Statistical properties of Evanescent Self propelled particles

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INTRODUCTION

Self-propelled particles (SPP) are active particles that take up energy from the environment and utilize some of the energy in directed motion allowing them to explore their surroundings [1]. One way of studying the statistical behavior of these particles is through the run-and-tumble movement. Statistical properties such as probability distributions with and without diffusion terms and first passage properties of these active particles were investigated in the previous studies [2-3]. However, the effects of death on the first passage properties were not considered. For example, *Escherichia coli* bacteria are self-propelling particles that move in a run-and-tumble way in order for them to navigate in search of food or nutrients and move away from chemical toxins [4]. These microorganisms may die in a process called programmed cell death (PCD), where the death of a cell happens as a result of events inside of it [5]. With this motivation, we define evanescent self-propelling particles as active particles that may die, decay, or disappear while moving towards or away from the boundaries. In this study, we investigate the effect of evanescence or mortality on the statistical properties of self-propelling particles moving in a run-and-tumble manner. We obtained the occupational probability in the Laplace space and the first passage characteristics of evanescent run-and-tumble particle systems under spatially symmetric and partially absorbing or reflecting boundaries.

EVANESCENT RUN-AND-TUMBLE PARTICLE

We consider an evanescent run-and-tumble particle that runs with velocity v, tumbles with rate α, and dies or evanesces with a rate λ. The system is confined to a partially reflecting (partially absorbing) region $a \le x \le b$ [6]. We represent the probability density function (PDF) for the right-oriented and left-oriented particles as $P_R(x, t)$ and $P_L(x, t)$ respectively

$$\frac{\partial P_R(x,t)}{\partial t} = -\frac{\partial (v(x)P_R(x,t))}{\partial x} - \frac{\alpha}{2}P_R(x,t) + \frac{\alpha}{2}P_L(x,t) - \lambda P_R(x,t) \qquad (1)$$

$$\frac{\partial P_L(x,t)}{\partial t} = \frac{\partial (v(x)P_L(x,t))}{\partial x} + \frac{\alpha}{2}P_R(x,t) - \frac{\alpha}{2}P_L(x,t) - \lambda P_L(x,t) \qquad (2)$$

From equations (1) and (2), the following steps are done, as shown in Figure 1, to obtain the probability distribution and conditional mean first passage time:

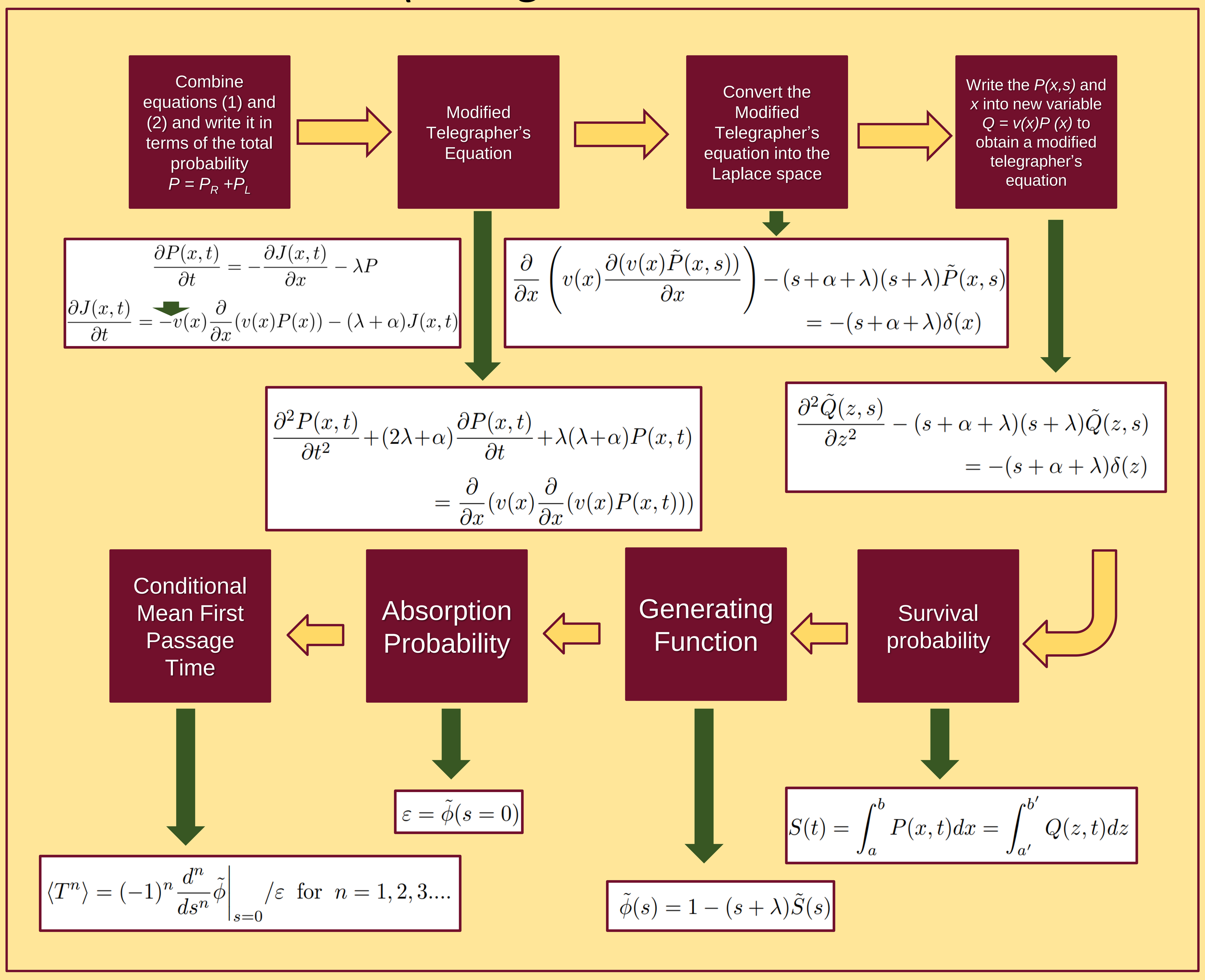

Figure 1: Schematic representation for obtaining the statistical properties

FIRST PASSAGE PROPERTIES

Here, we consider an equal and spatially symmetric boundary case where $a = -b$, $\epsilon_a = \epsilon_b = \epsilon$, and $v(x) = v(-x)$. The generating function is

$$\tilde{\phi}(s) = \frac{2\epsilon}{\left(\sqrt{\frac{s+\lambda}{s+\alpha+\lambda}}+\epsilon\right)e^{kb'} - \left(\sqrt{\frac{s+\lambda}{s+\alpha+\lambda}}-\epsilon\right)e^{-kb'}}$$

where $k^2 = (s+\alpha+\lambda)(s+\lambda)$ and ϵ is the reflecting or absorbing coefficient and the eventual absorption probability is

$$\varepsilon = \tilde{\phi}(s=0) = \frac{2\epsilon}{\left(\sqrt{\frac{\lambda}{\alpha+\lambda}}+\epsilon\right)e^{cb'} - \left(\sqrt{\frac{\lambda}{\alpha+\lambda}}-\epsilon\right)e^{-cb'}}$$

with $c^2 = \lambda(\alpha+\lambda)$.

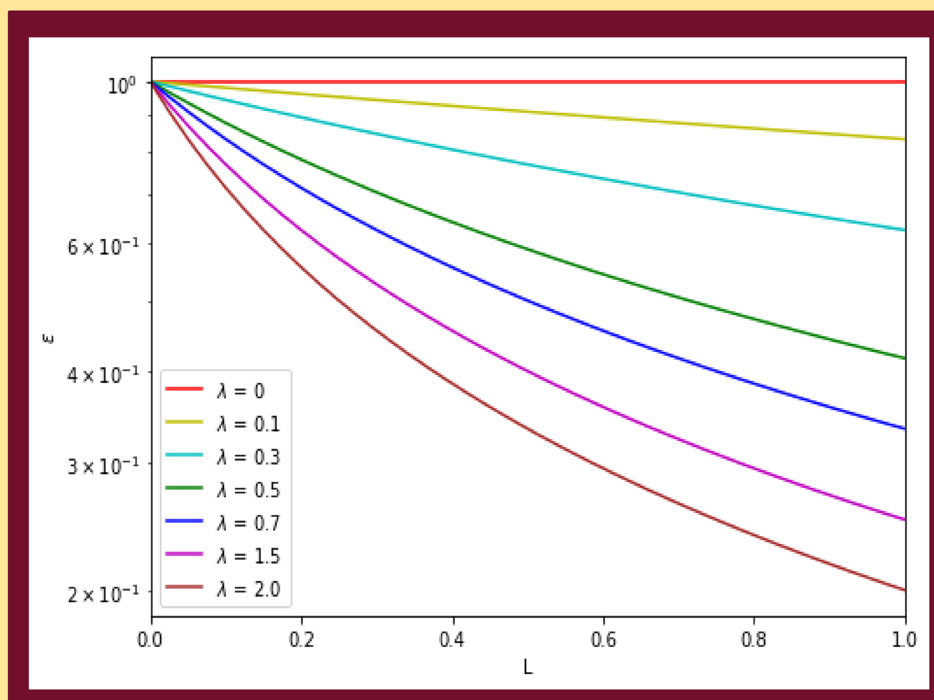

Figure 2: Plot of the absorption probability ε against length L with different values of death rate at constant speed v.

The eventual absorption probability of the particle when $v(x) = v_0$ is given by

$$\varepsilon = \frac{2\epsilon}{2\epsilon + 2(L/v_0)\lambda + \epsilon(L/v_0)^2\lambda(\alpha+\lambda)}$$

The eventual absorption probability of the evanescent run-and-tumble particle decreases as the death rate increases. This is because only a portion of the particles will successfully reach the boundaries as some will die or decay or disappear while in motion.

The obtained expression for the conditional mean first passage time of evanescent run-and-tumble particles when the boundaries are symmetric, $a = -b = L$ is given by

$$T = \frac{(L/v_0) + \frac{\epsilon}{2}(L/v_0)^2(\alpha+2\lambda)}{\epsilon + (L/v_0)\lambda + \frac{\epsilon}{2}(L/v_0)^2\lambda(\alpha+\lambda)}$$

The plot shows the decrease in the value of the conditional mean first passage time as the death rate increases. This means that evanescent run-and-tumble particles must move intelligently toward the boundary in order to escape death.

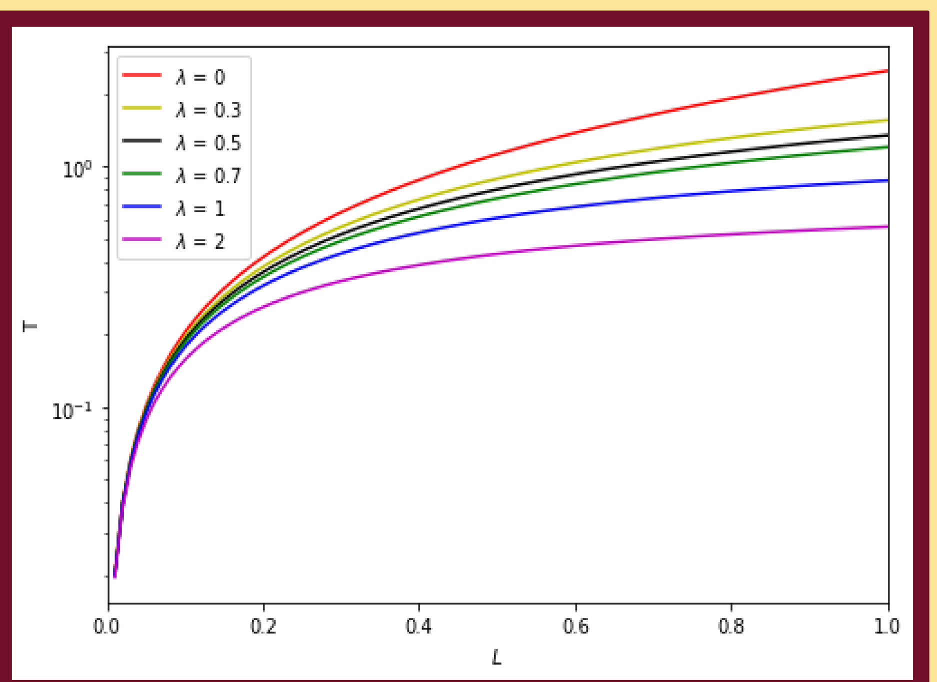

Figure 3: Conditional mean first passage time at constant velocity

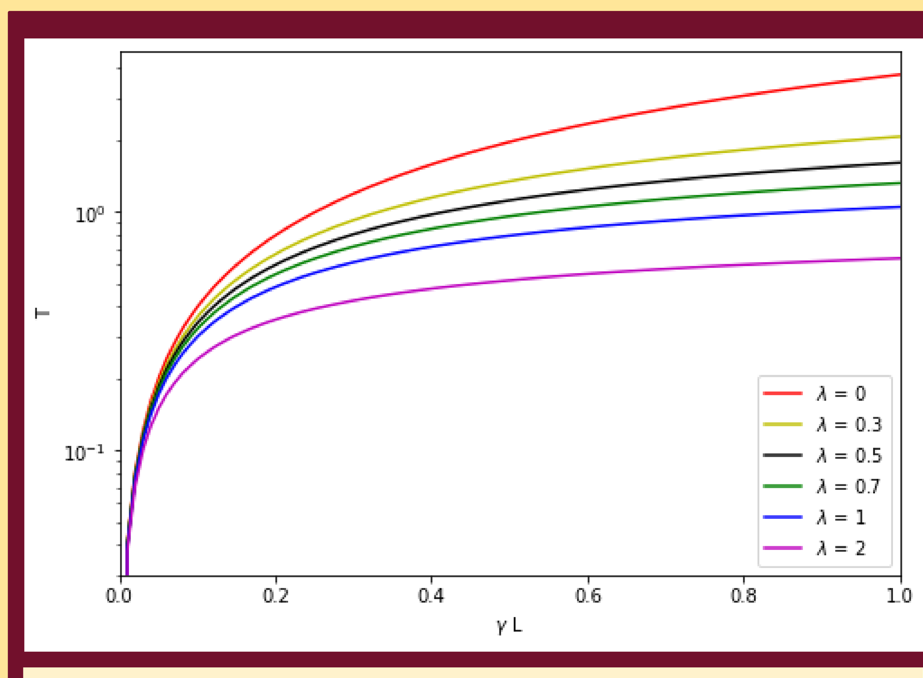

Figure 4: Conditional mean first passage time of evanescent run-and-tumble particles with an exponential form of speed as a function of length L and with different values of death rate λ

The plot shows the conditional mean first passage time as a function of length L, constant γ, and an increasing value of death rate λ.

$$T_{exp} = \frac{v_0\gamma(1-e^{-\gamma L}) + \frac{\epsilon}{2}(\lambda+\alpha)(1-e^{-\gamma L})^2}{\epsilon v_0^2\gamma^2 + \lambda v_0\gamma(1-e^{-\gamma L}) + \frac{\epsilon}{2}\lambda(\lambda+\alpha)(1-e^{-\gamma L})^2}$$

As the death rate increases, the time for the particle to reach the boundary decreases. The plot reflects that there is a much longer time for particles to reach the boundary at lower values of the death rate.

For symmetric boundaries where $a = -b$ and $\epsilon_a = \epsilon_b = \epsilon$, we derived the eventual absorption probability and the conditional mean first passage time for both uniform and exponentially increasing speeds. We observed that in both cases, there is a decrease in the eventual absorption probability as the death rate increases. Due to the decrease in value, we can say that not all evanescent run-and-tumble particles reached the boundaries, and some of them may have already died/decayed or disappeared. This gives us the information that the evanescent particles' success in reaching either or both walls happens at a decreasing time rate.

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