The standard model

G. Bélanger
LAPTH, Annecy-le-Vieux

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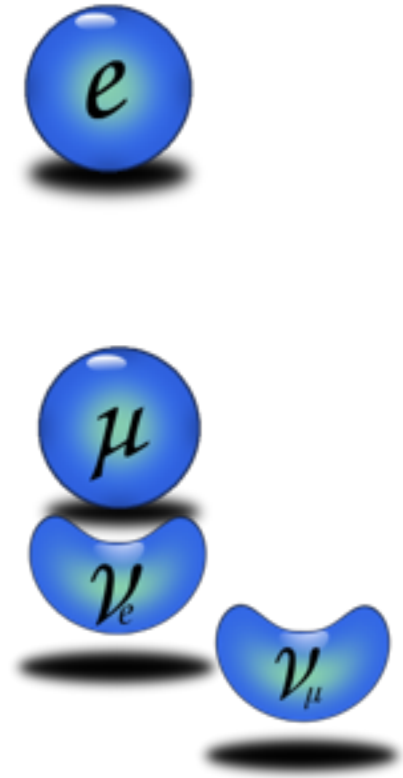
- Elementary particles and their interactions
- Building the Standard model
  - electroweak
  - strong
- Higgs physics
- After LHC first run

Discovery of elementary particles

- 1898 : electron (Thomson)
- 1930 : neutrino proposed (Pauli)
- 1932 : neutron (Chadwick) - not elementary
- 1936 : muon
- 1956 : neutrino (Cowan-Reines)
- 1957 : Parity violation (Wu)
- 1962 : muon neutrino
- 1964 : quark model -u,d,s (Gellman)



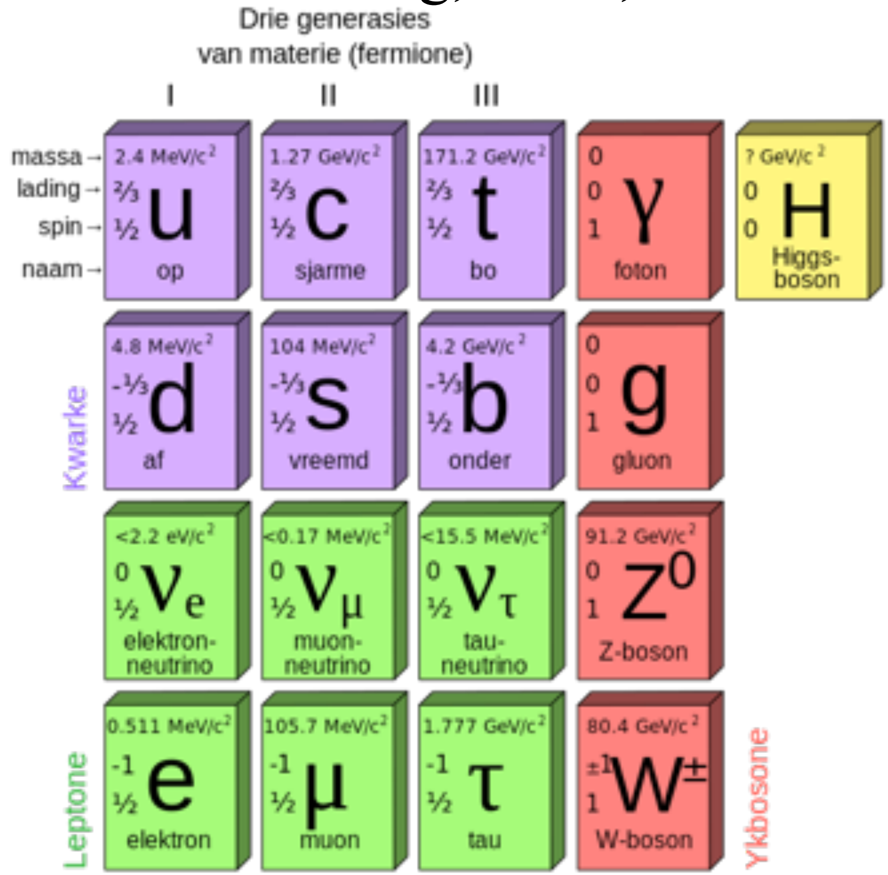
Discovery of elementary particles

- 1967 : Standard model : Glashow Weinberg Salam
  - Brout Englert Higgs mechanism
- 1969 : evidence quarks, u,d,s (SLAC)
- 1973 : neutral currents
- 1974 : c-quark (Richter/Ting)
- 1975 : tau
- 1977 : b-quark
- 1979 : gluons (DESY)
- 1983 : W,Z - UA1/2 (CERN)
- 1995 : top quark - Fermilab
- 2000 : tau-neutrino
- 2012 : Higgs boson (CERN)

Englert & Higgs
CERN 2012

Elementary particles

- What we know today : elementary quarks and leptons and four fundamental interactions : strong, weak, electromagnetic, gravity



fermions bosons

Fundamental interactions - 1960's

- Electromagnetic interactions

$$\mathcal{L} = e\bar{\psi}\gamma_\mu\psi A^\mu$$

- Weak interactions : described by contact interaction
- Observation Parity violation in electron scattering on nucleon (Wu), angular distribution shows a dependence on s.p -> electron is Left-Handed (positron is RH)
  - weak interactions are purely V-A

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu \qquad J_\mu = \bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_e \qquad G_F = 1.16 \times 10^{-5}\mathrm{GeV}^{-2}$$

- Then intermediate vector boson - weaknest of weak interactions due to "heavy" vector boson (W)

$$\mathcal{L} = J_\mu W^\mu \qquad J_\mu = \frac{g}{\sqrt{2}}\bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_e$$

- Problem with Fermi's theory : cross section becomes very large at high energies- can only be an effective theory valid at low scale
- Partially cured in IVB theory - at least in neutrino/electron scattering
- Still get large cross sections for scattering into W pairs.
  - Exercise : compute the cross-section for ν ν-> W^+ W^- at high energies

Towards the standard model

Local symmetry

- Lagrangian for free Dirac field

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi$$

$$\psi \to \mathrm{e}^{\mathrm{i}q\alpha}\psi \qquad \bar{\psi} \to \bar{\psi}\mathrm{e}^{-\mathrm{i}q\alpha}$$

- Invariance under local phase transformation ($\alpha(x)$) specifies interactions between gauge field and matter fields

$$D_\mu = \partial_\mu + \mathrm{i}qA_\mu$$

$$D_\mu\psi \to \mathrm{e}^{\mathrm{i}q\alpha(x)}D_\mu\psi \quad \text{if} \quad A_\mu \to A_\mu - \partial_\mu\alpha(x)$$

$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\gamma^\mu D_\mu - m\right)\psi$$

$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\gamma^\mu \partial_\mu - m\right)\psi - qA_\mu\bar{\psi}\gamma^\mu\psi$$

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- Including $\mathcal{L} = \bar{\psi}\left(\mathrm{i}\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

- Note that a mass term for the 'photon' $m^2 A_\mu A^\mu$ is not invariant under local 'gauge' transformations

Symmetry breaking

- Exact local symmetry lead to interactions with gauge bosons (could be used to describe electromagnetic and weak interactions) BUT these gauge bosons are massless
- Introducing scalar field can lead to spontaneous symmetry breaking
- Goldstone theorem : to each spontaneously broken continuous symmetry is associated a massless field (Goldstone boson)

Abelian Higgs model

- U(1) symmetry and complex scalar field

$$\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)$$

$$D_\mu = \partial_\mu + \mathrm{i e} A_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

- Invariance under local gauge transformation

$$\phi(x) \to \phi'(x) = \mathrm{e}^{\mathrm{i}\alpha(x)} \phi(x)$$

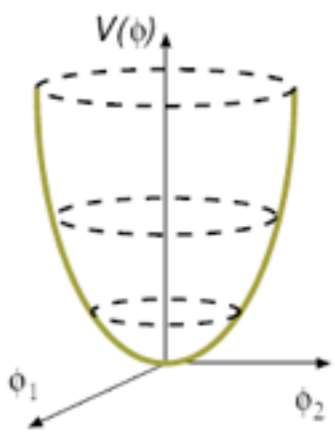
$$A_\mu(x) \to A_\mu(x) - \frac{1}{\mathrm{e}} \partial_\mu \alpha(x)$$

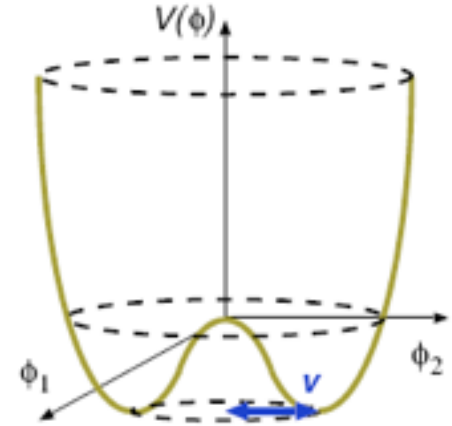
- Potential

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2} (\phi^* \phi)^2$$

- Note that μ^2 term has the wrong sign for a mass term if positive

Potential





- $\phi=(\phi_1+i\ \phi_2)/2$
- minimum of potential at $\phi = 0$ if $\mu^2 <0$ (massive scalar)
- min at $\phi = \phi_0=+/-\ v$ if $\mu^2>0$ where ϕ_0 minimizes the potential

$$\frac{\partial V}{\partial \phi} = -\mu^2 \phi + \lambda \phi^3 = 0 \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$

- v is the vacuum expectation value.
- degenerate minima

- Once a vacuum state is chosen, the symmetry is broken spontaneously -
- Choose $\phi_0 = v$ and make transformation $\phi' = \phi + \phi_0$
- In addition since L is invariant under U(1) can make a gauge transformation ($\phi' = e^{i\alpha(x)}\phi'$) and choose alpha such that ϕ' is real

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu \phi_1)^2 + \sqrt{2}e^2 v \phi_1 A_\mu A^\mu + e^2 v^2 A_\mu A^\mu + \frac{e^2}{2}\phi_1 \phi_1 A_\mu A^\mu - V(\phi)$$

$$V(\phi) = -\frac{\mu^4}{2\lambda} + \mu^2 \phi_1^2 + \frac{\lambda v}{\sqrt{2}}\phi_1^3 + \frac{\lambda}{8}\phi_1^4$$

- Once a vacuum state is chosen, the symmetry is broken spontaneously -
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$$V(\phi) = -\frac{\mu^4}{2\lambda} + \mu^2 \phi_1^2 + \frac{\lambda v}{\sqrt{2}}\phi_1^3 + \frac{\lambda}{8}\phi_1^4$$

mass term for ‘photon’

mass term for scalar

self-interactions for scalars

Symmetry breaking

- Massive gauge field and one massive scalar
- The ϕ_2 field has been eliminated - this ‘would-be’ Goldstone boson becomes the longitudinal component of the massive vector field after the gauge transformation
- Spontaneous symmetry breaking allows to build a model with massive vector field

Building the standard model

- Theoretical tools
  - Invariance under local symmetry
  - Spontaneous symmetry breaking

- Description of elementary particles and their interactions: electromagnetic, weak, strong
  - in particular weak interactions mediated by massive gauge boson
  - parity violation (V-A interactions)
  - problem with Fermi and IVB model : high energy behaviour

- Choice of symmetry group + choice of representations

SU(2)XU(1) : leptons

- Reproduce weak interactions : charged current V-A

$$\overline{(\nu \quad e)_L}\gamma_\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \overline{(\nu_L)}\gamma_\mu e_L = \bar{\nu}\gamma_\mu \frac{(1-\gamma_5)}{2} e$$

σ^+

$$\overline{(\nu \quad e)_L}\gamma_\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \bar{e}\gamma_\mu \frac{(1-\gamma_5)}{2} \nu$$

σ^-

- No symmetry group with only these 2 generators but ($\sigma^+ \sigma^- \sigma^3$) or ($\sigma^1\sigma^2\sigma^3$) are generators of SU(2) - minimal choice of symmetry
- New neutral current (not electromagnetism)

$$\overline{(\nu \quad e)_L}\gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \bar{\nu}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu e_L$$

SU(2)XU(1) : leptons

- Need also to incorporate electromagnetism with current

$$J_Q^\mu = \bar{e}\gamma_\mu e$$

$$J_Q^\mu = J_3^\mu + J_Y^\mu$$

$$J_Q^\mu = \frac{1}{2}\left(\bar{\nu}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu e_L\right) - \frac{1}{2}\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{e}_L\gamma_\mu e_L\right) - \bar{e}_R\gamma_\mu e_R$$

- The current J_Y is associated with a U(1) symmetry, Y is hypercharge $Y(e_L)=Y(\nu_L)=-1/2$ $Y(e_R)=-1$
- $Q=T_3+Y$ is the Gellman-Nishijima relation

- The symmetry group of the standard model is SU(2)XU(1)

SU(2) local symmetry

- SU(2) transformation , doublet

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\psi \to \exp\left(\frac{\mathrm{i}}{2}\sigma_i \alpha^i(x)\right)\psi$$

- Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad [\sigma_i, \sigma_j] = 2\mathrm{i}\epsilon_{ijk}\sigma_k$$

- Covariant derivative

$$D_\mu = \partial_\mu I + igA_\mu$$

$$A_\mu = \frac{1}{2}\sigma_i A^i_\mu = \frac{1}{2}\begin{pmatrix} A_{3\mu} & A_{1\mu} - \mathrm{i}A_{2\mu} \\ A_{1\mu} + \mathrm{i}A_{2\mu} & -A_{3\mu} \end{pmatrix}$$

- where g is the SU(2) coupling constant and

$$A'^i_\mu = A^i_\mu - \epsilon^{ijk}\alpha_j A_{\mu k} - \frac{1}{g}\partial_\mu \alpha^i$$

  – Exercise: check invariance of

$$\mathcal{L} = \bar{\psi}\gamma_\mu D^\mu \psi$$

  – (infinitesimal transformation)

SU(2)XU(1)

- Local gauge symmetry + spontaneous symmetry breaking
- Complex doublet scalar field

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \left(\phi_1 = \frac{\phi_a + \mathrm{i}\phi_b}{\sqrt{2}}, \quad \phi_2 = \frac{\phi_c + \mathrm{i}\phi_d}{\sqrt{2}}\right)$$

$$\phi(x) = \exp\left(\mathrm{i}\frac{\xi^i \sigma^i}{2v}\right)\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

- Covariant derivative

$$D_\mu \phi = \left(\partial_\mu - \mathrm{i}g\frac{A_\mu^i \sigma^i}{2} - \mathrm{i}g' Y B_\mu\right)\phi \quad \text{with} \quad Y = \frac{1}{2}$$

- A_μ^i : 3 vector bosons of SU(2), B_μ: vector boson of U(1)
- g, g' coupling constant of SU(2) and U(1)
- Complex scalar doublet can break SU(2) and lead to 3 massive gauge bosons

Mass of gauge bosons

- Scalar Lagrangian

$$\mathcal{L}_{\rm scalaire} = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

- Complex doublet scalar field

$$\phi(x) = \exp\left(\mathrm{i}\frac{\xi^i \sigma^i}{2v}\right)\left(\begin{array}{c} 0 \\ \frac{v+h}{\sqrt{2}} \end{array}\right)$$

- Gauge transformation $\phi(x) \to \exp\left(-\mathrm{i}\frac{\xi^i \sigma^i}{2v}\right)\phi(x)$

$$\phi(x) = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v + h(x) \end{array}\right)$$

- Writing down explicitly terms that contain gauge bosons masses (blackboard)

Mass of gauge bosons

$$D_\mu \langle\phi\rangle_0^\dagger D^\mu \langle\phi\rangle_0 = \quad \frac{1}{2}\frac{v^2}{4}\left(g^2(A_\mu^1 + \mathrm{i}A_\mu^2)(A_\mu^1 - \mathrm{i}A_\mu^2) + (-gA_\mu^3 + g'B_\mu)^2\right)$$

$$= \frac{g^2v^2}{4}(W_\mu^+ W^{\mu -}) + \frac{(g^2 + {g'}^2)v^2}{8} Z_\mu Z^\mu$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp \mathrm{i}A_\mu^2)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + {g'}^2}}(gA_\mu^3 - g'B_\mu)$$

- Three massive gauge bosons (W +,W-, Z) with masses

$$M_W = \frac{gv}{2}$$

$$M_Z = \sqrt{g^2 + g'^2}\frac{v}{2}$$

- Linear combination of A_μ, B_μ orthogonal to Z_μ corresponds to a massless gauge boson -- it will be the photon

$$A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2 + {g'}^2}}$$

Interactions fermions -bosons

- Rewrite covariant derivative using physical fields

$$
\begin{aligned}
D_\mu &= \partial_\mu - \mathrm{i} g A^a_\mu \frac{\sigma^a}{2} - \mathrm{i} g' Y B_\mu \\
&= \partial_\mu - \frac{\mathrm{i} g}{\sqrt{2}} (W^+_\mu T^+ + W^-_\mu T^-) \\
&- \frac{\mathrm{i}}{\sqrt{g^2 + {g'}^2}} Z_\mu (g^2 T_3 - {g'}^2 Y) \\
&- \frac{\mathrm{i} g g'}{\sqrt{g^2 + {g'}^2}} A_\mu (T_3 + Y)
\end{aligned}
$$

- To recover electromagnetic interaction, impose the relation

$$
e = \frac{g g'}{\sqrt{g^2 + g'^2}}
$$

Interactions fermions -bosons

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$$
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&= \partial_\mu - \frac{\mathrm{i}g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) \\
&\quad - \frac{\mathrm{i}}{\sqrt{g^2+g'^2}} Z_\mu (g^2 T_3 - g'^2 Y) \\
&\quad - \frac{\mathrm{i}gg'}{\sqrt{g^2+g'^2}} A_\mu (T_3 + Y)
\end{aligned}
$$

Q

- To recover electromagnetic interaction, impose the relation

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

- Mixing neutral gauge bosons

$$\begin{pmatrix} Z^0_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \quad \text{and} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

- Rewrite covariant derivative

$$D_\mu = \partial_\mu - \frac{\mathrm{i}g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - \frac{\mathrm{i}g}{\cos\theta_W} Z_\mu (T_3 - Q\sin^2\theta_W) - \mathrm{i}eA_\mu Q$$

- Couplings specified with two parameters e, $\sin\theta_W$

- Mass of W and Z are related

$$M_W = M_Z \cos^2\theta_W$$

- In gauge sector only 3 independent parameters
  - e, $\sin\theta_W$, M_W or M_Z

Leptons : interactions

- Having defined the covariant derivative, couplings of W and Z to fermions completely specified once representations for fermions are chosen
- Only LH leptons have charged current : doublet of SU(2)
- Choose different representation for LH and RH leptons (recall these two components are decoupled)

$$\bar{\psi} i\gamma^\mu \partial_\mu \psi = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R$$

- With Gellmann-Nishijima relation Q=T_3+Y, once SU(2) fixed, hypercharge also fixed

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad R = e_R$$

$$Y = -1/2 \quad Y = -1$$

Leptons : interactions

- SU(2)XU(1) Lagrangian for fermions

$$\mathcal{L}_{lepton} = \bar{L} \mathrm{i} \gamma_\mu D^\mu L + \bar{e}_R \mathrm{i} \gamma_\mu D^\mu e_R$$

- Invariance under SU(2)

$$L \to L' = \exp\left(\frac{\mathrm{i}\alpha^i \sigma^i}{2}\right) L$$

- and U(1)

$$L' = \exp\left(\frac{\mathrm{i}g'}{2} Y_L \alpha\right) L$$

$$R' = \exp\left(\frac{\mathrm{i}g'}{2} Y_R \alpha\right) R$$

- In term of physical fields

$$\mathcal{L}_{\mathrm{leptons}} = \bar{L} \mathrm{i} \gamma^\mu \partial_\mu L + \bar{e}_R \mathrm{i} \gamma^\mu \partial_\mu e_R + g\left(W^+_\mu J^{\mu+} + W^-_\mu J^{\mu-} + Z^0_\mu J^{Z_\mu}\right) - e A_\mu \bar{e} \gamma^\mu e$$

$$J^{\mu+}_W = \frac{1}{\sqrt{2}}\left(\bar{\nu}\gamma_\mu \frac{(1-\gamma_5)}{2} e\right) \quad J^\mu_Z = \frac{1}{\cos\theta_W}\left[\bar{l}\gamma_\mu\left(\frac{(1-\gamma_5)}{2} c^l_L + \frac{(1+\gamma_5)}{2} c^l_R\right) l\right]$$

$$c^l_L = T_3 - Q\sin^2\theta_W \quad c^l_R = -Q\sin^2\theta_W$$

- The existence of a new neutral current is the first prediction of the SM
- The SU(2)XU(1) model was proposed in 1964 independently by Glashow-Weinberg and Salam
- Neutral currents were discovered in 1973 - (neutrino scattering $\nu N \to \nu N$)
- Other predictions : Z mass and W mass are related
- Weinberg angle which drives the mixing of neutral gauge boson hence determines couplings of gauge bosons to fermions is the same that enters definition of weak coupling
- W and Z were discovered at CERN in 1984 in agreement with what was inferred from neutral current and strength of weak interactions

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8\sin^2\theta_W M_W^2}$$

- The Weinberg angle and the Z/W mass have been measured with great precision
- Measurements in agreement with SM (even test loop level)
- Specific examples

Lepton masses

- Direct mass term breaks SU(2) (and U(1)) symmetry

$$\bar{\psi}m\psi \to m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$\bar{L}e_R \to \bar{L}\mathrm{e}^{-\mathrm{i}\alpha Y_L}\mathrm{e}^{\mathrm{i}\alpha Y_{e_R}}e_R = \bar{L}e_R\mathrm{e}^{\mathrm{i}\alpha(Y_{e_R}-Y_L)} \quad \text{and} \quad Y_{e_R} - Y_L = -\frac{1}{2} \neq 0$$

- Use again the scalar field (Yukawa interactions)

$$\mathcal{L}_{\mathrm{Yuk}} = -\lambda_e \left(\bar{e}_R\phi^\dagger L + \bar{L}\phi e_R\right)$$

- Invariant under both SU(2) and U(1) (to check)

$$\begin{aligned}\mathcal{L}_{\mathrm{Yuk}} &= -\lambda_e\frac{h}{\sqrt{2}}\left(\bar{e}_Re_L + \bar{e}_Le_R\right) - \lambda_e\frac{v}{\sqrt{2}}\left(\bar{e}_Re_L + \bar{e}_Le_R\right)\\ &= m_e\frac{h}{v}\left(\bar{e}_Re_L + \bar{e}_Le_R\right) - m_e\left(\bar{e}_Re_L + \bar{e}_Le_R\right)\end{aligned}$$

$$\phi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

- Yukawa interactions
  - describe mass (need to introduce arbitrary Yukawa couplings)
  - specify interactions of physical scalar

Including 3 generations

- Simple extension : each generation in a different SU(2) representation

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L e_R, \quad \mu_R, \quad \tau_R$$

- Introduce independent Yukawa coupling for each charged lepton

$$\mathcal{L}_{Yuk} = -\lambda_e \left(\bar{e}_R \phi^\dagger L + \bar{L} \phi e_R \right) - \lambda_\mu \left(\bar{\mu}_R \phi^\dagger L_\mu + \bar{L}_\mu \phi \mu_R \right) - \lambda_\tau \left(\bar{\tau}_R \phi^\dagger L_\tau + \bar{L}_\tau \phi \tau_R \right)$$

$$m_\mu = \lambda_\mu v/\sqrt{2}, m_\tau = \lambda_\tau v/\sqrt{2}$$

- Values of fermion masses are not predicted in SM
- The neutrinos are massless (problematic since observation of neutrino oscillation imply small mass for neutrinos)
  - one possibility : add RH neutrinos with very small Yukawa couplings in order to have very light neutrinos
  - for problems and better solutions -> B. Kayser's lectures

SM - leptons

- Complete Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{gauge}}$$

$$\mathcal{L}_{\text{jauge}} = -\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk}A^j_\mu A^k_\nu$$

$$f_{\mu\nu} = \partial_\nu B_\nu - \partial_\nu B_\mu$$

- Self interactions (trilinear and quartic couplings) between gauge bosons - consequence of non-Abelian symmetry


γ
ν_e
ν_μ
ν_τ
W
H
e
μ
τ
Z

Quarks : electroweak interactions

- Universality of weak interactions (not quite)
  - measurements of weak interactions show that hadronic interactions are slightly weaker than leptonic ones, those involving s-quarks are much weaker
- Introduce an angle θ_c
- Form of weak current as observed :

$$J_\mu^W = \frac{g}{\sqrt{2}} \left(\bar{s}\gamma_\mu(1-\gamma_5)u \sin\theta_c + \bar{d}\gamma_\mu(1-\gamma_5)u \cos\theta_c \right)$$

- Choice of representation as for leptons LH quarks in doublets

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L u_R, d_R, c_R, s_R$$

  - c-quark was not present in initial formulation of SM
  - Each quark comes in 3 colors -> ignore strong interactions for now

Quark interactions

- Quantum numbers ($Q=T_3+Y$)

Charge

	T_3	Q	Y
u_L	1/2	2/3	1/6
d_L	-1/2	-1/3	1/6
u_R	0	2/3	2/3
d_R	0	-1/3	-1/3

- Lagrangian invariant under SU(2)XU(1)

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\mathcal{L}_{\rm quark} = \bar{Q} \mathrm{i} \gamma^\mu \left(\partial_\mu - \frac{\mathrm{i}g}{2} \sigma^i \cdot A^i_\mu - \frac{\mathrm{i}g'}{2} B_\mu Y \right) Q + \bar{u}_R \mathrm{i} \gamma^\mu \left(\partial_\mu - \frac{\mathrm{i}g'}{2} B_\mu Y \right) u_R + \bar{d}_R \mathrm{i} \gamma^\mu \left(\partial_\mu - \frac{\mathrm{i}g'}{2} B_\mu Y \right) d_R$$

- Following same steps done for leptons (only first generation)

$$\mathcal{L}_W = -\frac{g}{2\sqrt{2}} \left[\bar{u} \gamma_\mu (1-\gamma_5) d W^+_\mu + \bar{d} \gamma_\mu (1-\gamma_5) u W^-_\mu \right]$$

$$\mathcal{L}_{\rm NC} = -\frac{g}{2\cos\theta_W} \left[\bar{u} \gamma_\mu \left[c^u_L (1-\gamma_5) + c^u_R (1+\gamma_5) \right] + \bar{d} \gamma_\mu \left[c^d_L (1-\gamma_5) + c^d_R (1+\gamma_5) \right] d \right] Z^\mu$$

$$c^q_L = T_3 - Q \sin^2\theta_W \quad c^Q_R = -Q \sin^2\theta_W$$

- Problems : no angle - no us charged current

Quark masses

- Quark mass term (from Yukawa coupling)

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_u(\bar{Q}\bar{\phi}u_R + \bar{u}_R\bar{\phi}^\dagger Q) - \lambda_d(\bar{Q}\phi d_R + \bar{d}_R\phi^\dagger Q)$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \bar{\phi} = \mathrm{i}\sigma^2\phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$$

- After symmetry breaking $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\mathcal{L}_{\text{Yuk}} = -\lambda_u \frac{h}{\sqrt{2}}(\bar{u}_R u_L + \bar{u}_L u_R) - \lambda_u \frac{v}{\sqrt{2}}(\bar{u}_R u_L + \bar{u}_L u_R)$$
$$-\lambda_d \frac{h}{\sqrt{2}}(\bar{d}_R d_L + \bar{d}_L d_R) - \lambda_d \frac{v}{\sqrt{2}}(\bar{d}_R d_L + \bar{d}_L d_R)$$

$$m_u = \frac{\lambda_u v}{\sqrt{2}} \qquad m_d = \frac{\lambda_d v}{\sqrt{2}}$$

  – Masses not predicted in SM, given by Yukawa couplings
  – Couplings of the Higgs boson proportional to mass

Quark mixing

- Introducing a second generation - mixing between generations
- Quark doublets in SU(2) Lagrangian are not mass eigenstate

$$
\begin{aligned}
d' &= d\cos\theta_c + s\sin\theta_c \\
s' &= -d\sin\theta_c + s\cos\theta_c
\end{aligned}
$$

- Get the required form for weak current

$$\mathcal{L}_{=} -\frac{g}{2\sqrt{2}}\left[\bar{u}\gamma_\mu(1-\gamma_5)d\cos\theta_c W_\mu^+ + \bar{u}\gamma_\mu(1-\gamma_5)s\sin\theta_c W_\mu^+ + h.c.\right]$$

- Mixing : from weak to mass eigenstates (3 generations)

$$\mathcal{L}_{\text{quark}} = \bar{Q}\mathrm{i}\ \not{D}Q + \bar{u}\mathrm{i}\ \not{D}u + \bar{d}\mathrm{i}\ \not{D}d - \Lambda_u \bar{Q}'\bar{\phi}u' - \Lambda_d \bar{Q}'\phi d'$$

  – Q,u,d : now 3-component vector, Λ_u Λ_d : 3X3 matrices

$$u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \quad d = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Quark mixing

- After symmetry breaking (like for 1st generation)

$$\mathcal{L}_{\text{masse}} = \frac{v}{\sqrt{2}} \left[\overline{(u\,c\,t)}'_L \Lambda_\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}'_R + \overline{(d\,s\,b)}'_L \Lambda_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}'_R \right]$$

- Mu,Md : mass matrices

$$M_u = \frac{v}{\sqrt{2}} \Lambda_u \quad M_d = \frac{v}{\sqrt{2}} \Lambda_d$$

- Mass terms are indeed of required form $\bar{\psi}_L M \psi_R$
- Diagonalisation of mass matrix -> mass eigenstates
- Diagonalisation of non-hermitian matrix with biunitary transformation $U^+MV=M_{\text{diagonal}}$
  - U,V diagonalize MM^+ and M^+M

weak eigenstate ‘

$$\bar{\psi}'_L M \psi'_R = \bar{\psi}'_L U M_{\text{diagonal}} V^\dagger \psi'_R = \bar{\psi}_L M_{\text{diagonal}} \psi_R$$

$$\psi_R = V^\dagger \psi'_R \quad \psi'_R = V \psi_R$$

$$\bar{\psi}'_L U = \bar{\psi}_L \quad \psi'_L = U^\dagger \psi_L$$

mass eigenstate

Weak currents

- In terms of physical eigenstate

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}'_L \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}'_L W^+_\mu h.c. \right\}$$

$$= \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}_L \gamma_\mu (U_u^\dagger U_d) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + c.c. \right\}$$

- $K=U_u^+U_d$ is the Kobayashi-Maskawa matrix (unitary) observable
- Neutral current

$$\mathcal{L} = \frac{g}{2\cos\theta} \left\{ \overline{(u\,c\,t)}' \left[c_L^u \gamma_\mu (1-\gamma_5) + c_R^u \gamma_\mu (1+\gamma_5) \right] \begin{pmatrix} u \\ c \\ t \end{pmatrix}' \right.$$

$$\left. + \overline{(d\,s\,b)}' \left[c_L^d \gamma_\mu (1-\gamma_5) + c_R^d \gamma_\mu (1+\gamma_5) \right] \begin{pmatrix} d \\ s \\ b \end{pmatrix}' \right\}$$

Weak currents

- In terms of physical eigenstate

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}'_L \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}'_L W^+_\mu h.c. \right\}$$

$$= \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}_L \gamma_\mu (U_u^\dagger U_d) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + c.c. \right\}$$

- $K=U_u^{+}U_d$ is the Kobayashi-Maskawa matrix (unitary) observable
- Neutral current

$$\mathcal{L} = \frac{g}{2\cos\theta} \left\{ \overline{(u\,c\,t)}_L U_u^\dagger c_L^u \gamma_\mu U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L + \overline{(u\,c\,t)}_R V_u^\dagger c_R^u \gamma_\mu V_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \right.$$

$$\left. + \overline{(d\,s\,b)}_L U_d^\dagger c_L^d \gamma_\mu U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + \overline{(d\,s\,b)}_R V_d^\dagger c_R^u \gamma_\mu V_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \right\}$$

$$U_u^\dagger c_L^u U_u = c_L^u \quad U_d^\dagger c_L^d U_d = c_L^d$$

- Since all 3 generations have same couplings, c_L^u, c_L^u are diagonal

Weak currents

- In terms of physical eigenstate

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}'_L \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}'_L W^+_\mu h.c. \right\}$$

$$= \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c\,t)}_L \gamma_\mu (U_u^\dagger U_d) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + c.c. \right\}$$

- $K = U_u^+ U_d$ is the Kobayashi-Maskawa matrix (unitary) observable
- Neutral current

$$\mathcal{L} = \frac{g}{2\cos\theta} \left\{ \overline{(u\,c\,t)} \left[c_L^u \gamma_\mu (1-\gamma_5) + c_R^u \gamma_\mu (1+\gamma_5) \right] \begin{pmatrix} u \\ c \\ t \end{pmatrix} \right.$$

$$\left. + \overline{(d\,s\,b)} \left[c_L^d \gamma_\mu (1-\gamma_5) + c_R^d \gamma_\mu (1+\gamma_5) \right] \begin{pmatrix} d \\ s \\ b \end{pmatrix} \right\}$$

- No Flavour-changing Neutral Current (FCNC)
- Note : each quark comes in 3 color -> ignore strong interactions for now

Mixing

- Of the 4 unitary matrices only one combination (CKM matrix) is observable
- Convention : matrix for up-type quark is diagonal, only mixing in d-quark sector
- The special case of two generations : the Cabibbo matrix
- 2X2 unitary matrix --> one angle

$$\begin{pmatrix} d \\ s \end{pmatrix}' = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

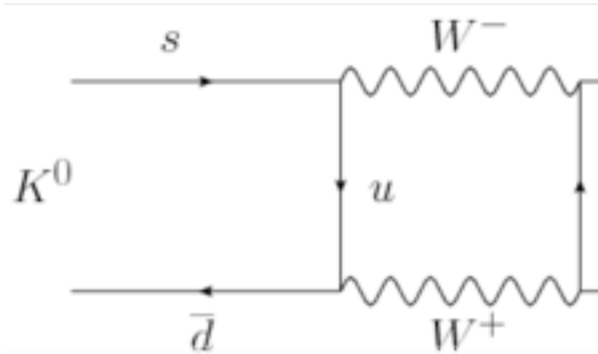
$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left\{ \overline{(u\,c)}_L \gamma_\mu \begin{pmatrix} d \\ s \end{pmatrix}'_L W^\dagger_\mu \;\; + h.c. \right\}$$

$$\begin{aligned}\mathcal{L}_W =& \frac{g}{2\sqrt{2}} (\overline{u}\gamma_\mu(1-\gamma_5)d\cos\theta_c + \overline{u}\gamma_\mu(1-\gamma_5)s\sin\theta_c \\ &- \overline{c}\gamma_\mu(1-\gamma_5)d\sin\theta_c + \overline{c}\gamma_\mu(1-\gamma_5)s\cos\theta_c)\, W^\dagger_\mu \;\; + h.c.\end{aligned}$$

- Recover observed weak neutral current AND current with c-quark
- existence postulated by Glashow,Iliopoulos, Maiani

c-quark

- Note c-quark is required to guarantee no FCNC
- Not enough : higher order charged current can induce FCNC



- Recover observed weak neutral current


1968: SLAC
u
up quark
1974: Brookhaven & SLAC
c
charm quark
1995: Fermilab
t
top quark
1979: DESY
g
gluon
1968: SLAC
d
down quark
1947: Manchester University
s
strange quark
1977: Fermilab
b
bottom quark
1923: Washington University*
γ
photon
1956: Savannah River Plant
ν_e
electron neutrino
1962: Brookhaven
ν_μ
muon neutrino
2000: Fermilab
ν_τ
tau neutrino
1983: CERN
W
W boson
1897: Cavendish Laboratory
e
electron
1937 : Caltech and Harvard
μ
muon
1976: SLAC
τ
tau
1983: CERN
Z
Z boson
H

Standard Model Lagrangian

- Complete Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{gauge}}$$

$$\mathcal{L}_{\text{scalaire}} = |D_\mu \phi|^2 + \underbrace{\mu^2 \phi^+ \phi - \lambda(\phi^+ \phi)^2}_{=-V(\phi)}$$

$$|D_\mu \phi|^2 = M_W^2 W_\mu^+ W^{\mu-} + \frac{1}{2} M_Z^2 Z_\mu^0 Z^{\mu 0} + \frac{vh}{2} \left(g^2 W_\mu^+ W^{\mu-} + \frac{g^2}{\cos^2 \theta_W} Z_\mu^0 Z^{\mu 0} \right)$$

$$+ \frac{g^2 h^2}{8} \left(W_\mu^+ W^{\mu-} + \frac{1}{\cos^2 \theta_W} Z_\mu Z^\mu \right) + \frac{1}{2} \partial_\mu h \partial^\mu h$$

Free parameters

- SU(2) XU(1) only (17 parameters)
  - coupling constants : g,g' or e,$\sin\theta_W$
  - scalar sector : v, m_h
  - Fermion masses (or Yukawa couplings) : 9
  - Quark mixing : 3 angles + 1 phase
- Gauge bosons masses and couplings only depend on g,g',v

$$M_W = \frac{gv}{2}$$
$$M_Z = \frac{\sqrt{g^2+g'^2}v}{2} = \frac{gv}{2\cos^2\theta_W}$$
$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

- Most precisely measured input parameters

$$G_\mu = (1.16637 \pm .00001) \times 10^{-5} GeV \quad \frac{\Delta G_\mu}{G_\mu} = 8.6 \times 10^{-6}$$
$$M_Z = 91.1876 \pm .0021 GeV \quad \frac{\Delta M_Z}{M_Z} = 2 \times 10^{-5}$$
$$\alpha^{-1} = 137.03599235(73) \quad \frac{\Delta\alpha^{-1}}{\alpha^{-1}} = 5 \times 10^{-9}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2\sin^2\theta_W}$$
$$e^2 = 4\pi\alpha$$
$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}$$

- These relations are modified by higher order corrections