THE STANDARD MODEL

Additional ingredients

- We want to build a renormalizable gauge theory describing:
  - **QED at low energy**
  - **weak interactions, β decays**.

- This is achieved by postulating:
  - a **U(1) "hypercharge"** local symmetry → **massless vector B**
  - a **$SU(2)_L$** local symmetry → **massless W^+,W^0,W^-**
  - a complex **scalar SU(2) doublet** and ϕ^4 potential → **the Higgs field**
  - In the ground state, the **v.e.v couples to the bosons** → **massive H and W's**
  - **B and W0** have same spin/parity and **mix** → **γ, Z; $m_\gamma = 0$; $m_Z \neq m_W$**

Additional ingredients

- We want to build a renormalizable gauge theory describing:
  - **QED at low energy**
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- Boundary conditions
  - γ – Z mixing parameter : $\sin^2\theta_W$
  - $U(1)_Y$ coupling g_1, $SU(2)_L$ coupling g_2 must satisfy

$$g_1 \sin\theta_W = g_2 \cos\theta_W = e$$

  - The Fermi interaction is now interpreted as W or Z exchange at *Q~0* and

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W^2} = \frac{e^2}{8\sin^2\theta_W \cos^2\theta_W M_Z^2}$$

Standard Model relations

- Predictions at the tree level (sufficient for now)

- The Mw / Mz relation is dictated by the weak mixing angle and the Higgs sector:

$$\rho_0 = \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2 \cos^2 \theta_{\mathrm{W}}^{\mathrm{tree}}}.$$

ρ_0 = 1 in models with only Higgs singlets and doublets (such as the SM).

- The left and right (or vector and axial) coupling strengths of the fermions are given by:

$$g_{\mathrm{L}}^{\mathrm{tree}} = \sqrt{\rho_0}\,(T_3^{\mathrm{f}} - Q_{\mathrm{f}} \sin^2 \theta_{\mathrm{W}}^{\mathrm{tree}}) \qquad g_{\mathrm{V}}^{\mathrm{tree}} \equiv g_{\mathrm{L}}^{\mathrm{tree}} + g_{\mathrm{R}}^{\mathrm{tree}} = \sqrt{\rho_0}\,(T_3^{\mathrm{f}} - 2Q_{\mathrm{f}} \sin^2 \theta_{\mathrm{W}}^{\mathrm{tree}})$$

$$g_{\mathrm{R}}^{\mathrm{tree}} = -\sqrt{\rho_0}\, Q_{\mathrm{f}} \sin^2 \theta_{\mathrm{W}}^{\mathrm{tree}}, \qquad g_{\mathrm{A}}^{\mathrm{tree}} \equiv g_{\mathrm{L}}^{\mathrm{tree}} - g_{\mathrm{R}}^{\mathrm{tree}} = \sqrt{\rho_0}\, T_3^{\mathrm{f}}.$$

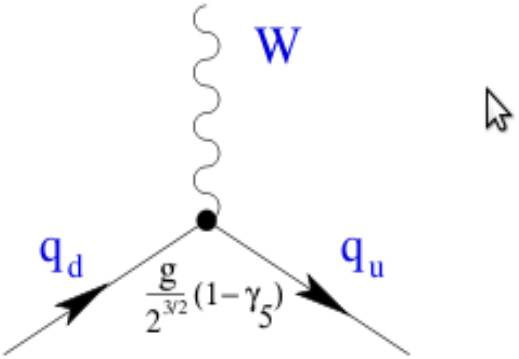
Standard Model relations

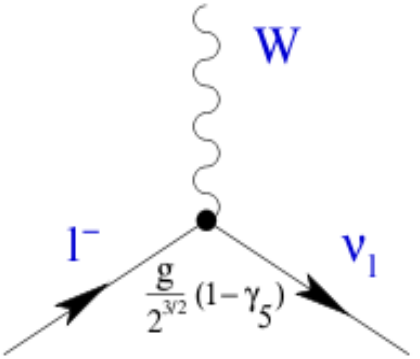
- Fermions and quantum number assignments:

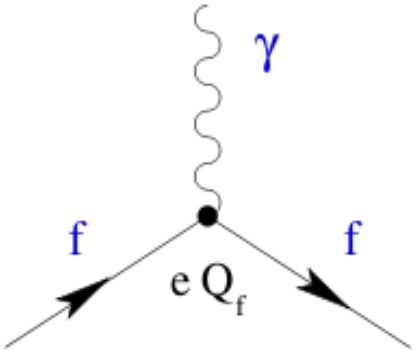
Family			T	T_3	Q
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$1/2$	$+1/2$ $-1/2$	0 -1
ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	0	0	0
e_R	μ_R	τ_R	0	0	-1
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$1/2$	$+1/2$ $-1/2$	$+2/3$ $-1/3$
u_R	c_R	t_R	0	0	$+2/3$
d_R	s_R	b_R	0	0	$-1/3$

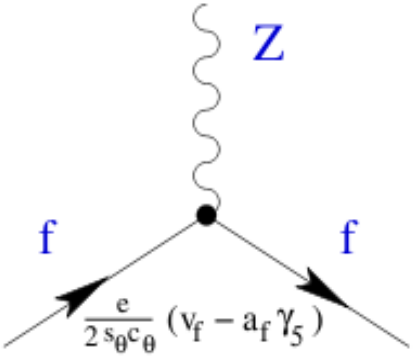
- Exercise : calculate g_V, g_A for all fermions

Vertices and couplings : fermion-boson



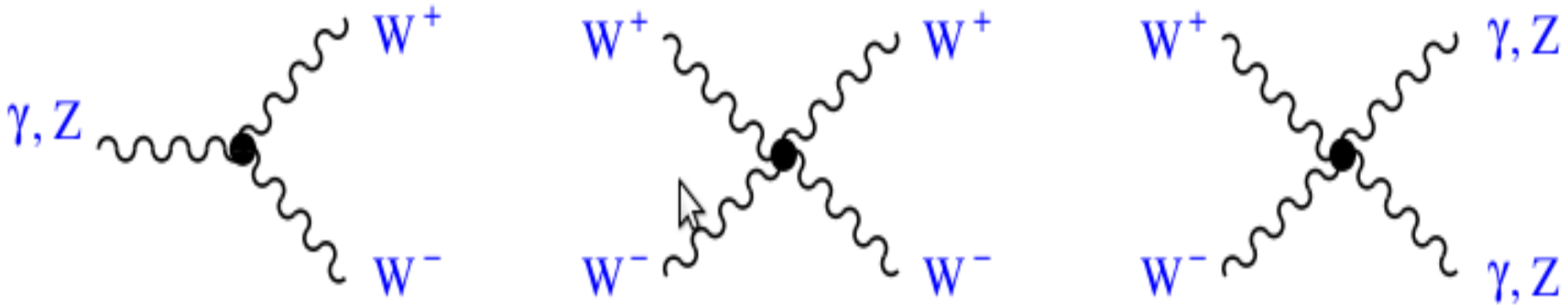






Vertices and couplings : boson-boson

- “TGCs” and “QGCs”

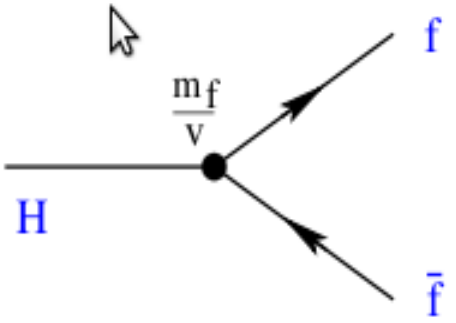


- The couplings are more complicated here (derivative, momentum dependent)

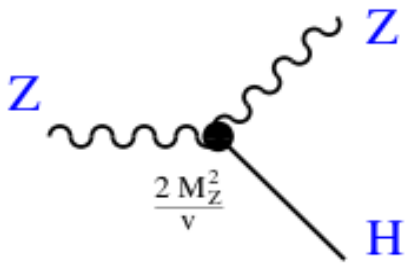
- We will not care about their magnitude, although we will discuss their contribution to the SM validation

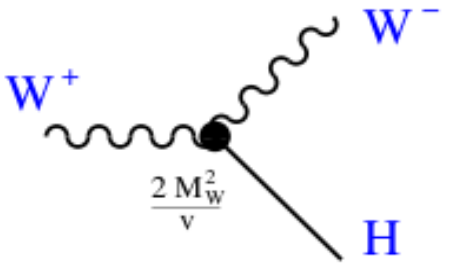
Vertices and couplings : Higgs sector

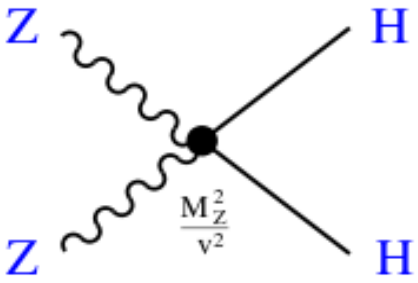
- To the fermions:

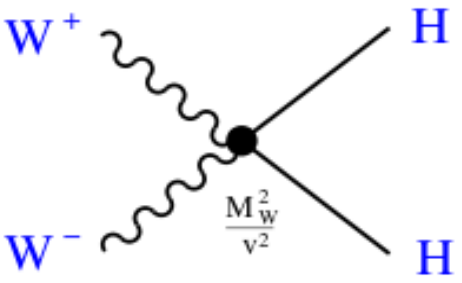


- To the vector bosons:









Summary of tree-level predictions (and post-dictions!)

- **Known, and embedded in the theory**
  - **QED. The photon is re-interpreted as a mixture of W0 and B**
  - **Charged currents (known since β and μ decays, also observed in collision).**
  - **Maximal parity violation in charged current sector. Technically possible via Dirac spinors and the (1-γ5) operator**

Summary of tree-level predictions (and post-dictions!)

- **Genuine new predictions**
  - **Neutral currents (with neutrinos!!)**
  - **Two new heavy resonances : one neutral, one charged (+/-)**
  - **The branching fractions of these resonances**
  - **Couplings among gauge bosons (TGC, QGC)**
  - **The Higgs boson, how to produce and detect it**

HIGHLY NON TRIVIAL!

- Agreed, the SM construction is “ad-hoc”. But it is still very predictive!

- Having derived the interaction from symmetry has important implications! Assuming the W bosons are the generators of a new symmetry group has had implications:
  - The smallest group having a pair of charged generators is SU(2)
  - SU(2) has THREE generators → Existence of a new heavy neutral boson
  - SU(2) is non-abelian → Couplings among gauge bosons
  - The weak mixing angle determines the W and Z mass ratio AND the fermion couplings to the Z
  - Existence of the Higgs boson.

Exercises

- Calculate the expected values of g_V, g_A for all fermions

- Calculate the Higgs boson branching fractions, assuming only light decay modes are open. Use:
  - b quark mass ~ 4.5 GeV
  - τ lepton mass = 1.8 GeV
  - c quark mass = 1.5 GeV
  - s quark mass = 200 MeV
  - photon, gluon mass = 0