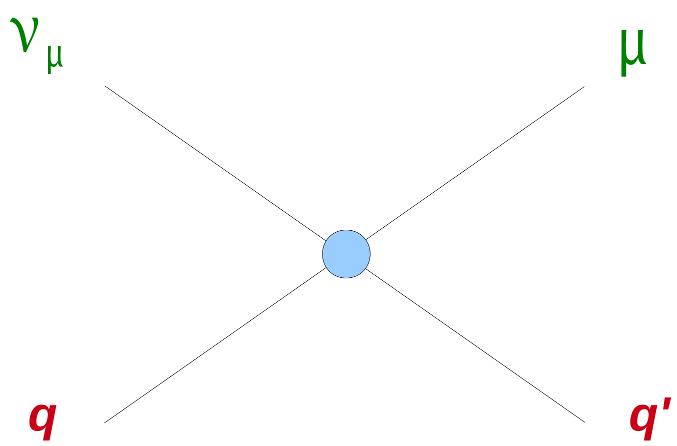
A FIRST MEASUREMENT OF **$\sin^2\theta_W$**

A first measurement of $\sin^2\theta_W$

- How to design a measurement that is specifically sensitive to $\sin^2\theta_W$?

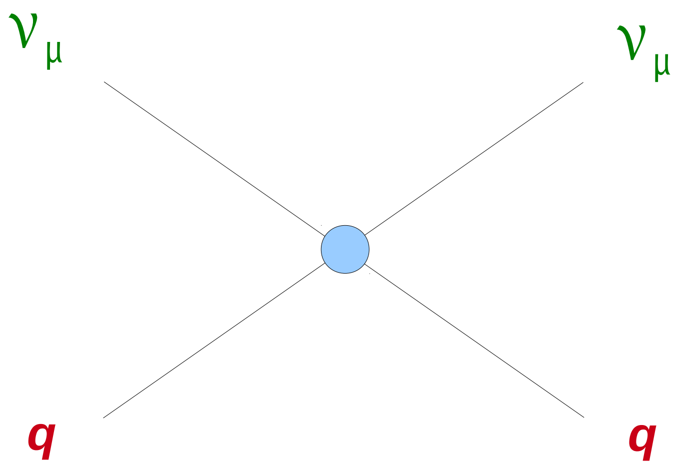
  Remember, the charged current itself does not allow to disentangle the weak mixing angle and the W boson mass:



$$\sim \frac{e^2}{8\sin^2\theta_W M_W^2} = \frac{G_F}{\sqrt{2}}$$

A first measurement of $\sin^2\theta_W$

- The neutral current IS sensitive to $\sin^2\theta_W$:



$$\sim \frac{e^2}{8\sin^2\theta_W M_Z^2} \times (g_V, g_A)$$

$$= \frac{G_F}{\sqrt{2}} \times (1 - \sin^2\theta_W) \times (g_V, g_A)$$

But there are many problems converting the observed number of events into a theoretical rate!

Interlude : cross-section measurements

- Physically, we want to determine the fundamental parameters of a theory
- We are measuring **cross-sections,** which are functions of these parameters
  = theoretical event rates
- In experiments we measure **numbers of events** which reflect
  - The running time of the experiment, and beam and target density, summarized by the **luminosity parameter, L**
  - Backgrounds, **B**
  - Detector **efficiency, ε**, and **finite acceptance A**
- Both are related by $$N = L \sigma \epsilon A + B$$

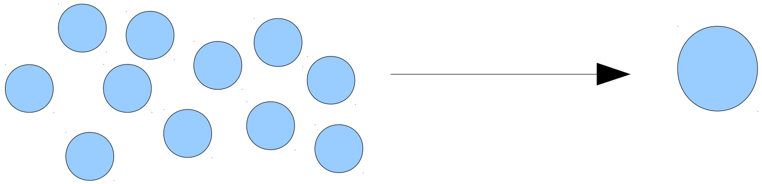
or

$$\sigma = \frac{N - B}{L \epsilon A}$$

… cross-section and luminosity

- Cross-sections are defined as **surfaces** (hence the name!)
  Hence Luminosity is defined as "**N per unit surface**" such that the number of collisions is $\mathbf{N_{coll} = \sigma \times L}$. Consider the problem of classic billiard balls:

- The process cross-section is $\mathbf{\sigma = 4\pi(r+R)^2}$
- Assume a beam of incoming balls:



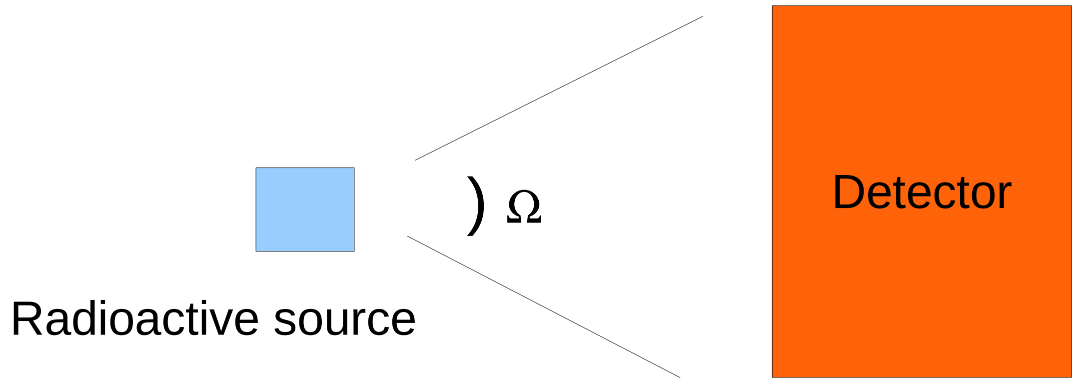
  One is easily convinced that the number of collisions is

$$N_{coll} = \sigma \times \frac{N_{beam}}{\sigma_{beam}} = \sigma \times \frac{dN_{beam}/dt}{\sigma_{beam}} \Delta t == \sigma \times L$$

- We keep the same definitions in particle physics, except $\boldsymbol{\sigma}$ is less trivial!

… acceptance and efficiency

- These parameters are related to the detector.
  Imagine the simple example of a detector measuring electrons from β decay.



- In the simple case (no polarization, …) the emission is isotropic. The detector will only catch those that come into its direction. The acceptance is the fraction of solid angle covered, as seen from the source:

$$A=\frac{\Omega}{4\pi}$$

- The efficiency is the fraction of recorded electrons that pass "quality cuts"

$$\epsilon=\frac{N_{accepted}}{N_{recorded}}$$

… uncertainty estimation

- Simple error propagation gives, in terms of relative uncertainties:

$$\frac{\delta\sigma}{\sigma} = \frac{\delta N \oplus \delta B}{N-B} \oplus \frac{\delta L}{L} \oplus \frac{\delta A}{A} \oplus \frac{\delta\epsilon}{\epsilon}$$

- Uncertainty on N : the “statistical uncertainty” (fluctuations, expt. by expt.) As we saw, event counting follows the Poisson distribution. At large N, this tends to the Gaussian distribution, of width: $\sigma = \sqrt{N}$

- Choosing the convention to quote “1σ” uncertainties (consistently throughout all terms above!) refers to the interval [μ–σ, μ+σ] of the gaussian function representing the Probability Density Value of the true value, containing 68% of its integral. In other words, we claim that the true value has 68% chance lying in the interval $[\sigma_{meas} - \delta\sigma_{meas}, \sigma_{meas} + \delta\sigma_{meas}]$

- Other conventions:

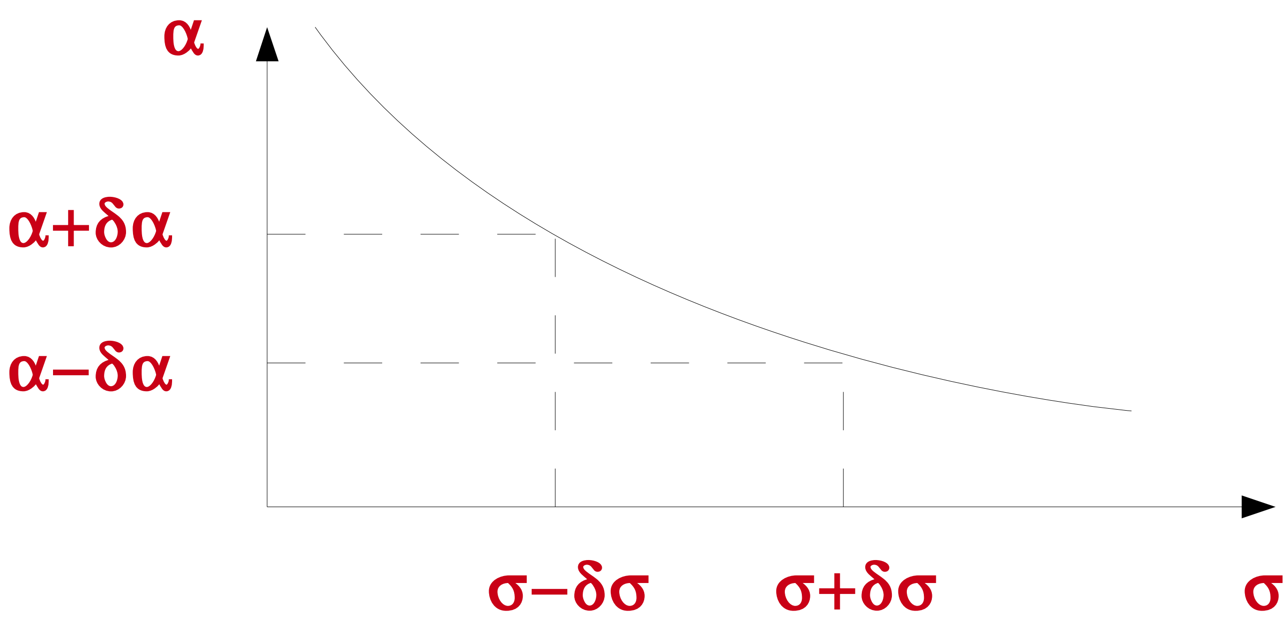
“1σ”	“2σ”	“3σ”	“5σ”
~68% CL	~95% CL	~99.7% CL	~99.994% CL

… link to the fundamental parameter

- Predicted cross-section as a function of the fundamental parameter:

$$\sigma = f(\alpha)$$

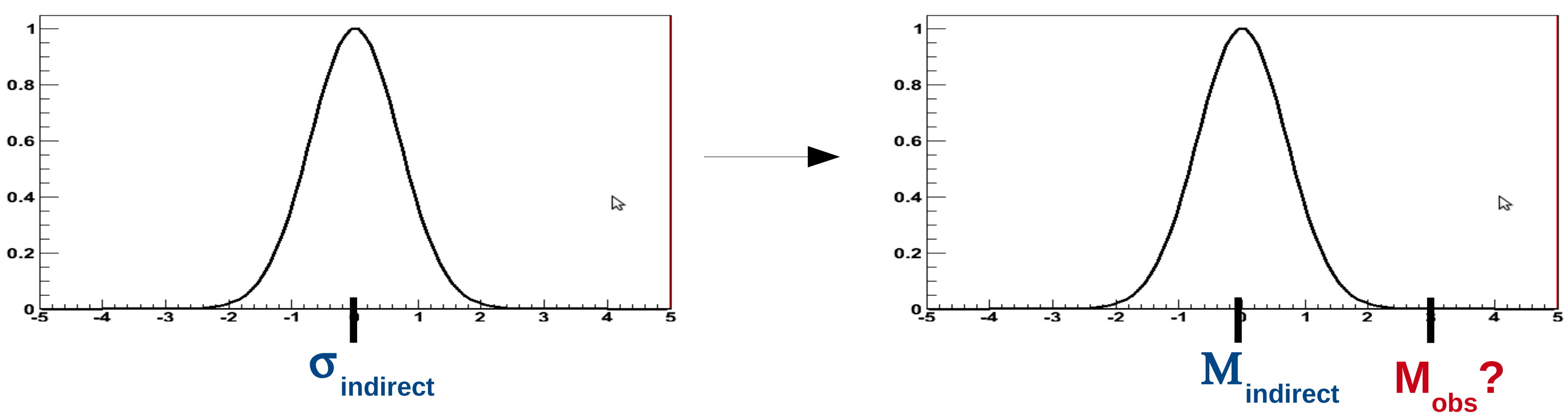
- Measured value: $\alpha = f^{-1}(\sigma)$ $\quad \delta\alpha = \frac{\partial f^{-1}}{\partial \sigma}\delta\sigma$



… link to the fundamental parameter

- Finally, imagine that α can be measured from elsewhere
  For example, it is a particle mass, **M.** We can search and discover this particle at some mass M_{obs}

- Then we can confront our two values of M:
  - extracted from the cross-section measurement, $M_{indirect}$
  - directly observed, M_{obs}



Coming back to our case

- Let us discuss the uncertainties affecting the neutral current cross-section measurement

  - **The backgrounds were shown to be small ($\nu_\mu e \rightarrow \nu_\mu e$) or under control (hadronic neutral currents)**, cf our previous discussion

  - **The detector acceptance is large, and efficiency ~ 100%**

  - Several parameters enter the Luminosity parameter:
    - **Running time : should be OK!**
    - Beam and target density : here are the problems.

      The neutrino flux is not well known, and the neutrino beam profile is uncertain

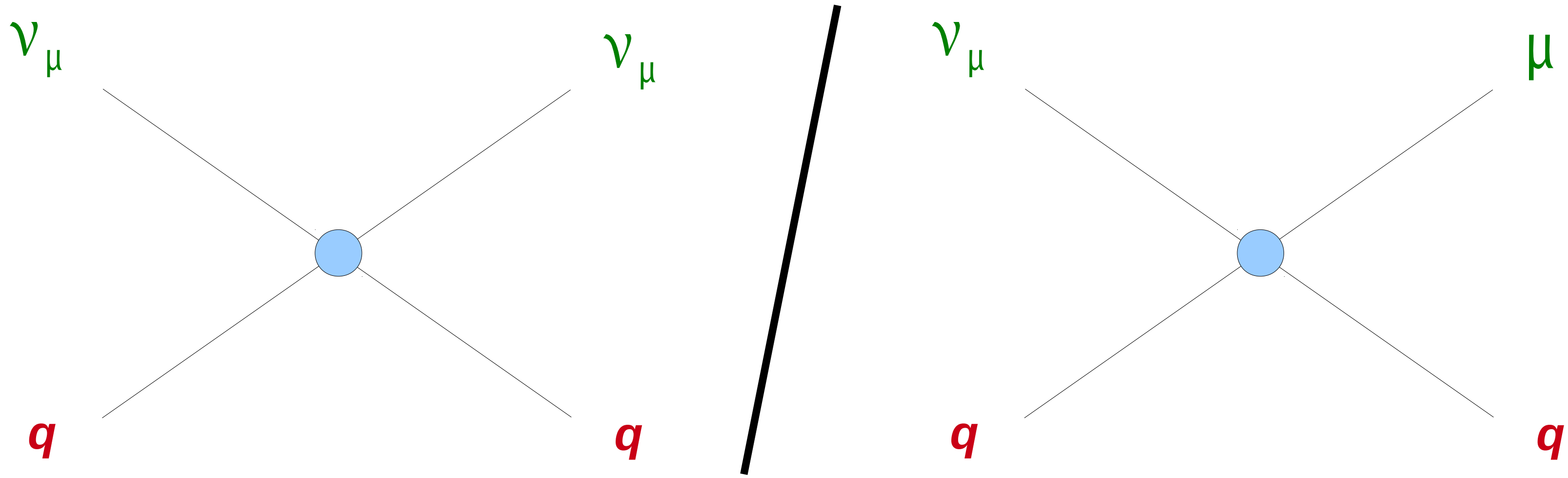
      The target density is well known in terms of atoms and molecules, and thus of nucleons **BUT the cross-section is computed in terms of quarks!**
      The measurement relies on the quark densities in the nucleons

- We will discuss the concept of nucleon PDFs and related problems later.

The measurement of $\sin^2\theta_W$

- We can imagine measuring the ratio:

***R* =**



$$\sim \frac{M_W^2}{M_Z^2}\,(\dots) \sim \left(1 - \sin^2\theta_W\right)\,(\dots)$$

- The cross-section ratio writes $$\frac{\sigma_{NC}}{\sigma_{CC}} = \frac{N_{NC}-B_{NC}}{L\epsilon A} \times \frac{L\epsilon A}{N_{CC}-B_{CC}} = \frac{N_{NC}-B_{NC}}{N_{CC}-B_{CC}}$$

- All uncertainties drop out! We are left will a simple counting experiment.

A first measurement of $\sin^2\theta_W$

- Another common approach in experimental measurements! Try to define the observable such that it is
  - **As sensitive as possible to the fundamental parameter to be measured**
  - **As insensitive as possible to other ingredients of the experiments.**
- The optimal combination of cross sections, that is only sensitive to the weak mixing angle, is

$$R = \frac{\sigma(\nu \to \nu) - \sigma(\bar{\nu} \to \bar{\nu})}{\sigma(\nu \to \mu) - \sigma(\bar{\nu} \to \bar{\mu})} = \frac{1}{2} - \sin^2\theta_W$$

- An early measurement of this quantity gives : **R = 0.3 +/- 0.06**

A first measurement of $\sin^2\theta_W$

- Exercise : what do we find for **$\sin^2\theta_W$? For M_W? For M_Z?**

- Exercise and discussion : suppose we proceed with beam-on-target experiments. Can we produce these particles? Do we need a new approach? Assume:

Beam; E ~100 GEV

Nucleon, M=1 GeV

Exercises

- Gauge boson mass estimation
  - Use **R = 0.3 +/- 0.06.** What do we find for **$\sin^2\theta_W$? For M_W? For M_Z?**
  - Reminder : $$G_F = \frac{\pi\alpha}{\sqrt{2} m_W^2 \sin^2\theta_W^{\text{tree}}}$$
- Which beam energy do we need, for a fixed target experiment, to produce a W or a Z boson? Is this realistic?