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# On the mass enhancement of black body background fluctuations

**Nguyen Van Hoa[1], Le Van Xuan[2], Nguyen Tri Lan[3], and Nguyen Ai Viet[3]**

[1]Hochiminh Pedagogical University, 270 An Duong Vuong, Hochiminh City, Viet Nam

[2]Hanoi College of Technology and Economics, Soc Son, Ha Noi, Viet Nam

[3]Institute of Physics,Vietnam Academy of Science and Technology, 10 Dao Tan, Ha Noi, Viet Nam

E-mail: `xuanlv.hn@gmail.com`

**Abstract.** Motivated by the mass enhancement in the toy model of moving particle in Boltzmann and Gaussian background fluctuations, the contribution of black body background fluctuation to the effective mass of massless and massive particles is considered in this work. As the black body radiation depends only on its temperature, the dependence of the effective mass on the temperature is obtained. The results, therefore, provide several physical insights for the research of complex systems where the interaction and equilibrium of a system and its surroundings are still not clear. By interpreting a characteristic parameter of the environmental contribution as its “effective temperature”, “the thermal equilibrium” condition for complex systems would be discussed in the context of a thermodynamic theory.

## 1. Introduction

Considering the toy models of Boltzmann and Gaussian fluctuations has shown that observing and analyzing the behaviors of a particle or a wave moving in the field of fluctuations would provide a lot of information of statistical properties of the field. “In physics, a black body would be defined as an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. A white body is one with a "rough surface [that] reflects all incident rays completely and uniformly in all directions. A black body in thermal equilibrium (that is, at a constant temperature) emits electromagnetic radiation called black body radiation. The radiation is emitted according to Planck’s law, meaning that it has a spectrum that is determined by the temperature alone, not by the body’s shape or composition.”[1] In the research of complex systems where the influence of surrounding environment is unknown, the fluctuation of the background would be characterized in terms of an effective temperature, that of a black body that would emit the same total flux of electromagnetic energy. As next stage of investigation, the black body radiation is considered as more real model of fluctuations.

## 2. Energy and Effective Mass in Planck's Fluctuations

### *2.1. Definitions of Energy and Effective Mass*

In conventional understanding of modern physics, motion of a particle is characterized by its dispersion law $\mathcal{E}(p)$ where the dependence of $\mathcal{E}(p)$ on $p$ reflects whether particle is massive or massless.In one hand, the massless particle has dispersion law in form

$$\mathcal{E}(p) = cp, \tag{1}$$

and in other hand, the massive particle $m_0$ has the dispersion law written in the relativistic form[2, 3] as

$$\mathcal{E}(p) = \sqrt{(mc^2)^2 + (cp)^2}, \tag{2}$$

which tends to additive law of energy

$$\mathcal{E}(p) = mc^2 + \frac{1}{2m}p^2 + \underbrace{V(p)}_{o(p^4)}, \tag{3}$$

when momentum $p$ approaches to zero, where $c$ is some maximal velocity of the massless motion in given media. It is important to think of a possibility that the coordinate representation of function $V(p)$

$$V(r) = \frac{1}{\sqrt{V_\Omega}} \int_\Omega \mathrm{d}p \; \mathrm{e}^{ipr} V(p), \tag{4}$$

would contain some physical insights of potential affecting on the motion of a massive particle in the medium, or in other words, interaction between massive particle and surrounding environment.

Following the standard mathematical routine found in textbooks, the mass of a moving particle with given dispersion law $\mathcal{E}(p)$ is determined by second order derivative of $\mathcal{E}(p)$ with respect to $p$ [4, 5] as

$$\frac{1}{m^*} = \left.\frac{\mathrm{d}^2}{\mathrm{d}p^2}\mathcal{E}(p)\right|_{p=0}. \tag{5}$$

When a particle moves under influence of some fluctuations which changes the dispersion law of the particle, energy of massive particle reads[6, 7, 8]

$$\mathcal{E}(p, \epsilon) = \sqrt{(mc^2)^2 + c^2p^2 + \epsilon^2}, \tag{6}$$

then the energy of moving particle is changed, and obtained by including all contributions of fluctuations via integrating out with fluctuation distribution function $\mathcal{D}(\epsilon)$

$$\mathcal{E}[cp; \mathcal{D}] = \int_\Omega \mathrm{d}\epsilon \mathcal{D}(\epsilon) \sqrt{(mc^2)^2 + c^2p^2 + \epsilon^2}. \tag{7}$$

The effective mass of moving particle will be determined by the definition (5)

### 2.2. The Energy Density Function of Black Body Fluctuations

The law of back body radiation developed by Max Planck in 1900 has shown that, expressed as an energy distribution, it is the unique stable distribution for radiation in thermodynamic equilibrium. This law can be rewritten in the term of energy density function as

$$\begin{aligned}\mathcal{D}(\epsilon, k_{\rm B}T) &= \frac{1}{\mathcal{C}}\frac{2\epsilon^3}{(2\pi\hbar)^3 c^2}\frac{1}{e^{\frac{\epsilon}{k_{\rm B}T}}-1} \\ &= \frac{15}{\pi^4 (k_{\rm B}T)^4}\frac{\epsilon^3}{e^{\frac{\epsilon}{k_{\rm B}T}}-1}\end{aligned}, \tag{8}$$

$$\begin{aligned}\mathcal{C} &= \int_0^\infty {\rm d}\epsilon \frac{2\epsilon^3}{(2\pi\hbar)^3 c^2}\frac{1}{e^{\frac{\epsilon}{k_{\rm B}T}}-1} \\ &= \frac{2}{(2\pi\hbar)^3 c^2}\int_0^\infty {\rm d}\epsilon\epsilon^3\frac{1}{e^{\frac{\epsilon}{k_{\rm B}T}}-1} \\ &= \frac{2}{(2\pi\hbar)^3 c^2}(k_{\rm B}T)^4\int_0^\infty {\rm d}\left(\frac{\epsilon}{k_{\rm B}{\rm T}}\right)\left(\frac{\epsilon}{k_{\rm B}T}\right)^3\frac{1}{e^{\frac{\epsilon}{k_{\rm B}T}}-1} \\ &= \frac{2}{(2\pi\hbar)^3 c^2}(k_{\rm B}T)^4\int_0^\infty {\rm d}x\frac{x^3}{e^x-1} \\ &= \frac{2}{(2\pi\hbar)^3 c^2}(k_{\rm B}T)^4\frac{\pi^4}{15} \\ &= \frac{1}{15}\frac{2\pi}{(2\hbar)^3 c^2}(k_{\rm B}T)^4\end{aligned} \tag{9}$$

where $\hbar\omega$ of electromagnetic radiation is replaced by the background fluctuation $\epsilon$, and $T$ is some effective temperature which would be generalized by an abstract temperature characterizing the equilibrium of background, in case of complex systems

### 2.3. Energy and Effective Mass

Performing the integrals (6) and (7) with energy density function (8) leads to the total energy

$$\begin{aligned}\varepsilon(cp, k_B T) &= \frac{15}{\pi^4 (k_{\rm B}T)^4}\int_0^\infty {\rm d}\epsilon\frac{\epsilon^3}{e^{\frac{\epsilon}{k_{\rm B}T}}-1}\sqrt{(m_0c^2)^2+c^2p^2+\epsilon^2} \\ &= \sqrt{(m_0c^2)^2+c^2p^2}\,\mathcal{I}_{1/2}\left(\frac{k_{\rm B}T}{\sqrt{(m_0c^2)^2+c^2p^2}}\right),\end{aligned} \tag{10}$$

$$\begin{aligned}\hbar(\omega-\omega_o) &= \tfrac{15}{\pi^4}\hbar\omega_0\left(\tfrac{k_{\rm B}T}{\hbar\omega_0}\right)\int_0^\infty {\rm d}x\tfrac{x^3}{e^x-1}\sqrt{\left(\tfrac{\hbar\omega_0}{k_{\rm B}T}\right)^2+x^2} - \tfrac{15}{\pi^4}\hbar\omega_0\int_0^\infty {\rm d}x\tfrac{x^3}{e^x-1} \\ &= \frac{C}{\pi^4}\hbar\omega_0\int_0^\infty {\rm d}x\frac{x^3}{e^x-1}\left(\sqrt{1+\left(\frac{k_{\rm B}T}{\hbar\omega_0}\right)^2 x^2}-1\right) = \hbar\omega_0\left(\mathcal{I}_{1/2}\left(\frac{k_{\rm B}T}{\hbar\omega_0}\right)-1\right)\end{aligned}, \tag{11}$$

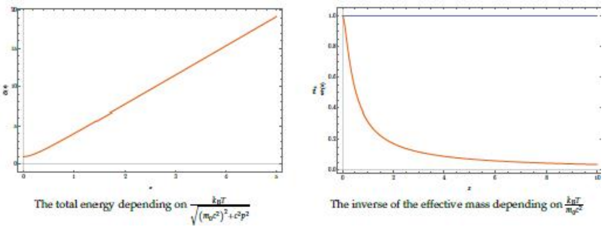


**Figure 1.** The total energy and effective mass depending on $k_BT$

and the inverse of the effective mass

$$\frac{1}{m^*} = \frac{15c^2}{\pi^4\,(k_\mathrm{B}T)^4}\int\limits_0^\infty \mathrm{d}\epsilon \frac{\epsilon^3}{e^{\frac{\epsilon}{k_\mathrm{B}T}}-1}\frac{1}{\sqrt{(m_0c^2)^2+\varepsilon^2}} = \frac{1}{m_0} I_{-\frac{1}{2}}(\frac{k_BT}{m_0c^2}), \tag{12}$$

respectively, where the auxiliary function $I_\gamma(\alpha)$ is analytically defined by an infinite series of $\alpha^2$ as

$$\begin{gathered} I_\gamma(\alpha) = \tfrac{15}{\pi^4}\int\limits_0^\infty dx(1+\alpha^2x^2)^\gamma \tfrac{x^3}{e^x-1} \\ = \tfrac{15}{\pi^4}\sum\limits_0^\infty \tfrac{(2i+3)!}{i!}\gamma(\gamma-1).......(\gamma-i+1)Li_{2i+4}(1)\alpha^{2i} \end{gathered} \tag{13}$$

It is obvious from the equation (12) that non–zero average value of fluctuation generates non–zero mass of a massless particle

$$m^*c^2 = (\frac{15}{\pi^4}(2\zeta(3)))^{-1}(k_BT) = 2.70118(k_BT), \tag{14}$$

### *2.4. The Frequency Shift and the de Broglie Wavelength*

In contrast to the particle formalism, in the wave formalism the frequency shift effect is obtained

$$\frac{\omega}{\omega_0} = I_{\frac{1}{2}}(\frac{k_BT}{\hbar\omega}) \tag{15}$$

As it is illustrated in the left panel of the above figure, function $I_{\frac{1}{2}}(\frac{k_BT}{\hbar\omega})$ is always greater than one.

This means that under the influence of black body fluctuation the frequency of initial wave increases,or in other words, the ultraviolet shift is observable. The de Broglie wavelength of wave propagating in the black body radiation is changed and determined by the expression

$$\frac{\lambda}{\lambda_0} = I_{\frac{1}{2}}(\frac{k_BT}{\hbar\omega}) \tag{16}$$

Also, as it is illustrated in the left panel of the above figure, the ratio $\frac{\lambda}{\lambda_0}$ gets maximum at zero temperature $T = 0$, and then it decreases while temperature is increasing.

## 3. Conclusion

In conclusion, by considering the probability density functions of background fluctuations modeled by black body radiation, the change of effective mass of a moving massive and massless particles has been studied. In the case of massless particle, the black body fluctuation generates non–zero mass which is linearly depending on background temperature. In the case of massive particle, the dependence of effective mass on the ratio of mass and temperature of background fluctuations, $k_B T$, has been analytically found. Furthermore, the frequency shift and the de Broglie wavelength are also analytically obtained.

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