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SFB TR9

One-loop calculations in the Standard Model

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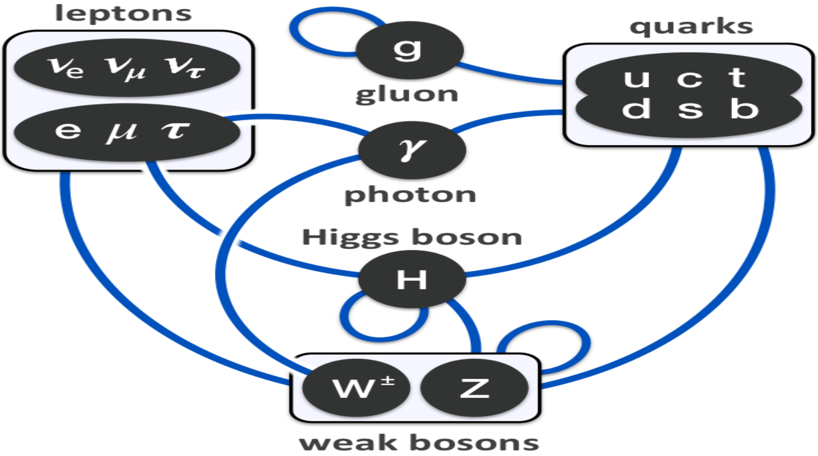
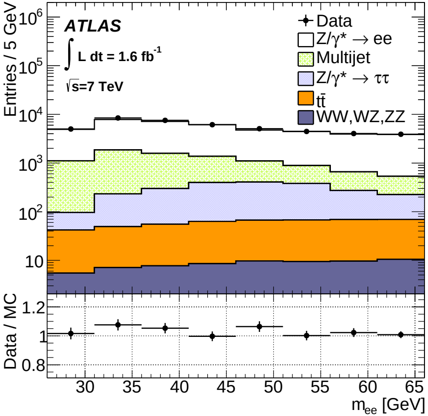


The SM is a correct theory!

- Q: How can we be sure of that?
- A: because everyone is saying that, because it has been well tested, all particles found, etc ...
- Q: how to test a theory? how to discover a particle?
- A: Do real experiments and real calculations. Then compare the two numbers.



Lagrangian $\rightarrow$ Monte Carlo simulations:

It is a long way to go. Therefore, we have to do it step by step.

- Lagrangian $\rightsquigarrow$ Feynman rules
- Cross section, distributions (differential cross sections)
- Helicity (polarized) amplitudes
- One-loop calculations, dimensional regularization
- ...

Do it YOUR way.

Quantum field theory

A mini review

Classical fields

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- scalar fields $\phi(x)$: spin-0 particles
- vector fields $A_\mu(x)$, $\mu = \overline{0,3}$ Lorentz index: spin-1 particles
- spinor fields $\psi_a(x)$, $a = \overline{1,4}$ spinor index: spin-1/2 particles

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

Lagrangian formalism

$$\mathcal{L} = \mathcal{L}(x, \dot{x}) \rightsquigarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \tag{1}$$

$$S[\phi] = \int d^4x \mathcal{L} \tag{2}$$

$$\delta S = S[\phi + \delta\phi] - S[\phi] = 0 : \quad \text{Hamilton's principle} \tag{3}$$

Euler-Lagrange equation:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{4}$$

Natural units: $\hbar = c = 1 \rightsquigarrow [x] = 1/E$.
The action is dimensionless: $[\mathcal{L}] = E^4$.

Free quantum fields: scalar

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$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) - m^2 \phi^\dagger \phi \tag{5}$$

$$(\Box + m^2)\phi(x) = 0 : \quad \text{Klein-Gordon equation} \tag{6}$$

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{2k_0} \left[a(k) e^{-ikx} + b^\dagger(k) e^{ikx} \right] : \quad \text{operators} \tag{7}$$

- a, $a^\dagger$: annihilation and creation operators for particle states.
- b, $b^\dagger$: annihilation and creation operators for anti-particle states.

Dimension: $[\phi] = E$.

Free quantum fields: spinor

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi, \quad \bar{\psi} = \psi^\dagger\gamma^0 \quad (8)$$

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0 : \quad \text{Dirac equation} \quad (9)$$

$$\psi(x) = \frac{1}{(2\pi)^{3/2}}\sum_{\sigma=\pm1}\int\frac{d^3k}{2k_0}\left[c_\sigma(k)u_\sigma(k)e^{-ikx} + d_\sigma^\dagger(k)v_\sigma(k)e^{ikx}\right] : \quad \text{operators} \quad (10)$$

$$(\not{k} - m)u_\sigma(k) = 0, \quad (\not{k} + m)v_\sigma(k) = 0. \quad (11)$$

Helicity index: $\sigma = \pm1$.

$$s_\sigma(k) = \begin{pmatrix} s_{\sigma,1}(k) \\ s_{\sigma,2}(k) \\ s_{\sigma,3}(k) \\ s_{\sigma,4}(k) \end{pmatrix}, \quad s = u, v. \quad (12)$$

$\psi(x)$: summed over two independent helicity states.
Dimension: $[\psi] = E^{3/2}$.

Free quantum fields: vector

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (13)$$

$$A^\mu(x) = \frac{1}{(2\pi)^{3/2}}\sum_\lambda \int \frac{d^3k}{2k_0}\left[a_\lambda(k)\epsilon^\mu_\lambda(k)e^{-ikx} + a^\dagger_\lambda(k)\epsilon^\mu_\lambda(k)^* e^{ikx}\right] : \quad \text{operators} \quad (14)$$

Polarization vectors ϵ^μ_λ, λ: helicity (polarization) index,

$$\epsilon_\lambda \cdot k = 0, \quad \epsilon^*_\lambda \cdot \epsilon_{\lambda'} = -\delta_{\lambda\lambda'}, \quad (15)$$

$$m > 0 : \quad \sum_{\lambda=1}^{3} \epsilon^\mu_\lambda(k)^* \epsilon^\nu_\lambda(k) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2} : \quad \text{3 helicity states} \quad (16)$$

$$m = 0 : \quad \sum_{\lambda=1}^{2} \epsilon^\mu_\lambda(k)^* \epsilon^\nu_\lambda(k) = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} : \quad \text{2 helicity states,} \quad (17)$$

where we have chosen the gauge $n \cdot \epsilon = 0$, n an arbitrary reference vector with $n^2 = 0$ and $k \cdot n \neq 0$.

- External A^μ: 2 (for $m = 0$) and 3 (for $m > 0$) physical states.
- Internal A^μ: Feynman propagator, unphysical states can occur (i.e. $\lambda > 3$).

Dimension: $[A^\mu] = E$.

Differential cross section

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$$d\sigma = \text{Flux}\left(\prod_{i=1}^{n}\frac{d^{D-1}\vec{p}_i}{2E_i(2\pi)^{D-1}}\right)(2\pi)^D\delta^D(p_{\text{in}} - \sum_{i=1}^{n} p_i)|M|^2, \tag{18}$$

$D = 4$ by default, but we need the general formula for dimensional regularization. M is the amplitude, which can be calculated using Feynman rules.
1) For scattering:

$$A + B \to f_1 + f_2 + \cdots f_n, \quad p_{\text{in}} = p_A + p_B, \tag{19}$$

$$\text{Flux} = \frac{1}{2E_A 2E_B|v_A - v_B|}, \tag{20}$$

$$|v_A - v_B| = |\frac{p_A^z}{E_A} - \frac{p_B^z}{E_B}| : \quad \text{the relative velocity of the incoming particles.} \tag{21}$$

2) For decay:

$$A \to f_1 + f_2 + \cdots f_n, \quad p_{\text{in}} = p_A, \tag{22}$$

$$\text{Flux} = \frac{1}{2m_A}. \tag{23}$$

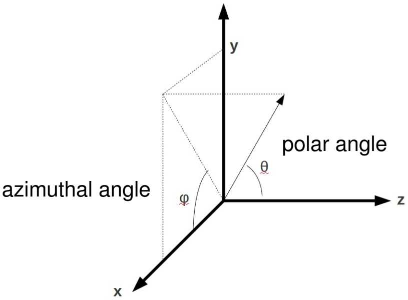
Problem: Assume $M = 1$, $D = 4$, calculate the decay width Γ for $A \to f_1 + f_2$ and the total cross section σ for $A + B \to f_1 + f_2$, $m_A = m_B$.

Integrated cross section, $D = 4$:

$$\sigma = \text{Flux} \int \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{2E_i(2\pi)^3} (2\pi)^4 \delta^4(p_{\text{in}} - \sum_{i=1}^{n} p_i) |M|^2 F_{\text{cut}}(p_i). \tag{24}$$

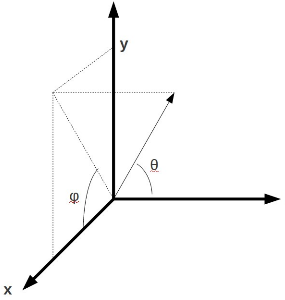
$F_{\text{cut}}(p_i)$: constraints on the final-state momenta (event selection).

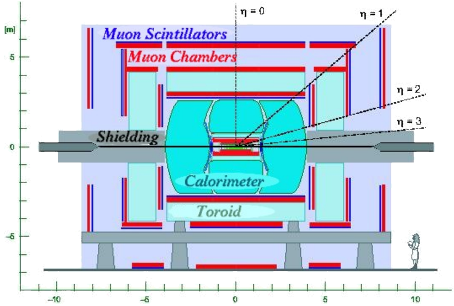
Number of events = $\sigma \times$ Luminosity



- Polar angle: $[0, \pi]$, azimuthal angle: $[-\pi, \pi]$.
- Transverse momentum: $p_{T,i} = \sqrt{p_{x,i}^2 + p_{y,i}^2}$.
- Rapidity: $y_i = \frac{1}{2} \log \frac{E_i + p_{z,i}}{E_i - p_{z,i}}$.

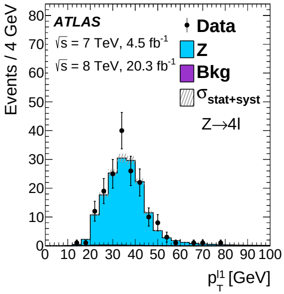
Kinematics





- Pseudo rapidity: $\eta_i = \frac{1}{2} \log \frac{|\vec{p}_i| + p_{z,i}}{|\vec{p}_i| - p_{z,i}}$.
  Problem: in 1 minute prove that $\eta_i = -\log \tan \frac{\theta_i}{2}$. Range: $-\infty < \eta_i < \infty$.
- Invariant mass: $m_{ij}^2 = (p_i + p_j)^2, \quad m_{ijk}^2 = (p_i + p_j + p_k)^2, \ldots$
- Pseudo-rapidity azimuthal-angle separation: $R_{ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}$.
- A typical cut: $p_{T,i} > 20 GeV$, $|\eta_i| < 2.5$, $R_{ij} > 0.4$.

Distributions (histograms)



Transverse momentum distribution of the leading (max p_T) lepton from the leading lepton pair (max m_{ll}).

Distributions tell us how events are distributed over the phase space. They are very useful to distinguish signal from background.
Example: signal $H \to e^+e^-\mu^+\mu^-$, background $Z \to e^+e^-\mu^+\mu^-$. Plot the $m_{e^+e^-\mu^+\mu^-}$ distribution.
Problem: Assume $M = 1$, $D = 4$, calculate the angular distribution $d\sigma/d\cos\theta$ for a $A + B \to f_1 + f_2$ process in the center of mass system, $m_A = m_B$. Try to calculate $d\sigma/dp_{T,1}$?

Distributions: how to do it?

$$\text{process}: \quad A + B \to f_1 + f_2 + f_3 + f_4 + f_5, \quad n = 5, \tag{25}$$

$$\sigma = \text{Flux} \int \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{2E_i(2\pi)^3} (2\pi)^4 \delta^4(p_{\text{in}} - \sum_{i=1}^{n} p_i) |M|^2 F_{\text{cut}}(p_i). \tag{26}$$

- We cannot do it analytically. We have to do it numerically.
- Key word: Monte Carlo method (the best option we have by now).

$$\sigma = \int_0^1 \prod_{i=1}^{d} dx_i f(x_i) \approx \frac{1}{N} \sum_{j=1}^{N} f(\vec{x}_j). \tag{27}$$

- The calculation error: $\sim 1/\sqrt{N}$.
- Advantage: very easy to get any distribution.
- Popular program: VEGAS.

References

1. A very useful reference in German, but you can read the equations: Übungen zu Strahlungskorrekturen in Eichtheorien (problems of radiative corrections in gauge theories): Matthias Steinhauser, Maria Laach, 2003. You can get a PDF file from google with key word: Steinhauser Maria Laach.
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5. Dimensional regularization and schemes for γ_5: 't Hooft and Veltman, Nucl. Phys. B44 (1972) 189, and Chanowitz, Furman and Hinchliffe, Nucl. Phys. B159 (1979) 225.
6. One loop integrals: for basic mathematical rules, see the Appendix D of Le Duc Ninh, arxiv: 0810.4078.