ICTP Asian Network School and Workshop on Complex Condensed Matter Systems

Quantum transport through charge Kondo circuits: Role of electron-electron interactions in Luttinger liquid

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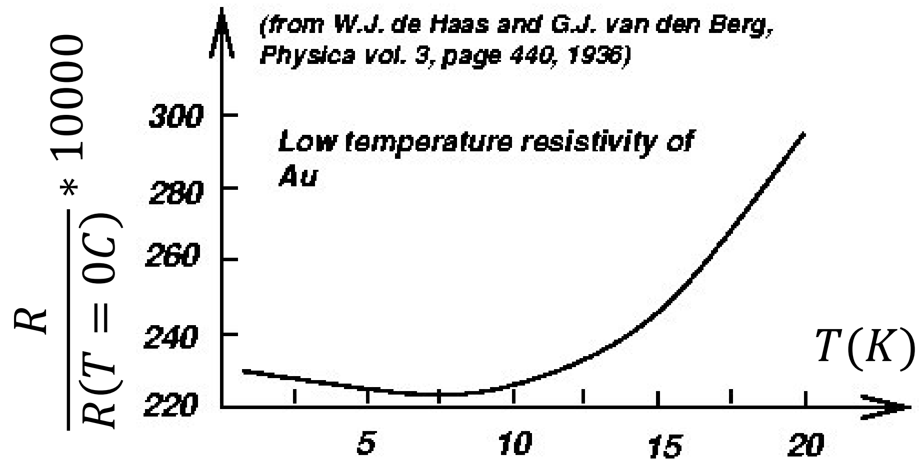
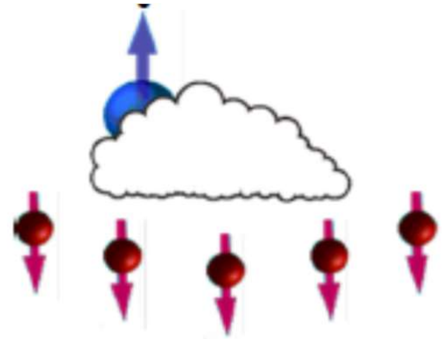
Outline

- Kondo effect
- Charge Kondo implementation
- Luttinger liquid
- Thermoelectric transport in a two-channel charge Kondo circuit
- Charge Kondo circuit as a detector for e-e interactions in a Luttinger Liquid
- Conclusion

Kondo effect





$$H_{\text{exchange}} = J\,(\mathbf{s}\cdot\mathbf{S})$$

conduction electrons — magnetic impurity

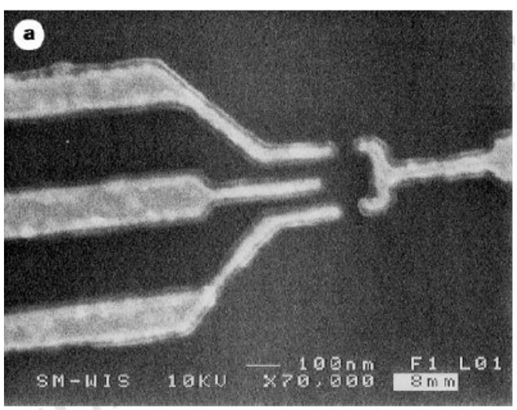
$$\Delta\rho \sim J^2 + J^3 \ln(\varepsilon_F / T)$$

Spin flip: Anderson explanation

Kondo theory: low T but T>TK

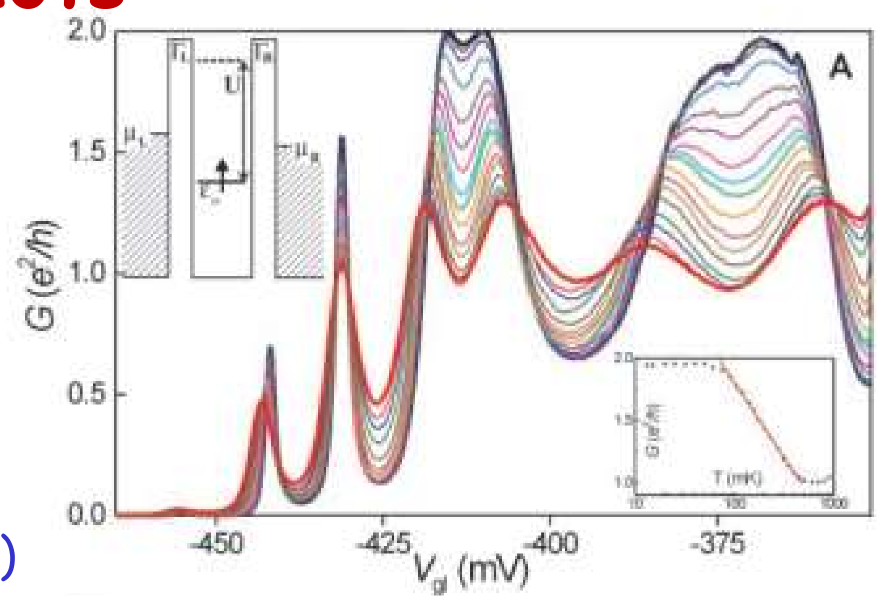
Kondo effect in quantum dots

QD with odd number of electrons: quantum impurity



D. Goldhaber-Gordon et al., Nature 391 (1998)

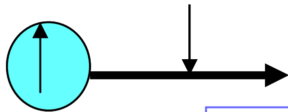
W. G. van der Wiel et al., Science 289 (2000)

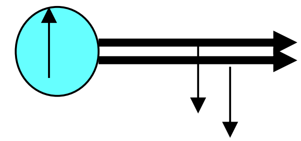


Fermi Liquid and non-Fermi Liquid behaviors

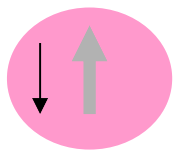
$$J c_\sigma^\dagger \vec{\sigma}_{\sigma\sigma'} c_{\sigma'} \vec{S}$$

$$J \sum_j c_{j\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$

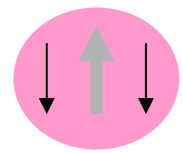




Kondo screening of the impurity spin



Complete screening below Tk

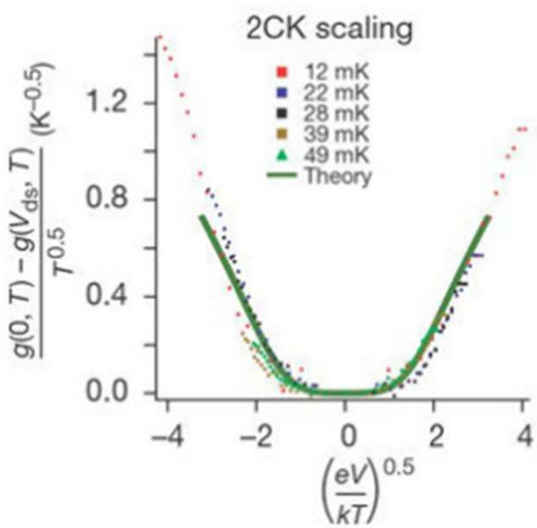


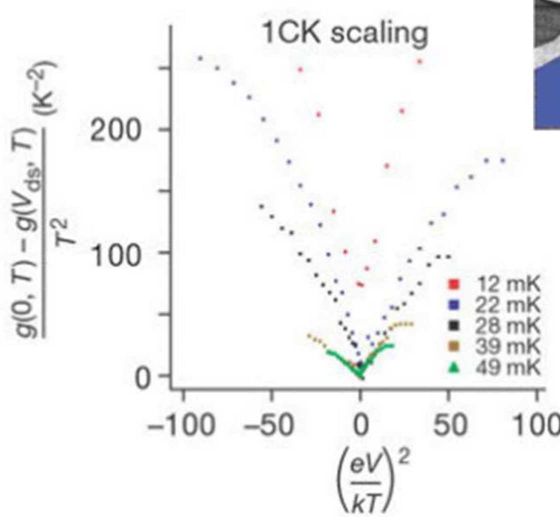
overscreening

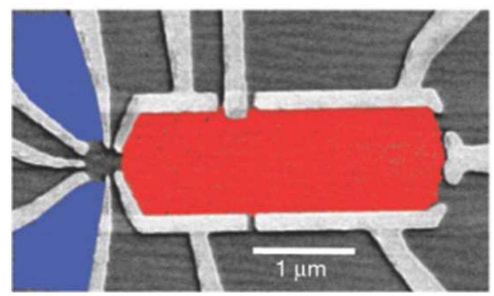
Fermi-liquid behavior

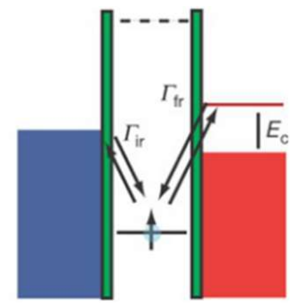
Non-Fermi-liquid behavior

$$G = G_0\left[1 - (\pi T/T_K)^2\right], \quad T \ll T_K$$





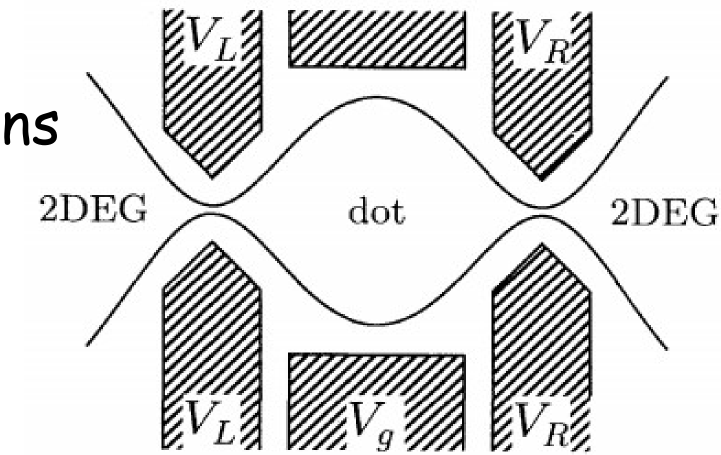


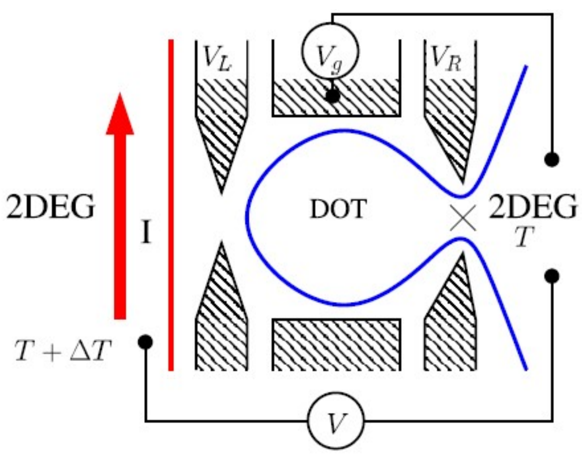


R. M. Potok et al., Nature 446 (2007)

Charge Kondo effect in Flensberg-Matveev-Furusaki model

- QD with CB effect: quantum impurity
- Electrons in the dot: up iso-spin; electrons out of the dot: down iso-spin
- Spin projections of electron: orbital channels





QD-QPC is a quasi-spin-1/2 orbital "impurity" +two spin up and down channels: 2-channel Kondo.

Spinless case: Fermi-liquid behavior **$S \sim T/E_C$**

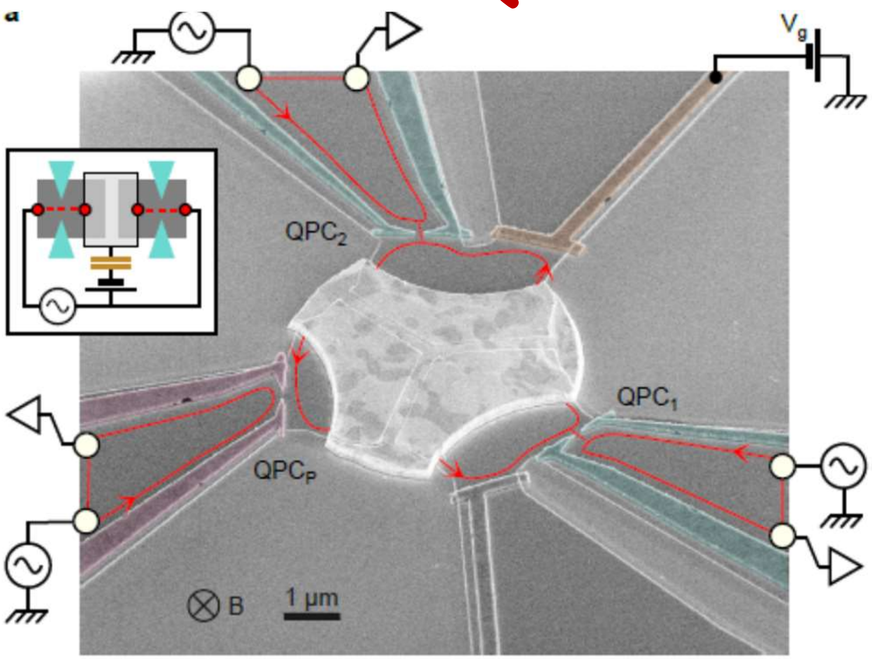
Spinful case: Non-Fermi liquid behavior

Matveev&Andreev, PRL 2001, PRB 2002

$S \sim (T/E_C)^{1/2} \ln(E_C/T)$

TKTN, MNK, VEK, PRB (2010): **any finite magnetic field induces NFL-FL crossover at temperature** $T_{min} \sim r_0^2 E_C (B/B_c)^2$

Charge Kondo implementation by Integer Quantum Hall edge currents



Two channel charge Kondo: Z. Iftikhar et al, Nature 526 (2015)

Three channel charge Kondo: Z. Iftikhar et al, Science 360 (2018).

Multi-channel charge Kondo: possible!

Conventional Kondo with S=1/2	Charge Kondo
Spin ½ of es	Real es' location
Interaction between spin of impurity and spin of conduction es	Backscattering at QPCs
Number of orbital channels	Number of QPCs
	Charge degree of freedom is blocked, charge quantization

Luttinger liquid

Luttinger liquid: systems of 1D interacting fermions

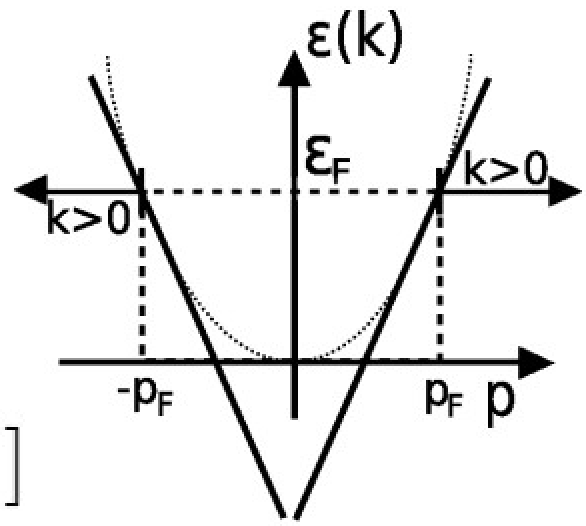
Fermi liquid	Luttinger liquid
Landau	Tomonaga, Luttinger
Elementary excitations are quasiparticles (fermions)	Elementary excitations are collective excitations (bosons)
Weak correlations	Strong correlations
1-1 correspondence between quasiparticles and excitations of FEG	

Bosonization technique:

Non-interacting spinless model:

$$H = \sum_{k;r=R,L} v_F(\epsilon_r k - k_F) c^\dagger_{r,k} c_{r,k}$$

$$\Longrightarrow H = \frac{1}{2\pi} \int dx v_F \left[(\pi \Pi(x))^2 + (\nabla \phi(x))^2 \right]$$

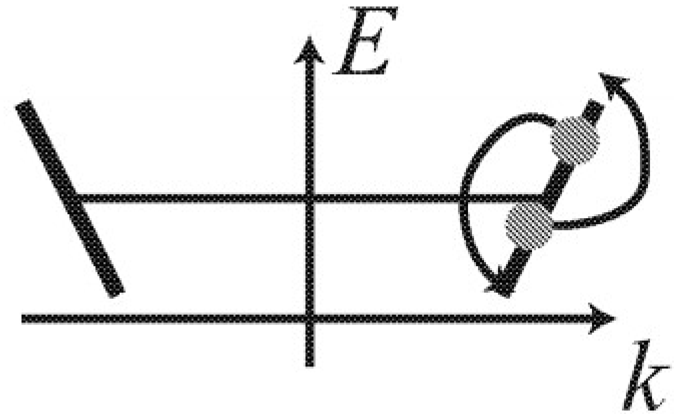


Luttinger liquid

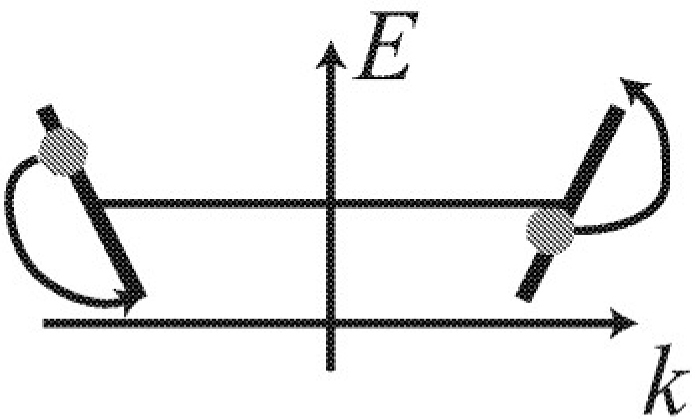
Interacting spinless model:

$$H = \sum_{k;r=R,L} v_F(\epsilon_r k - k_F) c^\dagger_{r,k} c_{r,k}$$

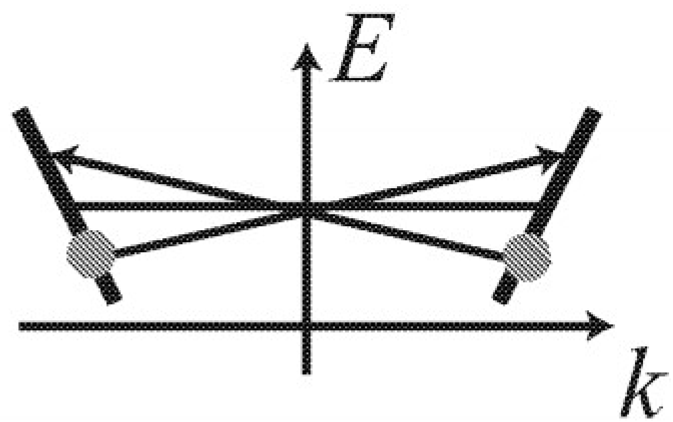
$$H_{\rm int} = \frac{1}{2\Omega} \sum_{k,k',q} V(q) c^\dagger_{k+q} c^\dagger_{k'-q} c_{k'} c_k$$



$g_4 = V(q{\sim}0)$



$g_2 = V(q{\sim}0)$



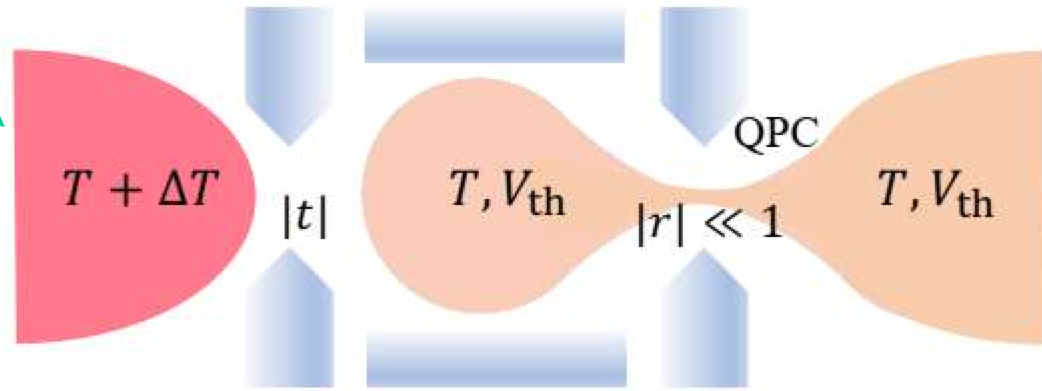
$g_1 = V(q{\sim}2k_F)$

T. Giamarchi,
Quantum Physics in One Dimension (Oxford University Press, Oxford, UK, 2003).

Our theoretical model 1

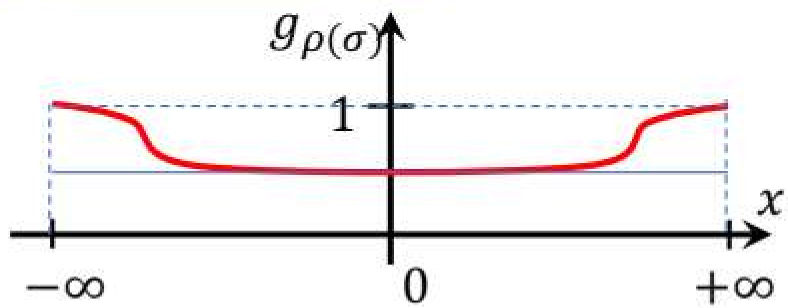
Noninteracting 2DEG

a)



Interacting 2DEG

b)



$$\phi_\rho = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$$

$$\phi_\sigma = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$$

Charge-spin separation

$$v_{F\alpha} = \sqrt{(2\pi v_F + g_{4\alpha})^2 - g_{12\alpha}^2}/2\pi,$$

$$g_\alpha = \sqrt{(2\pi v_F + g_{4\alpha} + g_{12\alpha})/(2\pi v_F + g_{4\alpha} - g_{12\alpha})}$$

$$g_{4,\alpha} = g_{4,\parallel} \pm g_{4,\perp},\ g_{12\alpha} = g_{1,\parallel} - g_{2,\parallel} \mp g_{2,\perp}$$

The action components

$$\mathcal{S}_0 = \mathcal{S}_0^{(\rho)} + \mathcal{S}_0^{(\sigma)},$$

$$\mathcal{S}_0^{(\rho)} = \frac{v_{F\rho}}{2\pi g_\rho} \int dx \int_0^\beta dt \left[\frac{(\partial_t \phi_\rho)^2}{v_{F\rho}^2} + (\partial_x \phi_\rho)^2 \right],$$

$$\mathcal{S}_0^{(\sigma)} = \int dx \int_0^\beta dt \left\{ \frac{v_{F\sigma}}{2\pi g_\sigma} \left[\frac{(\partial_t \phi_\sigma)^2}{v_{F\sigma}^2} + (\partial_x \phi_\sigma)^2 \right] + \frac{2 g_{1\perp} D^2}{(2\pi v_F)^2} \cos(\sqrt{8}\phi_\sigma(x,t)) \right\}.$$

$$\mathcal{S}_C = E_C \int_0^\beta dt [n_\tau(t) + \frac{\sqrt{2}}{\pi}\phi_\rho(0,t) - N]^2.$$

$$\mathcal{S}' = -\frac{2D}{\pi}|r| \int_0^\beta dt \cos[\sqrt{2}\phi_\rho(0,t)] \cos[\sqrt{2}\phi_\sigma(0,t)]$$

Perturbative Results: Massless spin case:

$$S \sim -\frac{|r^*|^2}{e}\sin(2\pi N)\log\left(\frac{E_C}{T}\right)\left(\frac{T}{g_\rho E_C}\right)^{g_\sigma - 1} \qquad g_\sigma \geq 1$$

$$|r^*| = |r|(g_\rho E_C/D)^{(g_\rho+g_\sigma)/2-1}$$

Perturbative Results: Massive spin case:

Mass of the spin field: $M = D(v_{F\sigma}/v_F)(|g_{1\perp}|/\pi v_{F\sigma})^{1/(2-2g_\sigma)}$

$$S \sim -\frac{1}{e}|r^*|\sin(2\pi N)\frac{T}{g_\rho E_C}\left(\frac{M}{2\sqrt{2}g_\rho E_C}\right)^{\frac{g_\sigma}{2}} \qquad g_\sigma < 1$$

The temperature scaling $T^{g_\sigma - 1}\log T$ is NFL behavior.

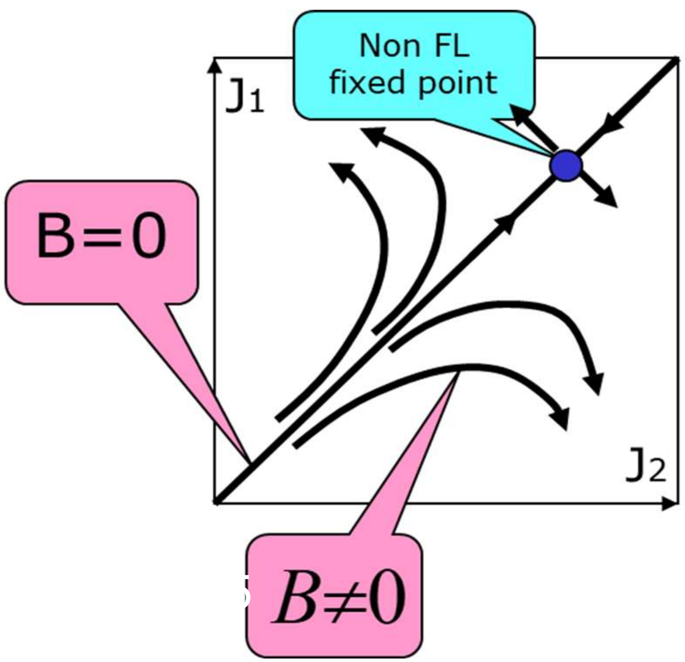
The temperature scaling $S \propto T$ is FL behavior.

The validity of the perturbation theory: $|r^*|^2 g_\rho E_C \ll T \ll g_\rho E_C$

What physical quantities do control the crossover from 2CK to 1CK?

In our PRB **82**, 113306 (2010): B field breaks the symmetry between ↑ and ↓ spins ⟶ 2CK - 1CK crossover with decreasing the temperature.

In this work: PRB **105**, L121405 (2022): $g_{1\perp}$ process in the LL induces the instant asymmetry of and ↑ spins ↓ ⟶ 2CK – 1CK crossover.



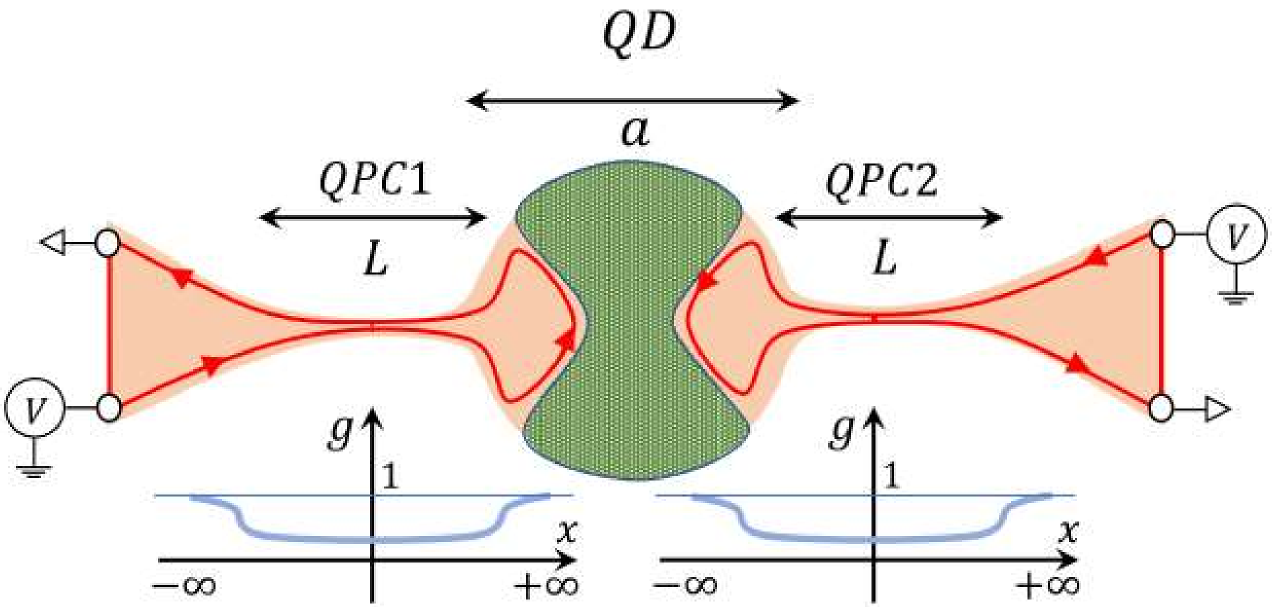
Alternative point of view: in the charge Kondo effect:

Charge mode is always blockaded (or gapped) locally.

If spin mode is unblockaded ⟶ gapless spin mode: NFL - 2CK.

If spin mode is additionally gapped (by either Zeeman effect or many-body effect in the LL) ⟶ the local NFL state is destroyed.

Our theoretical model 2



$L \ll a$

$\hbar v_F/L \gg \omega \gg \hbar v_F/a$

$g(x) = g_L \equiv 1,\ u(x) = v_F$ for $|x| > L/2$

$g(x) = g_W = g,\ u(x) = v$ for $|x| < L/2$

$$S = S_0 + S_C + S'$$

$$S_0 = \frac{1}{2\pi}\sum_{i=1,2}\int dx \int_0^\beta dt \frac{1}{g(x)}\left[\frac{[\partial_t \phi_i(x,t)]^2}{u(x)} + u(x)[\partial_x \phi_i(x,t)]^2\right]$$

$$S_C = \int_0^\beta dt \frac{E_C}{\pi^2}\left[\phi_2(0,t) - \phi_1(0,t) - \pi N\right]^2,$$

$$S' = \frac{D}{\pi}\int_0^\beta dt \left\{|r_1|\cos[2\phi_1(0,t)] + |r_2|\cos[2\phi_2(0,t)]\right\}$$

Perturbative result for electric conductance

$$G = G_0 \left[1 - |r_+|^2 C_1(g) \left(\frac{gE_C}{D}\right)^{2g-2} \left(\frac{T}{gE_C}\right)^{g-2} - |r_-|^2 C_2(g) \left(\frac{gE_C}{D}\right)^{2g-2} \left(\frac{T}{gE_C}\right)^{g}\right]$$

$$|r_\pm|^2 = [|r_1|^2 + |r_2|^2 \pm 2|r_1||r_2|\cos(2\pi N)], \quad G_0 = e^2/4\pi,$$

$$C_1(g) = (2\gamma)^g \pi^{-3/2} \Gamma[g/2] / 4\Gamma[1/2 + g/2]$$

$$C_2(g) = g(2\gamma)^g \pi^{5/2} \Gamma[1 + g/2] / 16\Gamma[3/2 + g/2]$$

$\Gamma(x)$ is Gamma function

$\gamma = e^C, \ C \approx 0.5772$

At 2CK fixed point: $|r_1| = |r_2| = |r|$. we obtain:

$$G = G_0[1 - 4C_2(g)|r^*|^2 (T/gE_C)^g],$$

$$|r^*| = |r|(gE_C/D)^{g-1}$$

accounts for both electron-electron interactions and 2CCK correlations.

At low temperatures: $G = G_0[1 - (T/T^*)^g]$

Conditions for perturbative solution: LL parameter: $0.6 \leqslant g < 1$

and temperature regime: $|r^*|^{2/(2-g)} gE_C \ll T \ll gE_C$

TN, AP, QN, MK, PRB 107, L201402 (2023)

Conclusions

- Thermoelectric transport in a Luttinger liquid based two-channel charge Kondo circuit:
  - *Massless spin field: 2CK: NFL:* $S \propto T^{g_\sigma - 1} \log T$
  - *Massive spin field: 1CK: FL:* $S \propto T$
- *The relevance of* $g_{1\perp}$ *process induces the universal crossover from NFL-2CK to FL-1CK.*

- Quantum transport through a 2CCK circuit: $L \ll a$ in the temperature range $|r^*|^{2/(2-g)} gE_C \ll T \ll gE_C$.
- Temperature scaling of the linear electric conductance:

$G_0 - G \propto (T/T^*)^g$ ⟶ NFL picture ⟶ determine the e-e interactions in the 2CCK - IQH setups.

Thank you for your attention!