Competing Laughlin state and Wigner crystal in Bernal-stacked bilayer graphene

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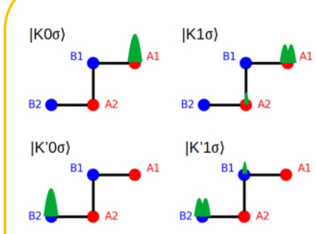
(Phys. Rev. B **107**, 125129 (2023))

Quantum Hall effect in Bernal-stacked Bilayer graphene

- Landau level energy:
  $$E_N = sign(N)\hbar\omega_c\sqrt{|N|(|N|-1)}$$
- Landau Levels N = 0 and N = 1 are nearly degenerate ($E_0 = E_1 = 0$)
- 8-fold nearly degenerate Zero-energy Landau level
- 3 degrees of freedom: spin $\sigma = \uparrow/\downarrow$, valley $\xi = K/K'$, orbital (Landau Level) N=0, 1
- By *neglecting all mixing with higher Landau levels*, the situation is *reminiscent of extreme Landau level mixing*

➔ Favor the formation of ***Wigner crystal*** [1]

Single-particle wavefunction

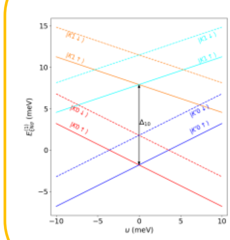


In the basis (A1,B2,A2,B1):

$$\Psi_0^K = (\phi_0, 0, 0, 0)$$
$$\Psi_1^K = (\sqrt{1-\gamma}\phi_1, 0, \sqrt{\gamma}\phi_0, 0)$$
$$\Psi_0^{K'} = (0, \phi_0, 0, 0)$$
$$\Psi_1^{K'} = (0, \sqrt{1-\gamma}\phi_1, 0, \sqrt{\gamma}\phi_0)$$

Parameter γ

Single-particle energy levels



- Single electron energy:
  $$E_{\xi N\sigma}^{(1)} = -\xi\alpha_N\frac{u}{2} + N\Delta_{10}^{(0)} - E_Z\sigma$$
  u = energy bias between the two layers (interlayer bias)
- The levels $|K0\uparrow\rangle$ and $|K1\uparrow\rangle$ cross at large value of u
- In the vicinity of the crossing, we define the *orbital splitting energy*:
  $$\Delta_{10} = E_{K0\uparrow}^{(1)} - E_{K1\uparrow}^{(1)} = -\Delta_{10}^{(0)} + (\alpha_1 - \alpha_0)\frac{u}{2}$$

Parameter Δ_{10}

Hamiltonian

We consider the regime in the vinicity of the crossing between the levels $|K0\uparrow\rangle$ and $|K1\uparrow\rangle$:

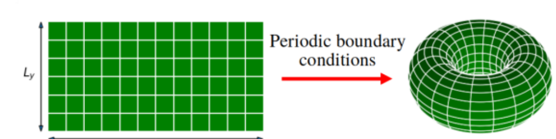
$$\mathcal{H} = \frac{1}{2}\sum_{N_1,N_2,N_3,N_4=0}^{1}\sum_{j_1,j_2,j_3,j_4=1}^{N_\phi}\mathcal{A}_{j_1j_2j_3j_4}^{N_1N_2N_3N_4}c_{N_1j_1}^{\dagger}c_{N_2j_2}^{\dagger}c_{N_3j_3}c_{N_4j_4} + \Delta_{10}\hat{N}_0$$

$$\mathcal{A}_{j_1j_2j_3j_4}^{N_1N_2N_3N_4} = \frac{1}{L_xL_y}\sum_{(s,t)\neq(0,0)}\delta_{q_x,s\frac{2\pi}{L_x}}\delta_{q_y,t\frac{2\pi}{L_y}}\frac{2\pi e^2}{\varepsilon q}e^{-\frac{q^2l^2}{2}}e^{i2\pi(j_1-j_3)\frac{s}{N_\phi}}F_{N_1N_4}(\boldsymbol{q})F_{N_2N_3}(-\boldsymbol{q})\delta'_{t,j_1-j_4}\delta'_{j_1+j_2,j_3+j_4}$$

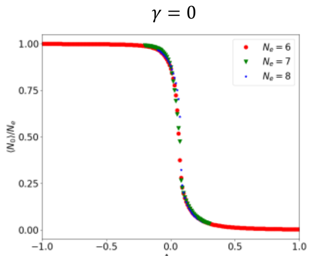
Form factors

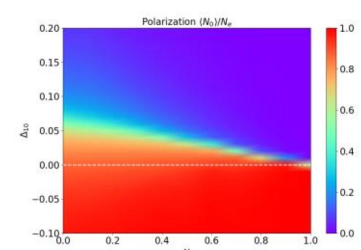
$$F_{00} = 1$$
$$F_{11} = \gamma + (1-\gamma)\left(1 - \frac{q^2l^2}{2}\right)$$
$$F_{01} = \sqrt{1-\gamma}\frac{(iq_x+q_y)l}{\sqrt{2}}$$
$$F_{10} = \sqrt{1-\gamma}\frac{(iq_x-q_y)l}{\sqrt{2}}$$

Method: exact diagonalization on the torus geometry



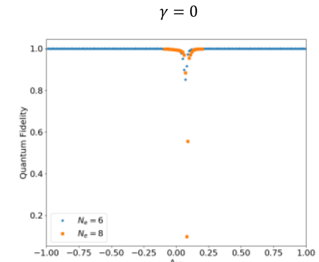
Orbital transition (filling factor 1/3)





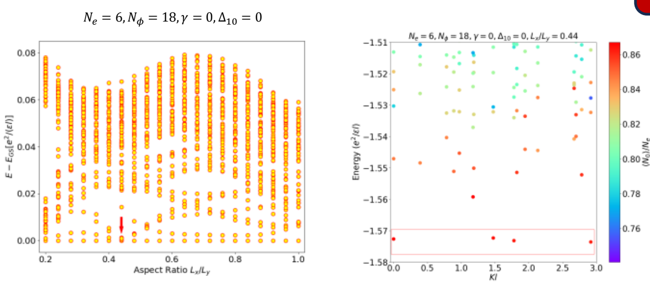
- Polarization in the orbital N = 0 of the ground state:
  $$\frac{\langle N_0\rangle}{N_e} = \frac{\langle\Psi_{GS}|\hat{N}_0|\Psi_{GS}\rangle}{N_e}$$
- An abrupt change in $\langle N_0\rangle/N_e$ when Δ_{10} increases while keeping γ unchanged
- Electrons transfer from orbital N = 0 to orbital N = 1

Quantum fidelity

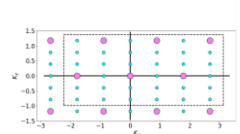


- Quantum fidelity:
  $$\mathcal{F} = |\langle\Psi_{GS}(\Delta_{10} + d\Delta_{10})|\Psi_{GS}(\Delta_{10})\rangle|^2$$
- A dip in $\mathcal{F}$ at the same value of Δ_{10} of the polarization transition

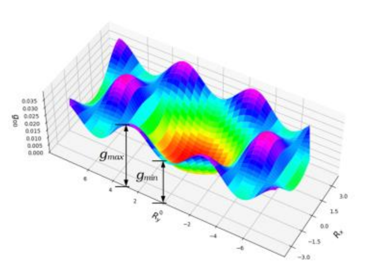
The Wigner crystal (filling factor 1/3)



Ground state degeneracy ➔Wigner crystal



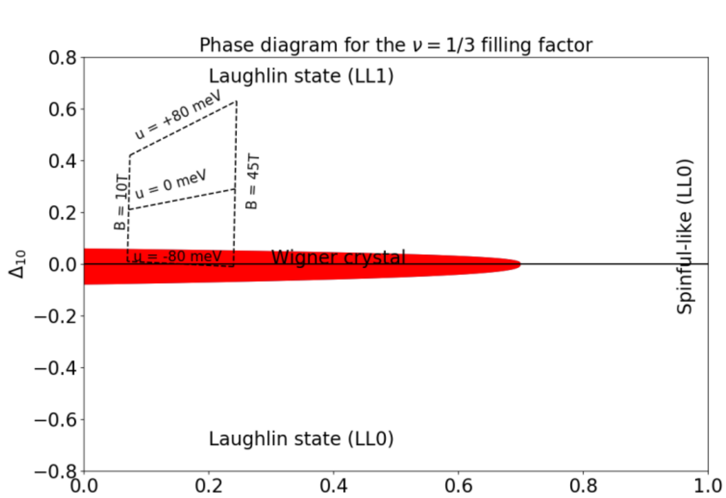
Many-body Brillouin zone



$N_e = 6, N_\phi = 18, \gamma = 0, \Delta_{10} = 0, \frac{L_x}{L_y} = 0.44$

- Pair correlation function between electrons in orbitals α and β separated by a distance ***R:***
  $$g_{\alpha\beta}(\boldsymbol{R}) = \frac{L_xL_y}{N_e^2}\sum_{i<j}\delta^2(\boldsymbol{r}_i - \boldsymbol{r}_j - \boldsymbol{R})(|\alpha\rangle\langle\alpha|)_i(|\beta\rangle\langle\beta|)_j$$
- The overdensities in g_{00} show the real-space distribution of Wigner crystal
- Situation reminiscent to extreme Landau level mixing
  ➔ favoring Wigner crystal formation
- In the Wigner crystal phase, electrons occupy the orbital N = 0

The phase diagram



Conclusion

- Competition between Wigner crystal and quantum Hall liquid in the vicinity of the crossing between the levels $|K0\uparrow\rangle$ and $|K1\uparrow\rangle$ of filling factor $\nu = 1/3$
- The phase transition can be controlled by tuning the electric energy bias u between the two layers (interlayer bias)
- Electrically-induced Wigner crystal

References

1. Phys. Rev. Lett. **121**, 116802 (2018)
2. Nature Communications, 8:948 (2017)
3. Phys. Rev. Lett. **124**, 097604 (2020)
4. Phys. Rev. B **107**, 125129 (2023)

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