General autocatalytic theory and simple model of financial markets

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General autocatalytic theory and simple model of financial markets

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Abstract. The concept of autocatalytic theory has become a powerful tool in understanding evolutionary processes in complex systems. A generalization of autocatalytic theory was assumed by considering that the initial element now is being some distribution instead of a constant value as in traditional theory. This initial condition leads to that the final element might have some distribution too. A simple physics model for financial markets is proposed, using this general autocatalytic theory. Some general behaviours of evolution process and risk moment of a financial market also are investigated in framework of this simple model.

1. Introduction

Autocatalyst system is a well-known concept in physics of complex systems [1], of which elements do not only exist or decay, but might have the possibility of forming new elements or destroying existed elements through recombination of already existing elements. This dynamics is already applied in many biological or sociological evolutions[2][3][4][5].

This paper deals with an economic system, in which each event is considered as a particle interacting with others and possibly producing new particles, therefore the system-environment transforms into a new one. In return, it opens the possibility for chemical reactions that have not been possible before the existence of the new product or the destruction of some existing ones. The concept of the "adjacent possible" [6] or "adjacent probable' '[7] needed to be introduced. This is the set of objects that can possibly become produced within a given time span into the future.

The formation process has been investigated with the cascades of creation and destruction. In this process, element exists in two states 0/1: existing or not existing. A set of elements is formed via an interaction matrix. An element can change its state depending on the states of the elements it is connected to.

This process could be used to investigate financial crisis period. Some information or behaviour could lead to the fact that investigators buy or sell their capitals. If most of investigators buy or sell the same capital then the market is forced to unstable state and there might create a financial crisis.

2. Autocatalyst system

In an evolution, each event i presents in a system with a frequency x_i - normalized abundance of the element i, such that $\sum x_i = 1$, n elements exist, if $x_i > 0$, and does not exist if $x_i = 0$.

Interaction matrix α is used to express inter-dependence between pairs of elements. If the combination of elements j and k are possible to form element i , then $\alpha_{ijk} = 1$, otherwise, $\alpha_{ijk} = 0$. Tensor α indicates the productive sets.

The dynamics of x_i is

$$\frac{d}{dt}x_i = \sum_{j,k} \alpha_{ijk} x_j x_k - x_i \sum_{lj,k} \alpha_{ljk} x_j x_k. \tag{1}$$

In general cases of the catalytic network, n elements are necessary to form a new one instead of two elements, then $j = (j_1, j_2, ..., j_n)$

The relative diversity of the system $a_t \in [0, 1]$ such that $a_t d$ is the number of existing elements at time t. Consider the productive sets in α, that $\alpha_{ijk} = 1$. Only the number r is known, such that rd is the number of such productive sets occurring in the system and that the entries 1 and 0 in α are distributed completely randomly. So r and n are system's characterized variables.

At time $t = 0$, there exist some initial element $a_0 d$, then at time $t \neq 0$ the produced elements are

$$rd \begin{pmatrix} a_t d \\ n \end{pmatrix} \begin{pmatrix} d \\ n \end{pmatrix}^{-1} \cong rda_t^n. \tag{2}$$

rda_{t-1}^n those elements already have been produced in the preceding time at $t-1$ and are certainly contained in a_t, only a fraction $1 - a_t$ of remaining elements were not be present in a_t and truly be newly created. Then

$$\begin{aligned} a_{t+1} &= a_t + \triangle a_t, \\ \triangle a_t &= r(1 - a_t)(a_t^n - a_{t-1}^n). \end{aligned} \tag{3}$$

As a_0 is the initial condition then $a_{-1} \equiv 0$. Define $c_t = \triangle a_{t+1}/\triangle a_t$, for long time limit $c \equiv lim_{t\to\infty} c_t$, then

$$c = nr(1 - a_\infty)a_\infty^{n-1}. \tag{4}$$

$$a_\infty = a_0 \sum_{t=0}^{\infty} c^t = \frac{a_0}{1-c} \tag{5}$$

The $(n + 1)th$ order equation

$$a_\infty - a_0 = nr(1 - a_\infty)a_\infty^n \tag{6}$$

It depends on n that equation (6) takes different forms as:

- $n = 1$ then $a_\infty - a_0 = r(1 - a_\infty)a_\infty$, there is no phase transition of this system,
- $n \geq 2$ there exits a phase transition of system complexity. For the simplest cases only $n = 2$ is considered in this work,

$$a_0 = a_\infty - 2r(1 - a_\infty)a_\infty^2. \tag{7}$$

At $r \sim 2$, there exists a phase transition.

Those solutions are mathematically identical to description of the real gases, i. e. van der Waals equation of state. By definding $V \equiv a_\infty$; $T \equiv a_0(nra_\infty^{n+1})^{-1}$; $P \equiv a_\infty^{-1} + (nra_\infty^{n+1})^{-1}$, then equation (6) can be mapped onto

$$\left(P - \frac{1}{V^2}\right) V = T. \tag{8}$$

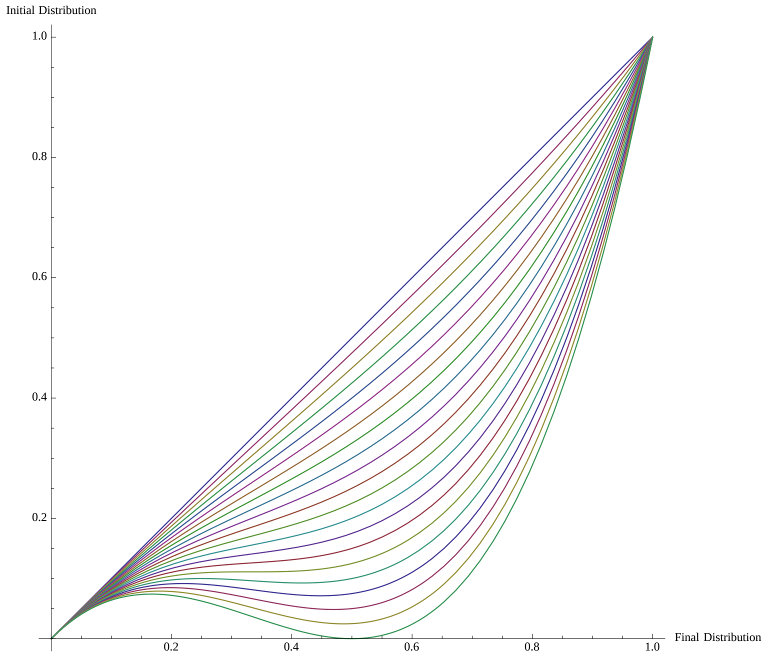

Figure 1. Phase diagram of the creative dynamic for $n = 2$ for various r. A phase transition starts at $r \sim 2$.

This description is a van der Waals gas of point–particles with constant internal pressure. In the context of this work, pressure and temperature are effective ones.

The equation shows the relationship between initial and final states of two elements in autocatalyst process. This equation is equivalent to van der Waals equation of real gas, so the phase transition theory of real gas might be useful to explore the phase transition phenomenon in autocatalyst system, and even in econophysical systems. Using the autocatalyst equation to investigate the transition from Boltzmann distribution to Gaussian distribution, it shows that the autocatalyst can not realize the transition as expected. The initial distribution conserve its form during the autocatalyst process. $\alpha_i j = 1, \alpha_i j = 0$

3. Autocatalyst with a specific complexity of the system

This part presents discussions about two specific cases of the initial complexity of the system, which follows Boltzmann and Gaussian distribution.

In our previous works[8][9], it has been showed that there exists a phase transition of return distributions in econophysics system, such as stocks. A simple model was proposed[10] based on market returns distribution, without any autocatalyst parameter. The model was investigated by using market data of Aluminium Company of America (Alcoa) and DJIA index.

Our previous results[10] showed that there exists the transformation process from Boltzmann-like distribution to Gaussian-like distribution, taking into account time parameter. This dynamic model worked quite good for several emperical data. Then we realised that this process is universal and could be explained by a non-dynamic model with contributions of white noise. Other results [8] showed non-dynamic process, where time parameter was ignored and only background noise contributions were considered. The study with real market data shows that under the influence of background fluctuations, most symmetric distributions tend to Gaussian-like ones before transforming to Gaussian one. These results made an empirical agreement with the central limit theorem.

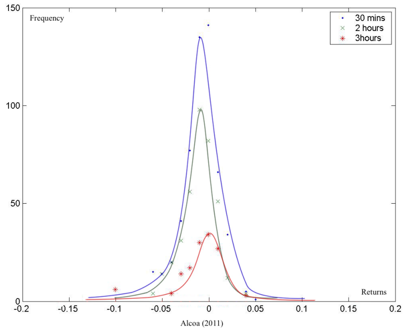

Figure 2. Distribution of Alcoa returns for short time period. The distribution collapsed from Boltzmann-like distribution to Gaussian-like distribution from thirty minutes to one day.

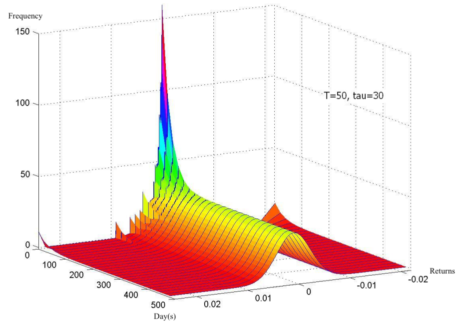

Figure 3. Distribution of DJIA index for medium time period. There is a fast collapse process during the first hundred days from Boltzmann-like distribution to Gaussian-like distribution

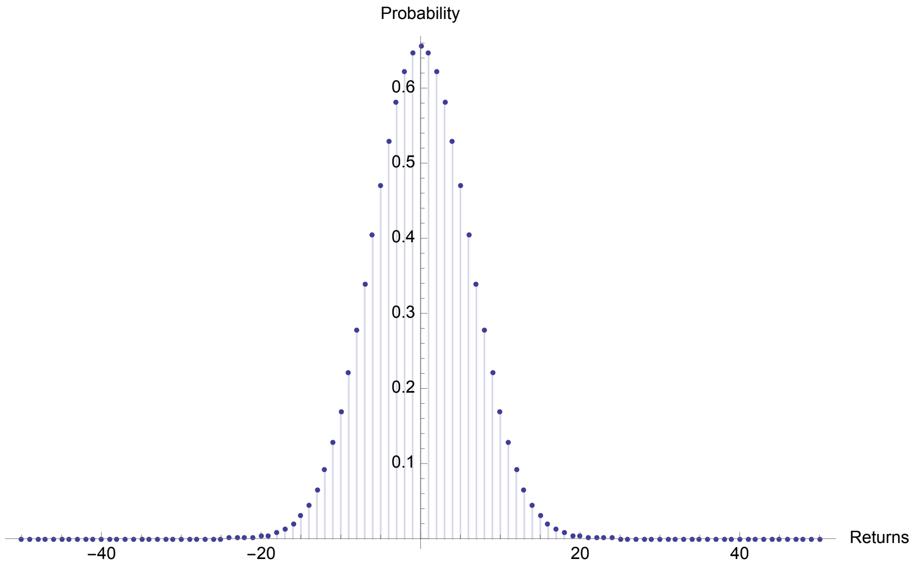

Figure 4. Boltzmann-like distribution collapses to Gaussian-like distribution under noises contribution.

Among various non-dynamic process, autocatalyst could be one of the very first system that could be considered due to analogy between particles interactions and economical interactions[11]. Each agent plays a role as an element in an autocatalyst system, interaction between two agents leads to a new behaviour. Each trading action is considered as an element in this autocatalyst system, represented by its returns. This process would be investigated as an autocatalyst process, with two type of initial returns distribution, Boltzmann and Gaussian distribution. The results are found on Fig. 4 and Fig. 5.

There is no strong emergence, returns distributions keep their type throughout the autocatalyst process. It means that autocatalyst emphasizes the phase transition but phase transition is not a direct result of autocatalyst process.

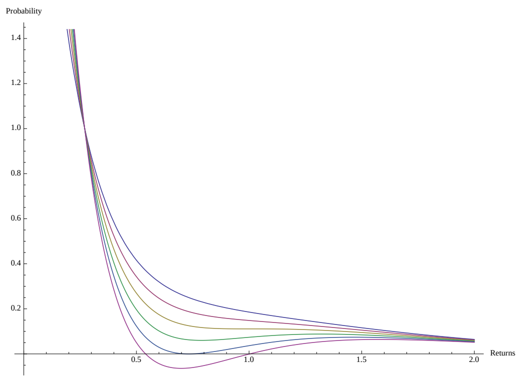

Figure 5. Initial complexity of the system follows Boltzmann distribution. Final distribution of the complexity is Boltzmann still.

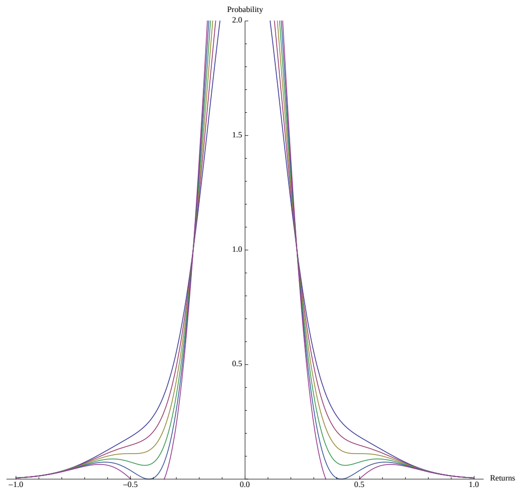

Figure 6. Initial complexity of the system follows Gaussian distribution. Final distribution of the complexity is Gaussian still.

4. Discussion

In this work, the formal equivalence between autocatalyst theory and van der Waals equation has been used to investigate the transition from Boltzmann distribution to Gaussian distribution within the concept of phase transition. As a part of our research in econophysics, returns distribution transformations were considered in order to find out the mechanics of this process. The autocatalyst theory of Hanel *et al.* was used to develop our non-dynamic transformation process, expecting that creation process could lead to a critical transformation in the system. It has been shown that no matter how the initial complexity of the system was, there always exist the phase transition. But it does not like noise contribution process, where no matter what type of initial distribution, general noises lead to Gaussian-like distribution, in this process, autocatalyst emphasizes the phase transition only, but the final distribution of the system does not depend on autocatalyst process.

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