LEP1 : INVESTIGATING THE Z RESONANCE

Resonance production cross sections

- **A general formula for vector resonance production:**

$$\sigma\left(f_1 f_2 \to f_3 f_4\right) = \frac{12\pi}{s} \frac{\Gamma_{f_1 f_2} \Gamma_{f_3 f_4}}{\left(s - M_0^2\right)^2 + s^2 \Gamma_{tot}^2 / M_0^2}$$

where: $$\Gamma_{f_i f_j} = \Gamma_{tot} \times BR\left(f_i f_j\right)$$

- **Tree level Standard Model formulae** for the widths into given fermion (of given color!):

$$\Gamma_{W \to f_i f_j} = \frac{G_F}{\sqrt{2}} \frac{M_W^3}{6\pi} \qquad \Gamma_{Z \to f \bar{f}} = \frac{G_F}{\sqrt{2}} \frac{M_Z^3}{6\pi} \left(v_f^2 + a_f^2\right)$$

Reminder : couplings v_f, a_f at the tree level

Family			T	T_3	Q
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	+1/2 −1/2	0 −1
ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	0	0	0
e_R	μ_R	τ_R	0	0	−1
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/2	+1/2 −1/2	+2/3 −1/3
u_R	c_R	t_R	0	0	+2/3
d_R	s_R	b_R	0	0	−1/3

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W^{tree}}.$$

$$g_V^{tree} \equiv g_L^{tree} + g_R^{tree} = \sqrt{\rho_0}\,(T_3^f - 2Q_f \sin^2 \theta_W^{tree})$$
$$g_A^{tree} \equiv g_L^{tree} - g_R^{tree} = \sqrt{\rho_0}\,T_3^f.$$

Numerical values, assuming
$\sin^2\theta_w = 0.23$

	ν	μ	u	d
gv	1/2	-0.04	0.19	-0.34
ga	1/2	-1/2	1/2	-1/2

Tree level cross-sections

- Compute all widths and keep them in mind! MeV's are the natural unit. What is the peak cross-section for:

  - $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$
  - $e^+e^- \rightarrow Z \rightarrow \nu_\mu\bar{\nu}_\mu$
  - $e^+e^- \rightarrow Z \rightarrow d\bar{d}$
  - $e^+e^- \rightarrow Z \rightarrow u\bar{u}$

  - What are the open modes? What is the total cross-section? What is the total width? What are the branching fractions?

Tree level cross-sections

- Widths (l = each lepton):

$\Gamma(Z \to ll)$	$\Gamma(Z \to \nu_l \nu_l)$	$\Gamma(Z \to dd)$	$\Gamma(Z \to uu)$	Γtot
83 MeV	165 MeV	361 MeV	282 MeV	2.390 GeV

- Branching fractions

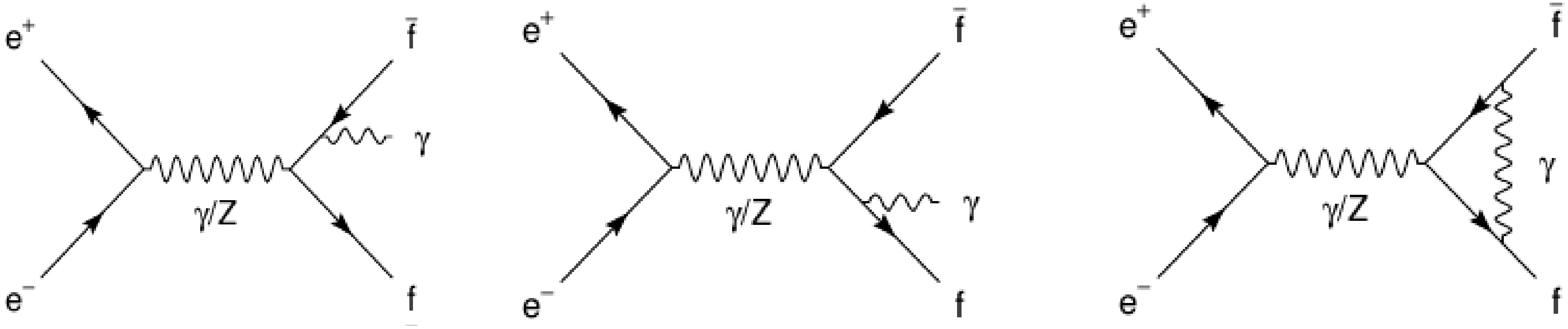
BR(Z → ll)	BR(Z → $\nu\nu$)	BR(Z → hadrons)
3.5%	20.6%	68.9%

- Peak cross-sections

σ(Z → ll)	σ(Z → hadrons)
2.1 nb	42 nb

QED and QCD radiative corrections

- QED corrections:



after computation, these corrections amount to a factor **$(1 + \alpha/\pi) \sim 1.0025$**, to be applied to the tree-level width.

- QCD corrections : the same graphs, just replacing photon by gluon. Only relevant for quarks
  The correction factor is **$(1 + \alpha_s/\pi) \sim 1.04$**

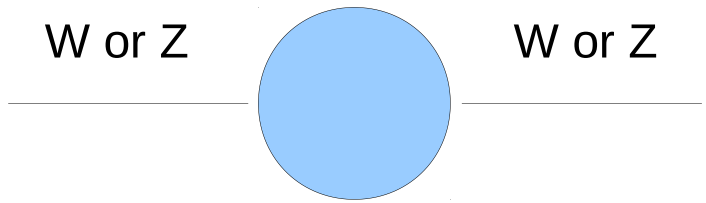
These factors should be applied to the partial widths (depending on flavour!), and the branching fractions recomputed

The EW radiative corrections in two slides

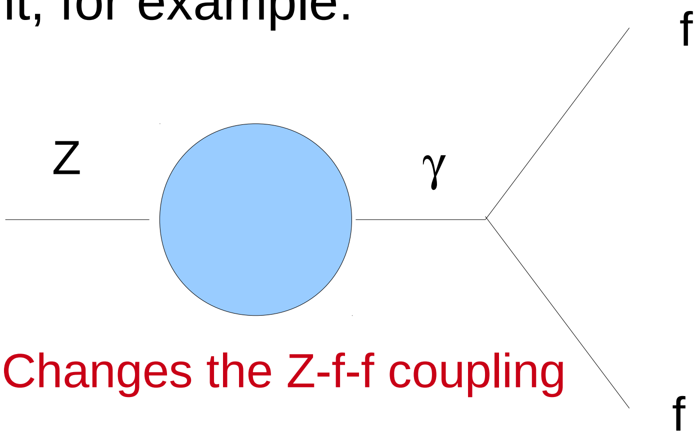
- The weak mixing angle appearing in the tree-level formulas:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \qquad v_f = \sqrt{1}\left(T_{3f} - 2Q_f \sin^2\theta_W\right)$$

can not be the same quantity when including radiative corrections, because the corrections to both relations are different, for example:



Changes the mass ratio



Changes the Z-f-f coupling

- One way to parametrize the corrections is to define $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$ as true to all orders, and to introduce an **effective weak mixing angle** controlling the couplings.

The EW radiative corrections in two slides

- We define the renormalised couplings:

$$v_f = \sqrt{1+\Delta\rho}\,\left(T_{3f}-2Q_f\sin^2\theta_W^{eff}\right)$$
$$a_f = \sqrt{1+\Delta\rho}\;T_{3f}$$

with

$$\sin^2\theta_W^{eff} = \kappa\,\sin^2\theta_W$$
$$\kappa = 1+\Delta\kappa$$

- **The tree-level expressions of the widths are still valid in terms of these modified couplings.**

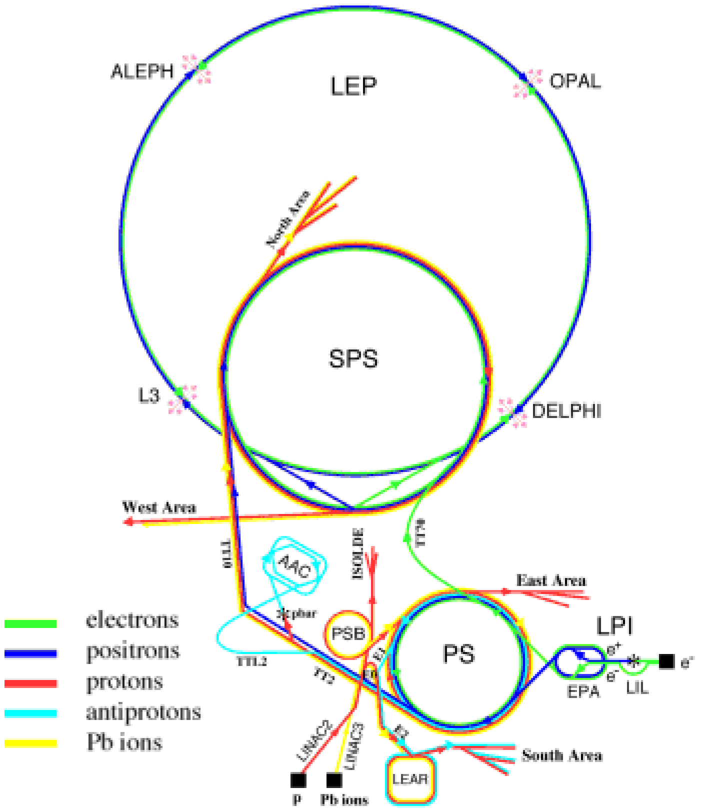
- And after calculations are done in this scheme we find:

$$\Delta\rho = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} - f(\ln(M_H)) \qquad \Delta\kappa = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\frac{\cos^2\theta_W}{\sin^2\theta_W} - f'(\ln(M_H))$$

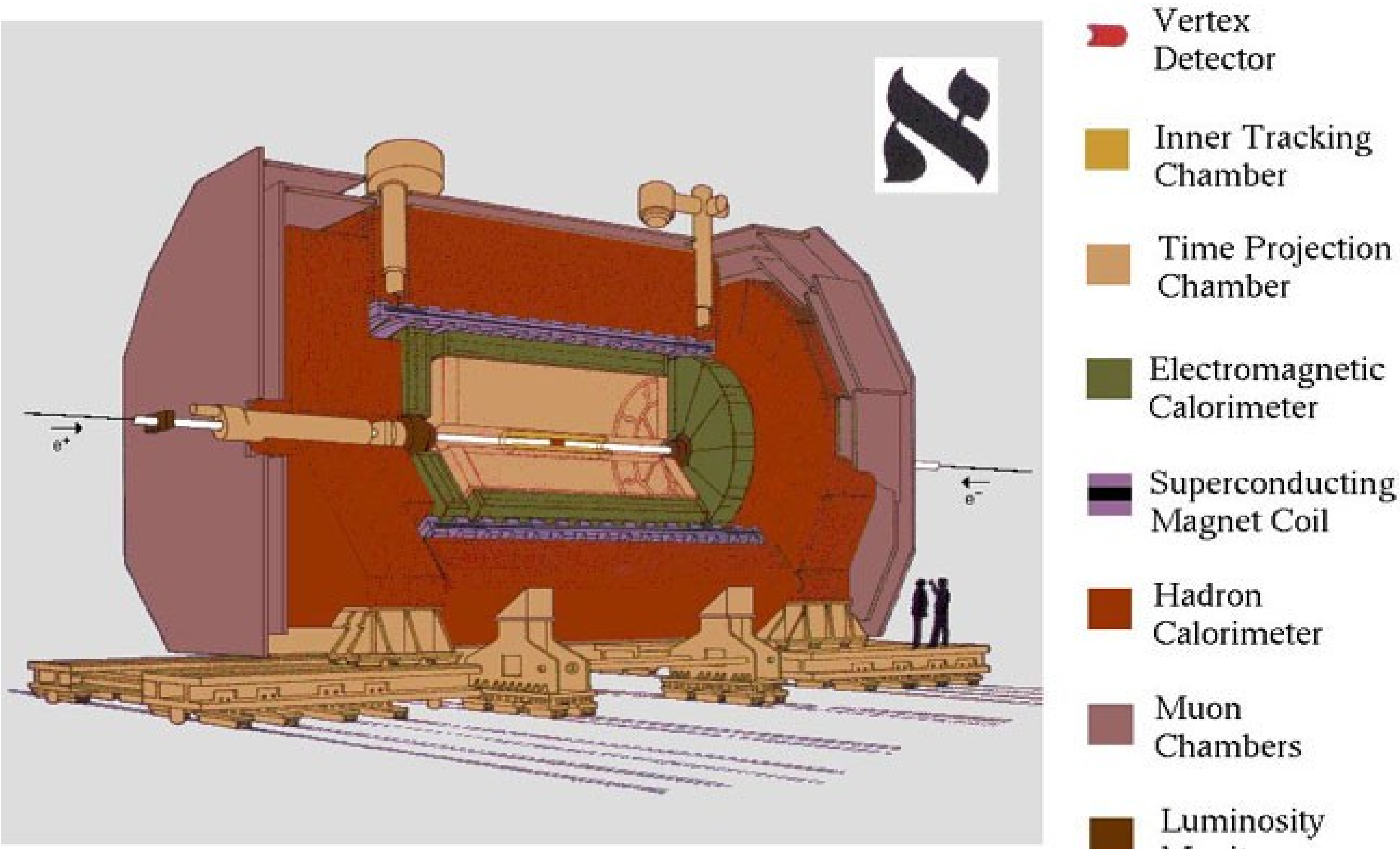
- So the observables, e.g $\Gamma_{Z\to f\bar{f}} \sim (v_f^2 + a_f^2)$ are sensitive to the top quark and Higgs boson masses.

The LEP accelerator and the experiments

- LEP1 : run around root(s) ~ Mz.
- Four experiments : ALEPH, DELPHI, L3, OPAL. Each experiment could accumulate about L = 200 pb-1



The experiments : ALEPH



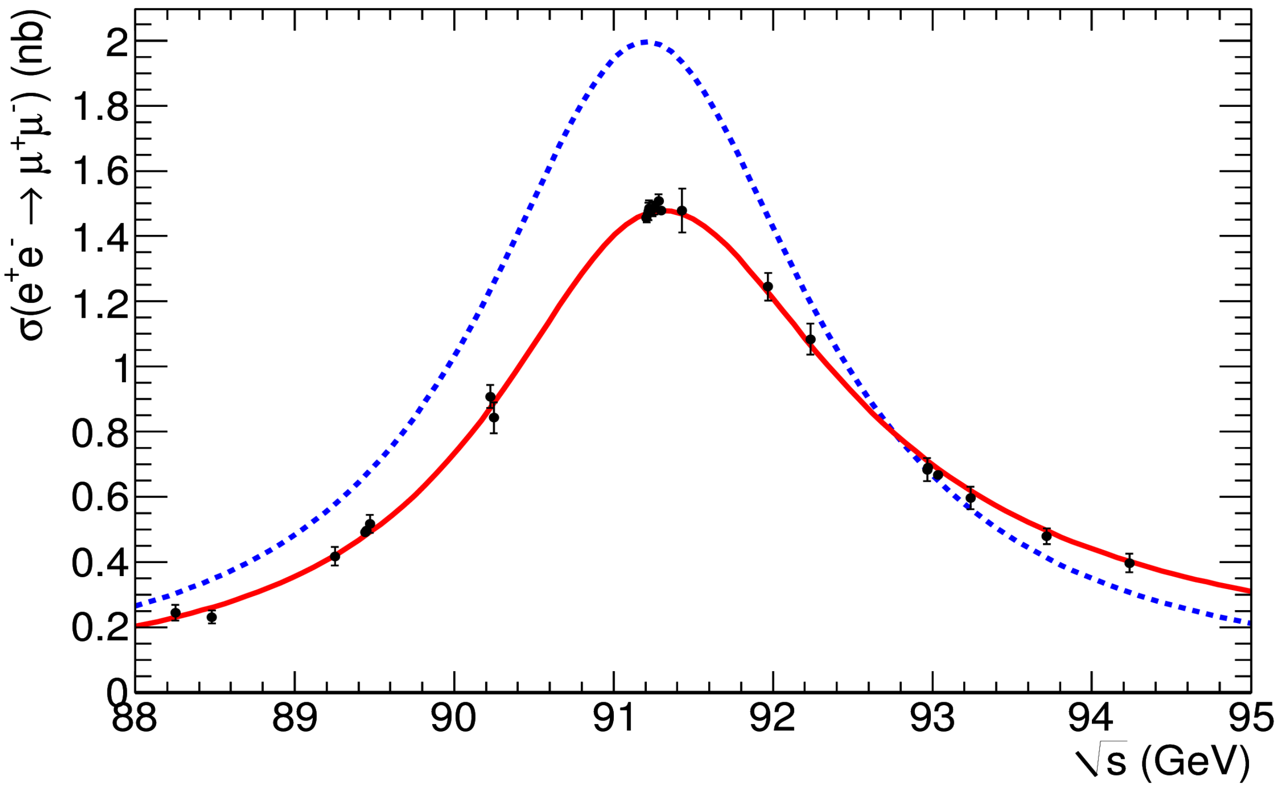
The ALEPH Detector

Measurement of the lineshape

- Why?
  - Contributes to the determination of the width, and thus of the couplings as discussed
  - The peak of the cross-section will allow to extract the Z boson mass.
- Measurement method:
  - The accelerator is operated at various energy points, and the cross-section is measurement at each point.
  - The theoretical model is then fitted to the points, i.e the relevant parameters of the theory (in this case, M_Z, Γ_Z) are varied until a good description of the data is found
  - The corresponding parameters are quoted as the measured values
- These data were taken between 1989 and 1995, with successive refinements of the scan points as the mass became more precise:

Year	Centre-of-mass energy range [GeV]	Integrated luminosity [pb^{-1}]
1989	88.2 – 94.2	1.7
1990	88.2 – 94.2	8.6
1991	88.5 – 93.7	18.9
1992	91.3	28.6
1993	89.4, 91.2, 93.0	40.0
1994	91.2	64.5
1995	89.4, 91.3, 93.0	39.8

Determination of the Z boson mass and width



$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$
$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

Measurement of the Branching fractions

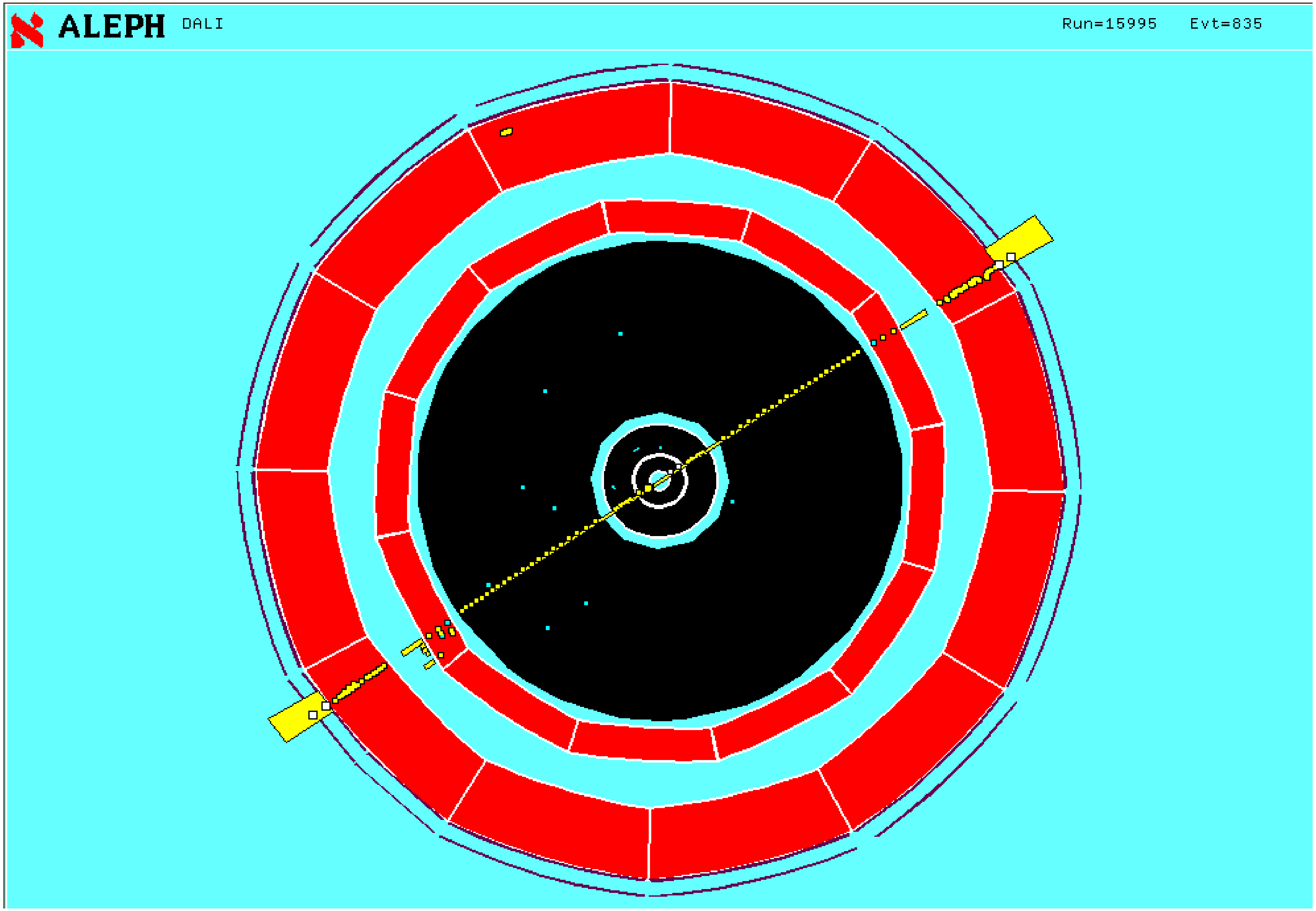
- Measuring brahching fractions means to measure the relative frequency for each final state, compared to the overall Z sample. We can identify:
  - Electrons
  - Muons
  - Tau leptons
  - Jets, and specifically b-quark jets

  For each flavour, we compute Nf/Ntot.

- Injecting the total width then gives the partial widths, which are related to our physical parameters:

$$\Gamma_{Z \to f\bar{f}} = BR(f\bar{f}) \times \Gamma_{tot}$$

Event selection and Identification – what is this?



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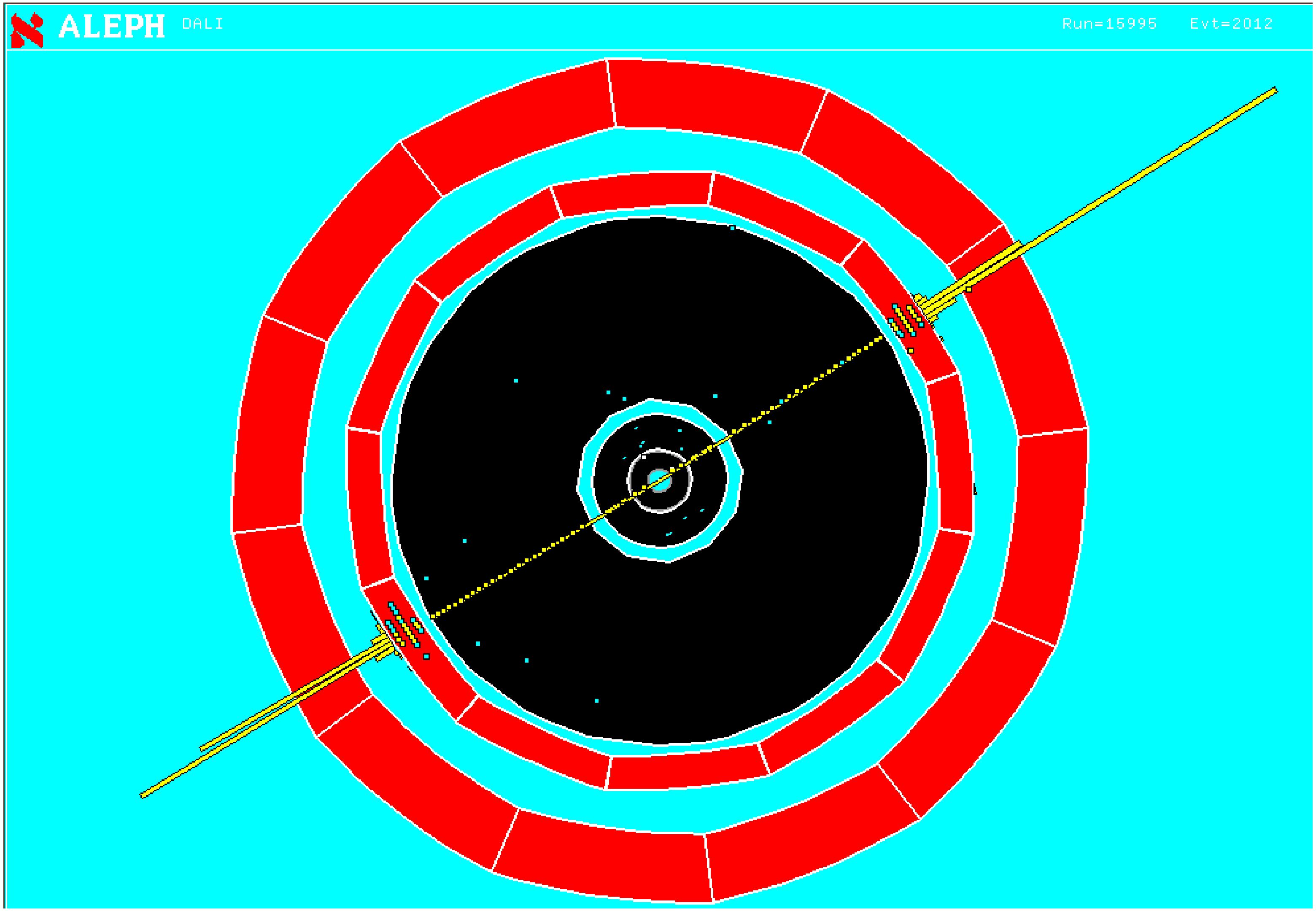

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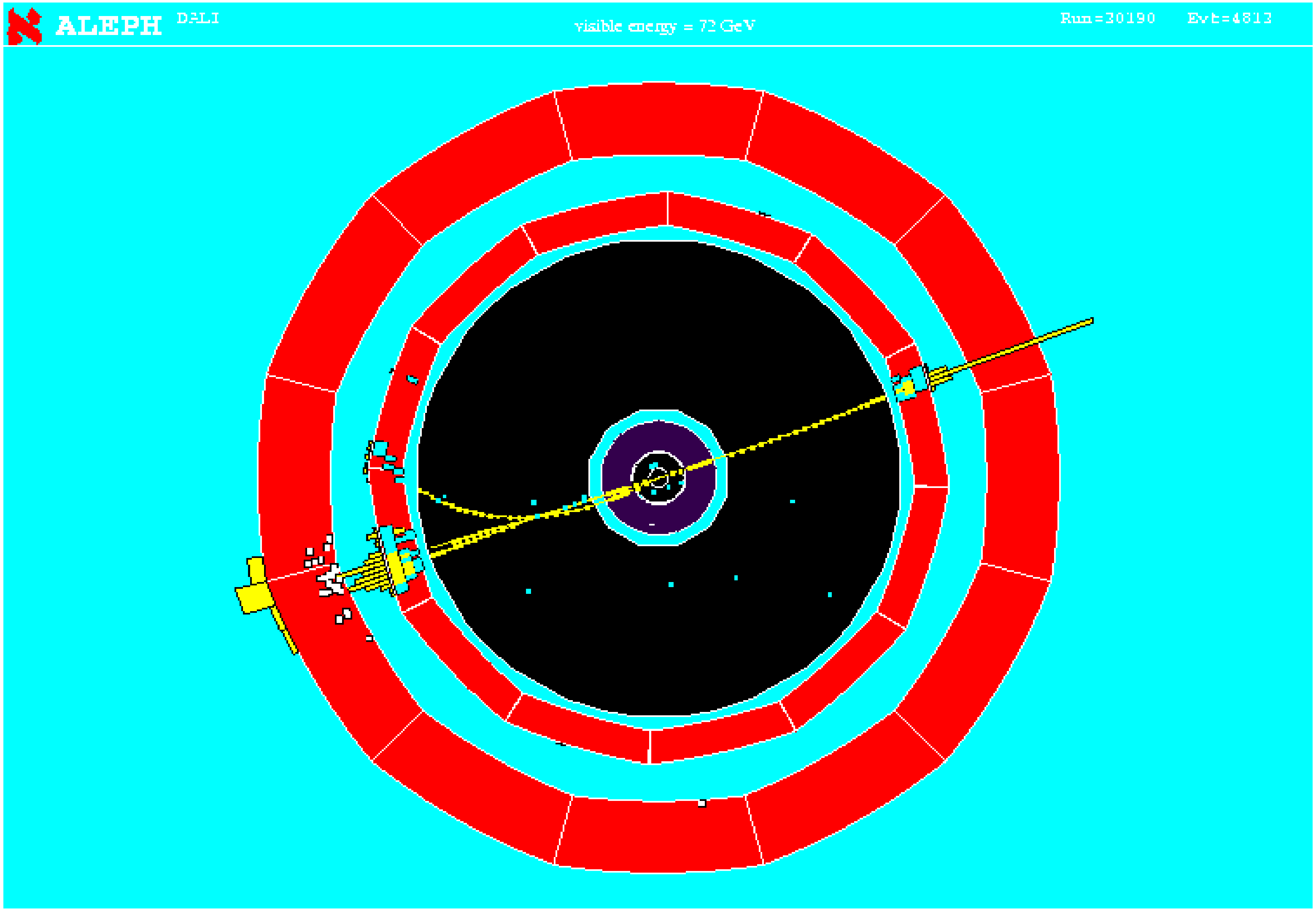
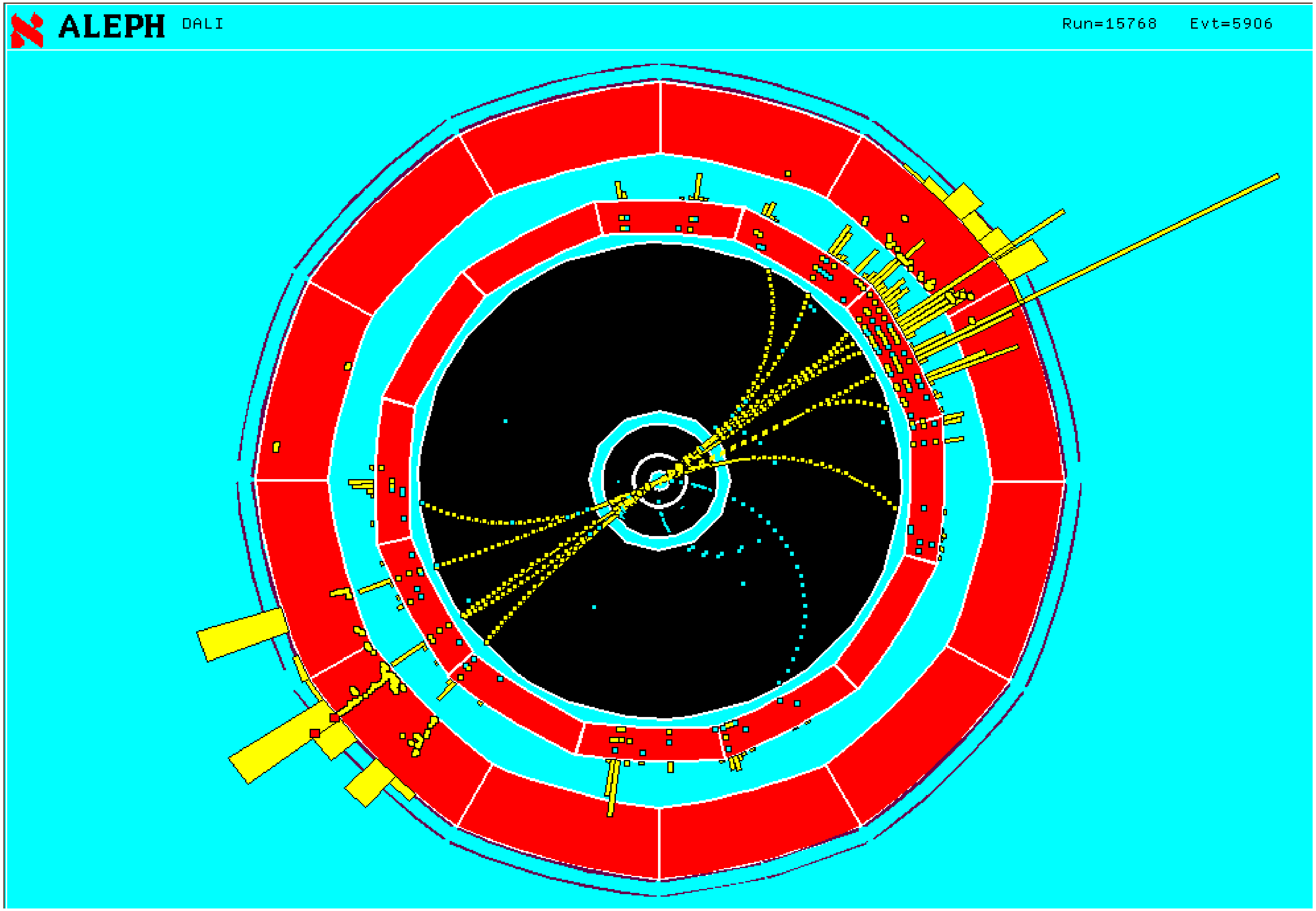


Event selection and Identification – what is this?



Partial width results, revisited

- Following the branching fractions, the partial widths can be computed as

$$Br(i) = \frac{\Gamma_i}{\Gamma_{tot}}$$

- It was found:

Γ(Z → ll)	Γhad	Γinv
83.98 +- 0.09 MeV	1744 +- 2 MeV	499 +- 1

- compared to our **radiatively corrected** predictions:

Γ(Z → ll)	Γhad	Γinv
83.25 MeV	1713 MeV	495

Interpretation!

- We have now included all non-ambiguous corrections, it is time to look at the electroweak corrections! Remember,

$$v_f = \sqrt{1+\Delta\rho}\,\left(T_{3f} - 2Q_f \Delta\kappa \sin^2\theta_W\right) \qquad a_f = \sqrt{1+\Delta\rho}\; T_{3f}$$

$$\Delta\rho = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} - f(\ln(M_H)) \qquad \Delta\kappa = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\frac{\cos^2\theta_W}{\sin^2\theta_W} - f'(\ln(M_H))$$

- Can you estimate the top mass? What is the simplest channel? Remember:

$$\Gamma_{Z\to f\bar{f}} = \frac{G_F}{\sqrt{2}}\frac{M_Z^3}{6\pi}\left(v_f^2 + a_f^2\right)$$

	ν	μ	u	d
gv	1/2	-0.04	0.19	-0.34
ga	1/2	-1/2	1/2	-1/2

Interpretation!

- Take the neutrino width (zero charge!). We can write the ratio of the radiatively corrected width to the tree-level width:

$$\frac{\Gamma_{\nu\nu}^{RC}}{\Gamma_{\nu\nu}^{0}} = 1+\Delta\rho$$

- The RC width should be our best description of the reality, so we put

$$\Gamma_{\nu\nu}^{RC} = \Gamma_{\nu\nu}^{obs}$$

- We thus find, for the top mass (let us neglect the small M_H contribution):

$$m_t^2 = \frac{8\sqrt{2}\pi^2}{3G_F}\left(\frac{\Gamma_{\nu\nu}^{obs}}{\Gamma_{\nu\nu}^{0}} - 1\right)$$

or

$$m_t = 161 \pm 20\, GeV$$

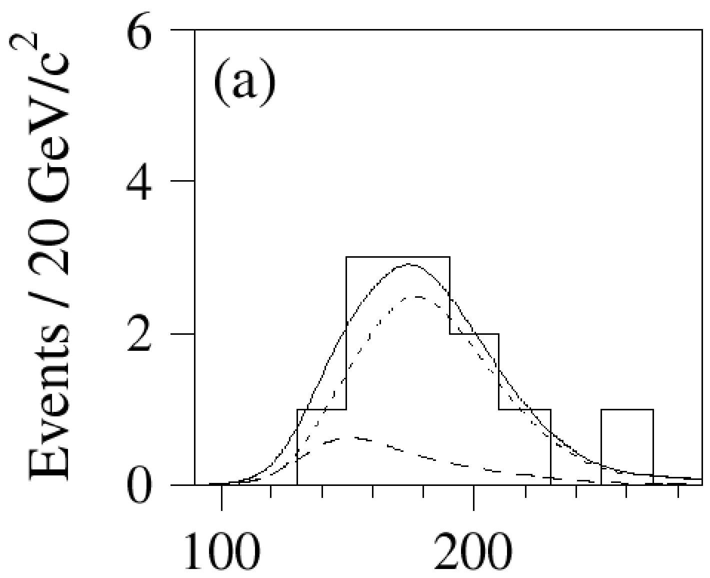
Z precision measurements : overall strategy

- The measurement campaign is much more involved than what I showed here. In particular:
  - It was desired to measure the v and a couplings of all identifiable final states individually, and check their consistence. For example, we want to verify that the analysis of the μμ and bb channels lead to the same top quark mass.
  - It was desired to verify assumptions like lepton universality, number of light neutrino families, etc.
  - Finally, all data need to be combined in order to predict the top quark mass as precisely as possible! The complete result is, including all channels::

$$m_t = 173 \pm 12\, GeV$$

The top quark discovery

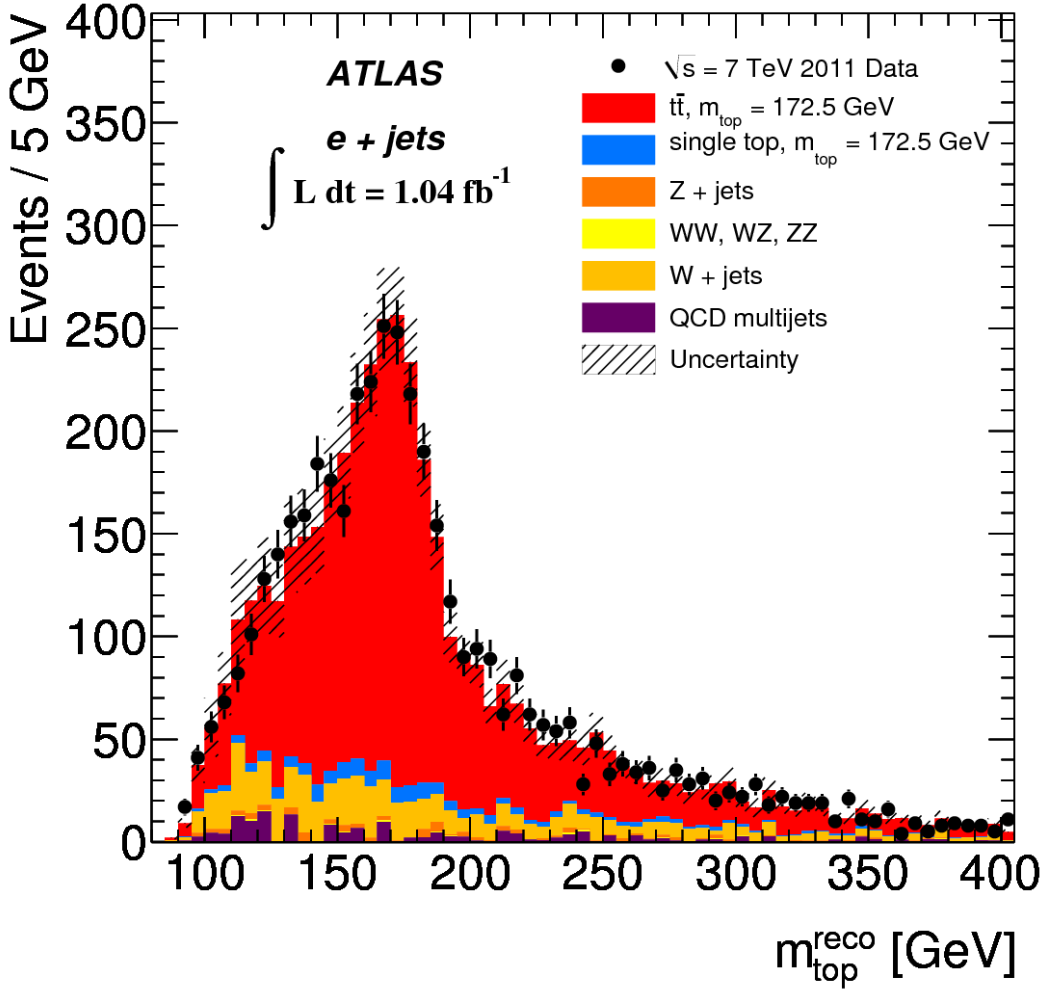
- … happened in 1994, at the Tevatron:
  - The production process is mainly $q\bar{q} \rightarrow t\,\bar{t}$
  - The decay mode is mainly: $t \rightarrow W\,b$
  - And all W decay modes are exploited.



M_{top} ~ 180 +/- 12 GeV

Confirmation of the electroweak theory at the quantum level!

The top quark today



M_{top} = 172 +/- 1.2 GeV

References

- Discovery and other interesting experimental papers

Phys.Rev.105:1413-1414,1957	discovery of parity violation
UA1, Phys.Lett.B126:398-410,1983 UA1, Phys.Lett.B122:103-116,1983 UA2, Phys.Lett.B129:130-140,1983 UA2, Phys.Lett.B122:476-485,1983	W and Z discoveries
Phys.Lett.B241:150-164,1990.	W mass measurement at UA2
Phys.Rept.427:257,2006	Summary of LEP1 precision measurements
Phys.Rev.Lett.74:2632-2637,1995 Phys.Rev.Lett.74:2626-2631,1995	Top quark discovery
Eur.Phys.J.C55:1-38,2008 Eur.Phys.J.C47:309-335,2006	W mass measurements at LEP

References

- Information on particle data: http://pdg.lbl.gov

  (ask your institutes to order booklets!)

- Lectures on statistical data analysis: http://www.hep.ph.rhul.ac.uk/~cowan

  (a colleague who gave many lectures on these issues)

- The VERY BEST resource on electroweak corrections: http://inspirehep.net/record/288139