Emergent holographic description for “strongly coupled” quantum field theories

Ki-Seok Kim (POSTECH)

The main message of the present talk

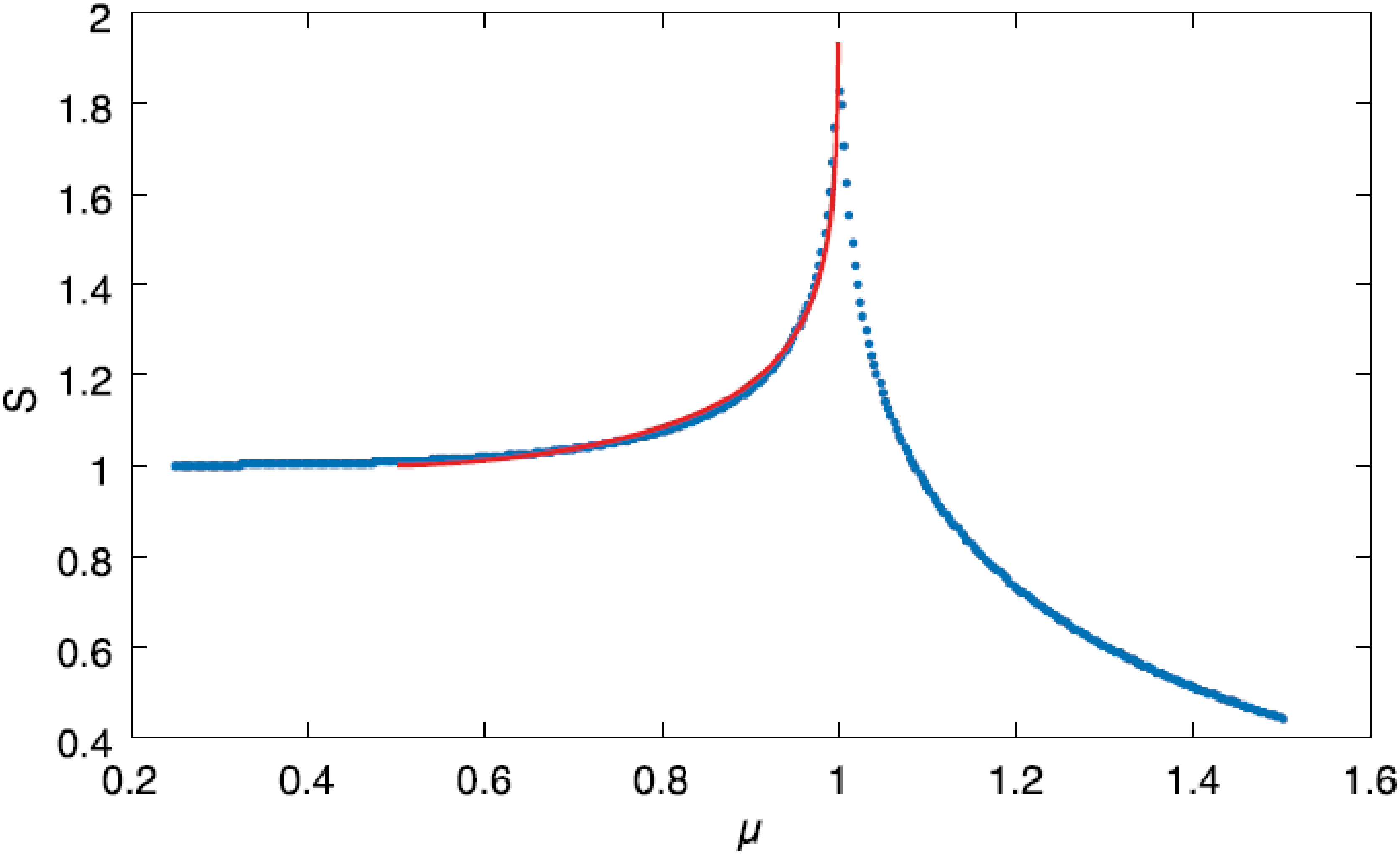
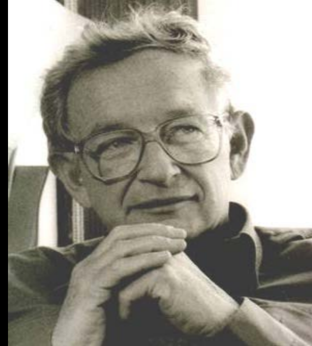
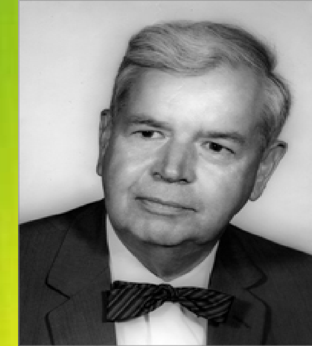
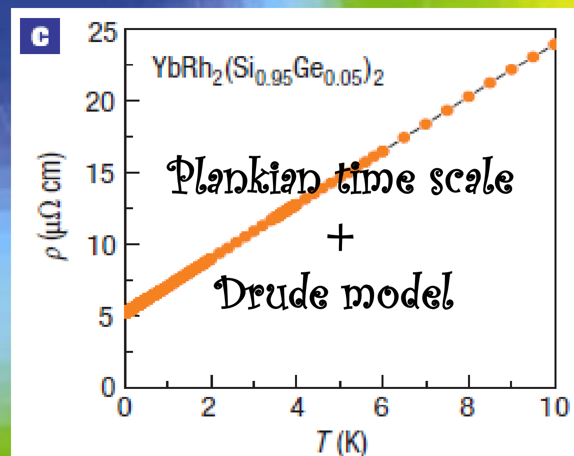


FIG. 2. Comparison between the Ryu-Takayanagi formula based on the emergent metric [Eqs. (31) and (32)] and the entanglement entropy based on the lattice model [Eq. (2)].

The final goal of the present theoretical framework







$$S = \frac{1}{4}\frac{c^3k}{G\hbar}A$$

AF

FL

DOI: 10.3938/PhiT.23.010 by J. D. Thompson

Why do we care about the AdS_{d+2}/CFT_{d+1} duality conjecture ?

AdS_{d+2}/CFT_{d+1} duality

International Journal of Theoretical Physics, Vol. 38, No. 4, 1999

The Large-N Limit of Superconformal Field Theories and Supergravity

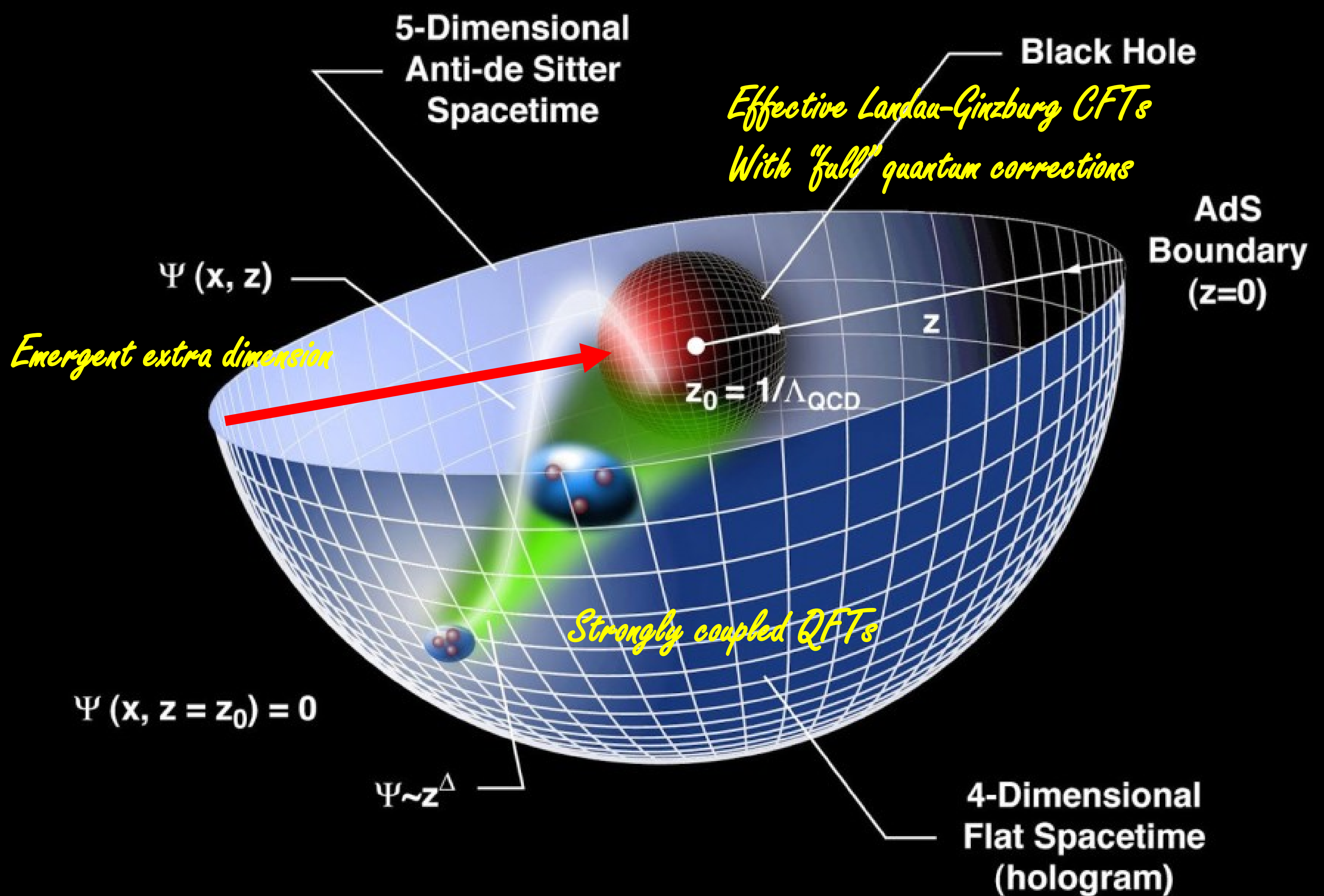
Juan Maldacena[1]

Received September 15, 1998

We show that the large-N limits of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of anti-de Sitter spacetimes, spheres, and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low-energy limit where the field theory on the brane decouples from the bulk. We observe that, in this limit, we can still trust the near-horizon geometry for large N. The enhanced supersymmetries of the near-horizon geometry correspond to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincaré group). The 't Hooft limit of 3 + 1 $\mathcal{N} = 4$ super-Yang–Mills at the conformal point is shown to contain strings: they are IIB strings. We conjecture that compactifications of M/string theory on various anti-de Sitter spacetimes is dual to various conformal field theories. This leads to a new proposal for a definition of M-theory which could be extended to include five noncompact dimensions.

- Solving strongly coupled quantum field theories = putting Landau-Ginzburg-Wilson-type order-parameter field theories on emergent curved spacetimes (black holes) with an extra dimension and solving classical equations of motion in order to find correlation functions in a non-perturbative way

cf. Solving "weakly coupled" quantum field theories = solving LGW semi-classical field theories



https://www.georgeshiber.com/a-sasaki-einstein-adscft-duality-susy-and-dp-brane-conic-orbifolding-of-gr/

Hydrodynamics in "metals"?

From AdS/CFT correspondence to hydrodynamics

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ABSTRACT: We compute the correlation functions of R-charge currents and components of the stress-energy tensor in the strongly coupled large-N finite-temperature $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, following a recently formulated minkowskian AdS/CFT prescription. We observe that in the long-distance, low-frequency limit, such correlators have the form dictated by hydrodynamics. We deduce from the calculations the R-charge diffusion constant and the shear viscosity. The value for the latter is in agreement with an earlier calculation based on the Kubo formula and absorption by black branes.

- Strong inelastic scattering → Fast thermalization → Effective hydrodynamics: AdS_{d+2} classical dual field theory

Hydrodynamic transport phenomena are quite difficult to realize in metals.

$$\tau_{el-el} < \tau < \tau_{el-ph}, \tau_{el-imp}$$

However, ……

Hydrodynamics in the Dirac fluid ?: Realization of the AdS_{d+2} semi-classical field theory ?

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4]

1. J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, K. C. Fong, *Science* **351**, 1058 (2016).
2. D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, M. Polini, *Science* **351**, 1055 (2016).
3. P. J. W. Moll *et al.*, *Science* **351**, 1061 (2016).

Negative local resistance caused by viscous electron bac
in graphene

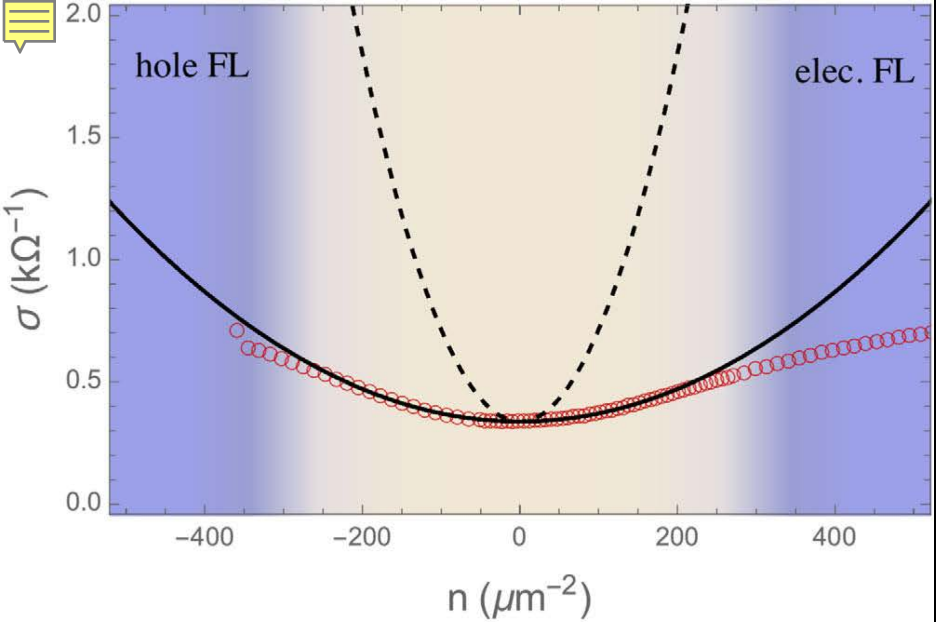
D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Be
A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Nov
L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]

Graphene hosts a unique electron system in which electro
weak but electron-electron collisions are sufficiently freque
the temperature of liquid nitrogen. Under these conditions
liquid and exhibit hydrodynamic phenomena similar to cla
strong evidence for this transport regime. We found that do
(negative) voltage drop near current-injection contacts, wh
of submicrometer-size whirlpools in the electron flow. The v
is found to be ~0.1 square meters per second, an order of m
in agreement with many-body theory. Our work demonstra
electron hydrodynamics using high-quality graphene.

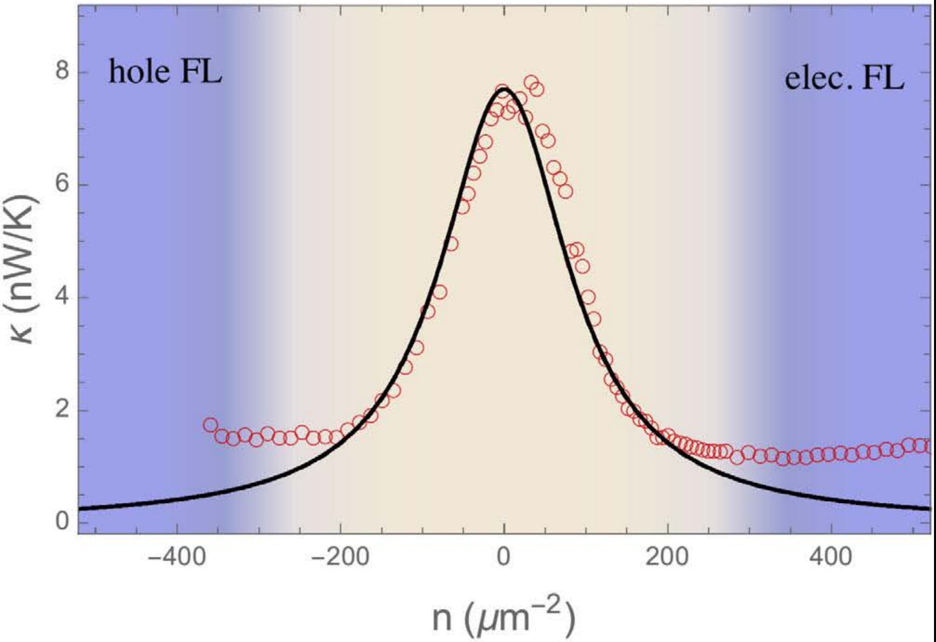
Evidence for hydrodynamic electron flow in $PdCoO_2$

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3] Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4*]

Electron transport is conventionally determined by the momentum-relaxing scattering of electrons by the host solid and its excitations. Hydrodynamic fluid flow through channels, in contrast, is determined partly by the viscosity of the fluid, which is governed by momentum-conserving internal collisions. A long-standing question in the physics of solids has been whether the viscosity of the electron fluid plays an observable role in determining the resistance. We report experimental evidence that the resistance of restricted channels of the ultrapure two-dimensional metal palladium cobaltate ($PdCoO_2$) has a large viscous contribution. Comparison with theory allows an estimate of the electronic viscosity in the range between 6×10^{-3} kg m^{-1} s^{-1} and 3×10^{-4} kg m^{-1} s^{-1}, versus 1×10^{-3} kg m^{-1} s^{-1} for water at room temperature.



(a)



(b)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge-neutrality point, which can form a strongly coupled Dirac fluid. This charge-neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, thanks to decoupling of charge and heat currents within hydrodynamics. Employing high-sensitivity Johnson noise thermometry, we report an order of magnitude increase in the thermal conductivity and the breakdown of the Wiedemann-Franz law in the thermally populated charge-neutral plasma in graphene. This result is a signature of the Dirac fluid and constitutes direct evidence of collective motion in a quantum electronic fluid.

PRL **118,** 036601 (2017) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2017

Holography of the Dirac Fluid in Graphene with Two Currents

Yunseok Seo,[1] Geunho Song,[1] Philip Kim,[2,3] Subir Sachdev,[2,4] and Sang-Jin Sin[1]
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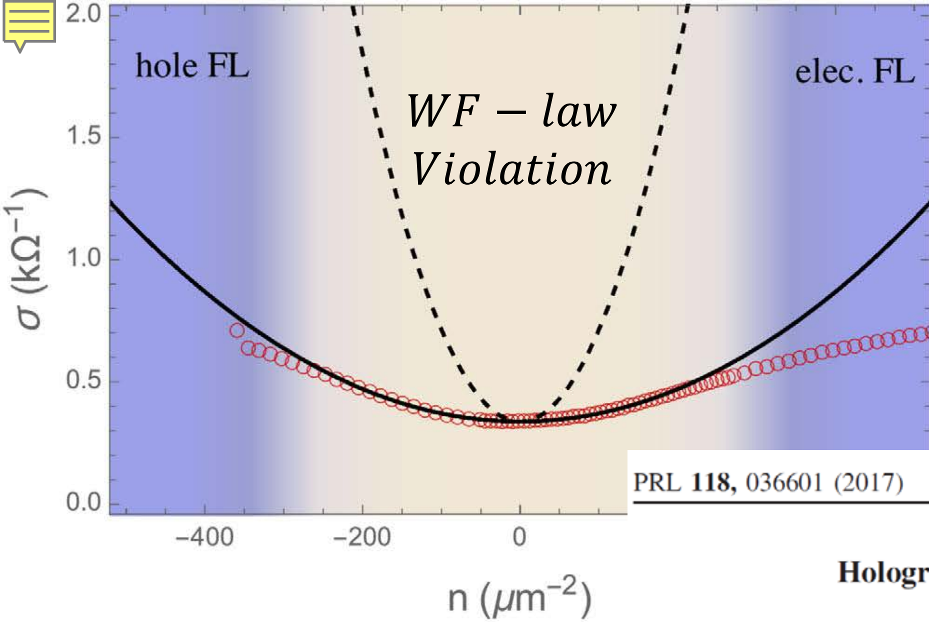
(Received 17 September 2016; published 18 January 2017)

Recent experiments have uncovered evidence of the strongly coupled nature of graphene: the Wiedemann-Franz law is violated by up to a factor of 20 near the charge neutral point. We describe this strongly coupled plasma by a holographic model in which there are two distinct conserved U(1) currents. We find that our analytic results for the transport coefficients for the two current model have a significantly improved match to the density dependence of the experimental data than the models with only one current. The additive structure in the transport coefficients plays an important role. We also suggest the origin of the two currents.

DOI: 10.1103/PhysRevLett.118.036601



(a)

PRL 118, 036601 (2017) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2017

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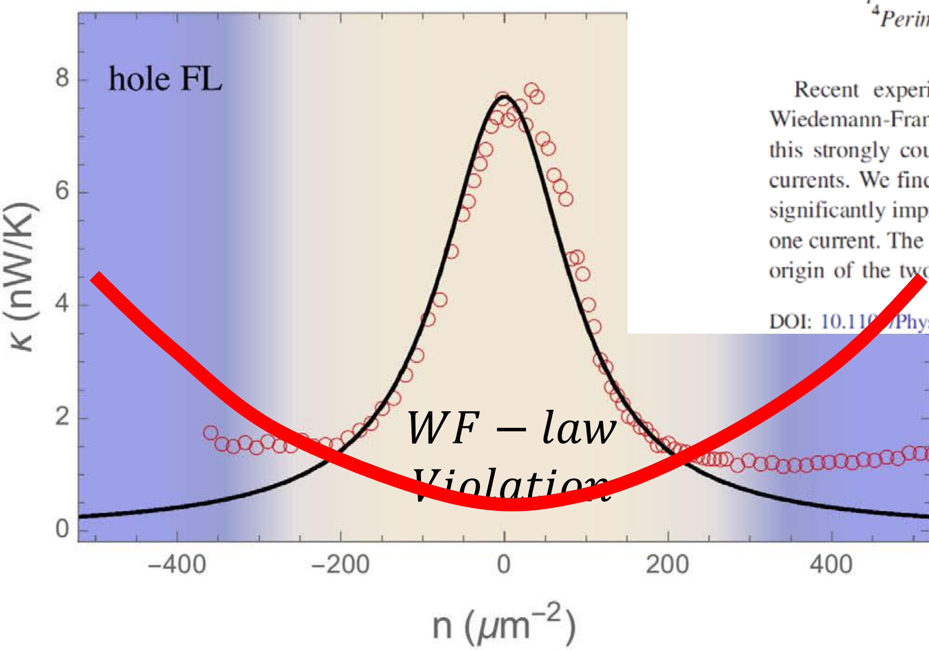


(b)

The Question

How can these emergent IR degrees of freedom be related with UV microscopics ?

Concept

- *Landau's Fermi-liquid theory for "weakly" correlated metals*

BCS theory

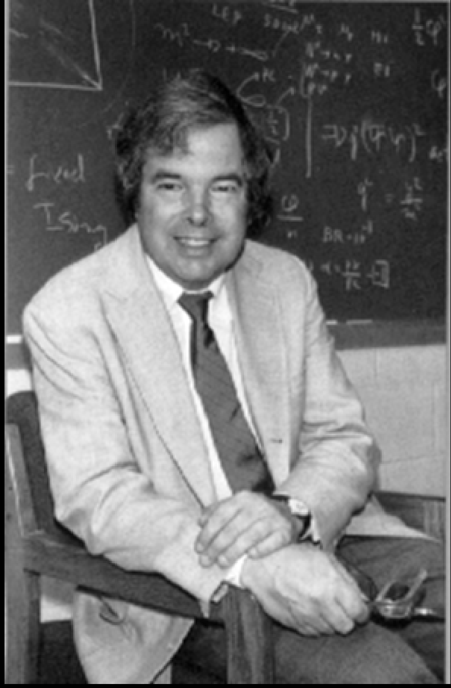
- *Landau-Ginzburg-Wilson theory for symmetry breaking phase transitions (Mean-field theory)*

Wilson's non-perturbative renormalization group

- *Strongly coupled conformal field theories for strongly correlated "metals"*

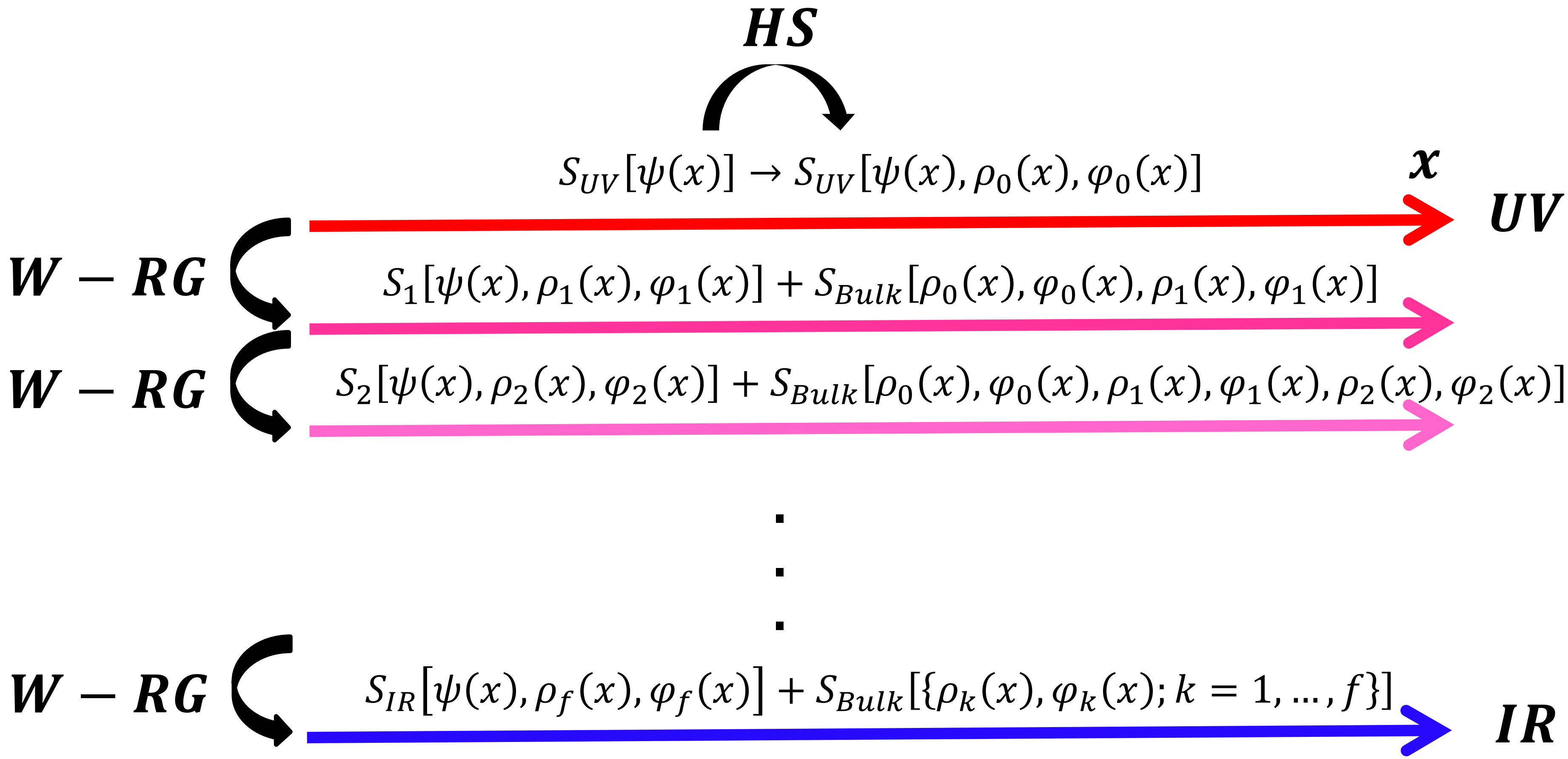
- AdS_{d+2}/CFT_{d+1} *effective dual field theories (Mean-field theory)*

Wilsonian renormalization group transformations



- To introduce order parameter fields for (interacting) quantum field theories
- To separate low- and high- energy degrees of freedom for all fluctuating fields
- To integrate out all high-energy degrees of freedom, controlled by the region of integrations $d\Lambda$
- To introduce order parameter fields once again for emergent effective interactions

Sung-Sik Lee, JHEP 2016, 44 (2016)


HS
$S_{UV}[\psi(x)] \to S_{UV}[\psi(x), \rho_0(x), \varphi_0(x)]$
x
UV
W − RG
$S_1[\psi(x), \rho_1(x), \varphi_1(x)] + S_{Bulk}[\rho_0(x), \varphi_0(x), \rho_1(x), \varphi_1(x)]$
W − RG
$S_2[\psi(x), \rho_2(x), \varphi_2(x)] + S_{Bulk}[\rho_0(x), \varphi_0(x), \rho_1(x), \varphi_1(x), \rho_2(x), \varphi_2(x)]$
W − RG
$S_{IR}[\psi(x), \rho_f(x), \varphi_f(x)] + S_{Bulk}[\{\rho_k(x), \varphi_k(x); k = 1, \ldots, f\}]$
IR

$$S_k[\psi(x), \rho_k(x), \varphi_k(x)]$$
$$+S_{Bulk}[\{\rho_j(x), \varphi_j(x); j = 1, \dots, k-1\}, \rho_k(x), \varphi_k(x)]$$

$$S_k[\psi_l(x), \rho_{kl}(x), \varphi_{kl}(x), \psi_h(x), \rho_{kh}(x), \varphi_{kh}(x)]$$
$$+S_{Bulk}[\{\rho_j(x), \varphi_j(x); j = 1, \dots, k-1\}, \rho_{kl}(x), \varphi_{kl}(x), \rho_{kh}(x), \varphi_{kh}(x)]$$

k

$$S_k[\psi_l(x), \rho_{kl}(x), \varphi_{kl}(x)] + \Delta S_k[\psi_l(x), \rho_{kl}(x), \varphi_{kl}(x)]$$
$$+S_{Bulk}[\{\rho_j(x), \varphi_j(x); j = 1, \dots, k-1\}, \rho_{kl}(x), \varphi_{kl}(x)]$$

HS

$$S_{k+1}[\psi(x), \rho_{k+1}(x), \varphi_{k+1}(x)]$$
$$+S_{Bulk}[\{\rho_j(x), \varphi_j(x); j = 1, \dots, k\}, \rho_{k+1}(x), \varphi_{k+1}(x)]$$

***k* + 1**

PHYSICAL REVIEW D **96,** 086015 (2017)

Emergent geometric description for a topological phase transition in the Kitaev superconductor model

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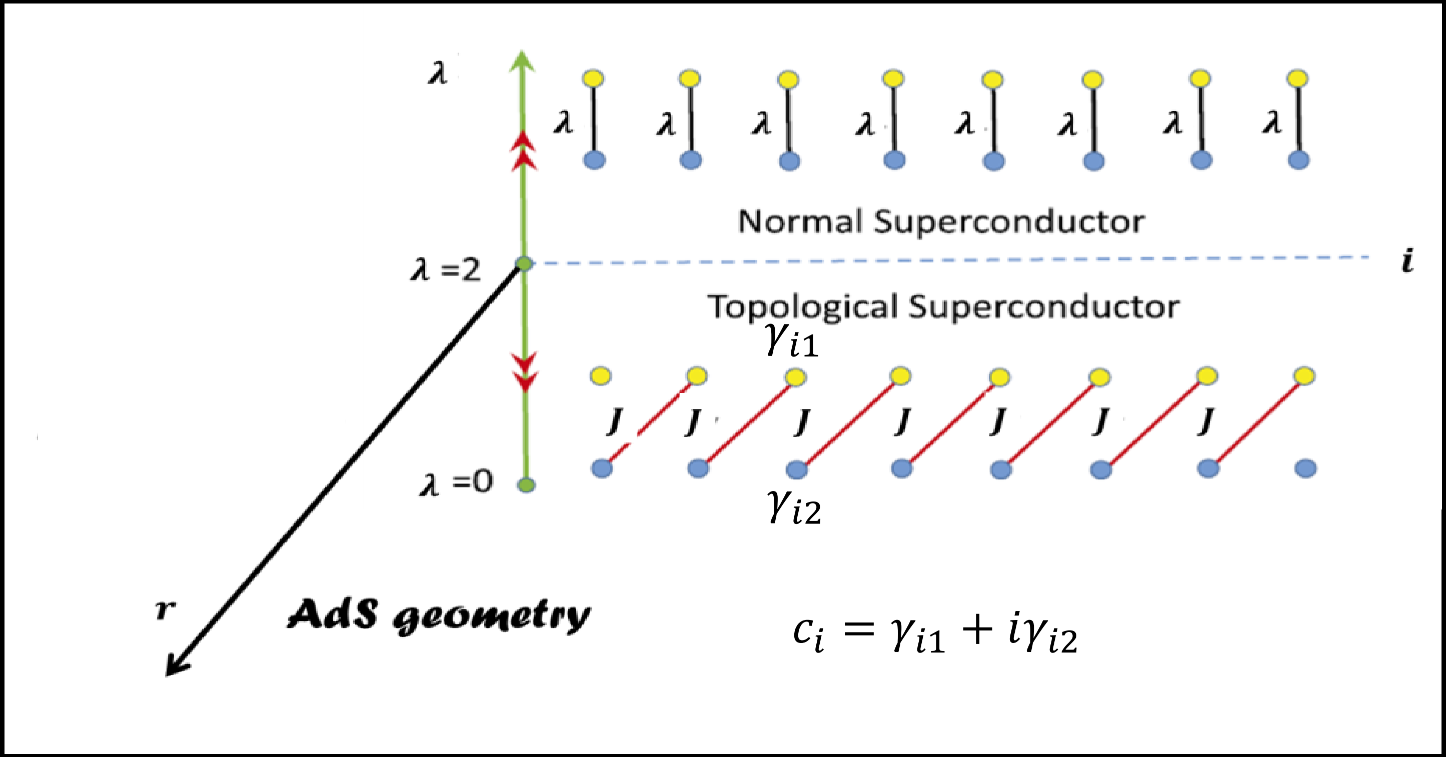
(Received 15 February 2017; revised manuscript received 12 June 2017; published 23 October 2017)

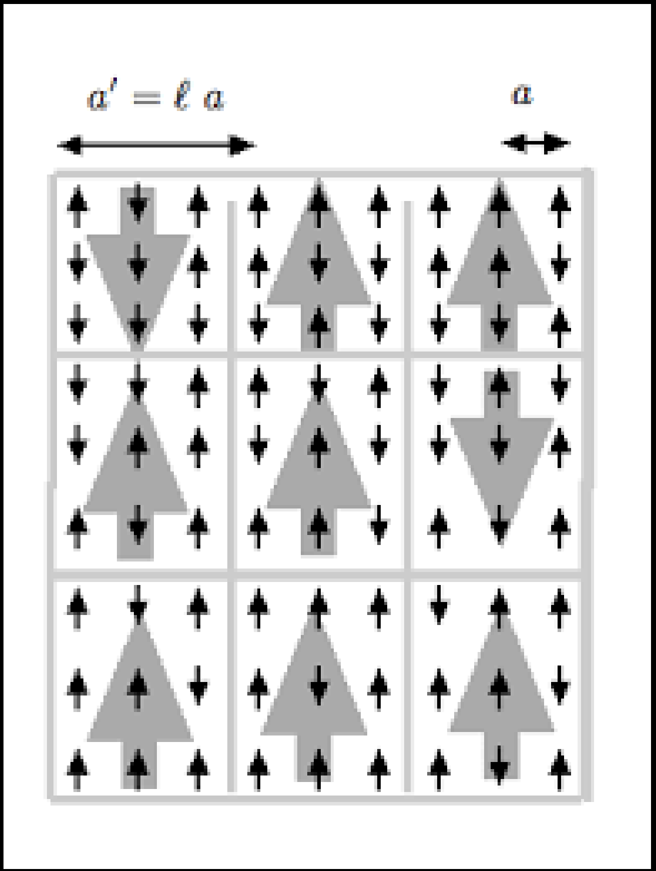
arXiv:1610.07312v9

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \frac{J}{2}\sum_{i=1}^{N}\left(c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger\right) + J\lambda\sum_{i=1}^{N} c_i^\dagger c_i$$



A. Y. Kitaev, Ann. Phys. (Amsterdam) 303, 2 (2003)

Renormalization group in real space



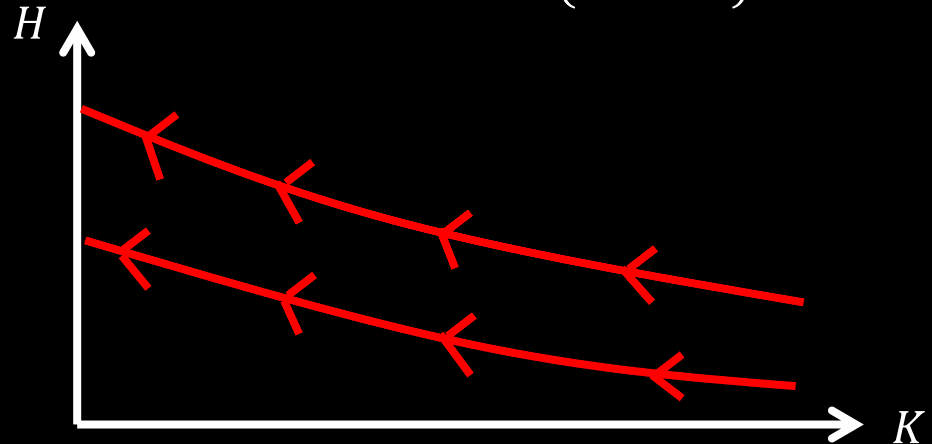
Equilibrium Statistical Physics (3rd Edition) by Michael Plischke

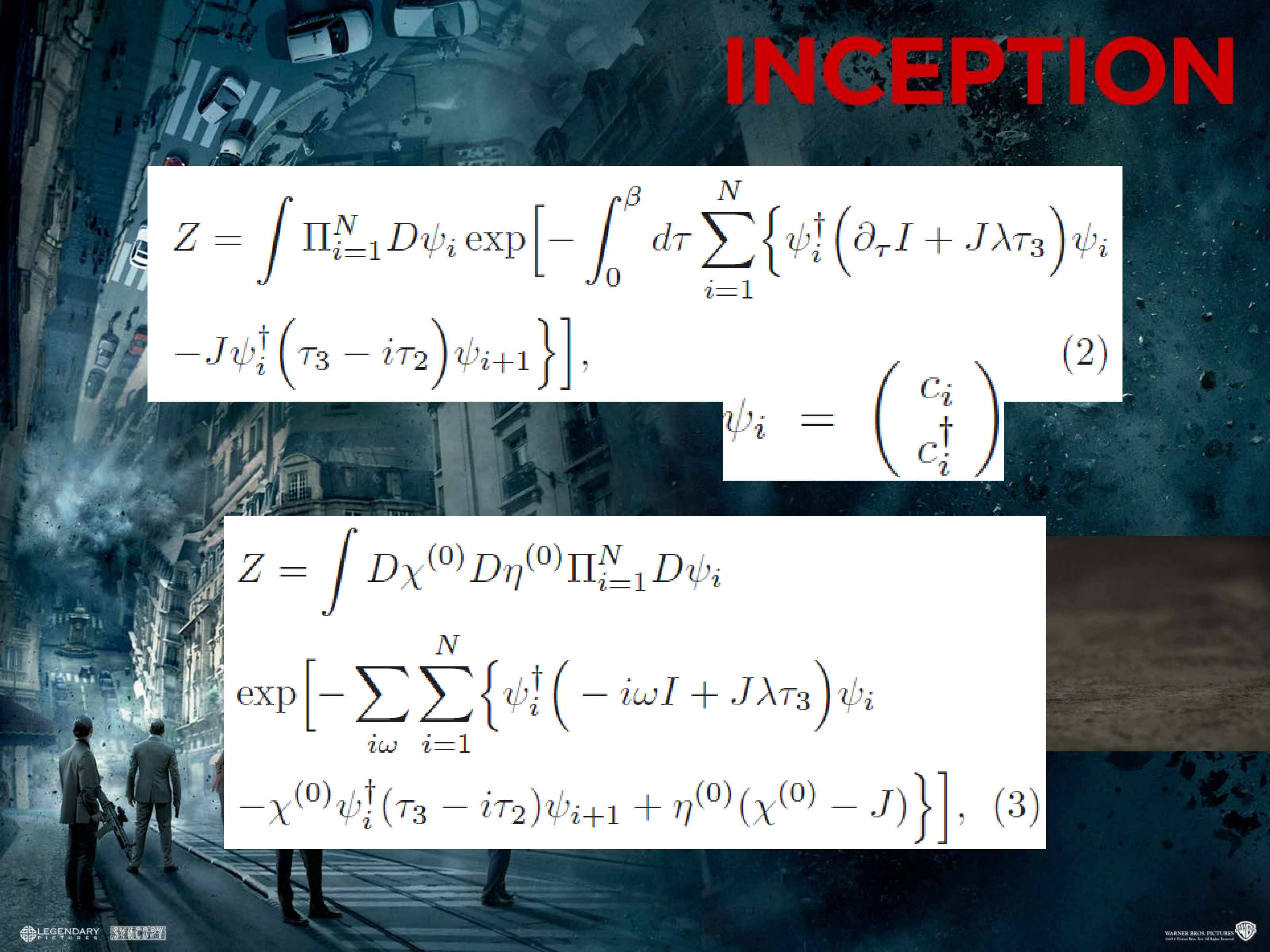
$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - h \sum_{i=1}^{N} \sigma_i$$

$$K^{(1)} = \frac{1}{4} ln \frac{\cosh(2K+H)\cosh(2K-H)}{cosh^2 H}$$

$$H^{(1)} = H + \frac{1}{2} ln \frac{\cosh(2K+H)}{\cosh(2K-H)}$$

$$K = \beta J$$
$$H = \beta h$$





$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\Big[-\int_0^{\beta} d\tau \sum_{i=1}^{N}\Big\{\psi_i^\dagger\Big(\partial_\tau I + J\lambda\tau_3\Big)\psi_i - J\psi_i^\dagger\Big(\tau_3 - i\tau_2\Big)\psi_{i+1}\Big\}\Big], \qquad (2)$$

$$\psi_i = \begin{pmatrix} c_i \\ c_i^\dagger \end{pmatrix}$$

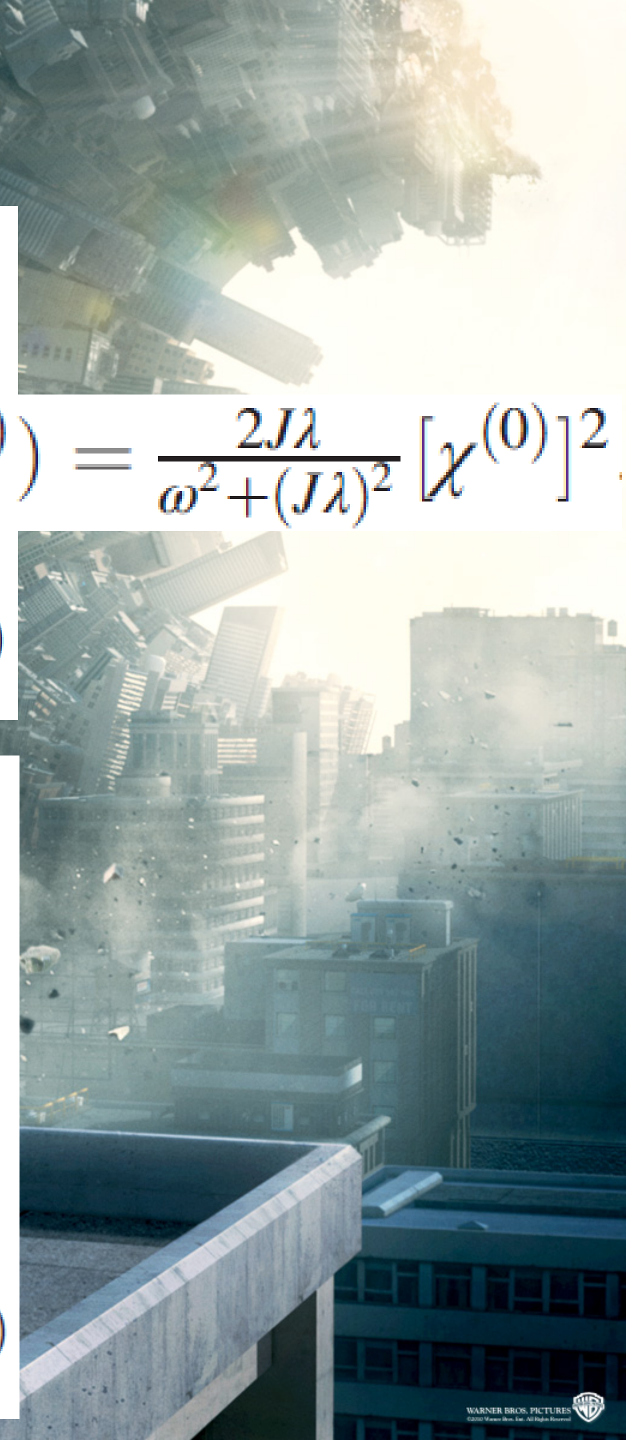
$$Z = \int D\chi^{(0)} D\eta^{(0)} \Pi_{i=1}^{N} D\psi_i \exp\Big[-\sum_{i\omega}\sum_{i=1}^{N}\Big\{\psi_i^\dagger\Big(-i\omega I + J\lambda\tau_3\Big)\psi_i - \chi^{(0)}\psi_i^\dagger(\tau_3 - i\tau_2)\psi_{i+1} + \eta^{(0)}(\chi^{(0)} - J)\Big\}\Big], \qquad (3)$$

INCEPTION

$$Z = \int D\chi^{(0)} D\eta^{(0)} \Pi_{i=1}^{N} D\psi_i$$
$$\times \exp\Big[-\sum_{i\omega}\sum_{i=1}^{N}\{\psi_i^\dagger(-i\omega I + J\lambda\tau_3)\psi_i$$
$$- f(\chi^{(0)})\psi_i^\dagger(\tau_3 - i\tau_2)\psi_{i+1} + \eta^{(0)}(\chi^{(0)} - J)\}\Big], \quad (4)$$

$$f(\chi^{(0)}) = \frac{2J\lambda}{\omega^2 + (J\lambda)^2}[\chi^{(0)}]^2$$

$$Z = \int D\chi^{(0)} D\eta^{(0)} D\chi^{(1)} D\eta^{(1)} \Pi_{i=1}^{N} D\psi_i$$
$$\times \exp\Big[\sum_{i\omega}\sum_{i=1}^{N}\{\psi_i^\dagger(-i\omega I + J\lambda\tau_3)\psi_i$$
$$-\chi^{(1)}\psi_i^\dagger(\tau_3 - i\tau_2)\psi_{i+1} + \eta^{(0)}(\chi^{(0)} - J)$$
$$+\eta^{(1)}(\chi^{(1)} - f(\chi^{(0)}))\}\Big], \quad (5)$$



$$
\begin{aligned}
Z = \int D\chi^{(0)} D\eta^{(0)} \Pi_{k=1}^{f} D\chi^{(k)} D\eta^{(k)} \Pi_{i=1}^{N} D\psi_i \\
\times \exp\Bigg[-\sum_{i\omega} \sum_{i=1}^{N} \{ \psi_i^\dagger (-i\omega I + J\lambda\tau_3) \psi_i \\
- \chi^{(f)} \psi_i^\dagger (\tau_3 - i\tau_2) \psi_{i+1} + \eta^{(0)} (\chi^{(0)} - J) \} \\
- N \sum_{k=1}^{f} \sum_{i\omega} \eta^{(k)} (\chi^{(k)} - f(\chi^{(k-1)})) \Bigg],
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
Z = \int \Pi_{i=1}^{N} D\psi_i(i\omega) D\chi(i\omega, z) D\eta(i\omega, z) \\
\times \exp\{ -\mathcal{S}_{\mathrm{UV}}[\eta(i\omega, 0), \chi(i\omega, 0)] \\
- \mathcal{S}_{\mathrm{Bulk}}[\eta(i\omega, z), \chi(i\omega, z)] \\
- \mathcal{S}_{\mathrm{IR}}[\psi_i(i\omega); \chi(i\omega, z_f)] \},
\end{aligned}
$$

$$\mathcal{S}_{\mathrm{UV}}[\eta(i\omega,0),\chi(i\omega,0)] = N\eta(i\omega,0)(\chi(i\omega,0)-J),$$

$$\mathcal{S}_{\mathrm{Bulk}}[\eta(i\omega,z),\chi(i\omega,z)] = N\int_0^{z_f} dz \sum_{i\omega} \eta(i\omega,z)\left(\frac{\partial\chi(i\omega,z)}{\partial z} + \chi(i\omega,z) - f[\chi(i\omega,z)]\right),$$

$$\mathcal{S}_{\mathrm{IR}}[\psi_i(i\omega);\chi(i\omega,z_f)] = \sum_{i\omega}\sum_{i=1}^{N}\{\psi_i^\dagger(i\omega)(-i\omega I + J\lambda\tau_3)\psi_i(i\omega) - \chi(i\omega,z_f)\psi_i^\dagger(i\omega)(\tau_3 - i\tau_2)\psi_{i+1}(i\omega)\}.$$

$$\frac{\partial \chi(i\omega, z)}{\partial z} = -\chi(i\omega, z) + f[\chi(i\omega, z)],$$

$$f[\chi(i\omega, z)] = \frac{2J\lambda}{\omega^2 + (J\lambda)^2}[\chi(i\omega, z)]^2$$

$$\chi(i\omega, z_f) = \frac{\{1 + (\frac{\omega}{J\lambda})^2\}J\lambda}{2 + \{\lambda - 2 + \lambda(\frac{\omega}{J\lambda})^2\}e^{z_f}}.$$

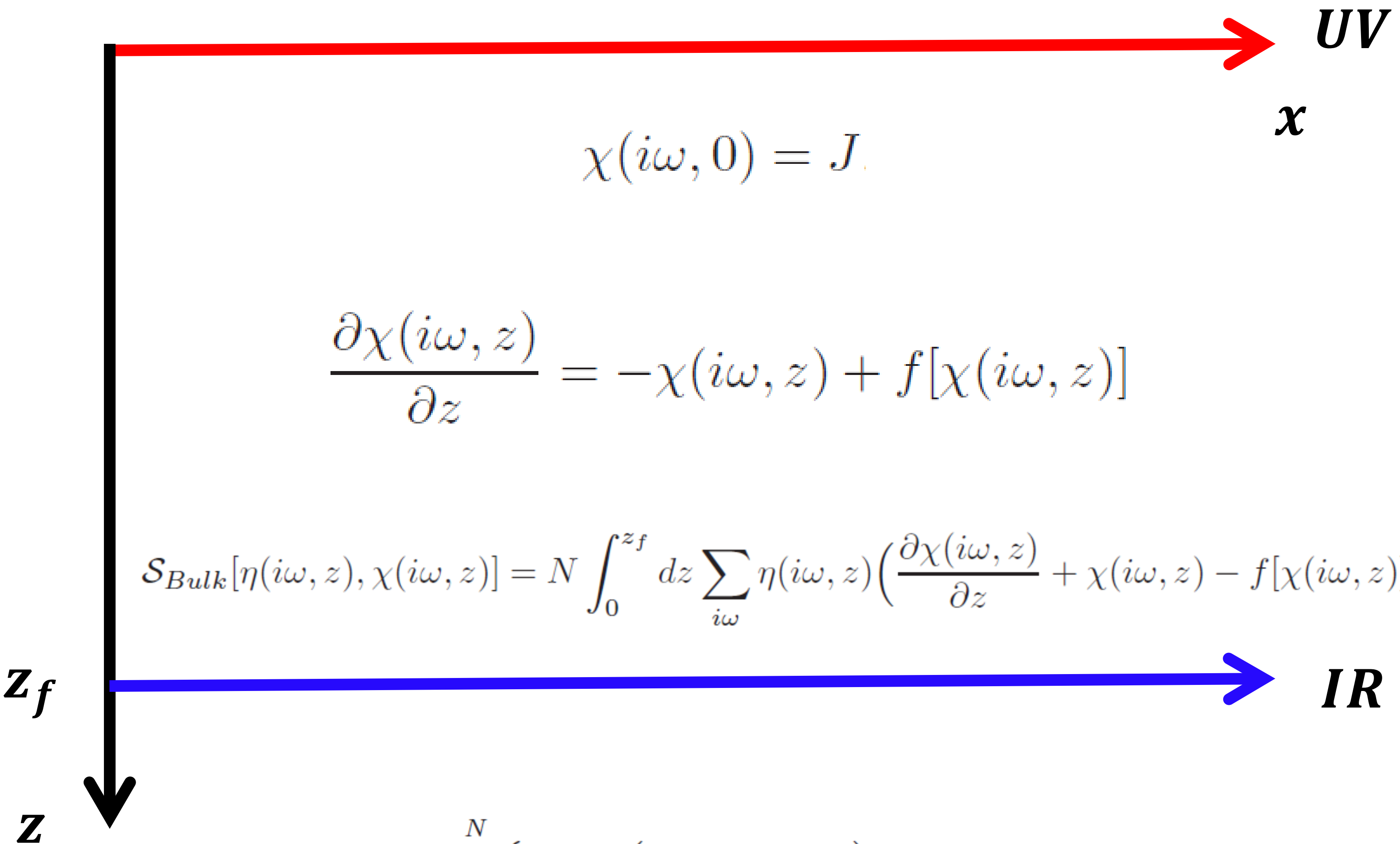
$$Z = \int D\chi(i\omega, z) D\eta(i\omega, z) \times \exp\{-\mathcal{S}_{\mathrm{UV}}[\eta(i\omega, 0), \chi(i\omega, 0)] - \mathcal{S}_{\mathrm{Bulk}}[\eta(i\omega, z), \chi(i\omega, z)] - \mathcal{S}_{\mathrm{IR}}[\chi(i\omega, z_f)]\}, \tag{14}$$

$$\mathcal{S}_{\mathrm{UV}}[\eta(i\omega, 0), \chi(i\omega, 0)] = N\eta(i\omega, 0)(\chi(i\omega, 0) - J), \tag{15}$$

$$\mathcal{S}_{\mathrm{Bulk}}[\eta(i\omega, z), \chi(i\omega, z)] = N \int_0^{z_f} dz \sum_{i\omega} \eta(i\omega, z) \left(\frac{\partial \chi(i\omega, z)}{\partial z} + \chi(i\omega, z) - f[\chi(i\omega, z)] \right), \tag{16}$$

$$\mathcal{S}_{\mathrm{IR}}[\chi(i\omega, z_f)] = -\frac{1}{2} \sum_k \sum_{i\omega} \ln\{(-i\omega)^2 - (2\chi(i\omega, z_f)\gamma_k - J\lambda)^2 - (2\chi(i\omega, z_f)\varphi_k)^2\}. \tag{17}$$

Physical picture



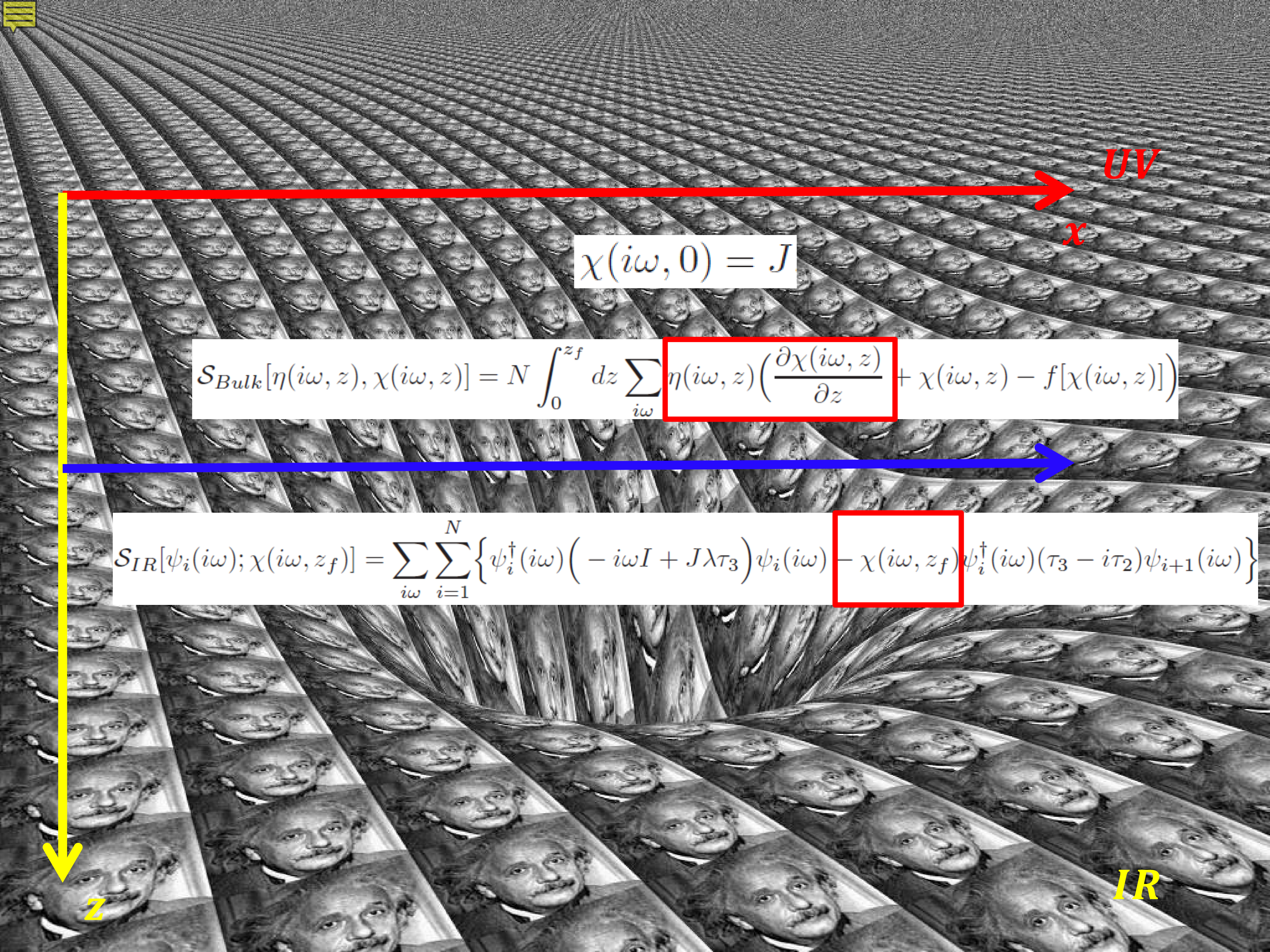
$$\chi(i\omega, 0) = J$$

$$\frac{\partial \chi(i\omega, z)}{\partial z} = -\chi(i\omega, z) + f[\chi(i\omega, z)]$$

$$\mathcal{S}_{Bulk}[\eta(i\omega, z), \chi(i\omega, z)] = N \int_0^{z_f} dz \sum_{i\omega} \eta(i\omega, z)\Big(\frac{\partial \chi(i\omega, z)}{\partial z} + \chi(i\omega, z) - f[\chi(i\omega, z)]\Big)$$

$$\mathcal{S}_{IR}[\psi_i(i\omega); \chi(i\omega, z_f)] = \sum_{i\omega} \sum_{i=1}^{N} \Big\{\psi_i^\dagger(i\omega)\Big(-i\omega I + J\lambda\tau_3\Big)\psi_i(i\omega) - \chi(i\omega, z_f)\psi_i^\dagger(i\omega)(\tau_3 - i\tau_2)\psi_{i+1}(i\omega)\Big\}$$

How to extract out an emergent metric structure ?



UV
x
$\chi(i\omega, 0) = J$
$\mathcal{S}_{Bulk}[\eta(i\omega, z), \chi(i\omega, z)] = N \int_0^{z_f} dz \sum_{i\omega} \eta(i\omega, z)\Big(\frac{\partial \chi(i\omega, z)}{\partial z} + \chi(i\omega, z) - f[\chi(i\omega, z)]\Big)$
$\mathcal{S}_{IR}[\psi_i(i\omega); \chi(i\omega, z_f)] = \sum_{i\omega} \sum_{i=1}^{N} \Big\{ \psi_i^\dagger(i\omega)\Big(-i\omega I + J\lambda\tau_3\Big)\psi_i(i\omega) - \chi(i\omega, z_f)\psi_i^\dagger(i\omega)(\tau_3 - i\tau_2)\psi_{i+1}(i\omega) \Big\}$
z
IR

$$\mathcal{S}_{\text{Bulk}}[\eta(i\omega,z),\chi(i\omega,z)]$$
$$=N\int_0^{z_f}dz\sum_{i\omega}\eta(i\omega,z)\left(\frac{\partial\chi(i\omega,z)}{\partial z}+\chi(i\omega,z)-f[\chi(i\omega,z)]\right),$$

$$\frac{d}{dz_f}\ln Z=0.$$

$$\mathcal{S}_{\text{IR}}[\psi_i(i\omega);\chi(i\omega,z_f)]$$
$$=\sum_{i\omega}\sum_{i=1}^{N}\{\psi_i^\dagger(i\omega)(-i\omega I+J\lambda\tau_3)\psi_i(i\omega)$$
$$-\chi(i\omega,z_f)\psi_i^\dagger(i\omega)(\tau_3-i\tau_2)\psi_{i+1}(i\omega)\}.$$

$$0=\sum_{i\omega}\Big\{-\eta(i\omega,z_f)\partial_{z_f}\chi(i\omega,z_f)$$
$$+\partial_{z_f}\chi(i\omega,z_f)\Big\langle\frac{1}{N}\sum_{i=1}^{N}\psi_i^\dagger(i\omega)(\tau_3-i\tau_2)\psi_{i+1}(i\omega)\Big\rangle\Big\}.$$

$$\mathcal{H}+\partial_f\mathcal{S}_{IR}=0.$$

$$\sum_{i\omega}\Big\{\gamma^{00}T_{00}+\gamma^{11}T_{11}+\beta_\chi\langle\mathcal{O}_\chi\rangle\Big\}=0.$$

$$\gamma^{00} T_{00} + \gamma^{11} T_{11} \equiv -\eta(i\omega, z_f)\partial_{z_f}\chi(i\omega, z_f),$$

$$\beta_\chi \equiv \partial_{z_f}\chi(i\omega, z_f) = -\chi(i\omega, z_f) + f[\chi(i\omega, z_f)],$$

$$\langle \mathcal{O}_\chi \rangle \equiv \left\langle \frac{1}{N}\sum_{i=1}^{N} \psi_i^\dagger(i\omega)(\tau_3 - i\tau_2)\psi_{i+1}(i\omega) \right\rangle$$
$$= \eta(i\omega, z_f).$$

$$T_{00} = -\frac{1}{N}\sum_{i=1}^{N}\{J\lambda\psi_i^\dagger(i\omega)\tau_3\psi_i(i\omega)$$
$$-\chi(i\omega, z_f)\psi_i^\dagger(i\omega)(\tau_3 - i\tau_2)\psi_{i+1}(i\omega)\},$$

$$T_{11} = \frac{1}{N}\sum_{i=1}^{N} i\chi(i\omega, z_f)\psi_i^\dagger(i\omega)\tau_2\psi_{i+1}(i\omega).$$

$$\chi(i\omega, z_f)(\gamma^{00} - \gamma^{11}) = \beta_\chi,$$

$$(J\lambda - \chi(i\omega, z_f))\gamma^{00} = -\beta_\chi.$$

$$\gamma^{00} = \frac{(\lambda-2)e^{z_f}}{[2+(\lambda-2)e^{z_f}][1+(\lambda-2)e^{z_f}]},$$

$$\gamma^{11} = \frac{(\lambda-2)e^{z_f}}{1+(\lambda-2)e^{z_f}},$$

$$z \to 2z, \qquad \tau \to \sqrt{\lambda-2}\,\tau, \qquad x \to \sqrt{\frac{\lambda-2}{2}}\,x,$$

$$ds^2 = dz^2 + g_{00} d\tau^2 + g_{11} dx^2$$

$$g_{00} = \frac{[2 + (\lambda - 2)e^{2z}][1 + (\lambda - 2)e^{2z}]}{2e^{2z}},$$

$$g_{11} = \frac{1 + (\lambda - 2)e^{2z}}{e^{2z}},$$

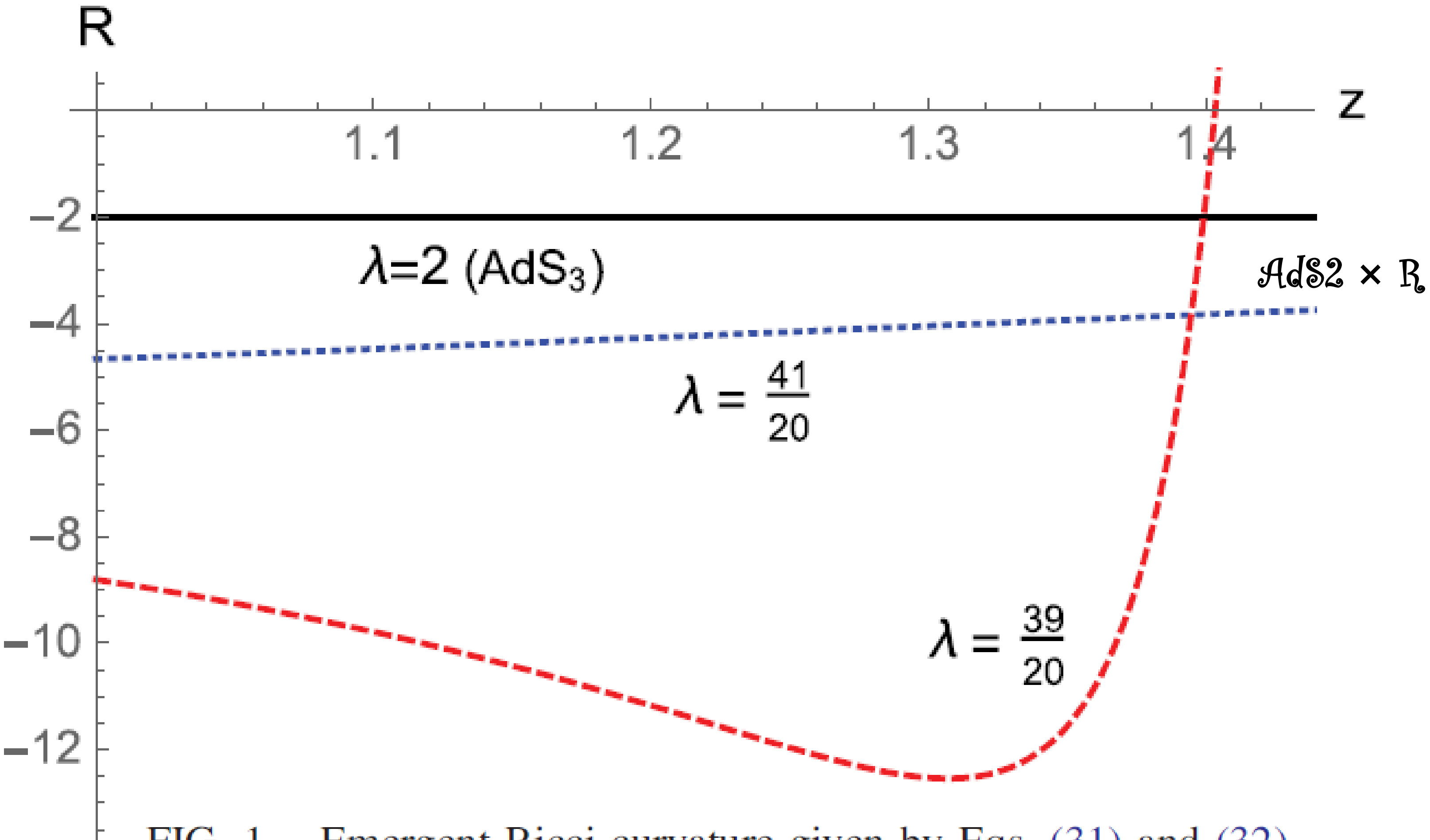


FIG. 1. Emergent Ricci curvature given by Eqs. (31) and (32) for the quantum critical point ($\lambda = 2.0$), the topologically trivial ($\lambda = 2.05$), and nontrivial ($\lambda = 1.95$) superconducting phases. We emphasize that the Ricci curvature diverges at $z = z_c$ in the topological superconducting phase, which may be identified with a horizon. The emergence of such a horizon in a dense phase is consistent with a recent study [23], where the existence of the horizon is a fingerprint of a quantum phase transition.

[23] Sung-Sik Lee, JHEP 2016, 44 (2016)

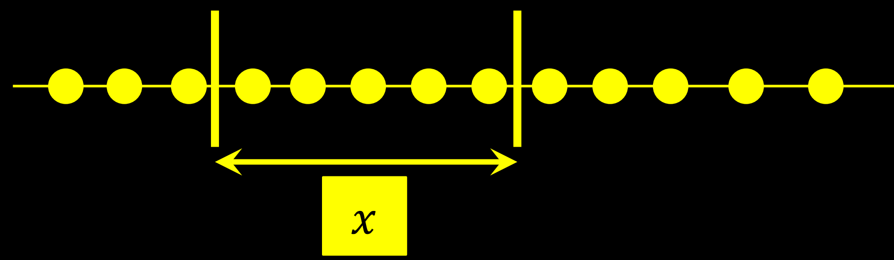
All equivalent equations

- UV/IR & bulk equations of motion
- Hamilton-Jacobi theory (Equation for counter terms, Callan-Symanzik equation for free energy, and equation to define energy-momentum tensor)
- The latter turns out to be just a reformulation of the former.
- Consistent with Einstein equation ➔ Under investigation

How to confirm this emergent metric structure ?

Entanglement entropy

- Von Neumann entropy of a reduced density matrix



System (S) *Environment (O)*

$$\rho_S = \mathrm{tr}_O \rho = \mathrm{tr}_O \frac{e^{-\beta H}}{Z}$$

$$S_S \equiv \langle \ln \rho_S \rangle = \mathrm{tr}_S \rho_S \ln \rho_S$$

Area Law

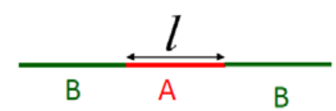
The leading divergent term of EE in a (d+1) dim. QFT is proportional to the area of the (d-1) dim. boundary ∂A:

$$S_A \sim \frac{\mathrm{Area}(\partial A)}{a^{d-1}} + (\text{subleading terms}).$$

where a is a UV cutoff (i.e. lattice spacing).

- There are two exceptions:

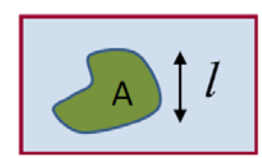
(a) 1+1 dim. CFT $S_A = \frac{c}{3} \log \frac{l}{a}.$



[Holzhey-Larsen-Wilczek 94, Calabrese-Cardy 04]

(b) QFT with Fermi surfaces ($k_F \sim a^{-1}$)

$$S_A \sim \left(\frac{l}{a}\right)^{d-1} \cdot \log \frac{l}{a} + \ldots$$

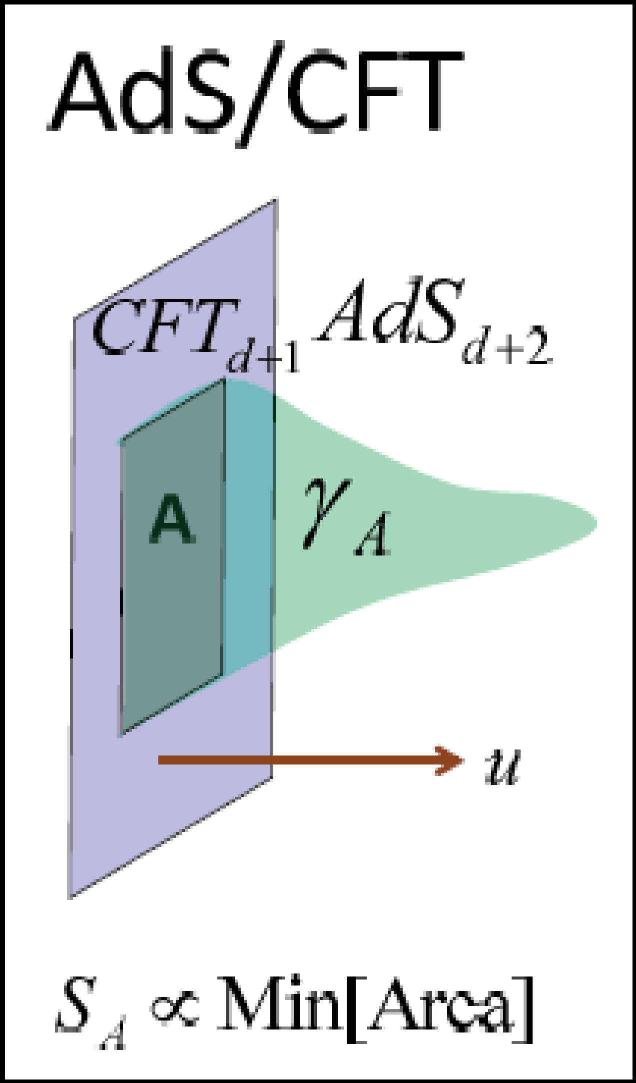


[Wolf 05, Gioev-Klich 05]

I. To evaluate the entanglement entropy based on Ryu-Takayanagi formula with the emergent metric tensor

II. To find the entanglement entropy of our system, solving the corresponding UV theory numerically

III. To compare these two entanglement entropies and confirm the emergent metric structure



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 181602 (2006);
S. Ryu and T. Takayanagi, JHEP 0608, 045 (2006)

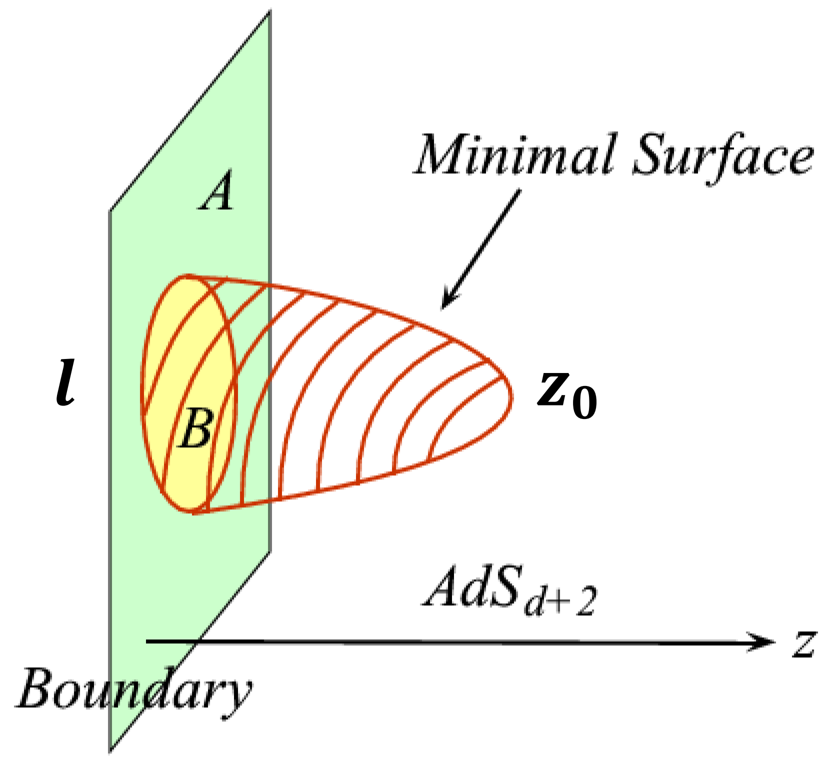
$$S_E = \frac{1}{4G}\int_{-l/2}^{l/2} dx \sqrt{g_{11}[z(x)] + \left(\frac{dz(x)}{dx}\right)^2},$$

$$S_E(z_0) = \frac{1}{2G}\int_0^{z_0} dz \sqrt{\frac{g_{11}}{g_{11} - g_{11}^0}},$$

$$\frac{dz(x)}{dx}\Big|_{x=0} = 0$$

$$g_{11}^0 = g_{11}(z_0)$$

$$l = 2\int_0^{z_0} dz \sqrt{\frac{g_{11}^0}{g_{11}(g_{11} - g_{11}^0)}}.$$



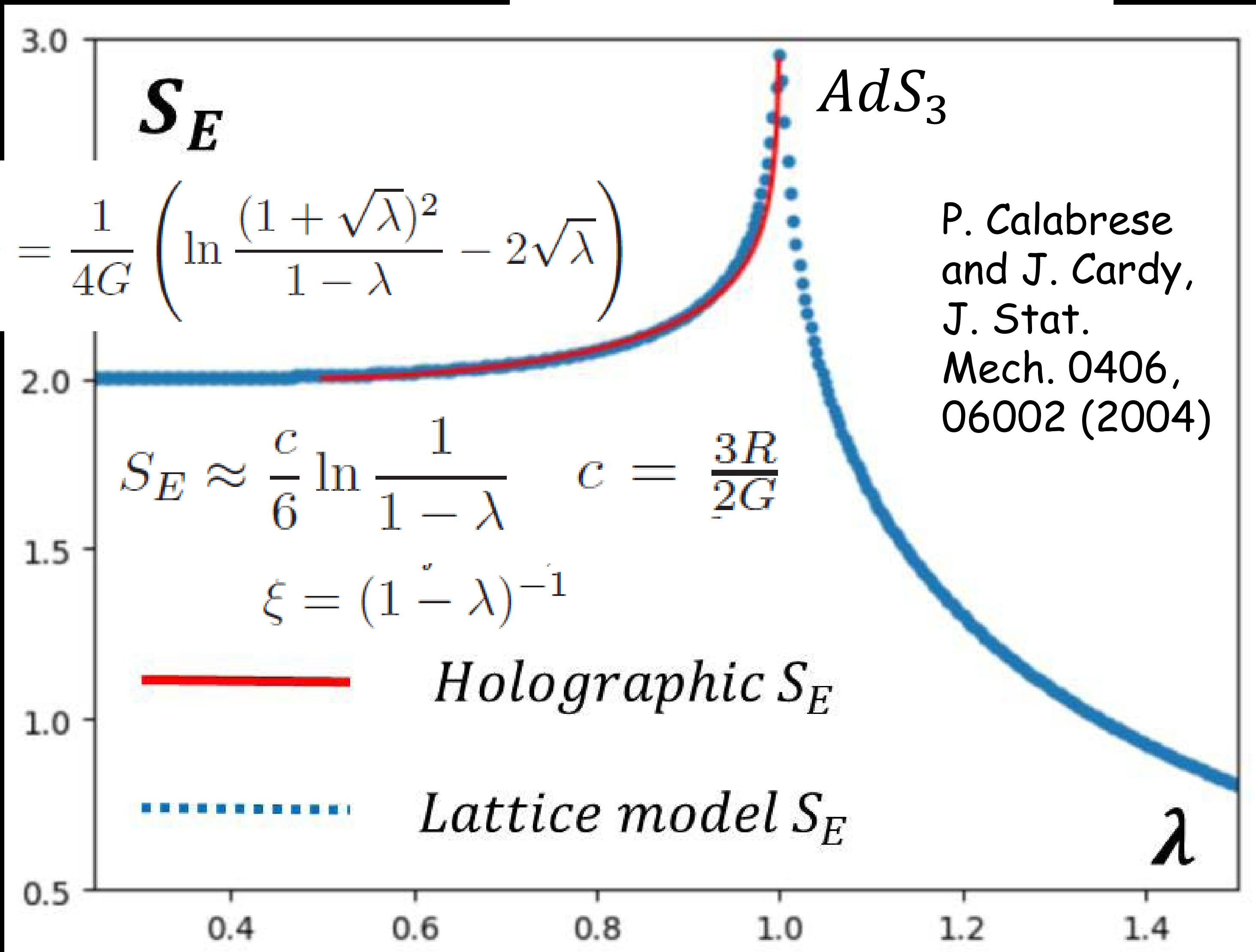
$$g_{00} = g_{11} = e^{-2z}$$

$$S_E = \frac{1}{2G} \ln\left(\frac{\sqrt{4+l^2}}{2} + \frac{l}{2}\right)$$

$$S_E = \frac{1}{4G}\left(\ln\frac{(1+\sqrt{\lambda})^2}{1-\lambda} - 2\sqrt{\lambda}\right)$$

$$S_E \approx \frac{c}{6}\ln\frac{1}{1-\lambda} \qquad c = \frac{3R}{2G}$$

$$\xi = (1-\lambda)^{-1}$$



The role of an emergent extra dimension in a non-perturbative description of interacting field theories: The Kondo effect

Ki-Seok Kim, Suk Bum Chung, and Chanyong Park, arXiv:1705.06571
"An emergent holographic description for the Kondo effect: The role of an extra dimension in a non-perturbative field theoretical approach"

THE ELECTRICAL RESISTANCE OF GOLD BELOW 1°K

by W. J. DE HAAS, H. B. G. CASIMIR and G. J. VAN DEN BERG

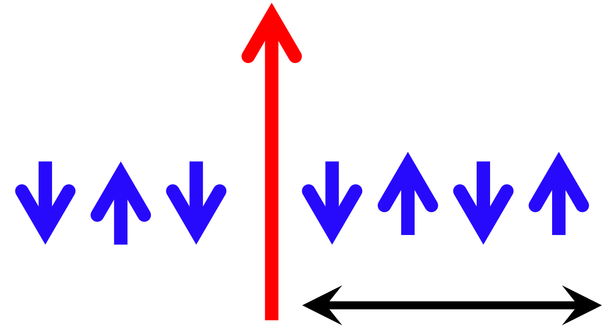
Communication No. 251c from the Kamerlingh Onnes Laboratory at Leiden

Dedicated to Professor Max Planck on the occasion of his eightieth birthday

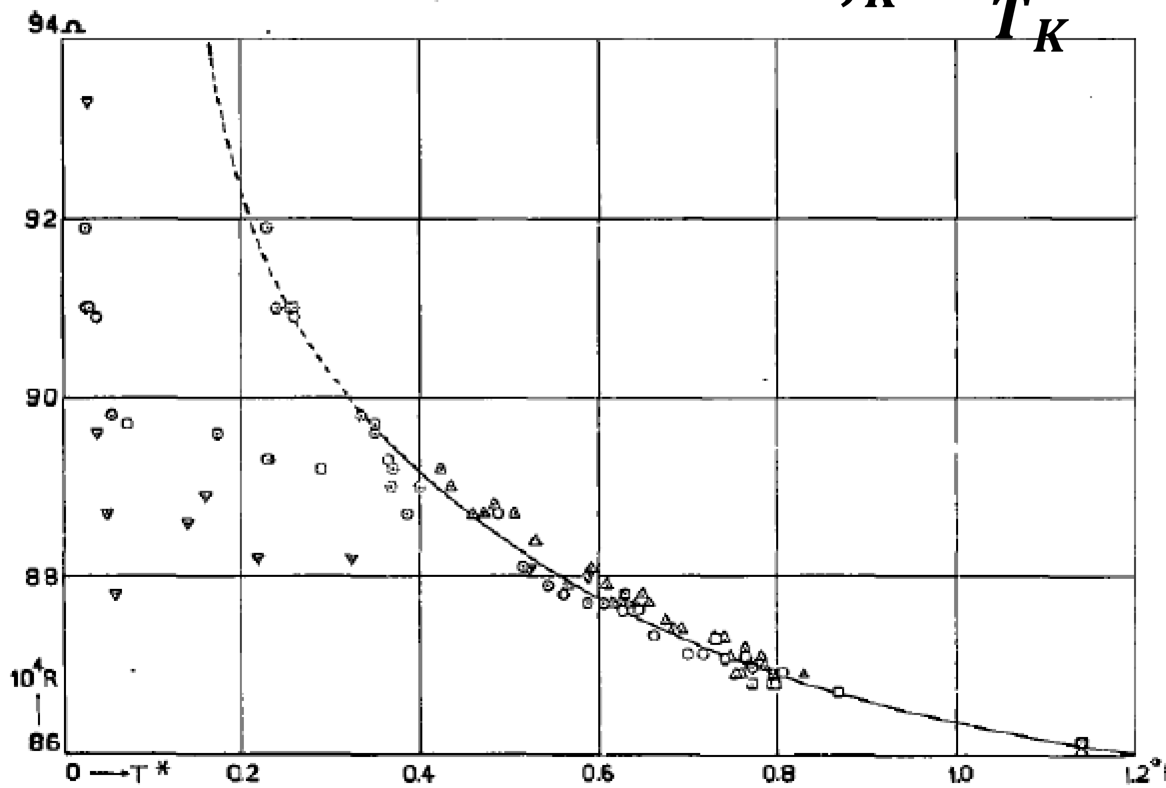
Summary

The resistance of gold was determined at temperatures below 1°K, obtained by adiabatic demagnetization of iron-ammonium-alum. The increase observed at ordinary liquid helium temperatures is much more pronounced below 1°K and our results suggest, that the resistance may become infinite at the absolute zero-point.

Physica **5**, 225 (1938)



$$\xi_K = \frac{v_F}{T_K}$$



H. v. Lohneysen, APCTP lecture on heavy − fermion quantum criticality (2016)

$$Z = \int Dc_\sigma(\boldsymbol{k},\tau) D\boldsymbol{S}(\tau) \exp\Big[- S_B[\boldsymbol{S}(\tau)]$$

$$- \int_0^\beta d\tau \Big\{ \int \frac{d^d\boldsymbol{k}}{(2\pi)^d} c_\sigma^\dagger(\boldsymbol{k},\tau)\Big(\partial_\tau - \mu + \frac{\boldsymbol{k}^2}{2m}\Big) c_\sigma(\boldsymbol{k},\tau)$$

$$+ J_K \int \frac{d^d\boldsymbol{k}}{(2\pi)^d} \int \frac{d^d\boldsymbol{k}'}{(2\pi)^d} c_\alpha^\dagger(\boldsymbol{k},\tau) \boldsymbol{\sigma}_{\alpha\beta} c_\beta(\boldsymbol{k}',\tau) \cdot \boldsymbol{S}(\tau) \Big\} \Big]$$

$$\boldsymbol{S}(\tau) = \frac{1}{2} f_\alpha^\dagger(\tau) \boldsymbol{\sigma}_{\alpha\beta} f_\beta(\tau)$$

$$f_\sigma^\dagger(\tau) f_\sigma(\tau) = NS$$

$$
Z = \int Dc_\sigma(\boldsymbol{k},\tau) Df_\sigma(\tau) D\lambda(\tau)
$$

$$
\exp\Big[-\int_0^\beta d\tau \Big\{ \int \frac{d^d\boldsymbol{k}}{(2\pi)^d} c_\sigma^\dagger(\boldsymbol{k},\tau)\Big(\partial_\tau - \mu + \frac{\boldsymbol{k}^2}{2m}\Big) c_\sigma(\boldsymbol{k},\tau)
$$

$$
-\frac{J_K}{N}\int \frac{d^d\boldsymbol{k}}{(2\pi)^d}\int \frac{d^d\boldsymbol{k}'}{(2\pi)^d} c_\sigma^\dagger(\boldsymbol{k},\tau) f_\sigma(\tau) \langle f_{\sigma'}^\dagger(\tau) c_{\sigma'}(\boldsymbol{k}',\tau)\rangle
$$

$$
+ f_\sigma^\dagger(\tau)\Big(\partial_\tau - i\lambda(\tau)\Big) f_\sigma(\tau) + iNS\lambda(\tau)\Big\}\Big] \qquad b^+(\tau) \qquad (4)
$$

$$
Z = Z_c \int Df_\sigma(\tau) \exp\Big[-\int_0^\beta d\tau \Big\{ \int_0^\beta d\tau' f_\sigma^\dagger(\tau) b(\tau,0) G_c(\tau-\tau') b^\dagger(\tau',0) f_\sigma(\tau')
$$

$$
+ f_\sigma^\dagger(\tau)\Big(\partial_\tau - i\lambda(\tau)\Big) f_\sigma(\tau) + iNS\lambda(\tau) + \frac{N}{J_K} b^\dagger(\tau,0) b(\tau,0)\Big\}\Big]
$$

Emergent gravity description for the Kondo effect

$$Z = Z_h^{z_f} Z_c \int Df_\sigma(\tau) Db(\tau,z) \exp\Big[- \int_{-\varepsilon}^{z_f+\varepsilon} dz \int_0^\beta d\tau \Big\{ \frac{1}{g_h^b} \Big(\partial_z b^\dagger(\tau,z)\Big)\Big(\partial_z b(\tau,z)\Big) + NSg_c b(\tau,z)\partial_\tau b^\dagger(\tau,z)$$

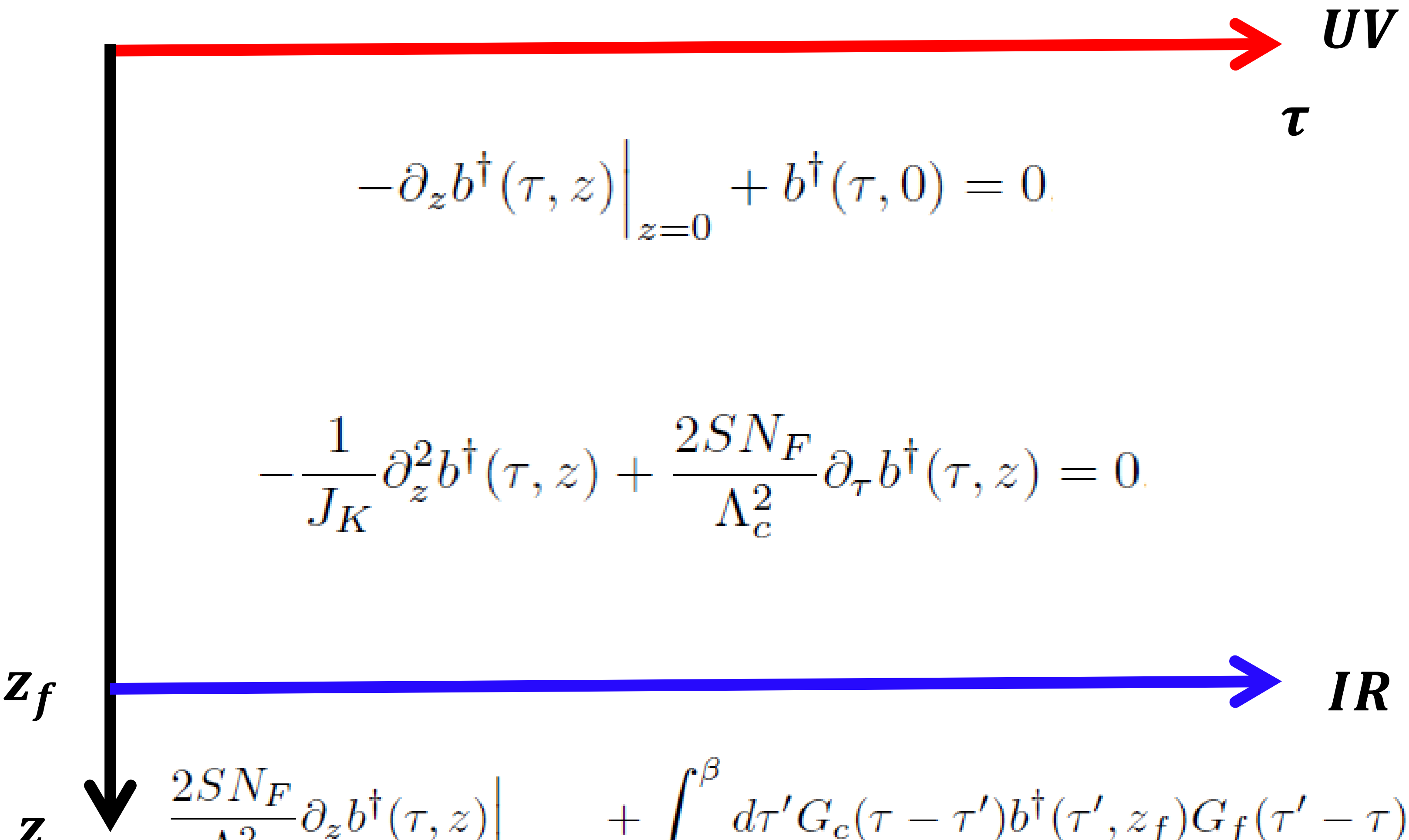
$$+\delta(z)\frac{N}{J_K} b^\dagger(\tau,z)b(\tau,z) + \delta(z-z_f)\Big(\int_0^\beta d\tau' f_\sigma^\dagger(\tau) b(\tau,z) G_c(\tau-\tau') b^\dagger(\tau',z) f_\sigma(\tau') + f_\sigma^\dagger(\tau)\Big(\partial_\tau - i\lambda(\tau)\Big) f_\sigma(\tau)$$

$$+iNS\lambda(\tau)\Big)\Big\}\Big]$$

$$Z = Z_c \int Df_\sigma(\tau) \exp\Big[- \int_0^\beta d\tau \Big\{ \int_0^\beta d\tau' f_\sigma^\dagger(\tau) b(\tau,0) G_c(\tau-\tau') b^\dagger(\tau',0) f_\sigma(\tau')$$

$$+ f_\sigma^\dagger(\tau)\Big(\partial_\tau - i\lambda(\tau)\Big) f_\sigma(\tau) + iNS\lambda(\tau) + \frac{N}{J_K} b^\dagger(\tau,0)b(\tau,0) \Big\}\Big]$$

Physical picture



$$-\partial_z b^\dagger(\tau, z)\Big|_{z=0} + b^\dagger(\tau, 0) = 0$$

$$-\frac{1}{J_K}\partial_z^2 b^\dagger(\tau, z) + \frac{2SN_F}{\Lambda_c^2}\partial_\tau b^\dagger(\tau, z) = 0$$

$$\frac{2SN_F}{\Lambda_c^2}\partial_z b^\dagger(\tau, z)\Big|_{z=z_f} + \int_0^\beta d\tau' G_c(\tau - \tau') b^\dagger(\tau', z_f) G_f(\tau' - \tau) = 0$$

Correspondence between a mean-field theory with 1/N quantum corrections and the emergent gravity description in the $z_f = dz \to 0$ limit

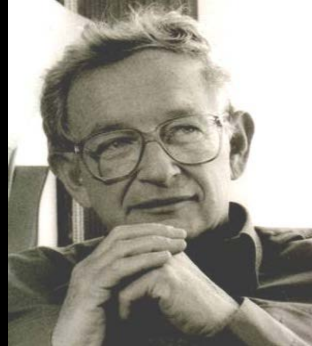
$$
\frac{1}{J_K} b^\dagger(\tau) + \int_0^\beta d\tau' G_c(\tau - \tau') b^\dagger(\tau') G_f(\tau' - \tau)
+ dz \Big\{ \frac{2SN_F}{\Lambda_c^2} \partial_\tau b^\dagger(\tau) - J_K \int_0^\beta d\tau' \int_0^\beta d\tau'' G_c(\tau - \tau') G_c(\tau' - \tau'') b^\dagger(\tau'') G_f(\tau'' - \tau') G_f(\tau' - \tau) \Big\} = 0
$$

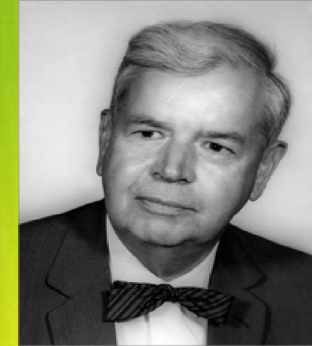
$$
\frac{N}{J_K} b^\dagger(\tau) + \int_0^\beta d\tau' G_c(\tau - \tau') b^\dagger(\tau') G_f(\tau' - \tau)
- \frac{J_K}{N} \int_0^\beta d\tau' \int_0^\beta d\tau'' G_c(\tau - \tau') G_c(\tau' - \tau'') b^\dagger(\tau'') G_f(\tau'' - \tau') G_f(\tau' - \tau) = 0
$$

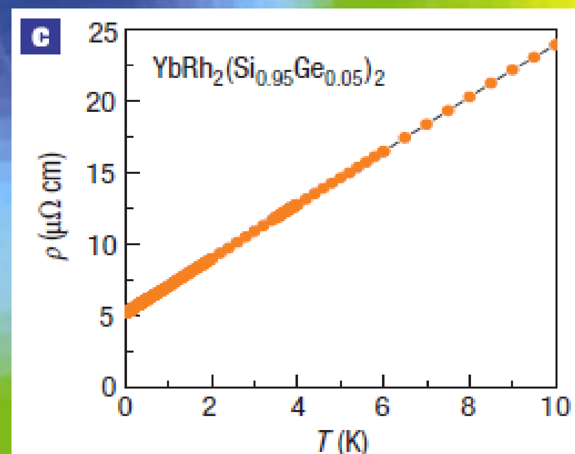
N. Read, J. Phys. C: Solid State Phys. 18, 2651 (1985)

Conclusion

- Mean-field (BCS) theory + "Full" quantum (vertex) corrections in a self-consistent way = Holographic Landau-Ginzburg theory on an emergent curved spacetime with an extra dimension
- Holographic LG theory = Bulk action + UV & IR boundary conditions
- UV B.C. + IR B.C. with $z_f = 0$ ➔ Mean-field theory
- UV B.C. + IR B.C. + Bulk eq. of motion with $z_f = dz$ ➔ Mean-field theory + 1/N quantum corrections
- Bulk action: RG equations for coupling functions
- IR boundary condition: Effective field theory with a fully renormalized coupling function
- The role of an extra dimension: Introduction of "full" quantum corrections in an iterative way
- Hamilton-Jacobi formulation = Callan-Symanzik equation: Emergent metric tensor ➔ Holographic entanglement entropy
- Holographic entanglement entropy = Field-theory entanglement entropy even away from criticality ??





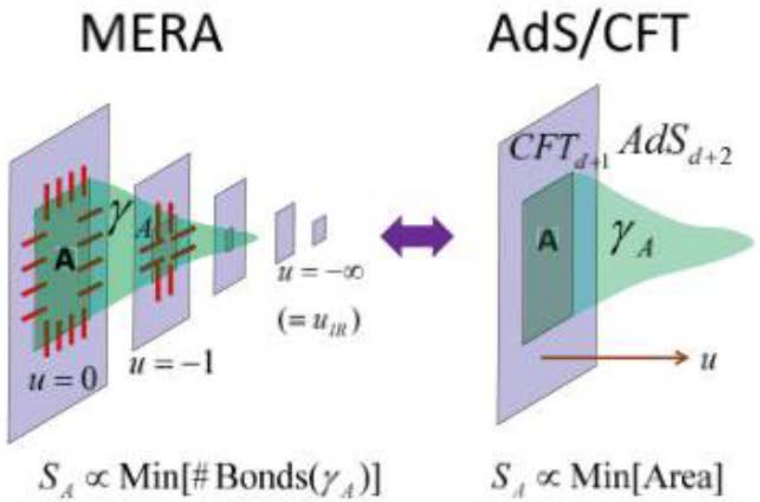


Emergent Einstein equation ??

DOI: 10.3938/PhiT.23.010 by J. D. Thompson

Connection to MERA (multi-scale entanglement renormalization ansatz as a tensor network variation approach)

Emergent space-time

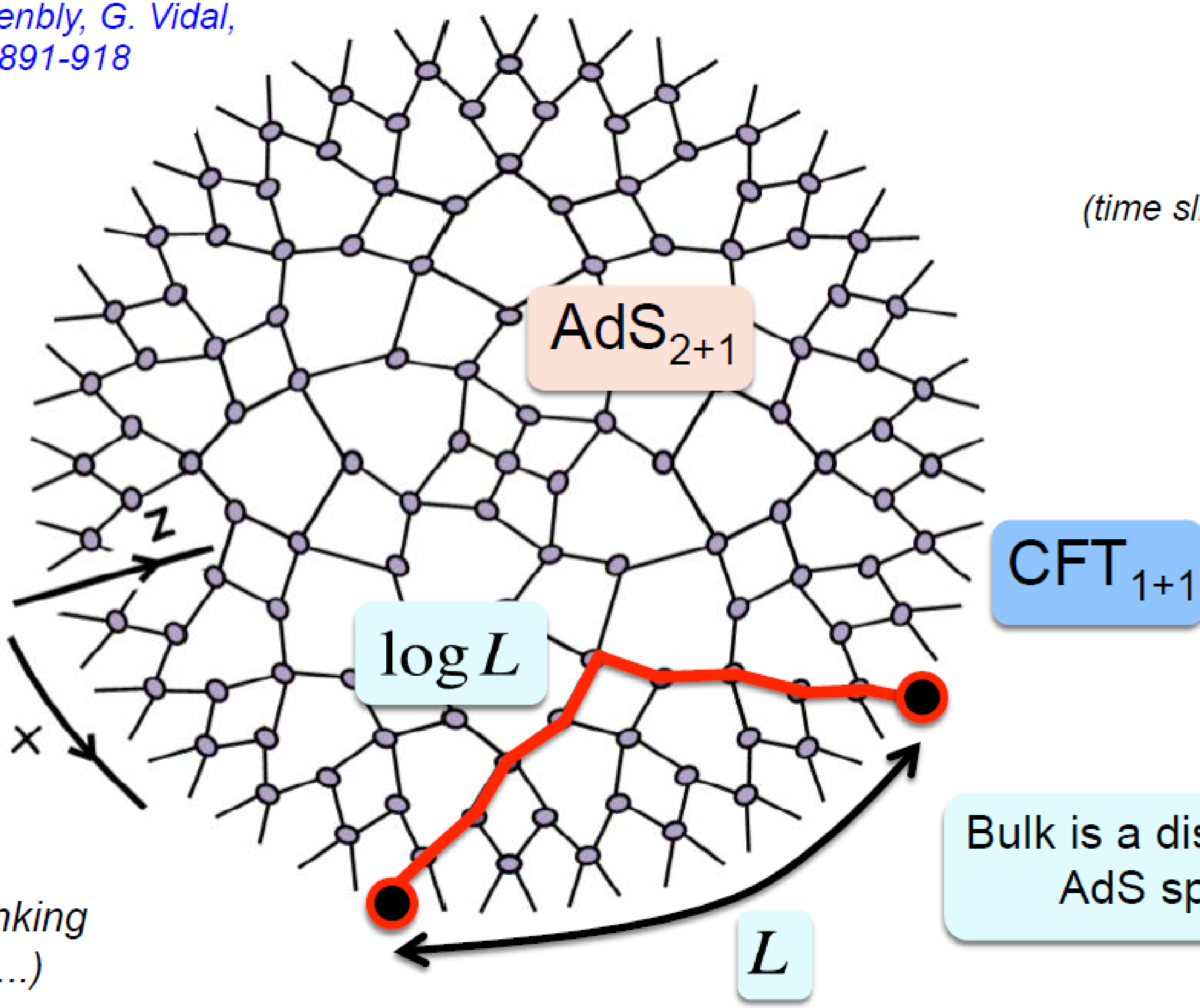


Picture from M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

MERA entropy ~ Ryu-Takayanagi prescription

R. Orus' lecture on entanglement in APCTP 2017

Picture from G. Evenbly, G. Vidal, (2011) JSTAT 145:891-918



(time slice)

Bulk is a discretized AdS space

(and we were not thinking about gravity at all...)

For a scale-invariant MERA, the tensors of a critical model with a CFT limit correspond to a „gravitational“ description in a discretized AdS space: „lattice“ realization of AdS/CFT correspondence

R. Orus' lecture on entanglement in APCTP 2017

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Ab initio holography

Horizon as critical phenomenon

Sung-Sik Lee

Peter Lunts,[a,b] Subhro Bhattacharjee,[c] Jonah Miller,[a,a] Erik Schnetter,[a,a]
Yong Baek Kim[e] and Sung-Sik Lee[a,b]

$$\mathcal{S} = m^2 \sum_i (\phi_i^* \cdot \phi_i) - \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2$$

$$\hat{H} = \sum_i \left[\frac{2}{m^2} \hat{\pi}_i^\dagger \cdot \hat{\pi}_i + i(\hat{\phi}_i \cdot \hat{\pi}_i + \hat{\phi}_i^\dagger \cdot \hat{\pi}_i^\dagger) \right]$$

$$Z = \left\langle S_0 \middle| e^{-z\hat{H}} \middle| t^{(0)} \right\rangle$$

$$\left| S_0 \right\rangle = \int D\phi \; e^{-m^2 \sum_i \phi_i^* \cdot \phi_i} \left| \phi \right\rangle$$

$$\left| t^{(0)} \right\rangle = \int D\phi \; e^{\sum_{ij} t_{ij}^{(0)} \phi_i^* \cdot \phi_j - \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2} \left| \phi \right\rangle$$