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NONLINEAR ABSORPTION LINE-WIDTHS IN CYLINDRICAL QUANTUM WIRES

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Abstract. *Applying the theory of nonlinear optical conductivity for an electron-phonon system that has appeared recently, we propose a new method to obtain the line-width for nonlinear optical conductivity in a system of electron-optical phonon in cylindrical quantum wires (CQW). General analytic expressions for the nonlinear absorption power (NLAP) are obtained in the presence of an intense field and the two photon process is included into the consideration. The graphic dependence of NAP on the photon energy and the size of CQW is achieved computationally for specific CQWs. From graphs of the NLAP we obtain line-width as profile of curves. The dependence of the line-width on the temperature and the parameter of a CQW is reasonable in comparison with the other theoretical and experimental results.*

I. INTRODUCTION

The problem of nonlinear conductivity of electron-phonon system has been studied recently by Nam Lyong Kang, Hyun Jung Lee and Sang Don Choi by using operator projection technique (OPT) [1, 2]. The results of this study is that when an intense laser field is applied, the system absorbs two photons with energy $\hbar\omega_1$ and $\hbar\omega_2$ in order to transit from the initial $|\alpha\rangle$ to the final state $|\beta\rangle$. Applying the OPT used in the work [1, 2], the above authors obtained the expression of the expectation value of current density in which the nonlinear term was included. This term has the form

$$\langle J_i\rangle_{ens} = \sum_j \sigma_{ij}(\omega)E_j(\omega) + \sum_{ijk} \sigma_{i,j,k}(\omega_1,\omega_2)E_j(\omega_1)E_k(\omega_2) + ..., \tag{1}$$

where the symbol "..." denotes the higher order nonlinear terms of the conductivity tensor, $E_j(\omega) = E_j e^{-i\omega t}$ is the laser field subjected to the system; $\sigma_{ij}(\omega)$ and $\sigma_{ijk}(\omega_1,\omega_2)$ respectively are the linear term for the incident wave of a frequency ω and the nonlinear term for the incident waves of frequencies ω_1 and ω_2. These quantities are expressed as (see Eq. (25) in Ref. [2])

$$\sigma_{ij}(\omega) = -e \lim_{\Delta\to 0^+} \sum_{\alpha,\beta} (r_j)_{\alpha\beta}(\mathrm{j}_i)_{\beta\alpha} \frac{f_\beta - f_\alpha}{\hbar\bar{\omega} - \varepsilon_{\beta\alpha} - \Gamma_0^{\alpha\beta}(\bar{\omega})} \tag{2}$$

is the linear term of the conductivity tensor. The nonlinear term of the conductivity tensor is defined as (see Eq. (42) in Ref. [2])

$$\sigma_{ijk}(\omega_1,\omega_2) = e^2 \lim_{\Delta\to 0^+} \sum_{\alpha,\beta}\sum_{\gamma,\delta}\sum_{\xi,\epsilon}(r_j)_{\alpha\beta}(r_k)_{\gamma\delta}(\mathrm{j}_i)_{\xi\epsilon}\frac{(f_\beta - f_\alpha)}{\hbar\bar{\omega}_2 - \varepsilon_{\beta\alpha} - \Gamma_0^{\alpha\beta}(\bar{\omega}_2)} \times \left[\frac{\delta_{\xi\beta}\delta_{\delta\alpha}\delta_{\epsilon\gamma}}{\hbar\bar{\omega}_{12} - \varepsilon_{\beta\gamma} - \Gamma_1^{\alpha\beta\gamma}(\bar{\omega}_{12})} - \frac{\delta_{\gamma\beta}\delta_{\epsilon\alpha}\delta_{\xi\delta}}{\hbar\bar{\omega}_{12} - \varepsilon_{\delta\alpha} - \Gamma_2^{\alpha\beta\delta}(\bar{\omega}_{12})}\right]. \tag{3}$$

The results obtained by this team of authors is clear and have important meaning for calculating linewidths of absorption power in which the nonlinear term is included. However, up to date these authors have not given any results concerning linewidths with nonlinear term. The reason for this is that this team has not obtained the expression of conductivity tensor in which nonlinear terms and external field are included. In fact, the linewidth is determined from the dependence of absorption power on photon energy. The absorption power is determined by

$$P(\omega) = \frac{E_0^2}{2}\mathtt{Re}[\sigma(\omega)], \tag{4}$$

where E_0 is the amplitude of the external field subjected to the system, $\sigma(\omega)$ is the conductivity tensor, Re[...] denotes the real part of [...]".

To overcome this difficulty, in this paper we suggest a new method for obtaining the explicit expression of conductivity tensor with nonlinear terms. This is the case in which an intense field is applied to the electron-phonon system, there occurs the simultaneous absorption of two phonons with the same energy $\hbar\omega$. From the expression of nonlinear conductivity tensor we obtain the expression of NLAP. From the graph describing the dependence of NLAP on photon energy $\hbar\omega$, we obtain the dependence of linewidths on temperature T and wire's radius R in which nonlinear terms are included.

II. NONLINEAR ABSORPTION POWER IN QUANTUM WIRES

II.1. Expression of conductivity tensor with the nonlinear terms

When the external field is intense we have the case of absorbing two photons with the same frequency ω, i.e., $\omega_1 = \omega_2 = \omega$, Eq. (1) becomes

$$\langle J_i\rangle_{ens} = \sum_{j=1}^{3}\left[\sigma_{ij}(\omega) + \sum_{k=1}^{3}\sigma_{ijk}(\omega)E_k(\omega)\right]E_j(\omega) = \sigma_{pt}(\omega)E_j(\omega). \tag{5}$$

The nonlinear conductivity can be rewritten as

$$\sigma_{pt}(\omega) = \sigma_{ij}(\omega) + \sum_{k=1}^{3}\sigma_{ijk}(\omega)E_k(\omega). \tag{6}$$

Suppose that the external electric field vector polarized along the horizontal direction, $E_\perp(\omega) = E_\perp e^{i\omega t}$, the transverse component of conductivity in which nonlinear terms are included can be written in the form

$$\sigma_{pt}(\omega) = \sigma_0(\omega) + \sigma_1(\omega)E_\perp(\omega), \tag{7}$$

where the first term and the second term correspond to the linear and nonlinear terms of the conductivity tensor. In this case, from Eqs. (2) and (3), we have

$$\sigma_0(\omega) = -e\sum_{\alpha,\beta}(r_\perp)_{\alpha\beta}(\mathrm{j}_\perp)_{\beta\alpha}\frac{f_\beta - f_\alpha}{\hbar\bar{\omega} - E_{\beta\alpha} - \Gamma_0^{\alpha\beta}(\bar{\omega})}, \tag{8}$$

$$\sigma_1(\omega) = e^2\sum_{\alpha,\beta}\sum_{\gamma,\delta}\sum_{\xi,\epsilon}(r_\perp)_{\alpha\beta}(r_\perp)_{\gamma\delta}(\mathrm{j}_\perp)_{\xi\epsilon}\frac{(f_\beta - f_\alpha)}{\hbar\bar{\omega} - E_{\beta\alpha} - \Gamma_0^{\alpha\beta}(\bar{\omega})} \times\left[\frac{\delta_{\xi\beta}\delta_{\delta\alpha}\delta_{\epsilon\gamma}}{2\hbar\bar{\omega} - E_{\beta\gamma} - \Gamma_1^{\alpha\beta\gamma}(2\bar{\omega})} - \frac{\delta_{\gamma\beta}\delta_{\epsilon\alpha}\delta_{\xi\delta}}{2\hbar\bar{\omega} - E_{\delta\alpha} - \Gamma_2^{\alpha\beta\delta}(2\bar{\omega})}\right]. \tag{9}$$

II.2. Linear term of conductivity tensor

The electron wave function in cylindrical quantum wire is given by [3]:

$$\psi_{n,\ell,\vec{k}}(r,\varphi,z) = \frac{1}{\sqrt{V_0}}\mathrm{e}^{ik_z z}\mathrm{e}^{in\varphi}\psi_{n,\ell}(r), \quad \psi_{n,\ell}(r) = \frac{J_n(A_{n,\ell}\frac{r}{R})}{J_{n+1}(A_{n,\ell})}, \tag{10}$$

where $V_0 = \pi R^2 L_z$ is the volume of the specimen; R is the wire's radius; $\vec{k} = (0, 0, k_z)$ is the electron wave vector, $A_{n,\ell}$ is the ℓ-th zero of Bessel function of the n-order, $n = 0, \pm1, \pm2, \ldots$; $\ell = 1, 2, \ldots$. For the lowest levels we have $A_{01} = 2.405, A_{11} = 3.832$. The electronic energy is [3]:

$$E_{n,\ell}(\vec{k}) = E(k_z) + E_{n,\ell}, \tag{11}$$

where $E(k_z) = \hbar^2 k_z^2/2m$ is the energy in z-direction, $E_{n,\ell} = \hbar^2 A_{n,\ell}^2/2mR^2$ is the energy quantized along the horizontal direction, m is the effective of electron.

Since $\overline{\omega} = \omega - i\Delta$, $(\Delta \to 0^+)$, the function $\Gamma_0^{\alpha\beta}(\overline{\omega})$ in Eq. (8) is complex and can be split into two parts $\Gamma_0^{\alpha\beta}(\overline{\omega}) = A_0^{\alpha\beta}(\overline{\omega}) + iB_0^{\alpha\beta}(\overline{\omega})$. For the weak scattering effect, $A_0^{\alpha\beta}(\overline{\omega}) \ll E_{\beta\alpha}$, we can ignore $A_0^{\alpha\beta}(\overline{\omega})$ in comparison with $E_{\beta\alpha}$. Considering the scattering process at the boundary of Brillouin ($k_z = 0$), we obtain the linear conductivity due to electron-LO phonon scattering.

$$\sigma_0(\omega) = \sum_{\alpha\beta}\frac{iA_0}{a_0 - ib_0}, \tag{12}$$

where we have denoted

$$A_0 = \frac{-e^2\hbar(2\pi)^6}{mV_0^2}I_{n_\alpha n_\beta}I'_{n_\beta n_\alpha}(f_\beta - f_\alpha), \quad a_0 = \hbar\omega - E_{\beta\alpha}, \quad b_0 = B_0^{\alpha\beta}(\omega), \tag{13}$$

with

$$I_{n_\alpha n_\beta} = \int_0^R \frac{J_0^*(A_{01}\frac{r}{R})J_1(A_{11}\frac{r}{R})}{J_1^*(A_{01})J_1(A_{11})}r^2 dr, \tag{14}$$

$$I'_{n_\beta n_\alpha} = \int_0^R \frac{J_1^*(A_{11}\frac{r}{R})}{J_2^*(A_{11})}\frac{\partial}{\partial r}\frac{J_0(A_{01}\frac{r}{R})}{J_0(A_{01})}r dr. \tag{15}$$

The quantity $B_0^{\alpha\beta}(\omega)$ is defined as

$$B_0^{\alpha\beta}(\omega) = \frac{1}{R(f_\beta - f_\alpha)} \sum_{n_\gamma,\ell_\gamma} \left[D_{01} \left(\frac{A^+_{\gamma\alpha 0}}{k^+_{\gamma\alpha 0}} + \frac{A^-_{\gamma\alpha 0}}{k^-_{\gamma\alpha 0}} \right) + D_{02} \left(\frac{A^+_{\gamma\beta 0}}{k^+_{\gamma\beta 0}} \frac{A^-_{\gamma\beta 0}}{k^-_{\gamma\beta 0}} \right) \right], \tag{16}$$

where

$$\begin{aligned}
D_{01} &= \frac{0.29e^2\omega_0}{\pi\varepsilon_0} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \frac{m}{\hbar}, \quad D_{02} = \frac{0.19e^2\omega_0}{\pi\varepsilon_0} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \frac{m}{\hbar}, && (17)\\
k^{\pm}_{\gamma i 0} &= \sqrt{\frac{1}{R^2}(2.405^2 - A_\gamma^2) \pm \frac{2m}{\hbar}(\omega \pm \omega_0)}, \quad A_\gamma = A_{n_\gamma,\ell_\gamma}, && (18)\\
f^{\pm}_{\gamma i 0} &= [e^{(E^{\pm}_{\gamma i 0} - E_F)/k_B T} + 1]^{-1}, \quad E^{\pm}_{\gamma i 0} = \frac{\hbar^2 (k^{\pm}_{\gamma i})^2}{2m} + E_{n\gamma}, && (19)\\
A^+_{\gamma\alpha 0} &= (1+N_q) f^+_{\gamma\alpha 0}(1-f_\alpha) - N_q f_\alpha (1 - f^+_{\gamma\alpha 0}), && (20)\\
A^-_{\gamma\alpha 0} &= N_q f^-_{\gamma\alpha 0}(1-f_\alpha) - (1+N_q) f_\alpha (1 - f^-_{\gamma\alpha 0}), && (21)\\
A^+_{\gamma\beta 0} &= (1+N_q) f_\beta (1 - f^+_{\gamma\beta 0}) - N_q f^+_{\gamma\beta 0}(1-f_\beta), && (22)\\
A^-_{\gamma\beta 0} &= N_q f_\beta (1 - f^-_{\gamma\beta 0}) - (1+N_q) f^-_{\gamma\beta 0}(1-f_\beta). && (23)
\end{aligned}$$

where χ_0 and χ_∞ is the static and the high-frequency dielectric constant, k_B being the Boltzmann constant and T - the temperature of the system, E_F is the Fermi level.

II.3. Nonlinear term of conductivity tensor

From the expression of nonlinear optical conductivity (3), calculating the sum of three indexes δ, ξ, ε for the first term and of γ, ξ, ε for the second term, and consider the process of simultaneously absorbing two photons with the same frequencies, we get

$$\sigma_1(\omega) = \sum_{n_\gamma,\ell_\gamma} \sum_{n_\delta,\ell_\delta} \frac{iA_1}{a_0 - ib_0} \left[\frac{A_2}{a_2 - ib_2} - \frac{A_3}{a_3 - ib_3} \right], \tag{24}$$

with

$$\begin{aligned}
A_1 &= \frac{e^3\hbar(2\pi)^9}{mV_0^3}(f_\beta - f_\alpha), \quad a_0 = \hbar\omega - E_{\beta\alpha}, \quad b_0 = B_0^{\alpha\beta}(\omega), && (25)\\
A_2 &= I_{n_\alpha n_\beta} I_{n_\eta n_\alpha} I'_{n_\eta n_\alpha}, \quad a_2 = 2\hbar\omega - E_{\beta\gamma}, \quad b_2 = B_1^{\alpha\beta\gamma}(2\omega), && (26)\\
A_3 &= I_{n_\alpha n_\beta} I_{n_\beta n_\delta} I'_{n_\delta n_\alpha}, \quad a_3 = 2\hbar\omega - E_{\delta\alpha}, \quad b_3 = B_2^{\alpha\beta\delta}(2\omega) && (27)
\end{aligned}$$

where $I_{n_\eta n_\alpha}, I'_{n_\eta n_\alpha}, I_{n_\beta n_\delta}, I'_{n_\delta n_\alpha}$ are calculated by the same way as Eq. (13). Doing the same calculation as in (16), we get

$$B_1^{\alpha\beta\gamma}(2\omega) = \frac{1}{R(f_\beta - f_\alpha)} \sum_{n_\eta,\ell_\eta} \left[D_{11} \left(\frac{A^+_{\eta\beta 1}}{k^+_{\eta\beta 1}} + \frac{A^-_{\eta\beta 1}}{k^-_{\eta\beta 1}} \right) + D_{22} \left(\frac{A^-_{\eta\gamma 1}}{k^-_{\eta\gamma 1}} + \frac{A^+_{\eta\gamma 1}}{k^+_{\eta\gamma 1}} \right) \right], \tag{28}$$

$$B_2^{\alpha\beta\delta}(2\omega) = \frac{D_{11}}{R(f_\beta - f_\alpha)} \sum_{n_\eta,\ell_\eta} \left[\frac{A^+_{\eta\alpha 2}}{k^+_{\eta\alpha 2}} + \frac{A^-_{\eta\alpha 2}}{k^-_{\eta\alpha 2}} + \frac{A^-_{\eta\delta 2}}{k^-_{\eta\delta 2}} + \frac{A^+_{\eta\delta 2}}{k^+_{\eta\delta 2}} \right], \tag{29}$$

where

$$D_{11} = \frac{0.19me^2\omega_0}{\pi\hbar\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right),\ D_{12} = \frac{0.073me^2\omega_0}{\pi\hbar\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right). \tag{30}$$

The other terms are calculated in the same way as in equations from (18) to (23).

II.4. Nonlinear absorption power in cylindrical quantum wire

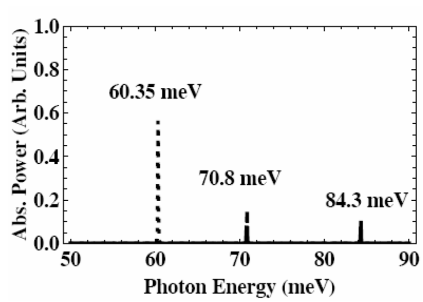
Inserting (12) and (24) into (7), we obtain the explicit expression of transverse component of the conductivity tensor

$$\sigma_{pt} = \sum_{\alpha,\beta,\eta,\gamma,\sigma}\left[\frac{iA_0}{a_0 - ib_0} + \frac{iA_1E_\perp(\omega)}{a_1 - ib_1}\left(\frac{A_2}{a_2 - ib_2} - \frac{A_3}{a_3 - ib_3}\right)\right]. \tag{31}$$

Taking into account the real part of conductivity tensor in Eq. (31), and inserting it into (4) we can express the nonlinear absorption power (NLAP).

III. NUMERICAL RESULTS AND DISCUSSIONS

The parameters used in the calculation of the absorption power for specific GaAs/AlAs CQW are as follow [4, 5, 6, 7, 8]: $\varepsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $m = 0.067m_0$ (m_0 being the mass of free electron), $\hbar\omega_0 = 36.2$ meV. We consider the transition with $n_\alpha = 0$, $\ell_\alpha = 1$; $n_\beta = 1$, $\ell_\beta = 1$.



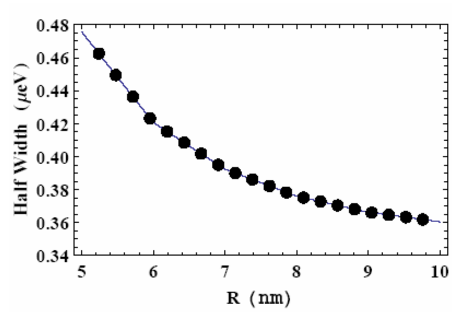


Fig. 1. On the left: Photon energy dependence of absorption power with different values of wire's radius. The solid, dashed, and dotted lines correspond to the radius of 5.5 nm, 6.0 nm, and 6.5 nm, respectively. On the right: wire's radius dependence of nonlinear linewidth at $T = 300$ K.

Figure 1 describes the dependence of NLAP on photon energy with different values of wire's radius at $T = 300$ K. The figure shows that as R increases the peak shifts to the lower energy (on the left hand side). With $R = 5.5$ nm, 6.0 nm, and 6.5 nm, the photon energy corresponding to resonance peaks are $\hbar\omega = 84.3$ meV, 70.8 meV, and 60.35 meV. Resonance photons with energy satisfying condition $2\hbar\omega = \Delta E_{\beta\alpha} + \hbar\omega_0$ describes the fact that one electron in the state $|\alpha\rangle$ absorbs two photons and concurrently emits one phonon then jumps to state $|\beta\rangle$. As R increases, $\Delta E_{\beta\alpha}$ decreases ($E_\alpha = \hbar^2 A^2_{n_\alpha,\ell_\alpha}/(2mR^2)$). As the result, the photon energy satisfied the resonance condition $2\hbar\omega = \Delta E_{\beta\alpha} + \hbar\omega_0$ decreases.

In order to find the dependence of NALW on the wire's radius R, we first plotted the graph showing the dependence of absorption power on the photon energy with different values of R in the case of taking into account the nonlinear term. Then we use the command FindRoot$[[P_{pt}(\omega)] = P_{max}(\omega)/2]$ of Mathematica software for finding two values of photon energy $\hbar\omega_1$ and $\hbar\omega_2$ corresponding to a half-maximum of the absorption power. One pair of $(R, \Delta\hbar\omega = \hbar\omega_2 - \hbar\omega_1)$ represents one point on the curve of the graph. Joining these points, we obtain the rule showing the dependence of NALW on R. The obtained results is shown on Fig. 1 (on the right hand side).

From the figure we can see that NALW decreases as the radius increases. This is because as the wire's radius increases the confinement of electron reduces, the probability of electron-phonon scattering decreases, so that NALW drops.

Figure 2 shows the dependence of NLAP on the photon energy with different value of temperature at $R = 6.5$ nm. It can be seen from the figure that resonance peaks appear at the same position. They describe the process in which one electron on the state $|\alpha\rangle$ simultaneously absorbs two photon with energy 60.35 meV to jump to the final state $|\beta\rangle$. In this process one phonon with energy $\hbar\omega_0$ is emitted. The transition process complied with the condition $2\hbar\omega = \Delta E_{\beta\alpha} + \hbar\omega_0$.

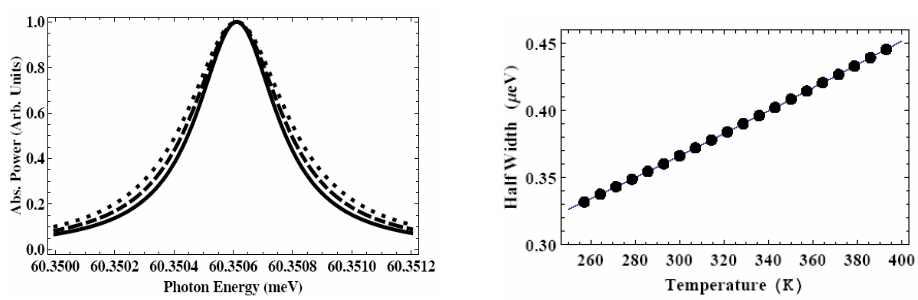


Fig. 2. On the left: Photon energy dependence of absorption power with different values of temperature. The solid, dashed, and dotted lines correspond to T=250 K, 300 K and 350 K. On the right: Temperature dependence of nonlinear linewidth at $R = 6.5$ nm.

In order to find the dependence of NALW on the temperature, we plot the graph showing the dependence of NLAP on the photon energy with different value of temperature in the case the nonlinear term is included. From the figure we can see that different values of NLAP corresponding to resonance peaks at different values of temperature are shifted to the same value (on the left hand side of Fig. 2). Doing the same way as that in the case of finding the dependence of NALW on R, we obtain the dependence of NALW on the temperature. This result is shows on the figure 2 (on the right hand side).

We can see from the figure that NALW increase with temperature. This is because as the temperature increases the probability of electron-phonon scattering rises, so does the NALW.

IV. CONCLUSION

In the present paper, based on the supposition that the electron system simultaneously absorbs two photon with the same energy $\hbar\omega$ we proposed a new method to obtain the explicit expression of NLAP, from which we can determine NALW. We also did numerical calculation and plotted the graph of NLAP for GaAs/AlAs cylindrical wire and determined NALW to illustrate the method. The dependences of NLAP on the photon energy with different value of temperature and wire's radius were considered. The energy of absorbed photons satisfied the resonance condition $2\hbar\omega = \Delta E_{\beta\alpha} + \hbar\omega_0$.

From the graph showing the dependence of NLAP on the photon energy, we derived the dependence of the NALW on the temperature and wire's radius. The obtained numerical results showed that the NALW increases with the temperature and decreases with the wire's radius. This result is consistent to our prediction and can be explained by means of physical meanings.

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