

# Introduction to the electronic properties of semiconductor nanostructures.

Yann Michel Niquet

(CEA/DRFMC/SP2M/L\_Sim, Grenoble, France)

Many Thanks to the VSOP organizing committee, and to the Franch embassy in Vietnam for a travel grant.



Chartreuse mountains

Belledonne mountains

ILL (Neutrons)

L\_Sim

ESRF (Synchrotron)

## General outline

---



- **Part I :**  
Introduction to semiconductor nanostructures
- **Part II :**  
Electronic structure methods
- **Part III :**  
Self-energy and excitonic corrections in nanostructures



---

## Part I

### Introduction to semiconductor nanostructures

## Outline

---



- **I.1 :**  
Electrons and holes
- **I.2 :**  
Semiconductor materials and nanostructures
- **I.3 :**  
Numerical simulation in nanosciences : Challenges and perspectives



---

## I.1 : Electrons and holes

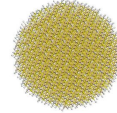
## Non-interacting systems : Ground-state (I)



- Consider  $N$  electrons moving in a one-body potential  $v(\mathbf{r})$ . We look for their ground-state energy  $E_0(N)$  and for their ground-state wavefunction  $\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  :

$$\hat{H}_N = \sum_{i=1}^N \hat{h}_i \quad \text{with} \quad \hat{h}_i = -\frac{\hbar^2}{2m_0} \Delta_{\mathbf{r}_i} + v(\mathbf{r}_i)$$

$$\hat{H}_N \Psi_0(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = E_0(N) \Psi_0(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$$



- Solution** : compute the spectrum of the one-particle Hamiltonian  $h...$

$$-\frac{\hbar^2}{2m_0} \Delta_{\mathbf{r}} \varphi_i(\mathbf{r}) + v(\mathbf{r}) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r}) \quad [\varepsilon_i \text{ twofold spin degenerate}]$$

... then build a Slater determinant with the  $N$  first  $\varphi_i$ 's :

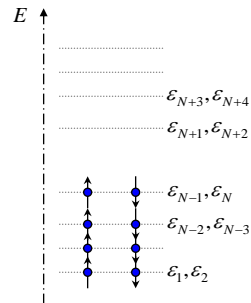
$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_N(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_N(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1(\mathbf{r}_N) & \varphi_2(\mathbf{r}_N) & \cdots & \varphi_N(\mathbf{r}_N) \end{vmatrix} \quad E_0(N) = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_N$$

## Non-interacting systems : Ground-state (II)



$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{N-1}, \mathbf{r}_N) = \begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_{N-1}(\mathbf{r}_1) & \varphi_N(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_{N-1}(\mathbf{r}_2) & \varphi_N(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_1(\mathbf{r}_{N-1}) & \varphi_2(\mathbf{r}_{N-1}) & \cdots & \varphi_{N-1}(\mathbf{r}_{N-1}) & \varphi_N(\mathbf{r}_{N-1}) \\ \varphi_1(\mathbf{r}_N) & \varphi_2(\mathbf{r}_N) & \cdots & \varphi_{N-1}(\mathbf{r}_N) & \varphi_N(\mathbf{r}_N) \end{vmatrix}$$

$$\text{Density } n_0(\mathbf{r}) = \sum_{i=1}^N |\varphi_i(\mathbf{r})|^2$$



$$E_0(N) = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_{N-1} + \varepsilon_N$$

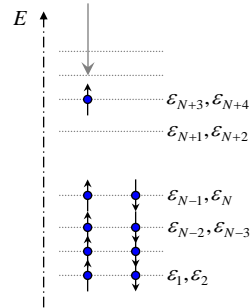
Fill the  $N$  lowest levels

## Non-interacting systems : Addition energies



$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N, \mathbf{r}_{N+1}) = \begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_N(\mathbf{r}_1) & \varphi_{N+3}(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_N(\mathbf{r}_2) & \varphi_{N+3}(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_1(\mathbf{r}_N) & \varphi_2(\mathbf{r}_N) & \cdots & \varphi_N(\mathbf{r}_N) & \varphi_{N+3}(\mathbf{r}_N) \\ \varphi_1(\mathbf{r}_{N+1}) & \varphi_2(\mathbf{r}_{N+1}) & \cdots & \varphi_N(\mathbf{r}_{N+1}) & \varphi_{N+3}(\mathbf{r}_{N+1}) \end{vmatrix}$$

$$n(\mathbf{r}) = \sum_{i=1}^N |\varphi_i(\mathbf{r})|^2 + |\varphi_{N+3}(\mathbf{r})|^2 = n_0(\mathbf{r}) + |\varphi_{N+3}(\mathbf{r})|^2$$



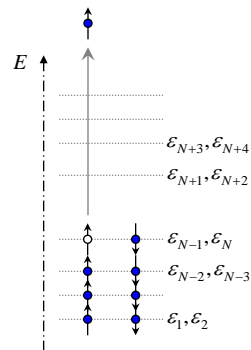
$$E = \epsilon_1 + \epsilon_2 + \dots + \epsilon_{N-1} + \epsilon_N + \epsilon_{N+3} \\ = E_0(N) + \epsilon_{N+3}$$

## Non-interacting systems : Ionization energies



$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{N-2}, \mathbf{r}_{N-1}) = \begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_{N-2}(\mathbf{r}_1) & \varphi_N(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_{N-2}(\mathbf{r}_2) & \varphi_N(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_1(\mathbf{r}_{N-2}) & \varphi_2(\mathbf{r}_{N-2}) & \cdots & \varphi_{N-2}(\mathbf{r}_{N-2}) & \varphi_N(\mathbf{r}_{N-2}) \\ \varphi_1(\mathbf{r}_{N-1}) & \varphi_2(\mathbf{r}_{N-1}) & \cdots & \varphi_{N-2}(\mathbf{r}_{N-1}) & \varphi_N(\mathbf{r}_{N-1}) \end{vmatrix}$$

$$n(\mathbf{r}) = \sum_{i=1}^{N-2} |\varphi_i(\mathbf{r})|^2 + |\varphi_N(\mathbf{r})|^2 = n_0(\mathbf{r}) - |\varphi_{N-1}(\mathbf{r})|^2$$



$$E = \epsilon_1 + \epsilon_2 + \dots + \epsilon_{N-2} + \epsilon_N \\ = E_0(N) - \epsilon_{N-1}$$

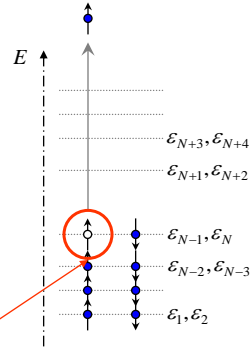
## Non-interacting systems : Ionization energies



$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{N-2}, \mathbf{r}_{N-1}) =$$

$$\begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_{N-2}(\mathbf{r}_1) & \varphi_N(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_{N-2}(\mathbf{r}_2) & \varphi_N(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_1(\mathbf{r}_{N-2}) & \varphi_2(\mathbf{r}_{N-2}) & \cdots & \varphi_{N-2}(\mathbf{r}_{N-2}) & \varphi_N(\mathbf{r}_{N-2}) \\ \varphi_1(\mathbf{r}_{N-1}) & \varphi_2(\mathbf{r}_{N-1}) & \cdots & \varphi_{N-2}(\mathbf{r}_{N-1}) & \varphi_N(\mathbf{r}_{N-1}) \end{vmatrix}$$

$$n(\mathbf{r}) = \sum_{i=1}^{N-2} |\varphi_i(\mathbf{r})|^2 + |\varphi_N(\mathbf{r})|^2 = n_0(\mathbf{r}) - |\varphi_{N-1}(\mathbf{r})|^2$$



$$E = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_{N-2} + \varepsilon_N$$

$$= E_0(N) - \varepsilon_{N-1}$$

Hole wavefunction and energy

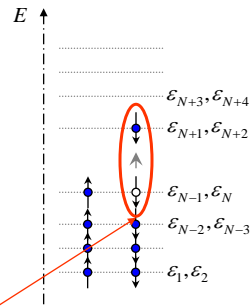
## Non-interacting systems : Excitation energies



$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{N-1}, \mathbf{r}_N) =$$

$$\begin{vmatrix} \varphi_1(\mathbf{r}_1) & \varphi_2(\mathbf{r}_1) & \cdots & \varphi_{N-1}(\mathbf{r}_1) & \varphi_{N+1}(\mathbf{r}_1) \\ \varphi_1(\mathbf{r}_2) & \varphi_2(\mathbf{r}_2) & \cdots & \varphi_{N-1}(\mathbf{r}_2) & \varphi_{N+1}(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_1(\mathbf{r}_{N-1}) & \varphi_2(\mathbf{r}_{N-1}) & \cdots & \varphi_{N-1}(\mathbf{r}_{N-1}) & \varphi_{N+1}(\mathbf{r}_{N-1}) \\ \varphi_1(\mathbf{r}_N) & \varphi_2(\mathbf{r}_N) & \cdots & \varphi_{N-1}(\mathbf{r}_N) & \varphi_{N+1}(\mathbf{r}_N) \end{vmatrix}$$

$$n(\mathbf{r}) = \sum_{i=1}^{N-1} |\varphi_i(\mathbf{r})|^2 + |\varphi_{N+1}(\mathbf{r})|^2 = n_0(\mathbf{r}) - |\varphi_N(\mathbf{r})|^2 + |\varphi_{N+1}(\mathbf{r})|^2$$



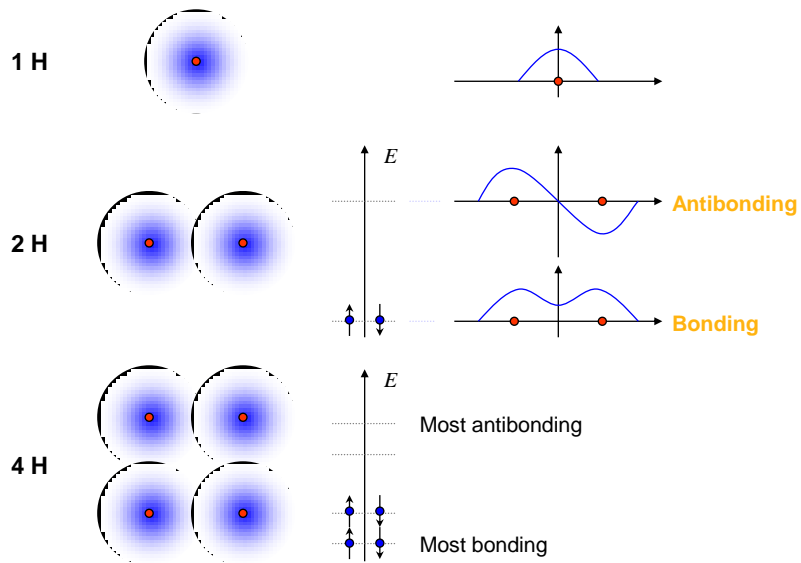
$$E = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_{N-1} + \varepsilon_{N+1}$$

$$= E_0(N) - \varepsilon_N + \varepsilon_{N+1}$$

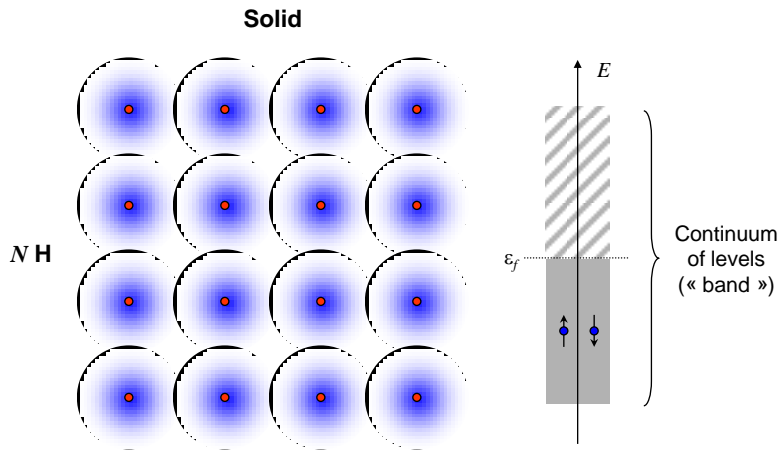
« Electron-hole pair »

## I.2 : Semiconductor materials and nanostructures

### From the atom to the solid (I)



## From the atom to the solid (II)

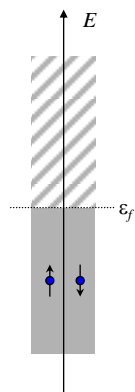


DSM/DRFMC/SP2M/L\_Sim

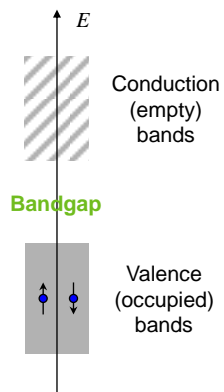
Quy Nhon, 31/12/2007

15

## Bulk metals, insulators, and semiconductors



**Metals**



**Semiconductors (low bandgap)  
Insulators (large bandgap)**

**No low energy electronic excitations**

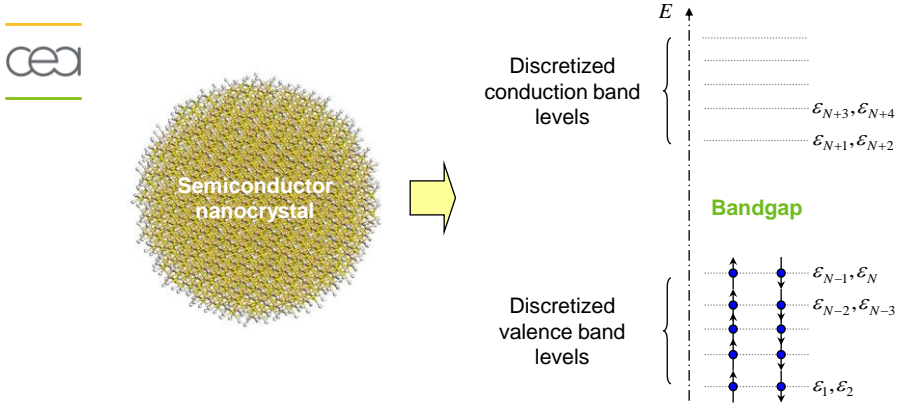
DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

16



## Semiconductor nanostructures



- The bandgap is actually larger than in bulk materials due to « quantum confinement » (see later examples on nanocrystals).

## Usual semiconductors (I)

**Periodic Table of the Elements**

1	IA																2	O																																																																																																																																																																																																																																																																														
1	H																He																																																																																																																																																																																																																																																																															
2	IIA																3	IIIA																4	IVA																5	VA																6	VIA																7	VIIA																8	F																9	Ne																																																																																																																																																																								
3	Na																Mg																13	Al																14	Si																15	P																16	S																17	Cl																18	Ar																																																																																																																																																																									
4	K																Ca																Sc																Ti																V																Cr																Mn																Fe																Co																Ni																Cu																Zn																Ga																Ge																As																Se																Br																Kr															
5	Rb																Sr																Y																Zr																Nb																Mo																Tc																Ru																Rh																Pd																Ag																Cd																In																Sn																Sb																Te																I																Xe															
6	Cs																Ba																*La																Hf																Ta																W																Re																Os																Ir																Pt																Au																Hg																Tl																Pb																Bi																Po																At																Rn															
7	Fr																Ra																+Ac																Rf																Ha																Sg																Ns																Hs																Mt																110																111																112																113																																																																																															
* Lanthanide Series																Ce																Pr																Nd																Pm																Sm																Eu																Gd																Tb																Dy																Ho																Er																Tm																Yb																Lu																																																																
+ Actinide Series																Th																Pa																U																Np																Pu																Am																Cm																Bk																Cf																Es																Fm																Md																No																Lr																																																																

- **Group IV elements** : Si, Ge, C (Diamond).  
Covalent systems.

## Usual semiconductors (II)



Periodic Table of the Elements

1	IA																O																																									
2	IIA																He																																									
3	IIIA																IVA						VA						VIA						VIIA																							
4	IIIB																IVB						VB						VIB						VIIB						VII						IIB						IIB					
5	III																IV						V						VI						VII						VIII						IX						X					
6	III																IV						V						VI						VII						VIII						IX						X					
7	III																IV						V						VI						VII						VIII						IX						X					

\* Lanthanide Series  
 + Actinide Series

- **III-V semiconductors** : GaAs, InAs, InSb...  
 Slightly ionic bonds (« Ga<sup>+</sup>As<sup>-</sup> »).

## Usual semiconductors (III)



Periodic Table of the Elements

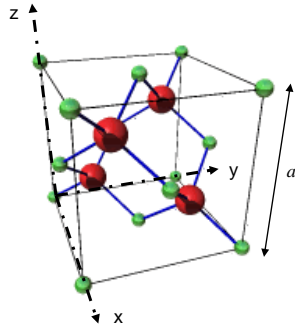
1	IA																O																																									
2	IIA																He																																									
3	IIIA																IVA						VA						VIA						VIIA																							
4	IIIB																IVB						VB						VIB						VIIB						VII						IIB						IIB					
5	III																IV						V						VI						VII						VIII						IX						X					
6	III																IV						V						VI						VII						VIII						IX						X					
7	III																IV						V						VI						VII						VIII						IX						X					

\* Lanthanide Series  
 + Actinide Series

- **II-VI semiconductors** : CdTe, ZnSe...

## Crystal structure

- Most usual semiconductors crystallize in the cubic **Diamond/Zinc-Blende** structure.



It is a **face-centered cubic** (FCC) lattice with a two atom unit cell :  
 – one at (0,0,0).  
 – the other at  $a(1,1,1)/4$ .

In the **Zinc-Blende** structure, each FCC sublattice is occupied by a different atom (e.g. Ga/As, In/P).

In the **Diamond** structure, the two sublattices are occupied by the same atom (e.g. Si, Ge, C).

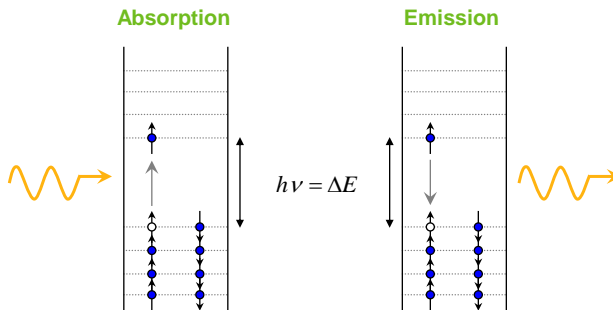
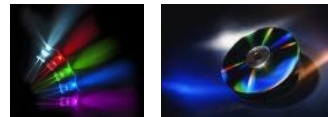
- Alloys can also be synthesized, e.g.  $\text{In}_{0.8}\text{Ga}_{0.2}\text{As}$ .  
 One FCC sublattice is occupied by the In/Ga atoms (80% In+20% Ga ~ randomly distributed), the other by the As atoms.

## Optical applications of semiconductors

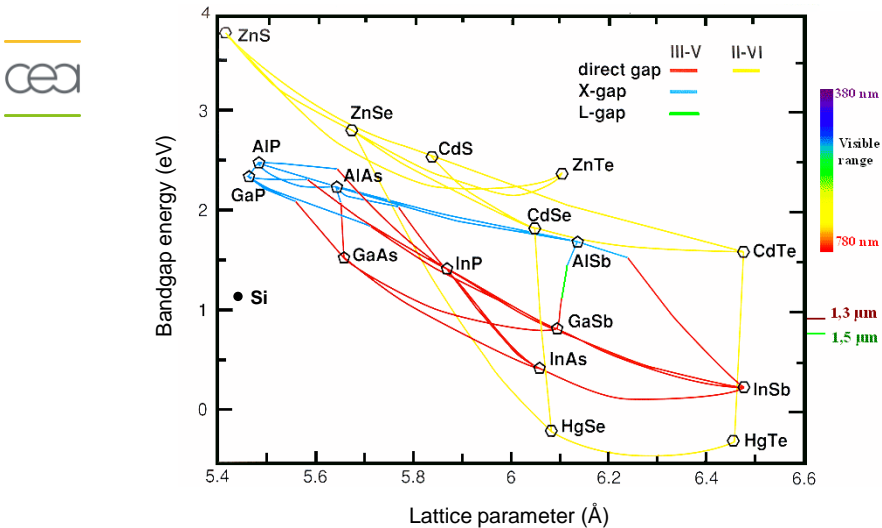
- Well defined bandgap energy :



- Light emission** : LEDs, lasers, ...
- Light absorption/detection** :  
Photovoltaics, ...



## Bandgap vs lattice parameter



DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

23

## Semiconductors for microelectronics

### ● Semiconductors vs metals :

- **Semiconductors** have no low-energy electronic excitations :
  - Highly resistive. ☹️
  - Incomplete screening of the electric fields :

$$V(r) = \frac{q}{\epsilon r}$$

Electric fields can be applied deep inside a semiconductor !.. 😊

- **Metals** have a large density of « free » electrons :
  - Highly conductive. 😊

But :

- Almost complete screening of the electric fields  $\Rightarrow$  Metals are just equipotentials. They can hardly be controlled by external electric fields ! ☹️

### ● « Doped semiconductors » combine the best of the two worlds !

DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

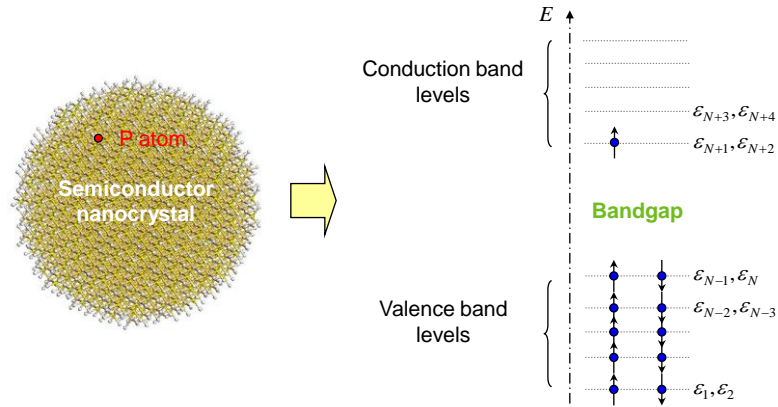
24

## Doping the semiconductors (I)

- Example : **n-type doping** of silicon.



Replace a few silicon atoms with phosphorous (one more electron) :



DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

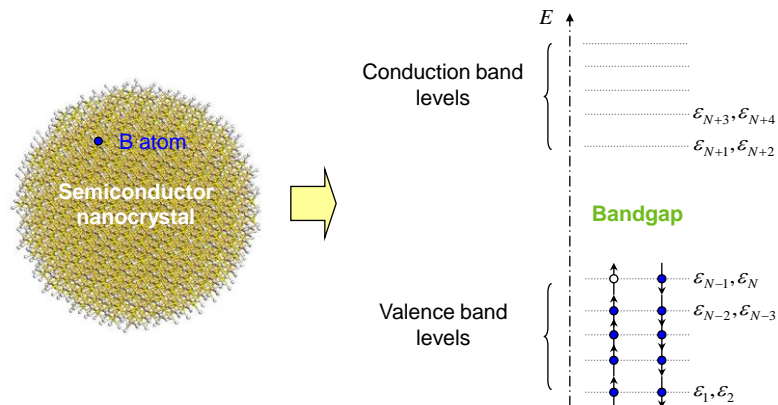
25

## Doping the semiconductors (II)

- Example : **p-type doping** of silicon.



Replace a few silicon atoms with boron (one less electron) :



DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

26

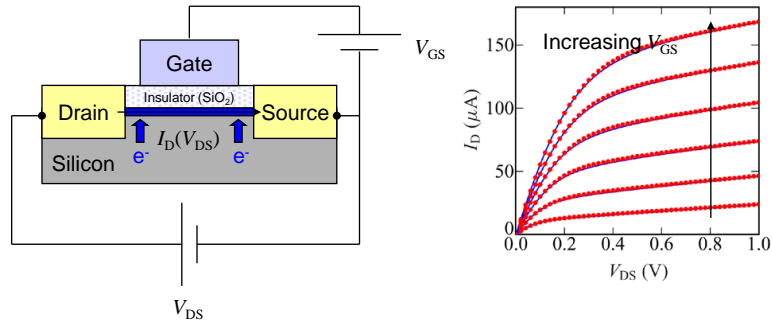
## Application : the field effect transistor

- Doped semiconductors :



- Are reasonably **conductive**.
- Can be **controlled by external electric fields**.

- **Example** : The field-effect transistor – Millions of them in this computer !!



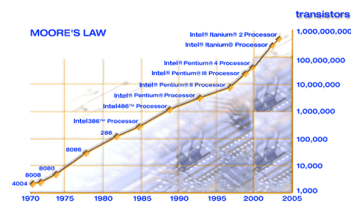
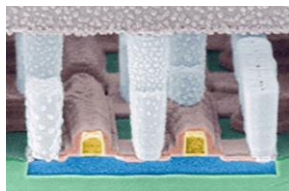
DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

27

## Towards nanoelectronics

- Continuous reduction in the characteristic size of the transistors (Moore's law).  
The number of transistors on a chip **doubles ~ every two years** !



Production : 65 & 45 nm « nodes ».

Research : 32 nm (2010), 22 nm, 16 nm (?)...

⇒ **Technological and physical limitations ?** ⇐

- « **Beyond CMOS** » : « Bottom-up » approaches based on the assembly of nanometer-scale building blocks.

⇒ **New devices** ⇐

DSM/DRFMC/SP2M/L\_Sim

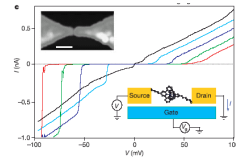
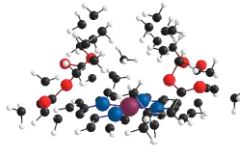
Quy Nhon, 31/12/2007

28

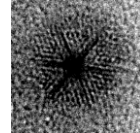
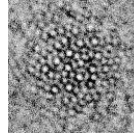
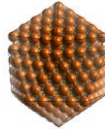
## Some building blocks in nanosciences



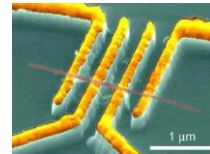
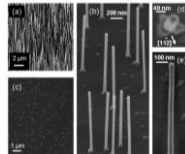
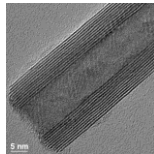
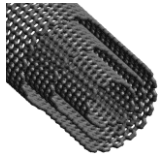
- **Molecules :**



- **Nanocrystals :**



- **Carbon nanotubes, semiconductor nanowires :**



DSM/DRFMC/SP2M/L\_Sim

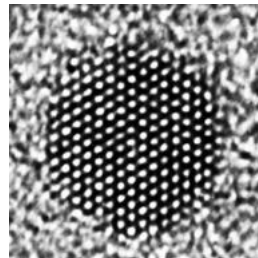
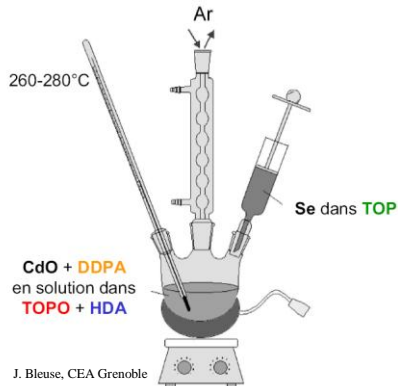
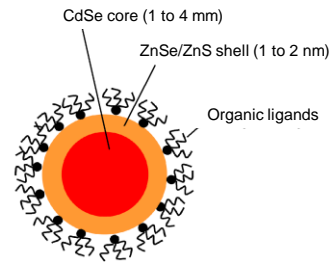
Quy Nhon, 31/12/2007

29

## Semiconductor nanostructures : Nanocrystals (I)



- Colloidal (liquid phase) synthesis of semiconductor **nanocrystals** (CdS, CdSe, ZnSe, InAs, InP...):



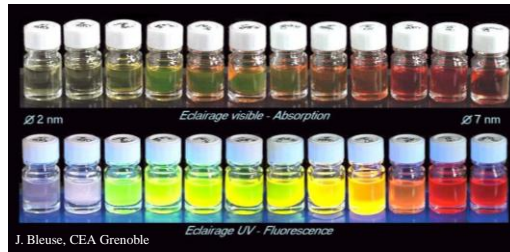
A. P. Alivisatos, J. Phys. Chem. **100**, 13226 (1996).

DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

30

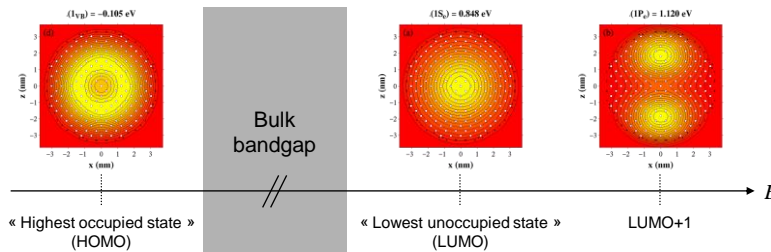
## Semiconductor nanostructures : Nanocrystals (II)



- Quantum confinement :

The bandgap energy increases with decreasing diameter. The absorption & emission shift from red to blue.

- Discrete, atomic-like set of states (« Artificial atoms ») :

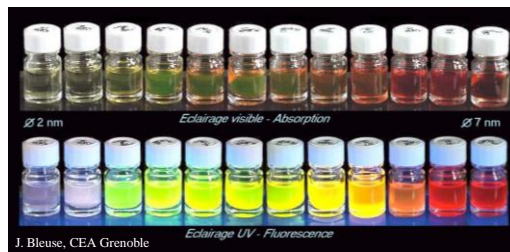


DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

31

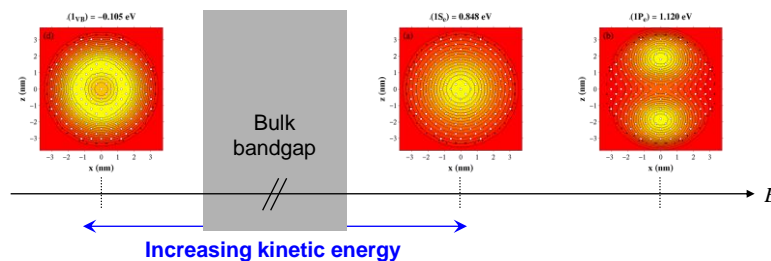
## Semiconductor nanostructures : Nanocrystals (III)



- Quantum confinement :

The bandgap energy increases with decreasing diameter. The absorption & emission shift from red to blue.

- The confinement increases the kinetic energy of the electrons and holes :



DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

32

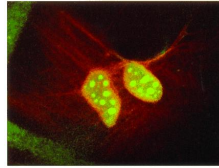
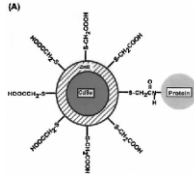


## Semiconductor nanostructures : Nanocrystals (IV)

- Applications (examples) :



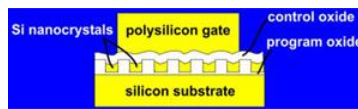
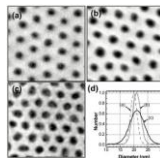
- Fluorescent labels for biology :



The nanocrystals are capped with molecules that bind to specific targets, such as tumor cells for example.

M. Bruchez Jr. et al., Science **281**, 2013 (1998) ; W. C. W. Chan and S. Nie, Science **281**, 2016 (1998)

- Few-electron memories (Si nanocrystals) :

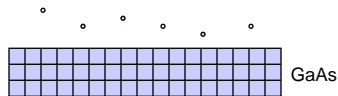


Electrons are stored in the nanocrystals.

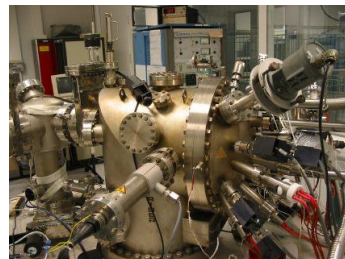
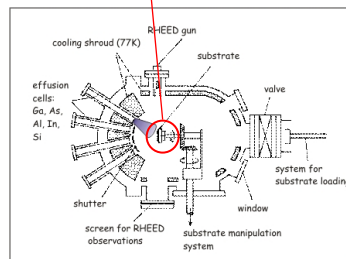
IBM research

## Semiconductor nanostructures : InAs/GaAs dots (I)

- Stransky-Krastanov growth :

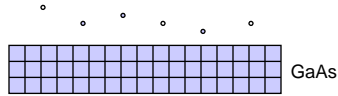


Semiconductor structures can be grown layer by layer using « molecular beam epitaxy ».

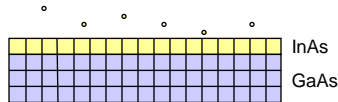


## Semiconductor nanostructures : InAs/GaAs dots (I)

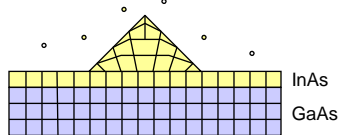
- Stransky-Krastanov growth :



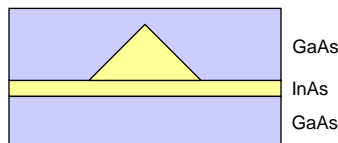
Semiconductor structures can be grown layer by layer using « molecular beam epitaxy ».



The bond length is 6.7% larger in InAs than in GaAs. The InAs layer is thus compressed by the thick GaAs substrate...



InAs pyramids finally grow onto the thin InAs « wetting layer ». The surfaces of the pyramids indeed help relaxing strains.



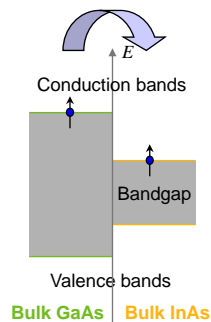
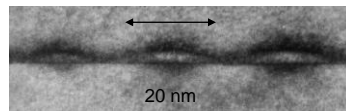
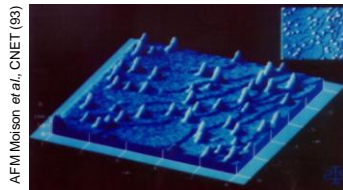
The pyramids can last be embedded in a thick, overgrown GaAs layer.

DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

35

## Semiconductor nanostructures : InAs/GaAs dots (II)



- The bandgap energy is lower in InAs (~0.5 eV) than in GaAs (~1.5 eV). The electrons and holes are thus confined in the InAs « quantum dots ».

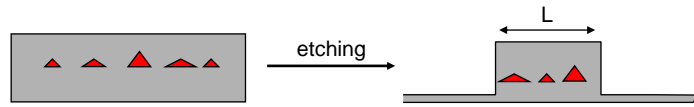
DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

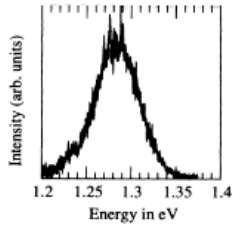
36

## Semiconductor nanostructures : InAs/GaAs dots (III)

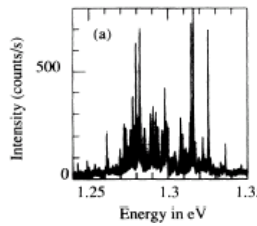
J.Y. Marzin *et al.*, Phys. Rev. Lett. **73**, 716 (1994)



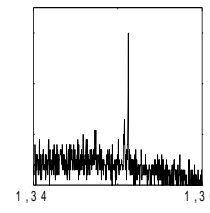
$L = 5 \mu\text{m}$  : **10000** QDs



$L = 0.5 \mu\text{m}$  : **100** QDs



$L \sim 50 \text{ nm}$  : **1** QD



- Very sharp emission from a single InAs quantum dot (QD).  
Applications : Quantum dots lasers...

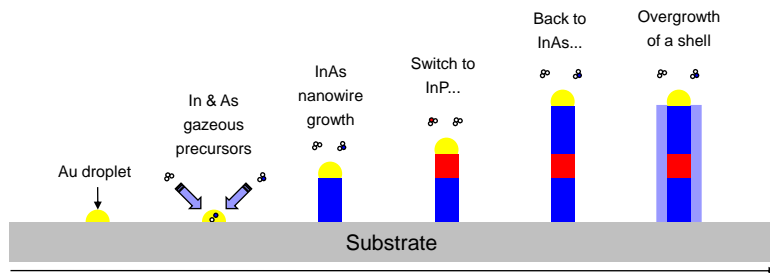
DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

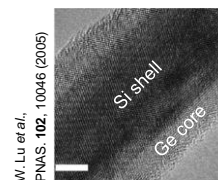
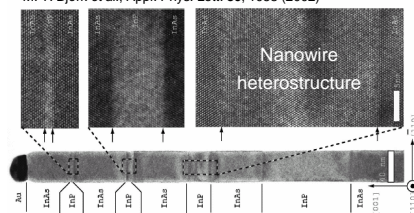
37

## Semiconductor nanostructures : Nanowires (I)

- Vapor-Liquid-Solid (VLS) growth :



M. T. Björk *et al.*, Appl. Phys. Lett. **80**, 1058 (2002)



W. Lu *et al.*,  
PNAS. **102**, 10046 (2005)

DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

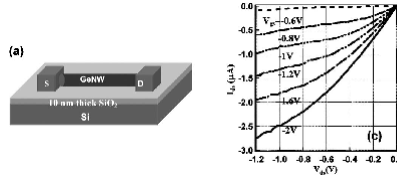
38

## Semiconductor nanostructures : Nanowires (II)

- Applications (examples) :



- Nanowire field effect transistor :

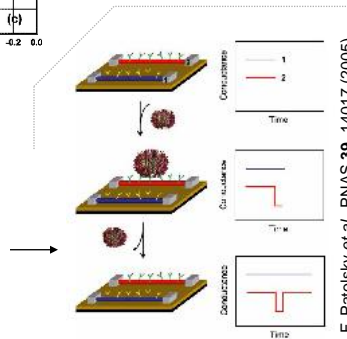


The current between the drain and source is controlled by the gate voltage.

D. Wang *et al.*, Appl. Phys. Lett. **83**, 2432 (2003)

- Detection of single molecules/viruses :

The surface of the nanowire is capped with molecules that bind to specific targets (DNA, viruses). The conductance of the nanowire changes each time the target binds to the wire.



F. Patolsky *et al.*, PNAS **39**, 14017 (2005)



## I.3 : Numerical simulation in nanosciences : Challenges and perspectives

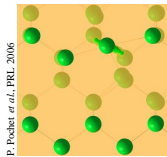
## Numerical simulation in nanosciences : Challenges

- **Atomistic simulation** is needed at the nanometer scale.

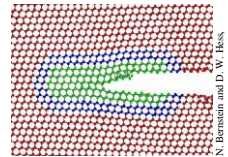


- Two challenges :

- Modelize the structural and electronic properties of the **materials** at the atomic scale, to :
  - Improve our understanding of the physics of present devices.
  - Anticipate the merits and limits of emerging technologies.



*How do vacancies migrate in silicon ?*



*Fracture propagation in silicon...*

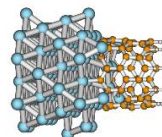
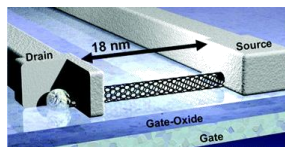
## Numerical simulation in nanosciences : Challenges

- **Atomistic simulation** needed at the nanometer scale.



- Two challenges :

- Modelize the structural and electronic properties of the **materials** at the atomic scale, to :
  - Refine our understanding of the physics of present devices.
  - Anticipate the merits and limits of emerging technologies.
- Modelize **nano-objects** to understand and optimize their :
  - Structural,
  - Optical, [Light sources, photovoltaics, ...]
  - Transport properties. [Nanoelectronics]



## Numerical simulation in nanosciences : Challenges

- **Atomistic simulation** needed at the nanometer scale.

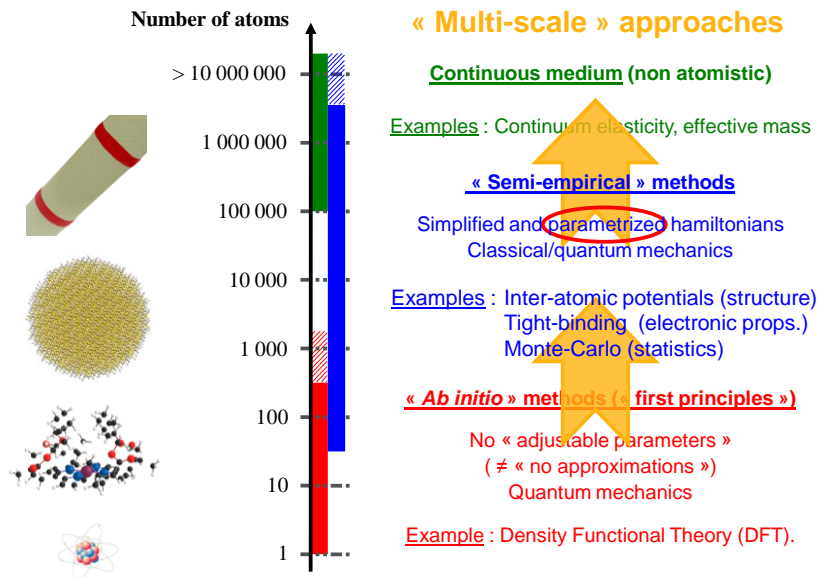


- Two challenges :

- Modelize the structural and electronic properties of the **materials** at the atomic scale, to :
  - Refine our understanding of the physics of present devices.
  - Anticipate the merits and limits of emerging technologies.
- Modelize **nano-objects** to understand and optimize their :
  - Structural,
  - Optical, [Light sources, photovoltaics, ...]
  - Transport properties. [Nanoelectronics]

**Towards a numerical “nanoscope/nanospectrometer”  
able to compute properties that are hardly  
accessible experimentally.**

## Methods



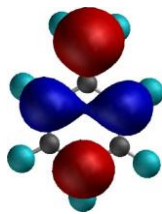
## Ab initio methods



- **No adjustable parameters (≠ no approximations).**

- **Example** : Density Functional Theory (DFT).

- The DFT allows the calculation of the **ground-state energy and properties of solids and molecules** (Hohenberg & Kohn 1964, Kohn & Sham 1965). It consists in replacing the system of interacting electrons with a fictitious system of non-interacting electrons moving in an effective potential  $v_{hxc}$ .



$$-\frac{\hbar^2}{2m_0} \Delta_r \psi(\mathbf{r}) + v(\mathbf{r})\psi(\mathbf{r}) + v_{hxc}[\psi](\mathbf{r})\psi(\mathbf{r}) = \varepsilon \psi(\mathbf{r})$$

**Hartree/Exchange/Correlation potential**  
 (accounts for the effects of the electronic interactions)  
**Approximations ! (LDA, GGA, ...)**  
 Depends on the  $\psi$ 's (« self-consistency ») !

- **Numerically intensive** (10 to ~1 000 atoms depending on the computer).

## Application : Vacancy diffusion in silicon (I)



- **Vacancy = Missing atom.**

- Semiconductors used in microelectronics are doped : Some silicon atoms have been replaced with e.g., boron or phosphorous to introduce extra electrons or holes.

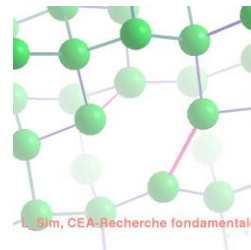
- These dopants are usually “implanted” in very specific locations on the chip.

- **Vacancy diffusion induces the migration of dopants**  
 ⇒ Dispersion of device characteristics.

- **The diffusivity of vacancies (related to the “migration energy”  $E_m$ ) is controversial :**

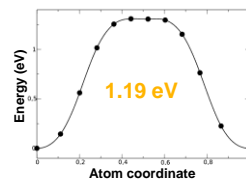
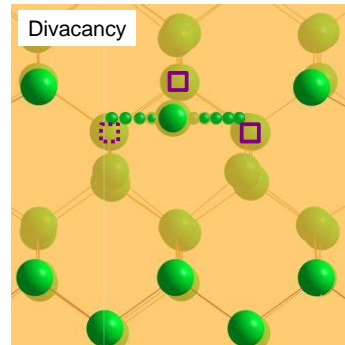
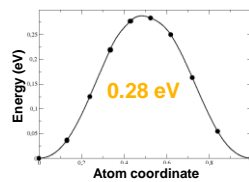
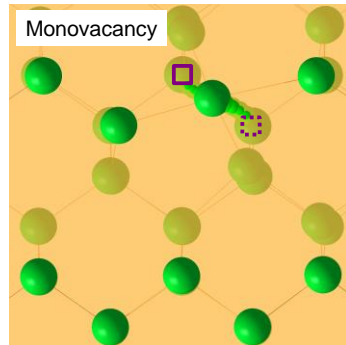
- **Watkins 1964** :  $E_m = 0.45$  eV.
- **Bracht 2003** :  $E_m = 1.80$  eV...

**Can atomistic simulation answer to this controversy ?**



## Application : Vacancy diffusion in silicon (II)

- Ab initio calculations (216 atoms) :



DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

47

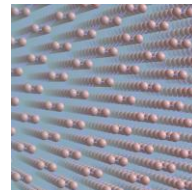
## Application : Vacancy diffusion in silicon (III)

- The *ab initio* calculations only provide a hint... What is indeed the exact dynamics of the vacancies ?



- Kinetic Monte-Carlo :

- **Two vacancies in a > 10 millions Si atoms box.**  
The vacancies can “jump” between neighboring sites. The configuration of the system is characterized by the position of the vacancies.
- **A simplified model for the transition energies,**  
parametrized on *ab initio* calculations.
- **An efficient sampling of the configurations space**  
(« Monte-Carlo » algorithms), that allows a fast calculation of the most probable trajectories of the vacancies.



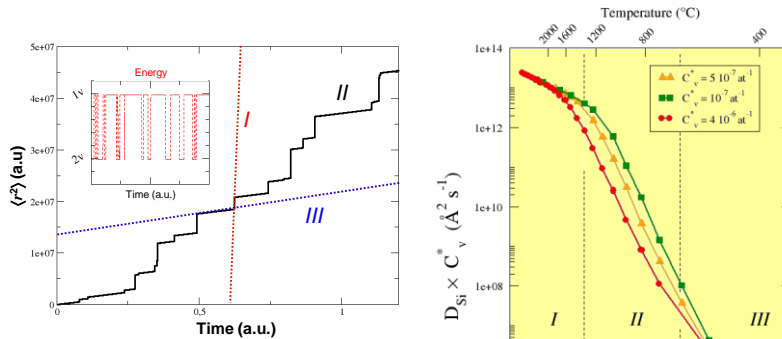
DSM/DRFMC/SP2M/L\_Sim

Quy Nhon, 31/12/2007

48



## Application : Vacancy diffusion in silicon (IV)



### • Three regimes :

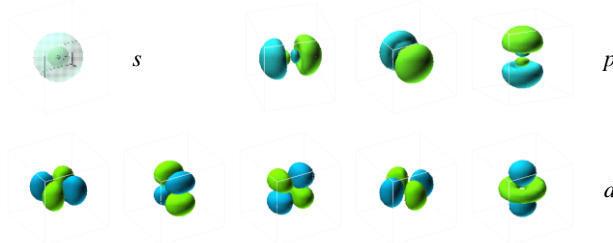
- Low temperature (III) :  
Slow divacancy diffusion.
- High temperature (I) :  
Complete divacancy dissociation.  
Fast monovacancy diffusion.
- Intermediate temperature (II) :  
Partial dissociation of divacancies  $\Rightarrow$  Average diffusivity.

D. Caliste et P. Pochet, *Vacancy-Assisted Diffusion in Silicon: A Three-Temperature-Regime Model*, Phys. Rev. Lett. **97**, 135901 (2006).

49

## Semi-empirical methods

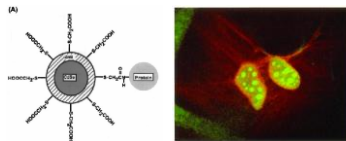
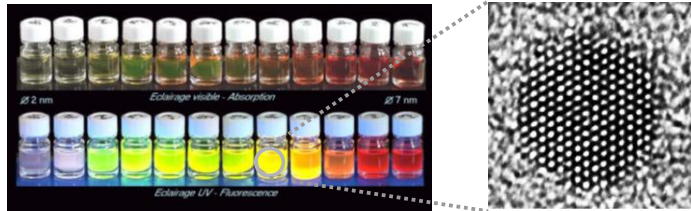
- Example : The tight-binding method.



- Principle : Write the wavefunctions as linear combination of atomic orbitals.

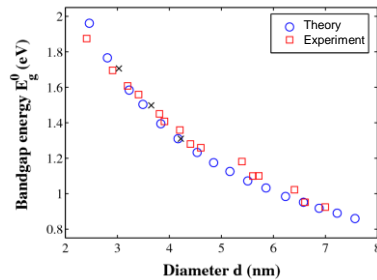
- The **range** of the model is limited to 1<sup>st</sup>, 2<sup>nd</sup> or 3<sup>rd</sup> nearest-neighbor atoms.
- The matrix elements of the hamiltonian are considered as adjustable parameters usually fitted to the bulk band structures then **transferred** to the nanostructures.
- The **computation time scales linearly with the number of atoms** (up to a few millions of atoms today).

## Application : Optical properties of nanocrystals

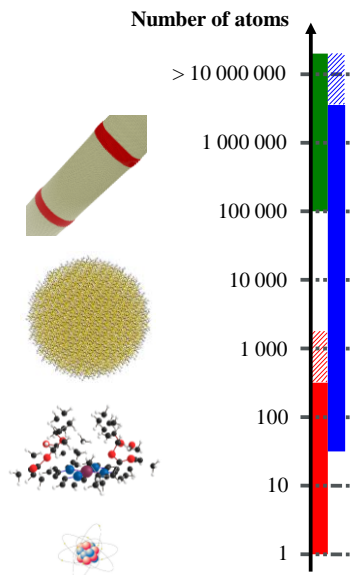


Application : Fluorescent labels in biology

M. Bruchez Jr. *et al.*, Science 1998  
W. C. W. Chan and S. Nie, Science 1998



## Methods



### « Multi-scale » approaches

#### Continuous medium (non atomistic)

Examples : Continuum elasticity, effective mass

#### « Semi-empirical » methods

Simplified and parametrized hamiltonians  
Classical/quantum mechanics

Examples : Inter-atomic potential (structure)  
Tight-binding (electronic props.)  
Monte-Carlo (statistics)

#### « Ab initio » methods ( « first principles » )

No « adjustable parameters »  
( ≠ « no approximations » )  
Quantum mechanics

Example : Density Functional Theory (DFT).