

## Boltzmann-Gaussian Transition under Specific Noise Effect

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 J. Phys.: Conf. Ser. 537 012005

(<http://iopscience.iop.org/1742-6596/537/1/012005>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

### Download details:

This content was downloaded by: vieta

IP Address: 113.190.228.94

This content was downloaded on 13/10/2014 at 03:47

Please note that [terms and conditions apply](#).

# Boltzmann–Gaussian Transition under Specific Noise Effect

**Chu Thuy Anh, Nguyen Tri Lan and Nguyen Ai Viet**

Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan, Ba Dinh, Ha Noi, Vietnam

E-mail: ctanh@iop.vast.ac.vn

**Abstract.** It is observed that a short time data set of market returns presents almost symmetric Boltzmann distribution whereas a long time data set tends to show a Gaussian distribution. To understand this universal phenomenon, many hypotheses which are spreading in a wide range of interdisciplinary research were proposed. In current work, the effects of background fluctuations on symmetric Boltzmann distribution is investigated. The numerical calculation is performed to show that the Gaussian noise may cause the transition from initial Boltzmann distribution to Gaussian one. The obtained results would reflect non–dynamic nature of the transition under consideration.

## 1. Introduction

Market returns have received much attention and have been studied for decades [7, 8, 9, 10, 3, 5]. Empirical data has been used to illustrate some dynamic models of market [4]. Gaussian distribution seems to be the most used approximation to describe real–valued random variables which tend to cluster around a single mean value [1, 11]. Theoretically, Gaussian distribution could be used to model market returns distribution when studying the market takes for long time enough, i.e. when the returns set is big enough, not for the not–big–enough data set, or for some assets that can not be described dynamically.

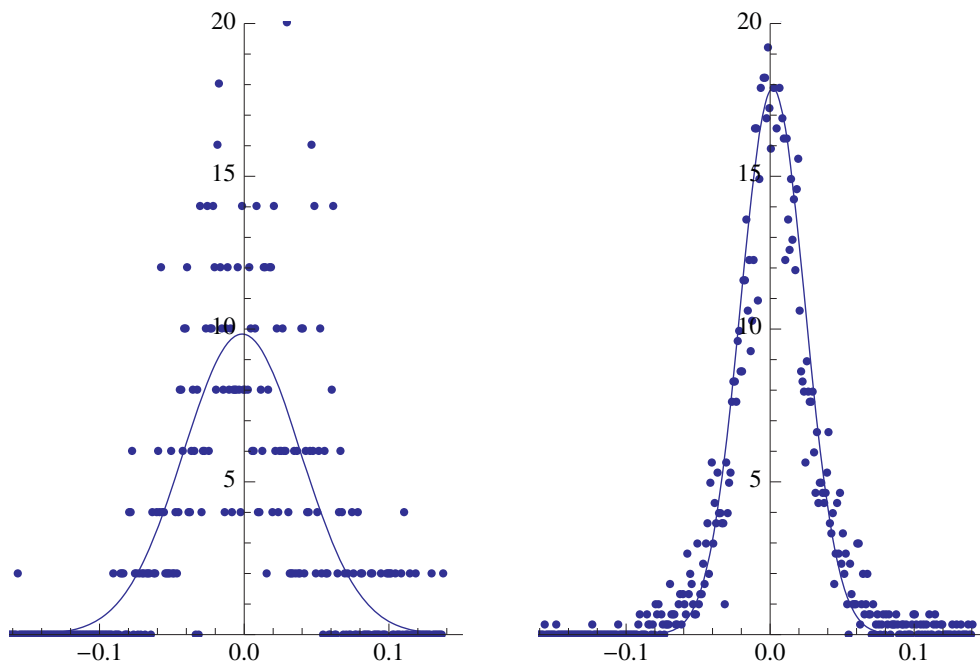
The model proposed in this paper is another view without taking into account the time role, but the sets of background fluctuations. In the ideal case, just as perfect condition in a physical system, there is no background fluctuation. However, for all real market data, there always are various parameters affecting more or less on each movement of the market, such as political situation in a country, or disaster, or information disturbance, . . . etc. These parameters could be considered as background fluctuations which will be describe as a set of random auxiliary variables. By including the background fluctuations into the data set, market returns tendency distribution can be studied independently to time value.

## 2. Standard Distributions

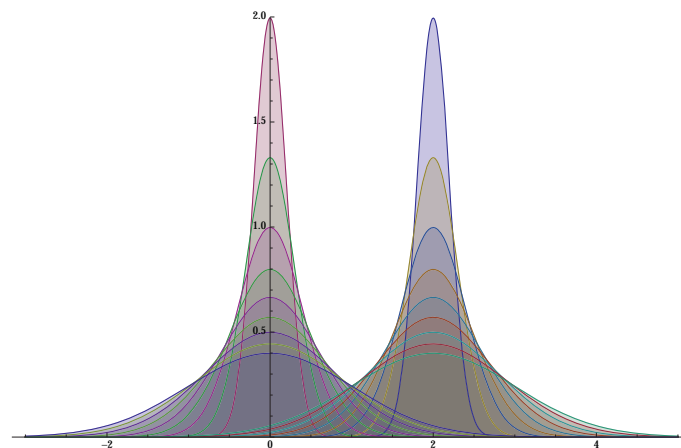
Conventionally, the return obtained from the long time data set is modeled by a Gaussian distribution, and the return from the short time data set is empirically presented by a symmetric Boltzmann distribution. As the initial stage these standard distributions are recalled.

Consider an asset with random variables in an ideal condition, means without background fluctuations, then probability distribution of returns reduces to exponential and Gaussian [6, 2].





**Figure 1.** The left graphic presenting data set of return of Apple Inc. in time period 2000–2002 is not able to be well fitted with a Gaussian distribution, whereas the right one presenting data set of return of the same stock in time period 2000–2012 is more fitted with a Gaussian distribution. Longer time of data set, the graphics is more well fitted with the Gaussian one.



**Figure 2.** Zero and Non-Zero Gaussian distributions with different values of variance  $\sigma$

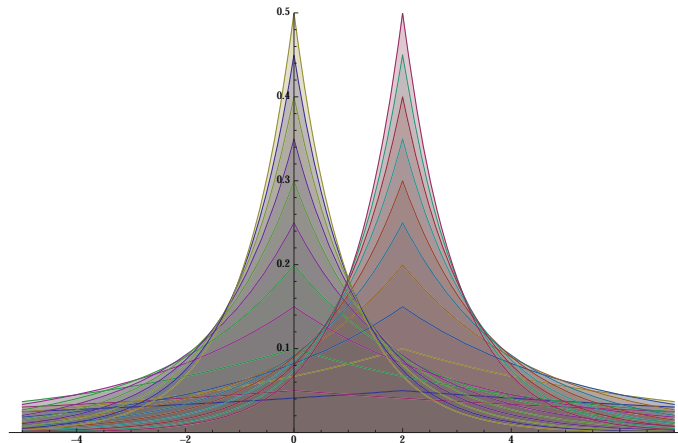
A normal distribution is written as

$$f_G(\sigma, x_0, x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \quad (1)$$

where  $x_0$  and  $\sigma$  are mean value and variance, respectively.

In other hand, a symmetric Boltzmann distribution is written under the form

$$f_B(\lambda, x_0, x) = \frac{1}{2} \lambda e^{-\lambda|x-x_0|} \quad (2)$$



**Figure 3.** Zero and Non-Zero Boltzmann distributions with different values of parameter  $\lambda$

where  $\lambda$  plays the role of inverse of market temperature, and  $x_0$  is central value of given distribution.

### 3. The contributions of background fluctuations

In most of the cases, a random variable is not absolutely independently random. In fact, the random variables are all affected by a set of other random variables that could be considered as background or environmental fluctuations, which should be described by a stochastic process. Using the same physical approach as in relativistic energy expressions, random quantities will be changed under background fluctuations just the same way as thermal fluctuations in thermodynamic systems, and it can be described

$$x^2 \rightarrow x^2 + \sum_i \epsilon_i^2. \quad (3)$$

For the sake of simplicity, the current consideration is limited in the case of  $i = 1$ . Hence, the distribution functions would be rewritten as

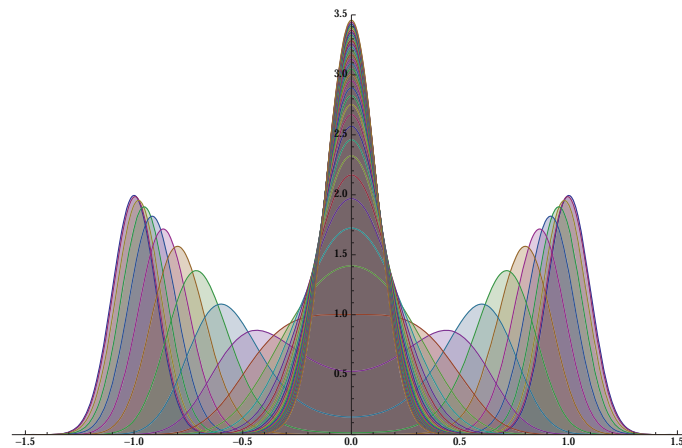
$$f(x) \rightarrow C(\epsilon) f\left(\sqrt{x^2 + \epsilon^2}\right), \quad (4)$$

in which  $C(\epsilon)$  is the normalization constant of the distribution function  $f\left(\sqrt{x^2 + \epsilon^2}\right)$ , i.e.

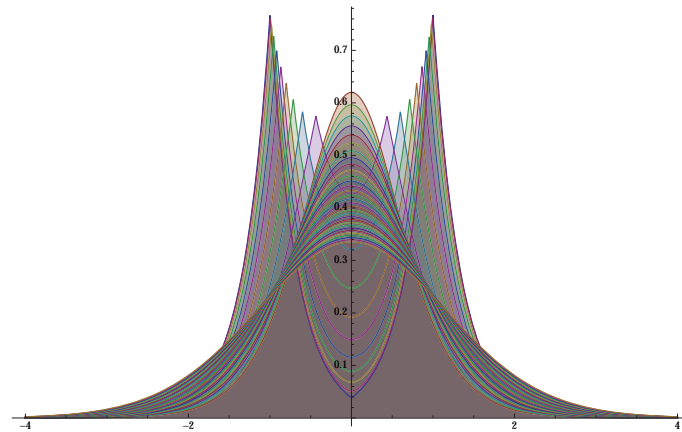
$$C(\epsilon) \int f\left(\sqrt{x^2 + \epsilon^2}\right) dx = 1. \quad (5)$$

It has been mentioned by Fokker-Plank function, under the influence of drag forces and random forces, the probability density function of a particle's velocity is described dynamically, as in Brownian motion, as well as all in this case. This is a dynamic description, in which the distribution tends to Gaussian one when time closes to infinity, means on this interval of time, there are enough fluctuations to make a tends-to-Gauss transition.

A close-to-infinity time series could be seen as a collection of fluctuations which is big enough to make a distribution to be a tends-to-Gauss one. The proposed model in this paper does not depend on a specific dynamic, and somehow a possibility to define relaxation time between two almost-balance-states is supplied. The proposed model can be used to describe not only equilibrium-equilibrium process but also non-equilibrium to equilibrium process.



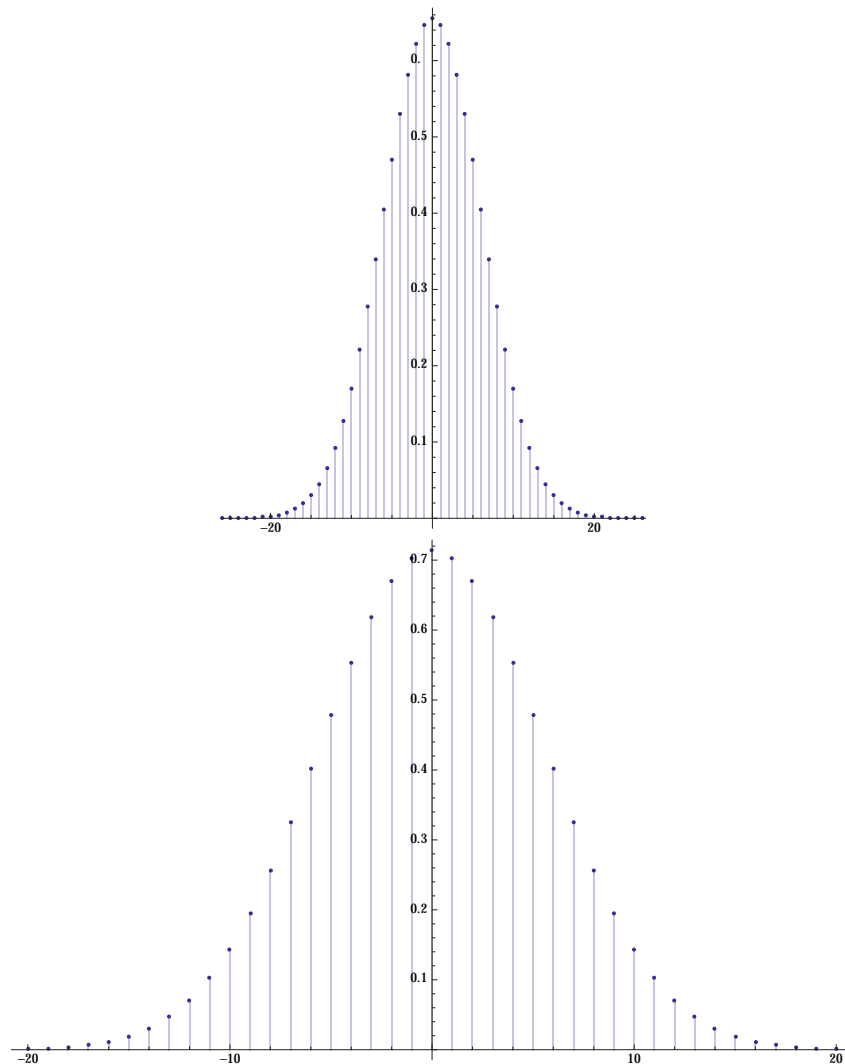
**Figure 4.** In the present of small noise, Gaussian distribution splits itself into two-pick distributions, and two picks get closer in the regime of large noise.



**Figure 5.** The same phenomenon happens with the Boltzmann distribution in present of various values of noise.

There are two types of symmetric distribution under consideration, zero centered distribution and non-zero centered one. Under background fluctuation effect, the non-zero centered distribution split into two symmetric distributions, whereas the zero centered one doesn't split into two distributions but degenerated and collapsed to a more stable Gaussian distribution. This phenomenon is quite the same as the picture of particle and antiparticle in a physics system. The zero-centered case could be treated as a physics system in an ideal condition, vacuum one. In the present of noise, the Gaussian and Boltzmann distributions split into two-pick distribution in the small value regime of noise, and bold single pick distribution in the large value regime of noise.

The transition of a symmetric or quasi-symmetric distributions to Gaussian distribution is a general phenomenon, which does not depend on a specific dynamic of noise, but depends on distribution of fluctuation only. The parameters of background fluctuations such as mean value and variation lead to the change of corresponding parameters of final Gaussian distribution. However, in real data set, it is impossible to directly observe and to classify the background fluctuations, their contributions are silently included into each element of data set. It can only be observed the transition of the initial distribution to final one under influence of background



**Figure 6.** The numerical summation over full range of Gaussian noise presents the final Gaussian distribution which does not depend on initial distribution.

fluctuations. Mathematically, the final distribution  $f_0(x)$ , which is obtained from real data set, is the sum of all possible contributions of background fluctuations with specific statistical weight  $f^G(\epsilon)$ , i.e.

$$f_0(x) \rightarrow \int f^G(\epsilon) C(\epsilon) f(\sqrt{x^2 + \epsilon^2}) d\epsilon. \quad (6)$$

Initial Boltzmann distribution affected by all possible background fluctuations then tends to Gaussian distribution. It can be shown analytically by taking integral of probability distribution function for all noises.

In general cases background fluctuations follow Gaussian law. Empirical data shows that under background fluctuations, most of the distributions tend to Gaussian-like ones before transforming to Gaussian one. As an inverse approach, by analyzing the long time data set the role of background fluctuations and their statistical parameters can be emerged and determined, and the obtained results provide a quantitative insight of background fluctuations in many phenomena of physics and interdisciplinary physics.

The condition where it is supposed to exist the Gaussian–distribution–form fluctuation is a kind of ideal one. In the case of econophysics assets, the data has been taken in specific time interval, means with incomplete Gaussian distribution fluctuations. Background fluctuations drive the initial distribution to a Levy distribution. This problem will be discussed numerically and analytically in next papers. In the context of this paper the Levy part is cut off.

#### 4. Conclusion

A non–dynamic approach has been proposed, taking into account the important role of background fluctuations. Background fluctuations in an economic system play the same role as in a physics one. Normal distribution fluctuations guide the transition process from exponential distribution to normal distribution. The collapse process has been draw numerically and also analytically in the paper.

Taking this result into economic view, it means no matter how many fluctuations it might have for an economic object, if all are random fluctuation, the probability distribution always tends to Gaussian one. If not, that mean some fluctuations are not considered yet.

For some special cases that background fluctuation does not follow Gaussian law, the probability distribution will be rearranged differently. Theses cases will be treated in our next works.

#### References

- [1] Victor M. Yakovenko A. Christian Silva, Richard E. Prange. Exponential distribution of financial returns at mesoscopic time lags: a new stylized fact. *Physica A*, (344), 2004.
- [2] V. M. Yakovenko A. Dr. *Quant. Finance* 2, (443), 2002.
- [3] V.M. Yakovenko A.C. Silva. *Physica A*, 324(303), 2003.
- [4] Thuy Anh Chu, Hong Lien Do, and Ai Viet Nguyen. Simple model for market returns distribution. *Communications in Physics*, 23(2), 2013.
- [5] Stauffer D. Phys. rev. e 60 (1999) 6519. *Physica A*, (336):1–5, 2004.
- [6] A. Silva et al. *Physica A* 344, (344):277–235, 2004.
- [7] M. Potters J.-P. Bouchaud. Theory of financial risks. *Cambridge University Press, Cambridge*, 2000.
- [8] H. Salunkay H.E. Stanley K. Matia, M. Pal. *Europhys. Lett.*, 66(909), 2004.
- [9] R. Riera L.C. Miranda. *Physica A*, 297(509), 2001.
- [10] L.N. Amaral M. Meyer H.E. Stanley V. Plerou, P. Gopikrishnan. *Phys. Rev. E*, 60(6519), 1999.
- [11] V. M. Yakovenko. Econophysics, statistical mechanics approach to. In *Encyclopedia of Complexity and System Science*. Springer, 2007.