Center for Theoretical Physics of Complex Systems

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Photoinduced supercurrent Hall effect in 2D superconductors

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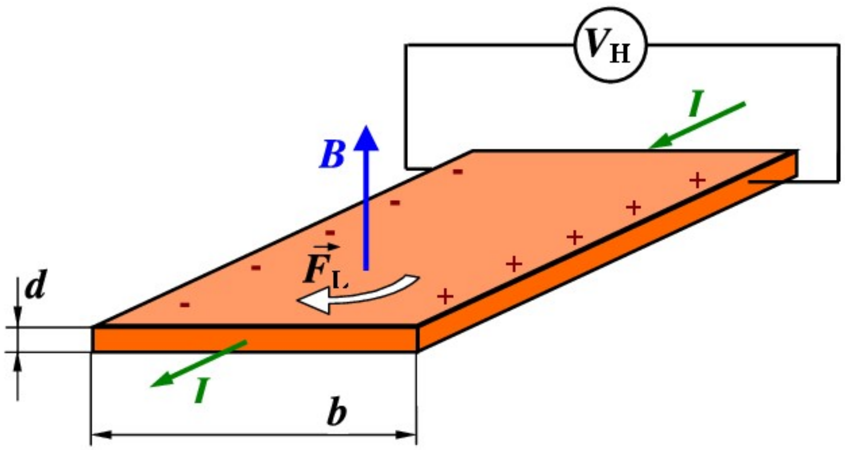
Asian Network School and Workshop on
Complex Condensed Matter Systems
Hanoi, 6-10 November 2023

Outline

- Main goal of project
- Brief historical introduction
- Model, Methods, Problems
- Results
- Summary

Main goal

Ordinary Hall effect



Lorentz force:

$$\mathbf{F}_L = q[\mathbf{v} \times \mathbf{B}]$$

$$\mathbf{F} = q\left(\mathbf{E} + [\mathbf{v} \times \mathbf{B}]\right) = 0$$

Hall voltage:

$$V_H = \frac{IB}{ned}$$

Is it possible to create Hall-like phenomena in 2D superconductors (SC film)?

Anomalous Hall effect

Spin-orbit interaction, Skew scattering, Berry phase, etc.

Photovoltaic Hall effect

PVE – effect of appearance of the dc current in homogeneous medium under uniform illumination.

$$j_\alpha = \sigma_{\alpha\beta} E_\beta + \underline{\sigma_{\alpha\beta\gamma}\left(E_\beta E_\gamma^* + h.c.\right)} + \chi_{\alpha\beta\gamma}\left(E_\beta E_\gamma e^{-i2\omega t} + h.c.\right) + \ldots$$

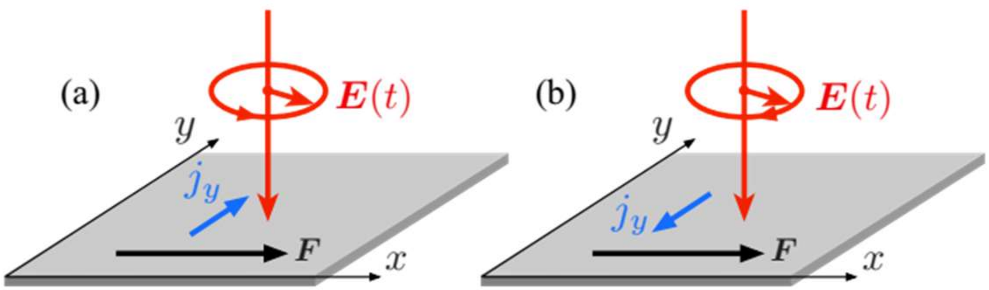
Boltzmann kinetic equation

$$\boxed{\frac{\partial f}{\partial t} + e\mathbf{E}(t)\frac{\partial f}{\partial \mathbf{p}} = -\frac{f - f_0}{\tau}}$$

$$\mathbf{E}(t) = \mathbf{E}e^{-i\omega t} + c.c.$$

$$f(\mathbf{p}, t) = f_0 + \tilde{f}_1(\mathbf{p}) + \left[f_1(\mathbf{p})e^{-i\omega t} + c.c.\right] + \tilde{f}_2(\mathbf{p}) + \left[f_2(\mathbf{p})e^{-2i\omega t} + c.c.\right] + \ldots$$

$$\mathbf{j}_{PVE} \sim \int d\mathbf{p}\, \mathbf{v} \tilde{f}_2(\mathbf{p})$$



Durnev, Phys. Rev. B **104**, 085306 (2021)

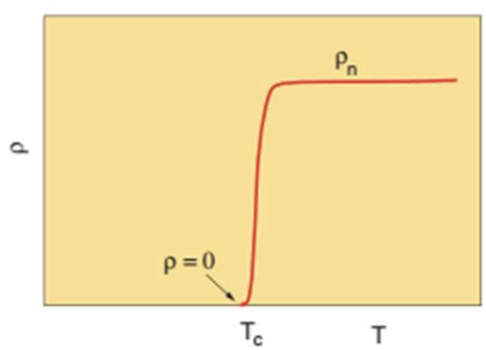
Isotropic 2DEG + EM field

$$\mathbf{j} = \gamma_1 |\mathbf{E}|^2 \mathbf{F} + \gamma_2 [\mathbf{E}^*(\mathbf{E} \cdot \mathbf{F}) + c.c.] + i\gamma_3 [\mathbf{F} \times [\mathbf{E}^* \times \mathbf{E}]]$$

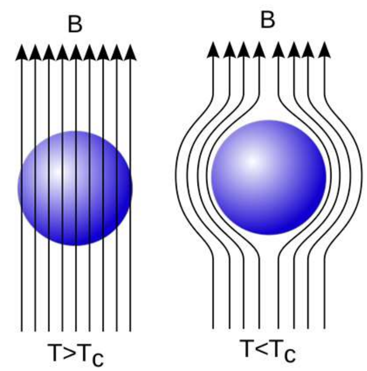
$$\delta f(\mathbf{v}, \mathbf{E}, \mathbf{F}) \propto (\mathbf{v} \cdot \mathbf{E})(\mathbf{v} \cdot \mathbf{E}^*)(\mathbf{v} \cdot \mathbf{F})$$

Superconductors

1) Zero-resistance



2) Meissner effect



$$H = E_0 + \sum_{\mathbf{p}} \left(a^\dagger_{p,\uparrow} \,,\; a_{-p,\downarrow} \right) \begin{bmatrix} \xi_p & \Delta \\ \Delta^* & -\xi_p \end{bmatrix} \begin{pmatrix} a_{p,\uparrow} \\ a^\dagger_{-p,\downarrow} \end{pmatrix}$$

Bogoliubov transformation:

$$c_{p\uparrow} = u_p a_{p\uparrow} + v_p a^\dagger_{-p\downarrow}$$

$$c_{p\downarrow} = -v_p a^\dagger_{p\downarrow} + u_p a_{-p\uparrow}$$

$$u_p^2, v_p^2 = \frac{1}{2}\left(1 \pm \frac{\xi_p}{\epsilon_p}\right)$$

$$u_p^2 + v_p^2 = 1$$

$$H = E_0 + \sum_p \epsilon_p c^\dagger_{p\sigma} c_{p\sigma}$$

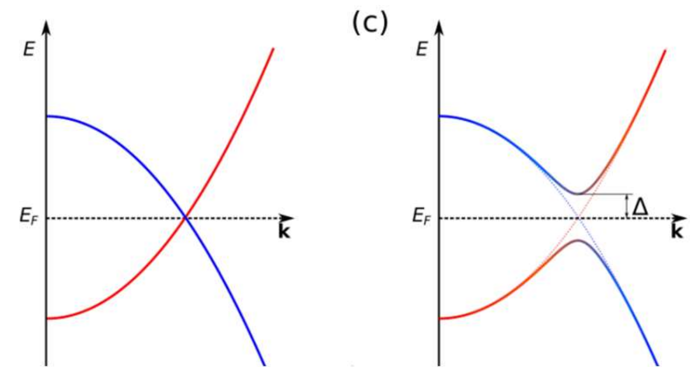
$$\epsilon_p = \sqrt{\xi_p^2 + |\Delta|^2}$$

Interacting Hamiltonian:

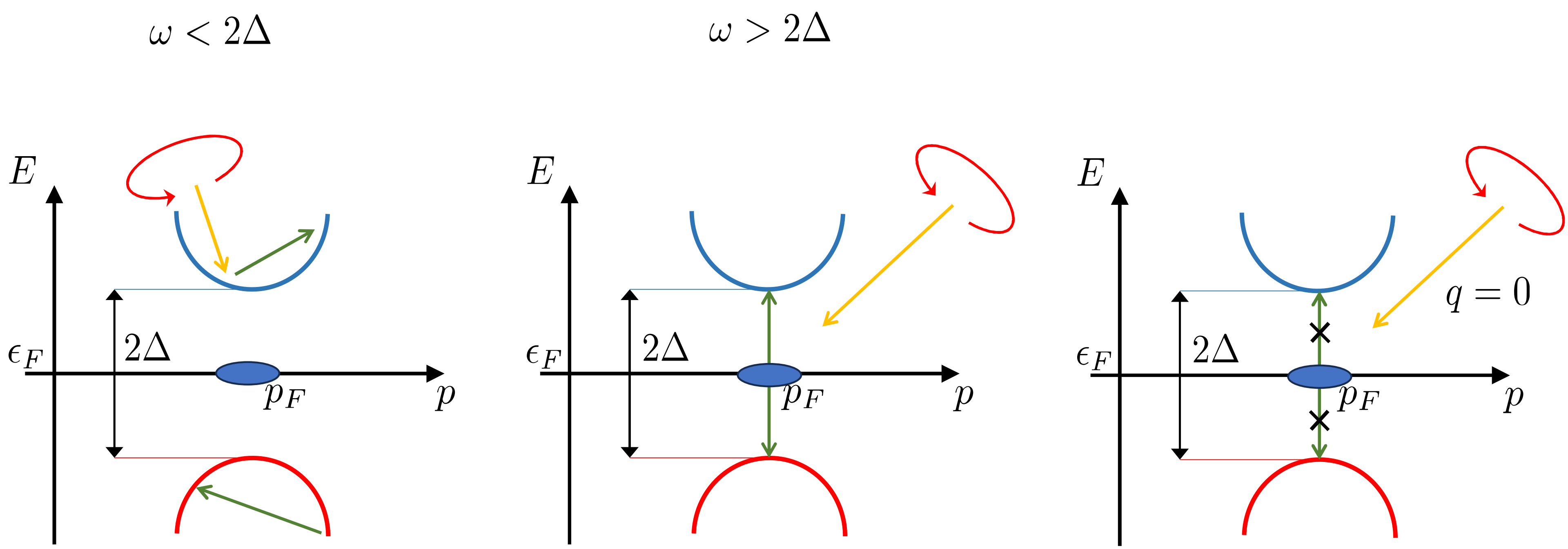
$$H = \sum_p \xi_p a^\dagger_{p\sigma} a_{p\sigma} - \frac{\lambda}{2} \sum_{pq} a^\dagger_{p\sigma} a^\dagger_{-p\tau} a_{q\tau} a_{-q\sigma}$$

$$\xi_p = \frac{p^2}{2m} - \epsilon_F$$

Mean field: $\Delta = \lambda \sum_q \langle a_{q\downarrow} a_{-q\uparrow} \rangle$



Light absorption in superconductors

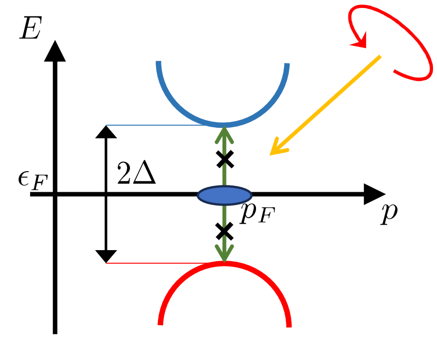


Mattis, Bardeen, *Theory of the Anomalous Skin Effect in Normal and Superconducting Metals*, Phys. Rev. **111**, 412 (1958)

Mattis-Bardeen theory

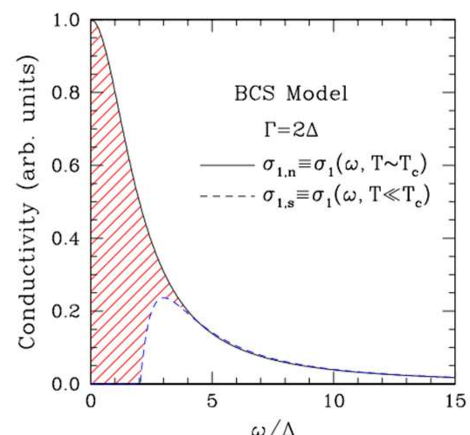
$$Q(\omega, \mathbf{q}) = Q_n(\omega, \mathbf{q}) + Q_a(\omega, \mathbf{q})$$

Mattis, Bardeen, *Theory of the Anomalous Skin Effect in Normal and Superconducting Metals*, Phys. Rev. **111**, 412 (1958)



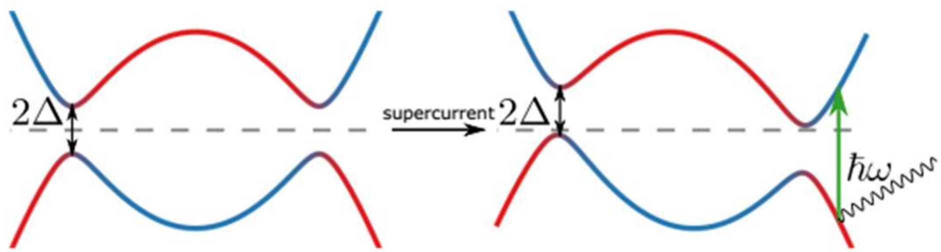
$$Q_n(\omega, \mathbf{q}) \propto \int_{-\infty}^{\infty} d\xi_{\mathbf{p}} \left(\frac{\epsilon_{\mathbf{p}}\epsilon_{\mathbf{p+q}} + \xi_{\mathbf{p}}\xi_{\mathbf{p+q}} + \Delta^2}{\epsilon_{\mathbf{p}}\epsilon_{\mathbf{p+q}}} \right) [f(\epsilon_{\mathbf{p}}) - f(\epsilon_{\mathbf{p+q}})] \times \left(\frac{1}{\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p+q}} + \omega + i\delta} + \frac{1}{\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p+q}} - \omega + i\delta} \right)$$

$$Q_a(\omega, \mathbf{q}) \propto \int_{-\infty}^{\infty} d\xi_{\mathbf{p}} \left(\frac{\epsilon_{\mathbf{p}}\epsilon_{\mathbf{p+q}} + \xi_{\mathbf{p}}\xi_{\mathbf{p+q}} - \Delta^2}{\epsilon_{\mathbf{p}}\epsilon_{\mathbf{p+q}}} \right) [1 - f(\epsilon_{\mathbf{p}}) - f(\epsilon_{\mathbf{p+q}})] \times \left(\frac{1}{\epsilon_{\mathbf{p}} + \epsilon_{\mathbf{p+q}} + \omega + i\delta} + \frac{1}{\epsilon_{\mathbf{p}} + \epsilon_{\mathbf{p+q}} - \omega + i\delta} \right)$$



$$\mathrm{Re}\left[\sigma(\omega)\right] \propto \frac{1}{\omega} \left[(\omega + 2\Delta) E\left(\frac{\omega - 2\Delta}{\omega + 2\Delta}\right) - 4\Delta K\left(\frac{\omega - 2\Delta}{\omega + 2\Delta}\right) \right] \Theta(\omega - 2\Delta)$$

Model



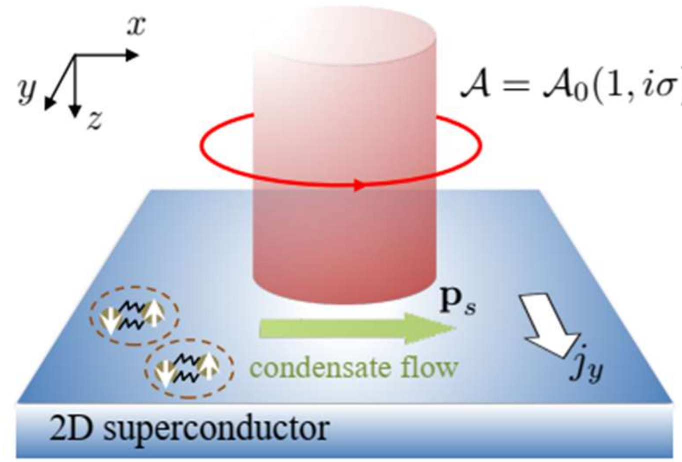
Breaking a spatial symmetry:

$$\epsilon_p = \mathbf{v}\mathbf{p}_s + \sqrt{\xi_p^2 + |\Delta|^2}$$

Hamiltonian of single band isotropic SC in Nambu space:

$$\hat{H} = \begin{pmatrix} \xi(\mathbf{p} - \mathbf{p}_s - e\mathcal{A}(t)) & \Delta \\ \Delta & -\xi(\mathbf{p} + \mathbf{p}_s + e\mathcal{A}(t)) \end{pmatrix}$$

$$\xi(\mathbf{p}) \equiv \xi_p = \frac{\mathbf{p}^2}{2m} - \epsilon_F$$



Circularly polarized EM wave:

$$\mathcal{A}(t) = \mathcal{A}e^{-i\omega t} + \mathcal{A}^* e^{i\omega t} \qquad \mathcal{A} = (1, i\sigma)\mathcal{A}_0$$

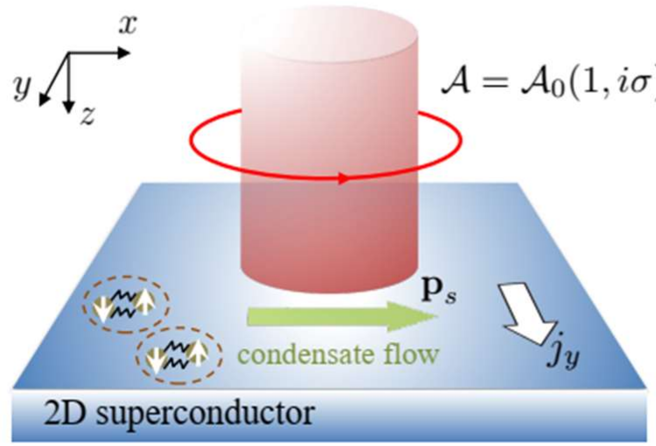
Linearly polarized EM wave: $\mathcal{A} = (\mathcal{A}_x, \mathcal{A}_y)$

Methods

Current operator:

$$\hat{\mathbf{j}} = -\frac{\delta \hat{H}}{\delta \mathcal{A}} = e\mathbf{v} - e\mathbf{v}_s \hat{\tau}_z - \frac{e^2}{m}\mathcal{A}(t)\hat{\tau}_z$$

$$\mathbf{j} = -i \sum_{\mathbf{p}} \mathrm{Tr}\left[\hat{\mathbf{j}}\, \mathcal{G}^{<}(t,t)\right]$$



Green’s function:

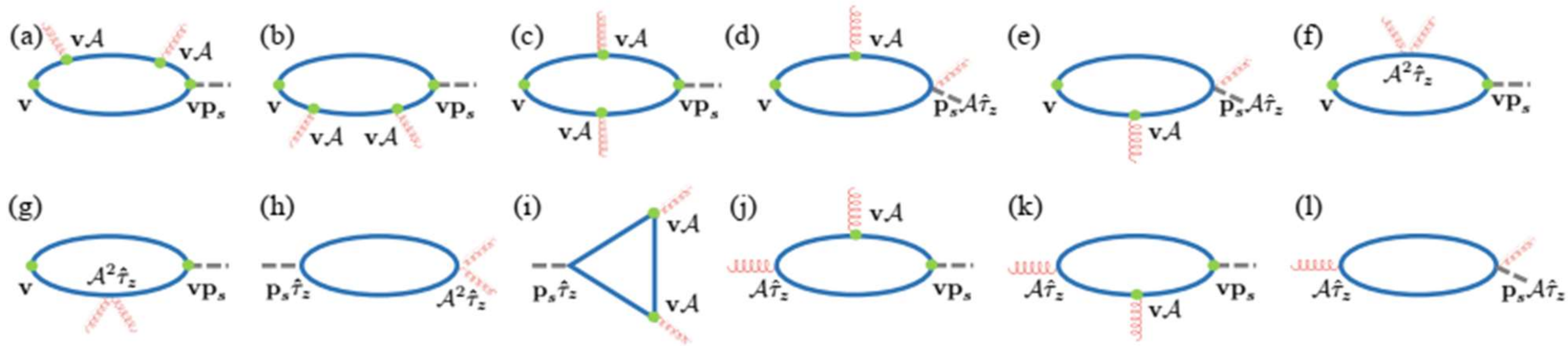
$$\left(i\frac{\partial}{\partial t} - \hat{H}\right)\mathcal{G}(t,t') = \delta(t-t') \qquad \hat{H} = \hat{H}_0 + \hat{V}(\mathbf{p}_s, \mathcal{A}(t)) = \begin{pmatrix} \xi(\mathbf{p}) & \Delta \\ \Delta & -\xi(\mathbf{p}) \end{pmatrix} + \hat{V}(\mathbf{p}_s, \mathcal{A}(t))$$

$$g_0^R = \frac{1}{\epsilon - \xi_p\hat{\tau}_z - \Delta\hat{\tau}_x} = \frac{\hat{A}_0}{\epsilon - \epsilon_p + i0^+} + \frac{\hat{B}_0}{\epsilon + \epsilon_p + i0^+} \qquad \hat{A}_0 = \begin{pmatrix} u^2 & uv \\ uv & v^2 \end{pmatrix} \qquad \hat{B}_0 = \begin{pmatrix} v^2 & -uv \\ -uv & u^2 \end{pmatrix}$$

$$\hat{A}_0^2 = \hat{A}_0 \qquad \hat{B}_0^2 = \hat{B}_0 \qquad \mathrm{Tr}[\hat{A}_0\hat{B}_0] = 0$$

Diagrams

Non-linear stationary photo-response (linear in $\mathbf{p}_s$ and quadratic in $\mathcal{A}_0$):



Optical absorption in SC is forbidden due to *Galilean invariance*!

Violation of Galilean invariance:

- Non-parabolicity of electronic band
- Multi-band superconductors
- Disorder

Disorder and relaxation

$$g_0^R = \frac{1}{\eta\epsilon - \xi_p\hat{\tau}_z - \eta\Delta\hat{\tau}_x}$$

$$\eta = 1 + \frac{\Theta[\Delta - |\epsilon|]}{2\tau_i\sqrt{\Delta^2-\epsilon^2}} + i\frac{\mathrm{sign}(\epsilon)\Theta[|\epsilon|-\Delta]}{2\tau_i\sqrt{\epsilon^2-\Delta^2}} \qquad \tau_i^{-1} \propto u_0^2$$

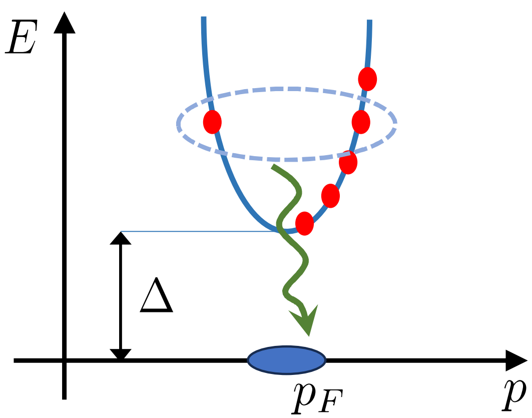
$$\frac{1}{\tau_p} = \frac{1}{\tau_i}\frac{|\xi_p|}{\epsilon_p} + \frac{1}{\tau_R}$$

$$g_0^R = \frac{\hat{A}_p}{\epsilon - \epsilon_p + \frac{i}{2\tau_p}} + \frac{\hat{B}_p}{\epsilon + \epsilon_p + \frac{i}{2\tau_p}}$$

$$\hat{A}_p = \hat{A}_0 + i\frac{1}{2\tau_i|\xi_p|}\begin{pmatrix} 1/2 & uv \\ uv & 1/2 \end{pmatrix}$$

$$\hat{B}_p = \hat{B}_0 - i\frac{1}{2\tau_i|\xi_p|}\begin{pmatrix} 1/2 & -uv \\ -uv & 1/2 \end{pmatrix}$$

Recombination



Clean superconductors
/Reizer, PRB (1998)

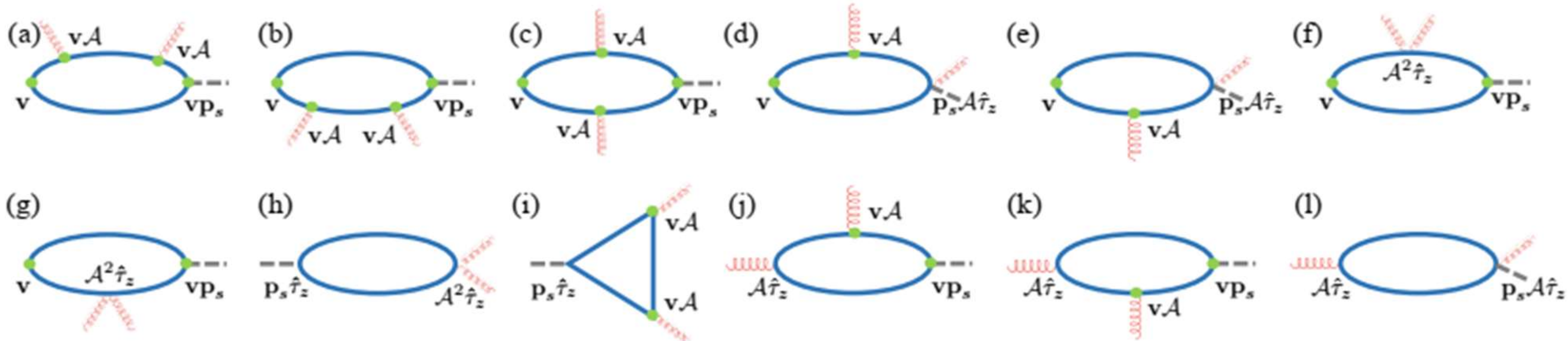
$$\frac{1}{\tau_R} \propto \frac{T}{\epsilon_F\tau_i}e^{-2\Delta/T}$$

Dirty superconductors
/Reizer, PRB (2000)

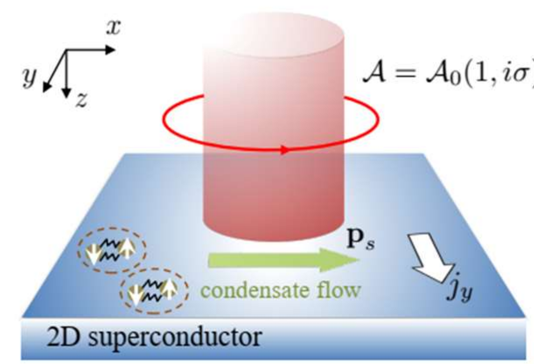
$$\frac{1}{\tau_R} \propto \frac{T\Delta}{\epsilon_F}e^{-2\Delta/T}$$

- High frequency, $\omega > 2\Delta \quad T \approx 0$
- Weak supercurrent, $v_F p_s/\Delta \ll 1$
- Weak disorder, $\Delta\tau_i \gg 1$
- Recombination across SC gap

Diagrams

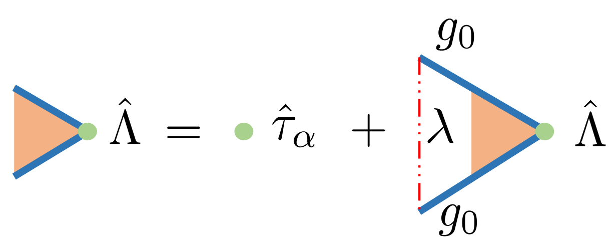


$$\mathbf{j} = a_\omega |\mathbf{A}_0|^2 \mathbf{p}_s + b_\omega [\mathbf{A}_0^*(\mathbf{A}_0 \cdot \mathbf{p}_s) + c.c.] + ic_\omega [\mathbf{p}_s \times [\mathbf{A}_0^* \times \mathbf{A}_0]]$$

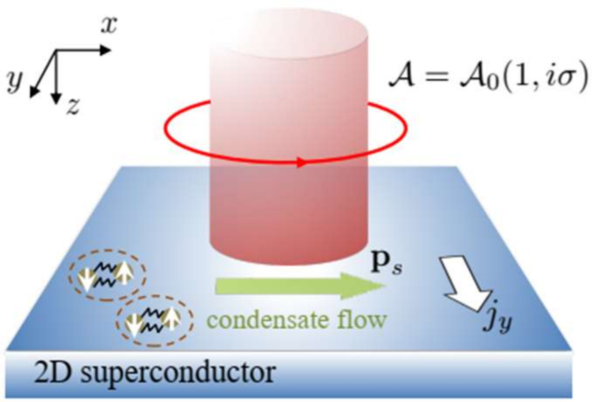


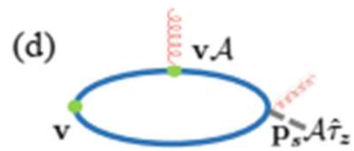
Gauge invariance problem in SC

Restoring the gauge invariance:

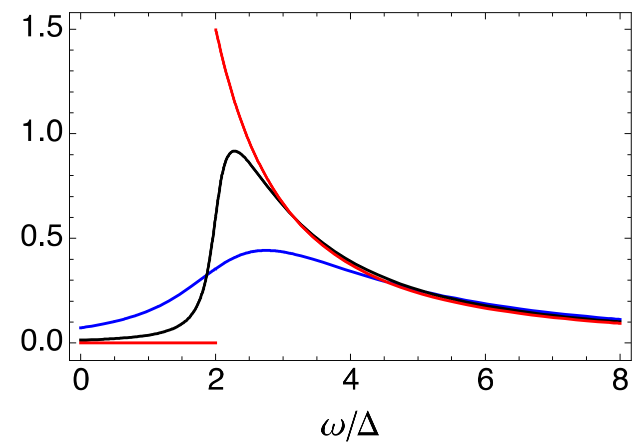


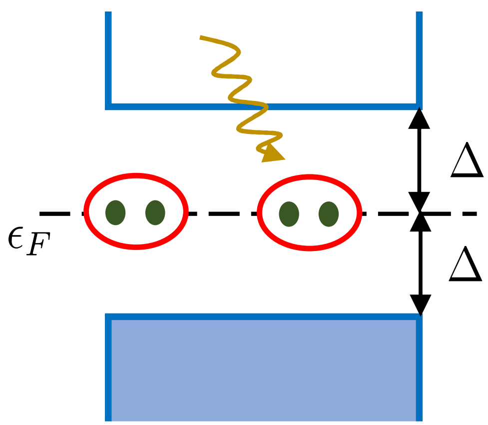
Photoinduced supercurrent Hall effect





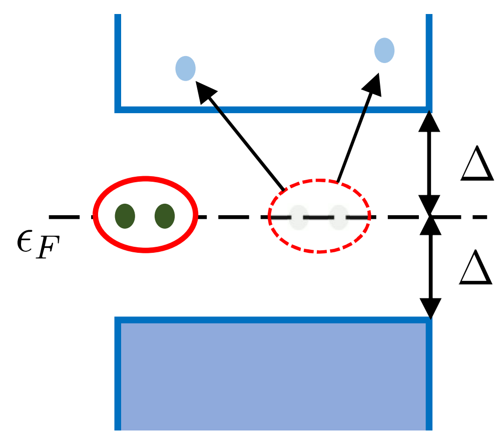
$$j_y = \frac{e^3}{2m\hbar^2} p_s \sigma \mathcal{A}_0^2 \frac{\tau_R}{\tau_i} \frac{\Delta^2}{\hbar^2\omega^2} \Theta(\hbar\omega - 2\Delta)$$





$$T \approx 0$$

$$\hbar\omega > 2\Delta$$



Nonequilibrium two-fluid model

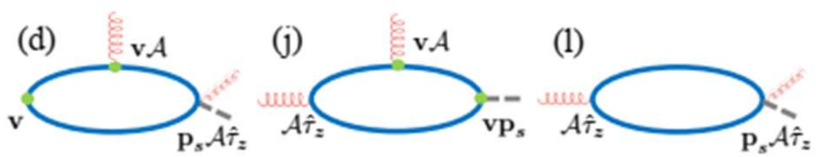
$$\mathbf{E} = 0$$

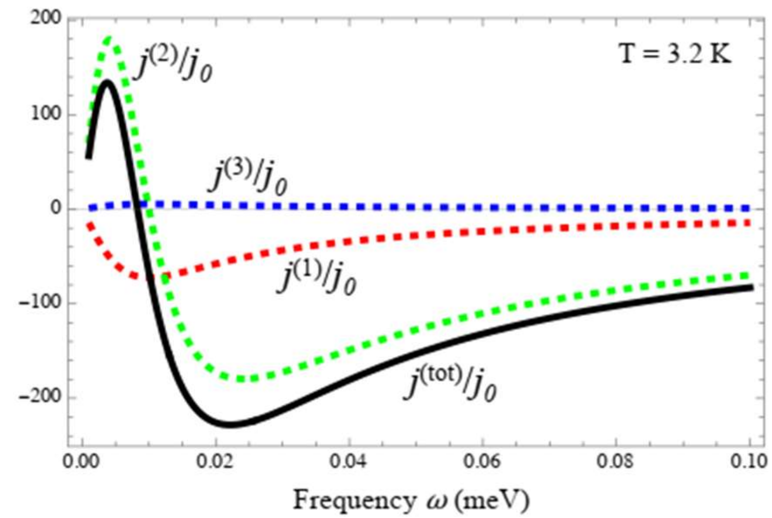
$$j_y + j_{sc} = 0$$

$$j_{sc} \propto \nabla \varphi_H$$

Photoinduced supercurrent Hall effect

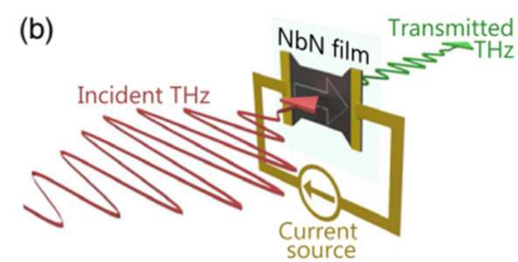
- Low frequency, $\omega < 2\Delta$
- "High" temperature, $T \sim T_c$





Importance:

- Fundamental reason (supercurrent coupling with light)
- Relaxation time spectroscopy
- Spectroscopy of various SC pairing mechanism



Nakamura et al., PRL (2019), PRL (2020)

Summary

• A theory of a nonlinear photoresponse in a single-band 2D isotropic SC with a built-in supercurrent, exposed to an external circularly-polarized EM field is developed. The theory accounts for the presence of impurities in the sample, which breaks the Galilean invariance for the transverse transport to take place.

• We predicted a photoinduced second-order transport phenomenon -- the emergence of a transverse (Hall-like) photoinduced supercurrent, and demonstrated, that its magnitude is determined by the quasiparticle recombination time.

• This photoinduced supercurrent Hall effect opens a way to manipulate the direction of SC condensate flow via optical tools without external magnetic fields.