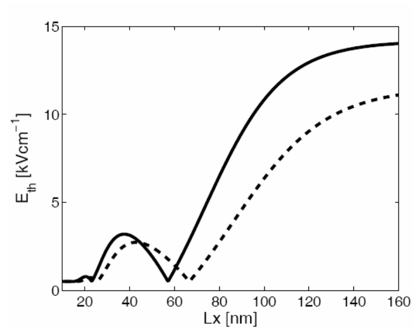


**Fig. 2.** $E_{th}$ (kVcm$^{-1}$) as a function of $L_x$ in two cases: confined-phonon (solid line) and bulk-phonon (dashed line). Here, $q_z = 2\times10^8$ m$^{-1}$

The dependence of the parametric amplification coefficient $F$ on the reduced wave number $q_z$, and the length $L_x$ of RQW is presented in Fig. 3 and Fig. 4. It is easy for us to recognize the shape of these curves are different from they are in above figures. $F$ has the both positive (amplification) and negative (absorption) values. This is consistent with the above consideration for the threshold amplitude. Apparently, there are two regions of wave number in which parametric amplification for acoustic phonons is achieved ($F > 0$). The amplification for wave number from $1\times10^8$ m$^{-1}$ to $3\times10^8$ m$^{-1}$ is stronger than it is

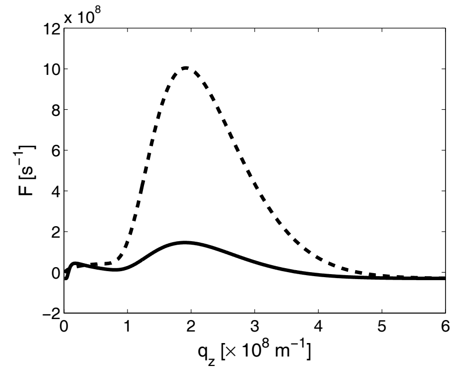


**Fig. 3.** $F$ ($\mathrm{s}^{-1}$) as a function of $q_z$ in two cases: confined-phonon (solid line) and bulk-phonon (dashed line). Here, $L_x = 40\mathring{A}$

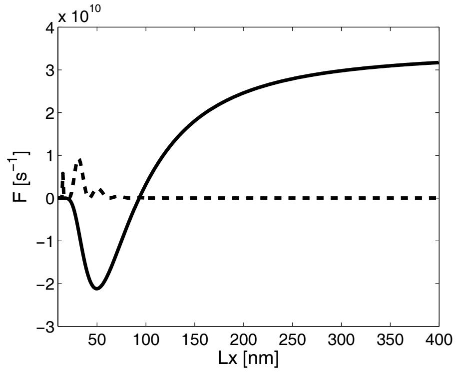


**Fig. 4.** $F$ ($\mathrm{s}^{-1}$) as a function of $L_x$ in two cases: confined-phonon (solid line) and bulk-phonon (dashed line). Here, $q_z = 2 \times 10^8\ \mathrm{m}^{-1}$

for the other wave numbers. In Fig. 4, $F$ is also sensitive to the length $L_x$ of RQW and it has the similar explanation.

## V. CONCLUSIONS

In this paper, we analytically investigated the possibility of parametric resonance of confined acoustic and confined optical phonons in RQWs. We obtained a general dispersion equation for parametric amplification and transformation of phonons. However, an analytical solution to the equation could only be obtained within some limitations. Using these limitations for simplicity, we obtained dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification. Analytical expressions show that the parametric amplification and threshold amplitude depend on the frequency $\omega_{m,n,q_z}$ and $\upsilon_{m,n,q_z}$ while $\omega_{m,n,q_z}$ and $\upsilon_{m,n,q_z}$ depend on $m, n$ quantum numbers and component of wave vector $q_z$. That is a basic difference between confined phonon mode and bulk phonon mode .

Numerical results for a specific quantum wire GaAs/GaAsAl: Be clearly showed the predicted mechanism. Parametric amplification for acoustic phonons and the threshold amplitude depended on the physical parameters of the system and were sensitive to the temperature in the region of low temperature but saturated in the region of high temperatures. These characteristics are similar to those in quantum wells, and maybe they are common properties of low dimensional systems.

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## REFERENCES

[1] V. P. Silin, *Parametric Action of the High-Power Radiation on Plasma*, 1973 Nauka Publisher, Moscow, in Russian.
[2] T. C. Phong, L. V. Tung, N. Q. Bau, *J. Korean Phys. Soc.* **53** (2008) 1971.
[3] N. Nishiguchi, *Phys. Rev. B* **52** (1995) 5279.
[4] Michael A. Stroscio, *Phys. Rev. B* **40** (1989) 6428.
[5] T. C. Phong, N. Q. Bau, *J. Korean Phys. Soc.* **42** (2003) 647.
[6] YU You-Bin, *Commun. Theor. Phys.* **49** (2008) 1615.
[7] X. F. Wang, X. L. Lei, *Phys. Rev. B* **49** (1994) 4780.
[8] V. B. Campos, S. Das Sarma, M. A. Stroscio, *Phys. Rev. B* **46** (1992) 3849.
[9] Y. He, Z. Yin, M. S. Zhang, T. Lu, Y. Zheng, *Mat. Sci. Eng. B* **75** (2000) 130; N. Morri, H. Momose, C. Hamaguchi, *Phys. Rev. B* **45** (1992) 4536.
[10] G. M. Shmelev, I. A. Chaikovskii, V. V. Pavlovich, E. M. Epshtein, *Phys. Stat. Sol. (b)* **80** (1977) 697; **82** (1977) 391.
[11] T. Brandes, A. Kawabata, *Phys. Rev. B* **54** (1996) 4444.
[12] H. Bruus, K. Flensberg, H. Smith, *Phys. Rev. B* **48** (1993) 11144.

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